CHAPTER –I

INTRODUCTION

INTRODUCTION TO CHAPTER –II :

We have dealt with some problems of flow of viscous stratified liquid over a porous bed under various natural boundary conditions.

To study the problems of viscous liquid over a porous bed, we divide the flow region into two zones. Zone –1 is the region occupied by the viscous fluid either from the rigid impermeable plate or from the free surface of the viscous liquid to the interface where the flow is governed by Navier-Stokes equations. Zone-2 lies below the interface where the flow is governed by modified Darcy law with the Beavers and Joseph boundary conditions. The study of viscous stratified fluid through a porous pipe has great importance because such type of flow has wide application in the field of engineering and chemical technology and also in geophysics.

The study of the stratified fluid through the porous medium is widely applicable to withdraw fluid from a region in which the fluid density and viscosity vary in vertical direction. Such type of flow is of great importance to the petroleum technology concerned with the movement of oil, gas and water through the reservoir of oil and gas field as the flow behaviour of the fluid in a petroleum reservoir depends to a large extent on the viscous stratification and depends also on the properties of the medium.

The aim of this study is to investigate the flow of a viscous stratified fluid past a permeable bed with a motivation that stratification may provide a technique for studying the pore size in a porous media. The physical reason is that stratification may retard or accelerate the flow, The magnitude of retardation or acceleration is related to the slip parameter(α), stratification factor (η) and porosity factor (σ) and Reynolds number R. Therefore these factors might be responsible to affect the motion and might provide a technique for studying the pore size in a porous medium, which is very useful in petroleum industry. These factors also play an important role on many atmospheric and oceanic geophysical phenomenons.

Owing to the increasing geophysical application the study of stratified fluid flows has received a considerable attention by a number of authors.

The problem of viscous incompressible stratified fluid over a porous bed have attracted many mathematicians regarding its application in engineering and chemical technology. Pioneer workers in this field are Saffman (1971), Beavers et al (1970), Barcilon and Pedlosky (1967) and Holton (1965) who may be regarded as the origin of the modern research on the above subject.

Gersten and Gross (1974) have studied the effect of transverse sinusoidal suction velocity on the flow and heat transfer over a porous plane wall. The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate due to non-torsional oscillation of the plate in its plane with the given frequency ω was first studied by Stokes (1851). It is also known as Rayleigh's problem with $\omega=0$ in the literature (Debnath & Bhatta).

Debnath (1974), Raja Sekhara, Rudraiah and Ramaiah (1975) considered the Couette flow over a naturally permeable bed. Channabassappa and Ranganna (1976) considered the flow of viscous stratified fluid past a porous bed with the anticipation that stratification may provide a technique for studying the pore size in a porous medium. Gupta and Sharma (1978) studied the flow of viscous stratified liquid of variable viscosity between a bed and moving impermeable plate under the action of body force. Hari Kishan and Sharma (1980) discussed the stratified viscous flow of variable viscosity between a porous bed and a moving impermeable plate under the action of body force. Bhattacharya (1980) studied the unsteady flow of viscous fluid in the channel under time dependent pressure gradient of exponentially decaying and periodic type.

In the second chapter we have considered two problems. In the first paper we have studied the effect of stratification factor, porosity factor and slip parameter on the slip velocity of the flow of viscous stratified liquid.

It is known that when a Newtonian fluid flows between two impermeable plates, the usual boundary condition with no slip condition on the boundary leads to a parabolic type of motion in the channel. Beavers and Joseph (1967) have shown that the flow between two porous beds is governed by Darcy law and no slip condition is replaced by a streamwise slip velocity at the nominal surface. In this paper, we have made a study of the slip velocity in presence of stratification factor with variable porosity factor under constant slip parameter. From this study it reveals that stratification accelerates the slip velocity when slip parameter (α) <! but it retards the slip velocity when slip parameter α=1. Thus we can conclude that stratification may accelerate or retard the motion depending on the values of slip parameter.

The second paper of this chapter we have studied the unsteady flow of viscous stratified fluid over a permeable bed under the action of body force. In the year (1980) Hari Kishan and Sharma considered the motion of stratified viscous fluid of variable viscosity between a porous bed and a moving impermeable plate under the action of body force.

In the second paper we have calculated the fractional increase $|\phi|$ in mass flow rate with the assumption of the velocity of Darcy law for laminar flow R<|. We have also seen that for very small body force η, σ and α have no effect on $|\phi|$, but for large body force $|\phi|$ increases with the increase of porosity factor σ for each set of value of η and α, but $|\phi|$ decreases with the increase of η (Stratification factor) for each set of values of σ and α. Thus we can predict that (σ) porosity factor is favourable to the fractional increase in the mass (ϕ) of the fluid and η (stratification factor) is not favourable to the fractional increase in the mass of the fluid.

INTRODUCTION TO CHAPTER –III :

The motion of viscous stratified fluid largely depends on the magnitude of stratification factor, porosity factor, slip parameter and Hartmann number, so they must have an effect on the boundary layer also. Density and stratification factor play an important role on many atmospheric and oceanic geophysical phenomena. Owing to the increasing geophysical applications, the study of stratified fluid flow has received a considerable attention by a number of authors.

Bathaiah (1980) has discussed the flow of a viscous incompressible slightly conducting fluid through a porous straight channel under a uniform transverse magnetic field. Bathaiah and Bhaskara Reddy (1982) have studied the effect of hall current on the flow of viscous incompressible slightly conducting fluid through a porous straight channel under a uniform transverse magnetic field. Again, Bathaiah and Bhaskara Reddy (1987) have studied the combined effects of free and forced free convection on the flow of an incompressible viscous conducting fluid between two horizontal insulated parallel walls, one of which is at rest and the other moving parallel to itself with a linear axial temperature variation under the uniform transverse magnetic field. Bathaiah and K. Sreenivasan (1993) considered the flow of a viscous conducting fluid between two parallel plates of uniform length, lower plate being stationary and upper plate moving with constant velocity under a periodic pressure gradient superimposed on a constant pressure gradient under the influence of a uniform transverse magnetic field. Manju Gupta and Sharma (1993) considered the unsteady flow of an electrically conducting elasto-viscous dusty liquid through a rectilinear pipe having its cross-section as a hyperbolic sector in the presence of a transverse magnetic field under the influence of an arbitrary time varying pressure gradient. Kumar, Prasad and Gupta (1990) considered the MHD flow of stratified fluid through a porous medium between two oscillating plates. Chawla (1972) has investigated the boundary layer flow of a micro polar fluid along an infinite plate when the plate performs impulsive motion in its own plane. Chiu (1962), Soo (1961), and Singleton (1965) investigated the problem of boundary layer flow of dusty fluid over a semi finite flat plate Srivastava and Maiti(1966) have solved the boundary layer equation for two dimensional flow of a second order fluid. Mathur and Nandanan (1972) have studied the laminar boundary layer flow of an oldroyed fluid under the influence of pressure gradient with and without suction through a wedge. Gersten and Gross (1974) have studied the three dimensional incompressible boundary layer flow past a porous flat plate. Lighthill (1954) studied the two dimensional boundary layer to the fluctuations in the oncoming stream. Sharma and Singh (1992) investigated three dimensional laminar boundary layer free convection flow past (i) a porous flat plate and (ii) a porous vertical plate. Jain and Singh (1992) considered the unsteady magneto-hydro-dynamics boundary layer flow past a porous flat plate with sudden change in suction. Rath and Parida (1982) considered the oscillating free convection boundary layer flow of a viscous fluid near an infinite vertical wall and investigated that (i) by increasing injection velocity the fluid layer very near the wall may be made to oscillate with an amplitude larger than that of the wall velocity (ii) the boundary layer temperature fluctuates with a phase which changes only with the magnitude but not the sign of fluid influx parameter.

To study the viscous stratified flow of variable viscosity between a porous bed and an impermeable plate in presence of transverse magnetic field, it is seen there exists a thin boundary just beneath the interface. It is of interest to find the expression for this boundary layer thickness. To study the problems of viscous liquid over a porous bed we divide the flow region into two zones. Zone-I is the region occupied by the viscous fluid from moving impermeable wall rigid plate to the interface when the flow is characterised by free flow and is governed by Navier-Stokes equations. Zone-2 lies below the interface where the flow is governed by modified Darcy law with the Beavers and Joseph boundary conditions.

In the third chapter, we have considered two problems. In the first paper we have studied the flow of viscous stratified liquid of variable viscosity between a porous bed and an impermeable plate in presence of transverse magnetic fluid. In this note we have also studied the effect of stratification factor β, porosity factor σ and slip parameter α on the boundary layer δ of such flow.

We have studied the variation δ with the variation of M, the Hartmann number and it is seen that δ decreases with the increase of M keeping n and α fixed. Also for fixed values of n and M, it is seen that δ increases with the increase of α. Also when the stratification factor n is increasing, it is seen δ the boundary layer also increases when α, M are kept fixed.

Thus starting from Prandtl's boundary layer equation, we can conclude that for a porous media when the porosity factor increases, the boundary layer δ also increases.

The Study of the flow of the viscous fluid past a porous medium without stratification has been studied by Beavers and Joseph (1967). Channabassappa and Ranganna (1976) considered the flow of viscous stratified fluid of porous bed with the anticipation that stratification may provide a technique for studying the pore size in a porous medium. In this paper he has shown that 'Slip velocity' is proportional to the pressure gradient. The boundary layer just beneath the permeable interface and the friction factor are also obtained.

The Second paper of this chapter we have studied the effect of stratification factor on the boundary layer of the flow of viscous stratified between a rigid impermeable bed and a permeable bed. While studying the stratified viscous flow of variable viscosity between a porous bed and an impermeable bed, it is seen that there exists a thin boundary layer just beneath the interface. It is also seen that the growth of the boundary layer increases with the increase of stratification factor and porosity factor respectively.

INTRODUCTION TO CHAPTER -IV

The aim of the study of stratified fluid of variable viscosity past a permeable circular tube is to investigate the flow of a viscous stratified fluid past a permeable bed with a motivation that stratification may provide a technique for studying pore size in a porous medium. The physical reason is that the stratification may retard or accelerate the flow depending on the magnitude of the stratification factor. The magnitude of retardation or acceleration is related to the slip parameter, stratification factor, the porosity factor and the Reynolds number R. Hence one would expect that these factors might provide a technique for studying pore size in a porous medium, which is very useful in petroleum industry in studying the factors which influence oil recovery from petroleum reservoirs. The study of the flow of the viscous fluid past a porous medium without stratification has been studied by Beavers and Joseph (1967). Surya Prakash (1961) has considered the periodic flow in annulus of two porous coaxial circular cylinders for ordinary viscous incompressible fluid in 1964. Devi Singh (1964) has discussed the motion of a visco-elastic Maxwell fluid through two concentric circular cylinders under the presence of exponential pressure gradient. Singh (1967) has considered the motion of visco-elastic Maxwell fluid through two porous concentric circular cylinders. In this problem he has chosen the pressure gradient to be of the form $Ke^{\alpha t}\,Cos\beta t$. In the year 1970 Gupta and Kulshrestha has studied the slow steady flow of a viscous liquid in an annulus with arbitrary suction and injection along the rough wall. In 1992 Usha Singh and G. C. Sharma (1992) has studied three dimensional MHD flow in a porous media with pressure gradient and fluid injection. Beavers et al (1970), Channabassappa and Ranganna (1975) considered the flow of viscous stratified fluid of variable viscosity past a porous bed. Raghavacharya (1985) considered the combined force and forced convection in vertical circular porous channel. Mukherjee et al (1986) considered the unsteady flow of a viscous stratified fluid in a rotating system. Sanyal and Jash (1992) considered the combined free and forced convection of a conducting fluid in a vertical circular tube.

The study of flow through porous media is of principal in rest due to its importance in petroleum engineering for studying the movement of natural gas, oil and water through the oil reservoirs and to study the underground water in river beds. The flows of viscous fluid through porous medium are of considerable importance for various purposes. Not only because of its significant flow situation and oil extraction but also for the rheometrical aspect, it has been dealt with by a number of researchers who presented solutions of the problems for various models of viscosity characteristics. The problem of unsteady flow of viscous incompressible fluid in an annulus of two porous co-axial circular cylinders subjected to suction or injection has been studied by Rao (1961), Bhattacharya (1980) considered slow steady flow of a viscous incompressible fluid between two porous walls at slightly variable distance from each other with distributed suction. Mukherjee, S. and Maiti, M.

(1986) has studied the flow of viscous stratified fluid with density and viscosity decaying exponentially with the vertical co-ordinates. Raghavacharyha (1985) considered the combined force and forced convection in vertical circular porous channel. Sanyal and Jash (1992) considered force and forced convection of a conducting fluid in a vertical circular tube.

Singh (1967) has studied the flow of visco-elastic Maxwell fluid in the annulus of two porous concentric circular tube under the influence of pressure gradient. Das (1977) considered the flow of viscous incompressible fluid between two porous concentric circular cylinder with the inner cylinder rotating. Recently, Gupta and Babu (1987) studied the flow of a viscous incompressible fluid through a porous medium near oscillating infinite porous plate in the slip flow regime. Mukesh Gupta and Shalini Sharma (1991) investigated the flow of viscous incompressible and electrically conducting fluid through a porous medium bounded by an oscillating porous infinite plate in slip flow regime under the influence of transverse magnetic field fixed relative to the fluid.

In the Fourth chapter, we have considered three problems. In the first paper we have studied the flow of viscous incompressible fluid through two porous concentric circular cylinders subjected to suction or injection under the influence of pressure gradient which is a function of time alone. The general solution of the problem is obtained by using Laplace transforms and it is believed that the general solution for this problem has not been done by any investigators. By putting suction parameter zero, we directly obtained the flow of viscous incompressible fluid through coaxial circular cylinders under the presence of pressure gradient which are functions of time. From the graph it is believed that in presence of suction the magnitude of velocity near the inner cylinder is greater than that of when there is no suction and magnitude of velocity is smaller than that of near the outer cylinder when there is no suction and in presence of injection the velocity near the inner cylinder is smaller than that of when there is no injection and greater than that of near the outer cylinder, when there is no injection.

The second paper of this chapter we have studied the stratified fluid of variable viscosity past permeable circular tube. In the present paper we have studied, the effects of stratification factor (n), porosity factor (σ) and slip parameter (α) on the slip velocity and velocity profile of the flow of viscous stratified fluid under the influence of pressure gradient.

It has been observed that if $\sigma = 2\alpha$, slip velocity (W_B) vanishes and slip velocity (W_B) increases with the increase in the value of R (Reynold's number) and also slip velocity (W_B) increases when $\sigma < 2\alpha$, but the back motion in the slip velocity occurs for values of sigma σ greater than that of 2α.

The distribution of velocity is numerical evaluated against σ for fixed values of η and α and we can conclude from the graph that Reynolds number, porosity factors ($\sigma < 2\alpha$) are favourable to the motion but retardation in the flow begins when $\sigma > 2\alpha$ and independently on R.

In the third paper of this chapter, we have studied the unsteady motion of viscous fluid through a straight porous channel due to pressure gradient with an initial arbitrary velocity distribution. The fluid is assumed to be Newtonian and incompressible. The general solution is obtained by using finite Hankel transform. Velocity profile for some particular types of pressure gradients namely (i) impulsive (ii) periodic are discussed.

INTRODUCTION TO CHAPTER -V

The study of the stratified fluid through the porous medium has a great importance in many engineering and technological fields. The motion of viscous stratified liquid largely depends on the magnitude of the stratification factor, porosity factor and slip parameter and magnetic parameter (M) and permeability parameter (H), so they must have an effect on the slip velocity also. The study of the flow of viscous fluid past a porous bed without stratification has been studied by Beavers and Joseph (1967). Beavers et al (1970) and Rudraiah N., Rajsekhara B.M. and Ramaiah B.J. (1975) studied the flow past a porous medium by the use of Beavers and Joseph slip condition in which transfer of momentum was considered. Musket (1946) and Scheidegger (1963) showed that the Darcy law is valid when Reynolds number R is low. However, in many cases the flow velocity is not always small in a porous medium. Brinkman (1947) suggested a model of boundary layer type equation for flow through porous medium. Kumar (1985) considered the flow between two permeable beds using Darcy's law in one permeable bed and in other bed he used Brinkman equation. The same problem was discussed by Chauhan and Vyas (1991). Singh (1995) discussed the MHD flow through porous medium of different permeabilities.

Yih (1959) studied the effect of density variation on the fluid flow. Dore (1969) has also studied the forced oscillation in a viscous stratified fluid in which the density and viscosity vary exponentially with vertical co-ordinate. Channabasappa and Ranganna (1976) discussed the flow of stratified fluid past a permeable bed with the anticipation that stratification may provide a technique for studying the pore size in a porous medium. Gupta and Sharma (1978) analysed the problem on stratified viscous flow of a variable viscosity between a porous bed and a moving impermeable plate. Varshney (1980) discussed the unsteady flow of a viscous fluid of a variable viscosity through a porous medium between two parallel plates with constant pressure gradient. Gupta (1983) has studied the flow of a stratified fluid through a porous medium between two plates in which the lower plate oscillates with time and upper stationary.

The problems of heat transfer in electrically conducting liquids permeated by electromagnetic field have been studied by number of investigators. Seth and Maite (1982) have studied MHD coutte flow and heat transfer in a rotating system. Pillai and Varma (1989) have studied flow of a conducting fluid between two co-axial rotating porous cylinders bounded by a permeable bed.

More recently, Kulshrestha and Singh (1993) considered the MHD flow of viscous incompressible fluid between two co-axial rotating porous cylinders and discussed the effects of injection, suction parameter and electro-magnetic field on velocity distribution.

In the fifth chapter, we have considered two problems. In the first paper, we have discussed the distribution velocity on the MHD plane Coutte flow of a viscous incompressible fluid in a channel with porous medium of different permeabilities.

Using the slip boundary condition of Beavers and Joseph, we divide the entire flow region into three zones. Zone-1 relates the free flow region above the bed which is governed by Navier-Stoke's equations and Zone-2 relates to highly permeable region where the flow is governed by Brinkman's equation and zone-3 the region of low permeability where the flow is governed by Darcy law.

The distribution of velocities in the three zones have been calculated in exact form separately. For different set of values of Magnetic parameter M, the distribution of velocity in Zone-1 have been calculated. It is seen that the velocity in Zone-1 increases with the increase of M and velocity increases exponentially with the increase of y.

The Skin friction on the plate y=1 has been calculated for different values of M and it is seen that drag on the plate y=1 increase with the increase in the value of M.

The second paper of this chapter, we have studied the MHD flow of a viscous incompressible fluid between two coaxial rotating porous cylinder bounded by permeable bed. The cylinders are composed of insulated material. The problem is divided into 3 zones. Zone-1 and Zone-3 are permeable zone where the flow is governed by Darcy Law and Zone – 2, consists of free flow region where the flow is governed by magneto-hydro-dynamic equations. Using boundary conditions of Beavers and Joseph (1967) an exact solution for the velocity distribution of the fluid in different zones are calculated in dimensionless form. From fig.2, in case of zone-1 it is seen that velocity distribution decreases with the increase of suction(S) for any fixed values of r and magnitude of velocity increases with the increases of injection for any fixed values of r. Also it is noted that for any fixed value of suction, the velocity distribution gradually decreases with increase of r and for any fixed values of injection the distribution of velocity increases with the increase of r.

In zone-2, it is seen that for any fixed values of r, when s is increasing in values, the magnitude of velocity is also increasing and in zone-3, it is seen that for any fixed values of r, velocity decreases when s is increasing in values.

INTRODUCTION TO CHAPTER -VI

The solution to the problem of steady flow of a viscous liquid through the region bounded by two concentric circles is well-known. Citron (1962) investigated the problem of slow steady viscous flow between two rotating concentric infinite cylinders with axial roughness. Khamrui (1963) has studied the problem of slow steady flow of a viscous incompressible liquid through a circular tube when the radius varies axially. Gupta and Kulshreshtha (1970) have studied the slow steady flow of a viscous liquid in an annulus with orbitrary injection and suction velocity along the rough wall. Gaur and Mehta (1981) have studied slow unsteady flow of a viscous incompressible fluid between two co-axial circular cylinders with axial roughness. Vasudevaiah and Majhi (1982) have studied matched solutions of slow viscous flow past a rotating of sphere.

In the last chapter, we have considered two problems. In the first paper, we have studied the problem of slow steady flow of a viscous incompressible fluid between two infinite co-axial circular cylinders with axial roughness. In the present note, we have considered the motion of viscous incompressible fluid through the annulus of two smooth concentric infinite circular cylinders with axial roughness. The motion is originally set up by pressure gradient acting in the direction of the common axis of the cylinders. The solution for the problem is obtained by applying complex Fourier transform.

The distribution of velocity in the radial and axial direction have been calculated. It is found for roughness of the wall, the nature of distribution of velocity seems to be oscillatory in nature.

The wave phenomenon is a very common occurrence of oscillatory motion in nature. It draws a greater attention to the scientists and research-workers due to its wide important role in the field of viscous and non-viscous, compressible and incompressible fluid. The waves are varieties in nature, they are depressive and non-depressive according to the depth of the fluid where they are generated.

The present study of wave motion is based on some assumptions and boundary value conditions. The problem is considered a two-dimensional one. The two–dimensional surface-wave in a viscous incompressible fluid generated due to the pressure applied on the free surface of the fluid has been studied by several classical investigators.

Sneddon (1951) studied the problem of semi-infinite viscous incompressible fluid under the action of radially symmetric pressure distribution.

Pramanik (1972) considered the two-dimensional problem of waves generated by moving oscillatory pressure distribution which is applied on the free surface of an infinitely deep viscous incompressible fluid.

Lamb (1932), Basset (1888), Besant and Ramsay(1929) dealt also with the problem of two-dimensional wave in viscous incompressible fluid in their classical treatises.

Bhattacharya (1968) considered the irrotational flow of semi-infinite viscous fluid by an impulsive velocity prescribed within a circular region on the surface of the fluid.

Datta (2000) considered the problem of slow rotation of a sphere with source at its centre in viscous fluid.

An elastic half-space problem which involves axisymmetric normally applied surface leads that expands is of interest to study the ground motion due to the surface blast. The problem which involves loads that suddenly emanates from a point on the surface and expands radially at a constant rate. These loads are so chosen so that they exert a constant force on the surface of the half-space as they expand.

Here an attempt has been to study the surface wave in semi-infinite viscous incompressible fluid. The motion is generated due to a load F_0 (ring and disc) acting normally to the surface emanates from the origin and expands radially at a constant rate C over the surface.

The second paper of this chapter we have studied generation of surface-wave in a semi-infinite viscous incompressible fluid due to ring load and disc load. The motion is generated due to a load F_0 (ring and disc) acting normally to the surface emanates from the origin and expands radially at a constant rate C over the surface.

An exact solution has been obtained for surface elevation and the motion has been studied graphically. In case of ring load $\frac{F_0}{2\pi r}\delta(ct-r), \quad t>\frac{r}{c}$ with the property of Dirac Delta function $\int_0^\infty f(r)\,\delta\,(ct-r)dr = f(ct)$, it has been found that the surface elevation increases asymptotically with the variation of time, when the radius vector remains constant. But in case of the disc-load $\frac{F_0}{\pi(ct)^2}H(ct-r), \; t>\frac{r}{c}$, the surface elevation diminishes slowly with the variation of radius vector when the time component remains unchanged. It is convenient to use Hankel transform with respect to the surface co-ordinate and Laplace transform with respect to the time co-ordinate to get the solution of the problem. The Hypergeometric function has been used to obtain the exact solution of the problem.