PART - TWO

TO STUDY THE FLOW OF VISCOUS MHD FLUID BETWEEN TWO ROTATING CYLINDERS

Introduction : Gaur and Mehta (1981) have studied slow unsteady flow of a viscous incompressible fluid between two coaxial circular cylinders with axial roughness. Vasudevaiah and Majhi (1982) have studied matched solutions of slow viscous flow past a rotating of sphere.

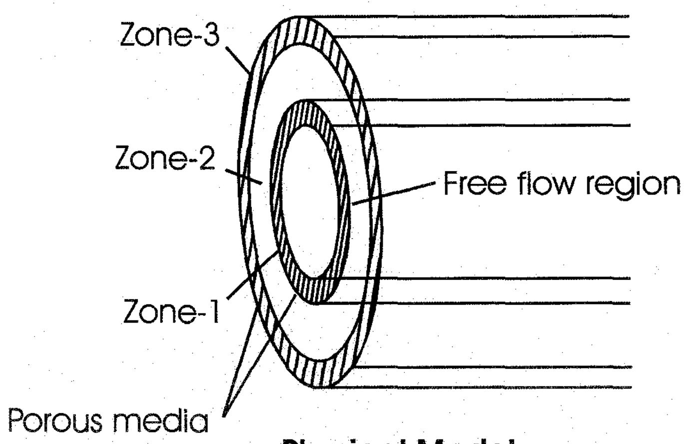
Yih (1959) studied the effect of density variation on the fluid flow. Dore (1969) has also studied the forced oscillations in a viscous stratified fluid in which the density and viscosity vary exponentially with vertical co-ordinate. Channabasappa and Ranganna (1976) discussed the flow of stratified fluid past a permeable bed with the anticipation that stratification may provide a technique for studying the pore size in a porous medium. Gupta and Sharma (1978) analysed a problem on stratified viscous flow of a variable viscosity between a porous bed and a moving impermeable plate. Varshney (1980) discussed the unsteady flow of a viscous fluid of a variable viscosity through a porous medium between two parallel plates with constant pressure gradient. Gupta (1983) has studied the flow of a stratified fluid through a porous medium between two plates in which the lower. plate oscillates with time and upper stationary.

The problems of heat transfer in electrically conducting liquid permeated by electromagnetic field have been studied by number of investigators. Seth and Maite (1982) have studied MHD coutte flow and heat transfer in a rotating system. Pillai and Varma (1989) have studied flow of a conducting fluid between two coaxial rotating porous cylinders bounded by a permeable bed.

Thus, our purpose of the present paper is to investigate the MHD flow of a viscous incompressible fluid between two coaxial rotating porous cylinders bounded by permeable beds. Using boundary condition of Beavers and Joseph (1967), an exact solution for the velocity distribution of the fluid in free region and porous region is evaluated in dimensionless form and the result obtained is discussed with the help of graphs.

Mathematical Formulation of the Problem :

Let us consider the steady flow of a conducting viscous incompressible fluid between two coaxial rotating porous cylinders composed of an insulated material. The cylinders terminate at perfect electrodes which are connected through a load. The walls of the inner and outer cylinders are porous bounded by a permeable bed (Fig. 1.) The problem is divided into three zones. In zone-1, which is porous zone, where the flow is governed by Darcy law and Beavers and Joseph condition. In zone-2, the zone of free flow region where the flow is governed by magnetohydrodynamic equations and in zone-3 which is made of permeable bed, the flow is governed by Darcy law.



Physical Model
Fig. -1

Assuming the slip velocity boundary condition of Beavers and Joseph (1967) the solution in different zones are matched at the interface between zone-1 and zone-2 and zone-3. A uniform magnetic field H_0 is applied in the radial direction throughout the flow region. Here K_1 is the permeability of porous bed of inner cylinder, m is the co-efficient of viscosity, s_2 is electrical conductivity of the fluid, m_2 is magnetic permeability of the fluid and E_Z is electric field and Ω_1, Ω_2 are the angular velocity of the inner and outer cylinder respectively.

Under present configuration, the governing equations for the steady viscous incompressible fluid flow in different zones are :

Zone–1

$$u_1 = -\frac{K_1}{\mu}\left(\frac{dp}{dr} - 2\rho\Omega_1\upsilon_1\right)\text{........................}(1)$$

$$v_1 = -\frac{K_1}{\mu}\left(2\rho\Omega_1 u_1 - \sigma_e\mu_e H_r E_z + \sigma_e\mu_e^2 H_r^2 \upsilon_1\right)\text{..................}(2)$$

$$\frac{d(ru_1)}{dr} = 0\text{..}(3)$$

i.e. ru_1 = constant = $-s$ (say)....................(4)

Where $S > 0$ is the suction parameter and $S < 0$ is the injection parameter.

Zone–2

$$u\frac{du}{dr} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{dp}{dr} + \upsilon\left(\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}\right) \text{.............(5)}$$

$$u\frac{du}{dr} - \frac{uv}{r} = \gamma\left(\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{v}{r^2}\right) + \frac{1}{\rho}\left(\sigma_e\mu_e H_r E_z - \sigma_e\mu_e^2 H_r^2 \upsilon\right) \text{.......(6)}$$

$$\frac{d(ru)}{dr} = 0 \text{..(7)}$$

i.e. ru = constant = $-S$ (Say)..........................(8)

Where $S > 0$ is the suction parameter and $S < 0$ is the injection parameter.

$$\frac{d(rH_r)}{dr} = 0 \text{..(9)}$$

$$\frac{d(E_Z)}{dr} = 0 \text{..(10)}$$

Zone–3

$$u_2 = -\frac{K_2}{\mu}\left(\frac{dp}{dr} - 2l\Omega_2\upsilon_2\right) \text{.....................................(11)}$$

$$v_2 = -\frac{K_2}{\mu}(2\rho\Omega_2\mu_2 - \sigma_e\mu_2 H_r E_Z + \sigma_e\mu_e^2 H_r^2 v_2) \text{........(12)}$$

$$\frac{d(rv_2)}{dr} = 0 \text{..(13)}$$

i.e. rv_2 = constant = $-S$ (say)............................(14)

Where $S > 0$ is the suction parameter and $S < 0$ is the injection parameter.

The boundary conditions are :

$$\left.\begin{aligned} u = u_a, \frac{du}{dr} = \frac{\alpha_1}{\sqrt{K_1}}(u_a + u_1) \\ v = v_a - r_1\Omega_1, \frac{dv}{dr} = \frac{\alpha_1}{\sqrt{K_1}}(u_a + v_1) \end{aligned}\right\} \qquad \text{......................}(15)$$

$$\left.\begin{aligned} u = u_b, \frac{du}{dr} = \frac{\sigma_2}{\sqrt{K_2}}(u_b - u_2) \\ v = v_a - r_1\Omega_1, \frac{dv}{dr} = \frac{\alpha_1}{\sqrt{K_1}}(v_b - v_2) \end{aligned}\right\} \qquad \text{......................}(16)$$

The following non-dimensional quantities are used

$$r^1 = \frac{r}{r_1},\quad u_1^1 = \frac{u_1 r_1}{\nu},\quad v_2^1 = \frac{v_2 r_2}{\nu},\quad u_b^1 = \frac{u_b r_2}{\nu},$$

$$u^1 = \frac{u r_1}{\nu},\quad v_1^1 = \frac{v_1 r_1}{\nu},\quad u_a^1 = \frac{u_a\, r_1}{\upsilon},\quad u_b^1 = \frac{v_b r_2}{\nu},$$

$$v^1 = \frac{v r_1}{\nu},\quad u_2^1 = \frac{u_2 r_2}{\nu},\quad v_a^1 = \frac{v_a r_1}{\nu},\quad \delta = \frac{r_2}{r_1},$$

$$\omega_1 = \frac{\Omega_1 r_1^2}{\nu},\quad \omega_2 = \frac{\Omega_2 r_2^2}{\upsilon},\quad p^1 = \frac{p r_1^2}{\rho\tau^2},\quad E_Z^1 = \frac{E_Z}{E_o}.$$

where $E_0 = \frac{\mu_0 H_0 \nu}{r_1}$, $H_r^1 = \frac{H_r}{H_o}$, $\sigma_1 = \frac{r_1}{\sqrt{K_1}}$, $\sigma_2 = \frac{r_2}{\sqrt{K_2}}$, $M^2 = \frac{\sigma_e \mu_e^2 H_0^2 r_1^2}{\mu}$,

ω_1 = dimensionless angular velocity of inner cylinder and ω_2 = dimensionless angular velocity of outer cylinder.

Using the above non-dimensional quantities equations (1) to (13) reduces to :

Zone–1

$$u_1^1 = -\frac{1}{\sigma_1^2}\left(\frac{dp^/}{dr^/} - 2\omega_1\upsilon_1^1\right) \qquad \text{......................}(17)$$

$$v_1^1 = \left(\frac{M^2 H_r^1 E_z^1 - 2\omega_1 u_1^1}{\sigma_1^2 + MH_r'^2}\right) \qquad (18)$$

$$\frac{d(r^1 u_1^1)}{dr^1} = 0 \qquad (19)$$

i.e. $r_1^1 u_1^1$ = constant = – S (say) ....(20)

where $S > 0$ is the suction parameter and $S < 0$ is the injection parameter.

Zone–2

$$u^1 \frac{du^1}{dr^1} - \frac{v^{12}}{r^1} = \frac{dp^1}{dr^1} + \frac{d^2u^1}{dr^{12}} + \frac{1}{r^1}\frac{du^1}{dr^1} - \frac{u^1}{r^{12}} \qquad (21)$$

$$r^{1^2}\frac{du^1}{dr^{12}} + r^1(1-S)\frac{dv^1}{dr^1}(1+M^2+S)\nu = -r^1 M^2 E_Z^1 \qquad (22)$$

$$\frac{d(r^1 u^1)}{dr^1} = 0$$

i.e. $r^1 u^1$ = constant = – S (say)................................(23)

where $S > 0$ is the suction parameter and $S < 0$ is the injection parameter.

$$\frac{d(r^1 H_r^1)}{dr^1} = 0, \quad \frac{d(E_Z^1)}{dr^1} = 0$$

Zone–3

$$u_2^1 = -\frac{1}{\sigma_1^2}\left(\delta^2 \frac{dp'}{dr'} - 2\omega_1 \upsilon_2^1\right) \qquad (24)$$

$$v_2^1 = \frac{M^2\delta^2 H_r^1 E_Z^1 - 2\omega_2 u_2^1}{\sigma_2^2 + M^2\delta^2 H_r^{12}} \qquad (25)$$

$$\frac{d(r^1 u_2^1)}{dr^1} = 0 \qquad (26)$$

i.e. $r^1 u_2^1$ = constant = – S (say)............................(27)

where $S > 0$ is the suction parameter and $S < 0$ is the injection parameter.

The boundary conditions (15) and (16) reduce to

$$\left.\begin{aligned} u' &= u'_a, \frac{du^2}{dr'} = \alpha_1\sigma_1\left(u_a^1 + u_1^1\right) \\ v' &= v'_a - \omega_1, \frac{dv'}{dr'} = \alpha_1\sigma_1\left(v_a^1 + v_1^1\right) \end{aligned}\right\} \quad r^1 = 1 \text{.............(28)}$$

$$\left.\begin{aligned} u' &= \frac{u'_b}{\delta}, \frac{du'}{dr'} = \frac{\alpha_2\sigma_2}{\delta^2}\left(u'_b - u'_2\right) \\ v' &= \left(\frac{v'_b + \omega_2}{\delta}\right), \frac{dv'}{dr'} = \frac{\alpha_2\sigma_2}{\delta^2}\left(v'_b - v'_2\right) \end{aligned}\right\} \quad r^1 = \delta \text{.............(29)}$$

Solution of the problem :

Zone–1

Velocity distribution u_1^1 and v_1^1 using condition (28) is

$$u_1^1 = \frac{-S}{\alpha_1\sigma_1}(1+\alpha_1\sigma_1) \text{..(30)}$$

$$v_1^1 = \frac{r^1M^2}{\tau^2\sigma_1^2 + M^2} + \frac{2\omega_1 r^{12} S(1+\alpha_1\sigma_1)}{\alpha_1\sigma_1(r^{1^2}\sigma_1^2 + M^2)} \text{........................(31)}$$

Zone–2

Velocity distribution u^1 and v^1, on solving the equations (23) and (22) is

$$u' = \frac{s}{r'} \text{...(32)}$$

$$v^2 = Ar^{1^{K_1}} + Br^{1^{K_2}} + \frac{M^2E_Z^1r^1}{2S+M^2} \quad \text{...............................(33)}$$

Zone–3

Velocity distribution u_2^1 and v_2^1 using condition (29) is

$$u'_2 = \frac{S(1 \_ \alpha_2\sigma_2)}{\alpha_2\sigma E'_z} \text{...(34)}$$

$$v_2^1 = \frac{r^l M^2 \delta^2 E_z^l}{\left(r^{l^2}\sigma_2^2 + M^2\delta^2\right)} - \frac{2\omega_2 S(1+\alpha_2\sigma_2)r^{l^2}}{\alpha_2\sigma_2\left(r^{l^2}\sigma_2^2 + M^2\delta^2\right)} \quad \text{.............}(35)$$

Discussion :

In Zone-1, the following conclusion can be made from the fig-2. It is seen that the velocity distribution decreases with the increase in amount of suction for each fixed value of r, but the velocity distribution increase with the increase in the amount injection for each fixed value of r. Thus in porous Zone suction retards the motion while injection accelerates it.

In Zone-2, the following conclusion can be made from the fig-3 for fixed value of suction parameter, the magnitude of the components of the velocity increases when r increases and in case of fixed value of injection parameter, the magnitude of the velocity decreases when r increases and for a fixed value of r, s increases from negative to positive, it is found that velocity also increases.

In Zone-3, it is observed from fig-4 that for any fixed value of suction, the magnitude of velocity increases for every increasing value of r. Also for any fixed value of injection parameter the magnitude of velocity decreases for every increasing value of r.

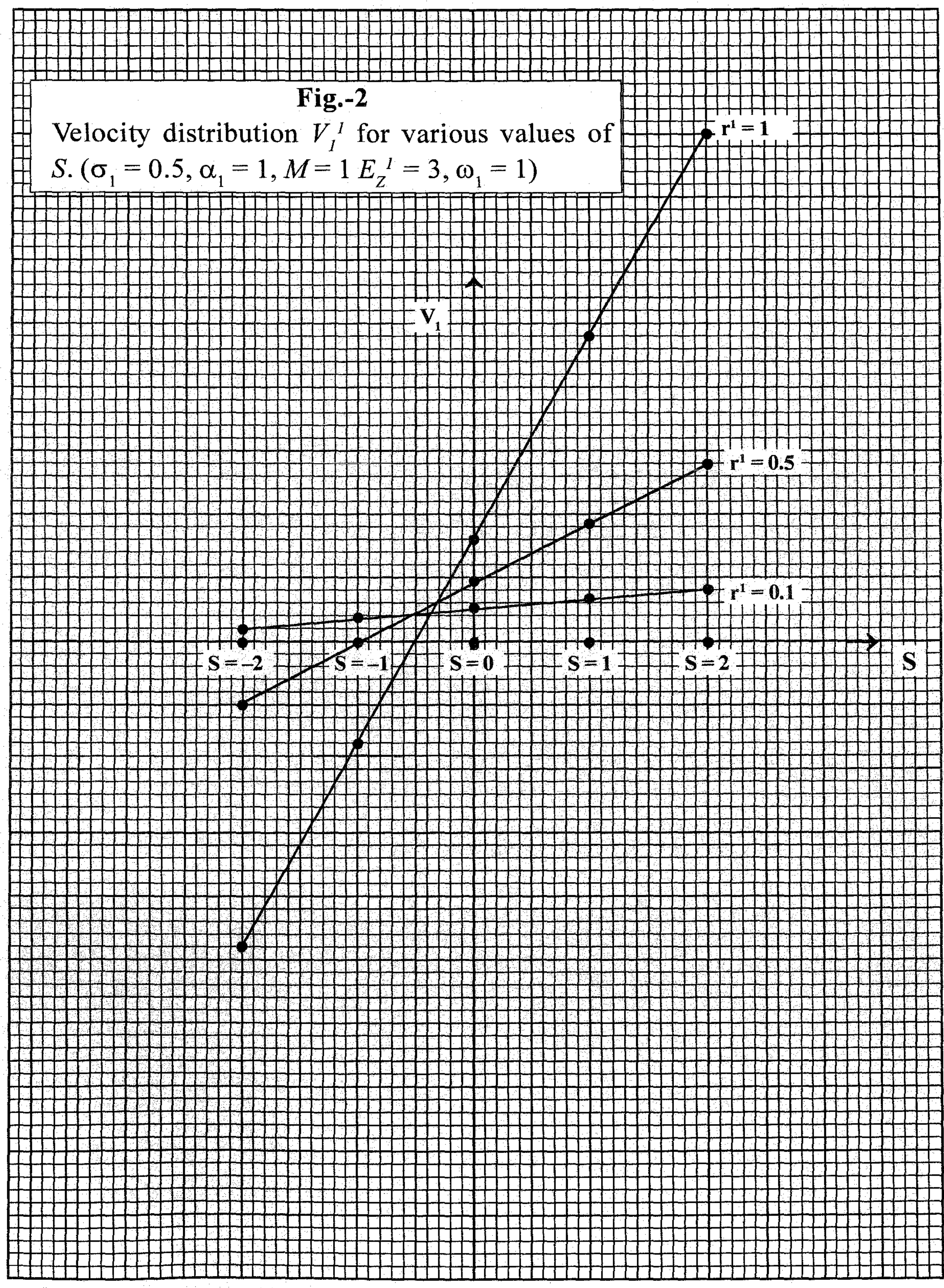
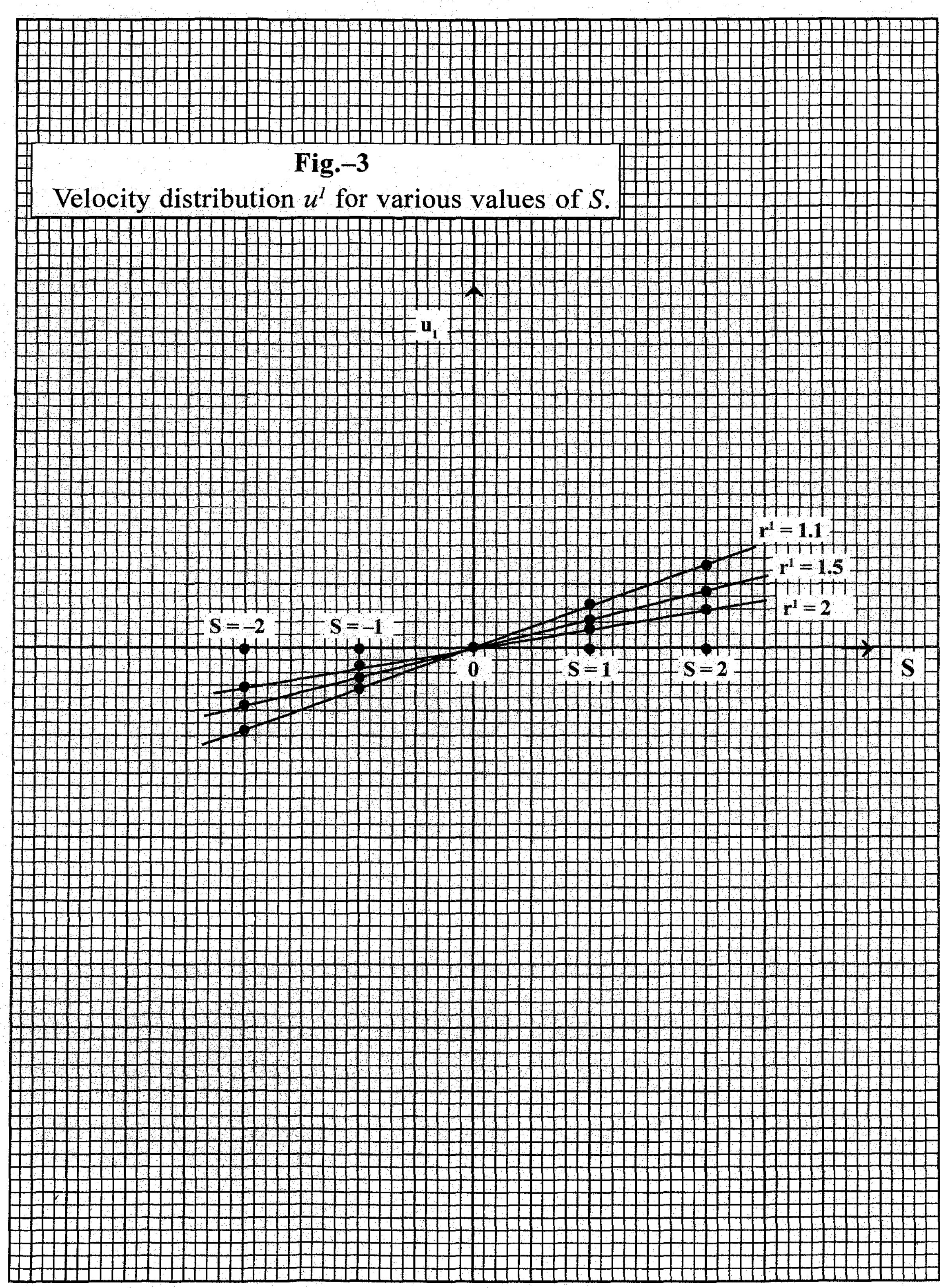


Fig.-2
Velocity distribution V_1^1 for various values of S. ($\sigma_1 = 0.5, \alpha_1 = 1, M = 1\ E_Z^1 = 3, \omega_1 = 1$)
V_1
$r^1 = 1$
$r^1 = 0.5$
$r^1 = 0.1$
S = –2
S = –1
S = 0
S = 1
S = 2
S

Fig.–3

Velocity distribution u^1 for various values of S.



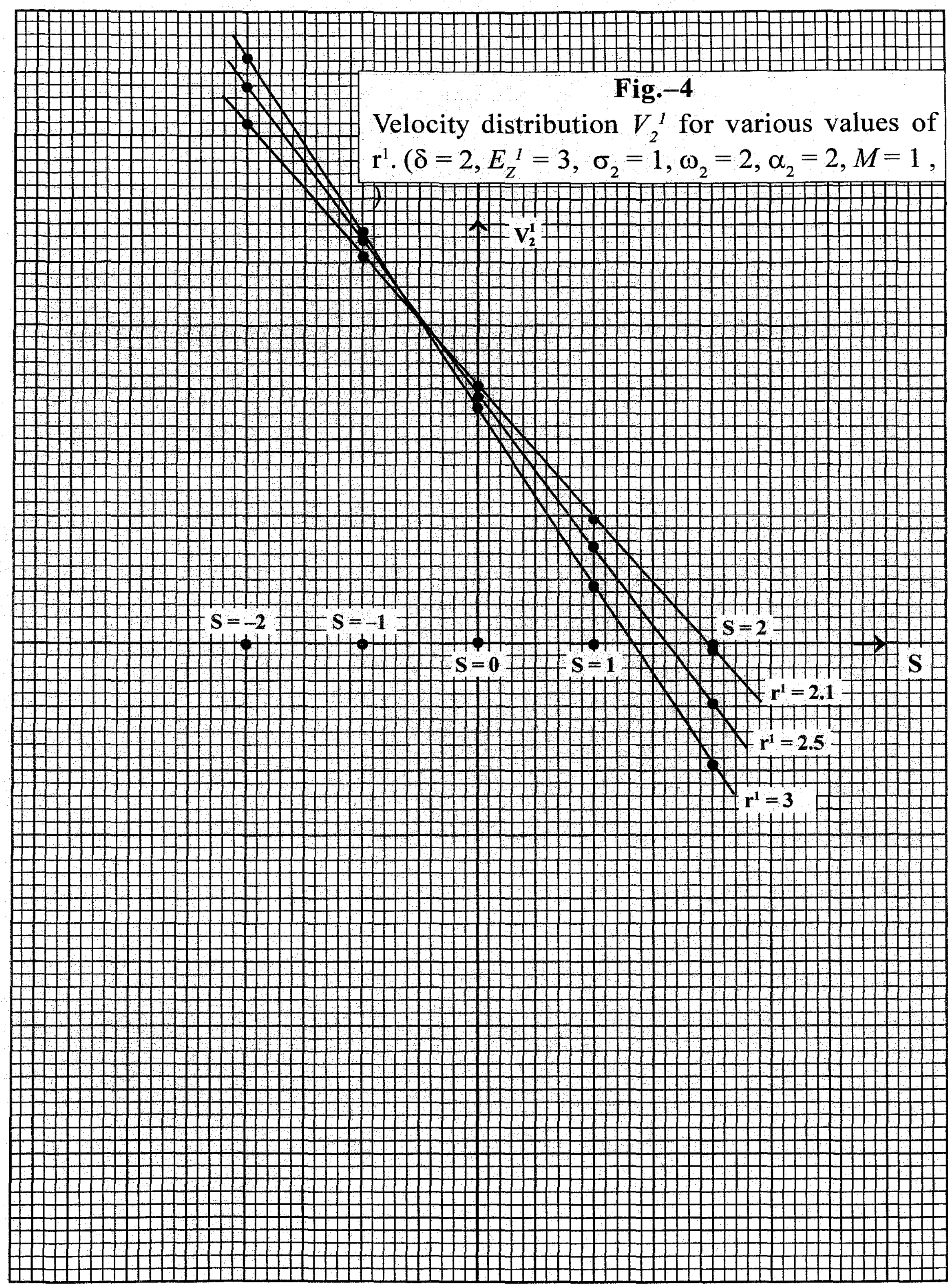


Fig.–4

Velocity distribution $V_2^{\,1}$ for various values of r^1. ($\delta = 2$, $E_Z^{\,1} = 3$, $\sigma_2 = 1$, $\omega_2 = 2$, $\alpha_2 = 2$, $M = 1$,)