PART – TWO

STRATIFIED FLUID OF VARIABLE VISCOSITY PAST A PERMEABLE CIRCULAR TUBE

Introduction

The aim of this study is to investigate the flow of a viscous stratified fluid past a permeable bed with a motivation that stratification may provide a technique for studying pore size in a porous medium. The physical reason is that the stratification may retard or accelerate the flow depending on the magnitude of the stratification factor. The magnitude of retardation or acceleration is related to the slip parameter, stratification factors, the porosity factor and the Reynolds number R. Hence one would expect that these factors might provide a technique for studying pore size in a porous medium., which is very useful in petroleum industry in studying the factors which influence oil recovery from petroleum reservoirs. The study of the flow of the viscous fluid past a porous medium without stratification has been studied by Beavers and Joseph (1967). Beavers et al (1970), Channabassappa and Ranganna (1975) considered the flow of viscous stratified fluid of variable viscosity past a porous bed. Raghavacharya (1985) considered the combined force and forced convection in vertical circular porous channel. Mukherjee *et al* (1986), considered the unsteady flow of a viscous stratified fluid in a rotating system. Sanyal and Jash (1992) considered the combined free and forced convection of a conducting fluid in a vertical circular tube.

In the present paper we have studied the effects of stratification factor, porosity factor and slip parameter on the slip velocityand velocity profile of the flow of viscous stratified fluid under the influence of pressure gradient .

It has been observed that if $\sigma = 2\alpha$, W_B velocity vanishes and W_B increases with the increase in the value of R (Reynolds's number) and also W_B increases when $\sigma < 2\alpha$. but the back motion in the slip velocity occurs for values of sigma σ greater than that of 2α.

The distribution of velocity is numerical evaluated against σ for fixed values of n and α. and we can conclude from the graph that Reynolds number, porosity factors($\sigma < 2\alpha$), is favourable to the motion but retardation in the flow begins when $\sigma > 2\alpha$. and independently on R.

Mathematical Formulation of the Problem :

Applying cylindrical co-ordinates (r, θ, z) with z-axis along the axis of the cylinders. Let us denote by u, v and w the components of velocity along r, θ and z increasing respectively. Assuming that the motion is symmetrical about the z-axis. We have $\delta / \delta\theta = 0$ and the nature of motion gives u=v=0.

Then for slow steady motion of viscous stratified fluid the Navier-Stokes equations in absence of external forces are

$$\frac{1}{r}(\mu r w')' = \partial p / \partial r \qquad \text{......(1)}$$

The prime denoting the differentiation with respect to r.

$$\mu = \mu_0 e^{\beta(r-a)}, \quad \rho = \rho_0 e^{\beta(r-a)} \qquad \text{.......(2)}$$

and $$\frac{\partial p}{\partial z} = -g\rho \qquad \text{.......(3)}$$

where μ_0 and ρ_0 are the co-efficient of viscosity and density respectively at the interface r=a and $\beta > 0$ represents the stratification factor.

$$Q = Q_0 \, e^{-\beta(r-\alpha)} \qquad \text{.......(4)}$$

$$Q_0 = -k / \mu_0 \, (\partial p / \partial z) \qquad \text{.......(5)}$$

Boundary conditions for the problem are

$$W = \text{Finite at } r = 0 \qquad \text{.......(6)}$$

$$W' = \alpha / \sqrt{K}\,(W_B - Q_0) \text{ at } r=a \qquad \text{.......(7)}$$

W_B is the slip velocity at the nominal surface r = a, α is the slip parameter, K is the permeability co-efficient (has the dimensional of the length square) and Q_0 is given by (5).

Let us now make equation (1) non-dimensional by using the dimensional less quantities.

$$\left.\begin{array}{l} \overline{w} = w/w_m,\ \overline{w}_B = w_B/w_m,\ \sigma = a/\sqrt{K},\ \overline{z} = z/a \\ n = \beta a,\ \ \overline{r} = r/a,\ \overline{p} = p/\rho_0 w_m^2,\ \ \nu_0 = \mu_0/\rho_0 \end{array}\right\} \text{......(8)}$$

where w_m is some characteristic velocity that is mean velocity.

Equation (1) now reduces to (dropping bars)

$$\frac{d^2w}{dr^2} + \left(n + \frac{1}{r}\right)\frac{dw}{dr} = R\, e^{(n-nr)} \frac{\partial p}{\partial z} \qquad \text{......(9)}$$

where R represents the Reynolds number $\frac{w_m a}{\nu_0}$

The corresponding boundary conditions in terms of the non-dimensional quantities become

$$w = \text{finite at } r = 0 \qquad \text{......(10)}$$

$$w' = \alpha\sigma\left(w_B + \frac{R}{\sigma^2}\frac{\partial p}{\partial z}\right) \text{ at } r = 1 \qquad \text{......(11)}$$

Solution of the problem :

Let us assume that the flow is driven by constant pressure gradient.

$$-\frac{\partial p}{\partial z} = g_0 \qquad \text{......(12)}$$

Solution of (9) which satisfies the conditions (10) and (11) is

$$W = W_B + P_1/n\,\{1/n(e^{-n} - e^{-nr}) + (e^{-n} - r\,e^{-nr})\} \qquad \text{......(13)}$$

$$\text{where } P_1 = -\frac{g_0}{2Re^n}$$

where W_B is the dimensionless slip velocity at the nomial surface.

W_B is given by

$$W_B = RAg_0 \qquad \text{......(14)}$$

and $$A = \frac{1}{\sigma} \times \left(\frac{1}{\sigma} - \frac{1}{2\alpha}\right) \qquad \text{......(15)}$$

This enables us (*i*) to visualize directly the effect of Reynolds number on the slip velocity and so on the entire velocity distribution (*ii*) to interpret that slip velocity is directly proportional to the pressure gradient (*iii*) to find out the condition involving σ and α which is responsible for the occurrence of back flow in the slip motion.

If the surface of the circular tube had been impermeable, the velocity profile can be derived from equation (13) making $\sigma \to \infty$.

$$W = \frac{P_1}{n}\{(1/n\,(e^{-n} - e^{-nr}) + (e^{-n} - re^{-nr})\} \qquad \text{.....(16)}$$

Satisfying the conditions

$$W_B = 0 \quad \text{on } r = 1 \qquad \text{.....(17)}$$

In this case the slip velocity indicated by (14) tends to zero satisfying the no-slip condition of the boundary.

Discussion :

From equation (14) and equation (!5) it follows that when $\sigma = 2\alpha$. W_B the slip velocity reduces to zero and W_B increases with the increase in the value of R (Reynold number) and also W_B increases when $\sigma < 2\alpha$, but back motion in slip velocity occurs for value of σ greater than 2α.

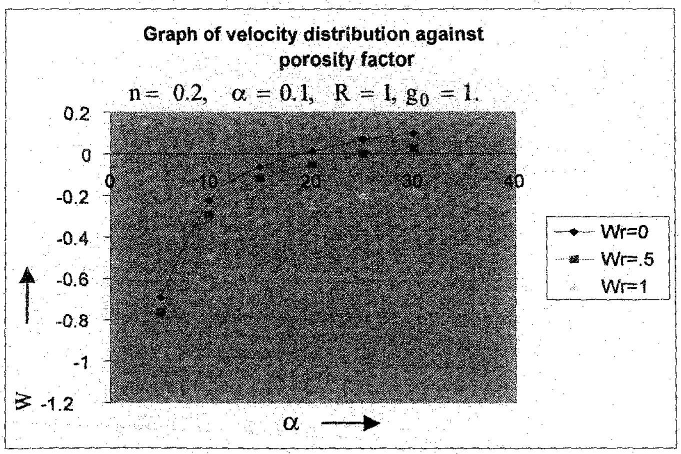
The distribution of velocity have been numerically evaluated against σ for fixed value of (*i*) $n= 0.2, \quad \alpha = 0.1$

(*ii*) $n= 1, \quad \alpha = 0.1$

(*iii*) $n= 1, \quad \alpha = 1$

(*ii*) $n= 0.2, \quad \alpha = 1$ and depicted in figures (1), (2), (3) and (4).

Thus we can conclude that Reynolds number, porosity factor $(\sigma < 2\alpha)$ is favourable to the motion but retardation in the flow begins when $\sigma > 2\alpha$ and independently on R.



	5	10	15	20	30
$W]_{r=0}$	-69×10^{-2}	-22×10^{-2}	-5×10^{-2}	-2×10^{-2}	10×10^{-2}
$W]_{r=0}$	-76×10^{-2}	-29×10^{-2}	-12×10^{-2}	-5×10^{-2}	3×10^{-2}
$W]_{r=0}$	-96×10^{-2}	-49×10^{-2}	-32×10^{-2}	-25×10^{-2}	-17×10^{-2}

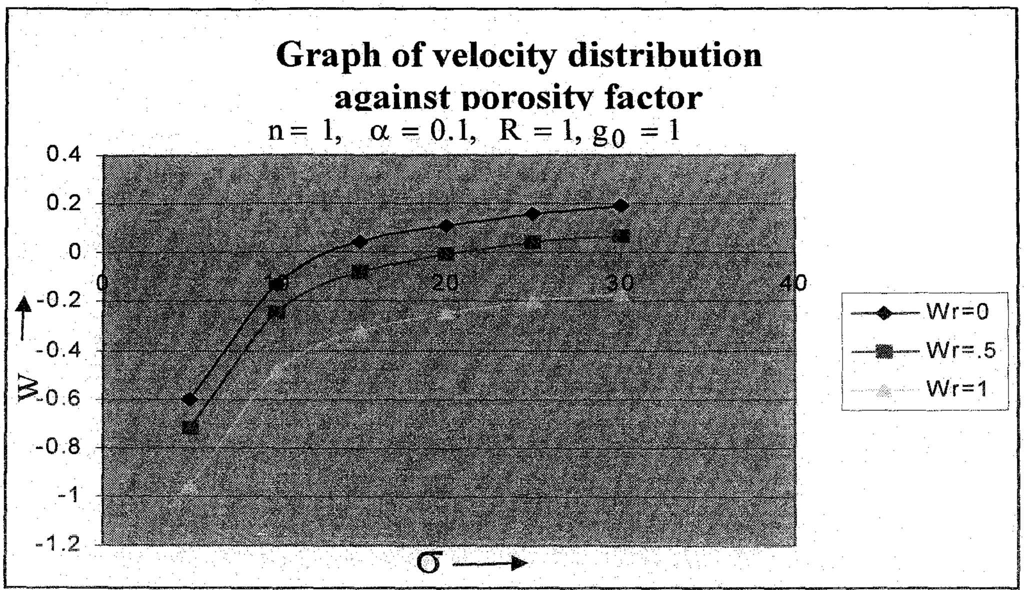


Fig. 2

$\sigma \rightarrow$	5	10	15	20	25	30
$W]_{r=0}$	-60×10^{-2}	-13×10^{-2}	4×10^{-2}	11×10^{-2}	16×10^{-2}	19×10^{-2}
$W]_{r=0.5}$	-72×10^{-2}	-25×10^{-2}	-8×10^{-2}	-1×10^{-2}	4×10^{-2}	7×10^{-2}
$W]_{r=1}$	-96×10^{-2}	-49×10^{-2}	-32×10^{-2}	-25×10^{-2}	-20×10^{-2}	-17×10^{-2}

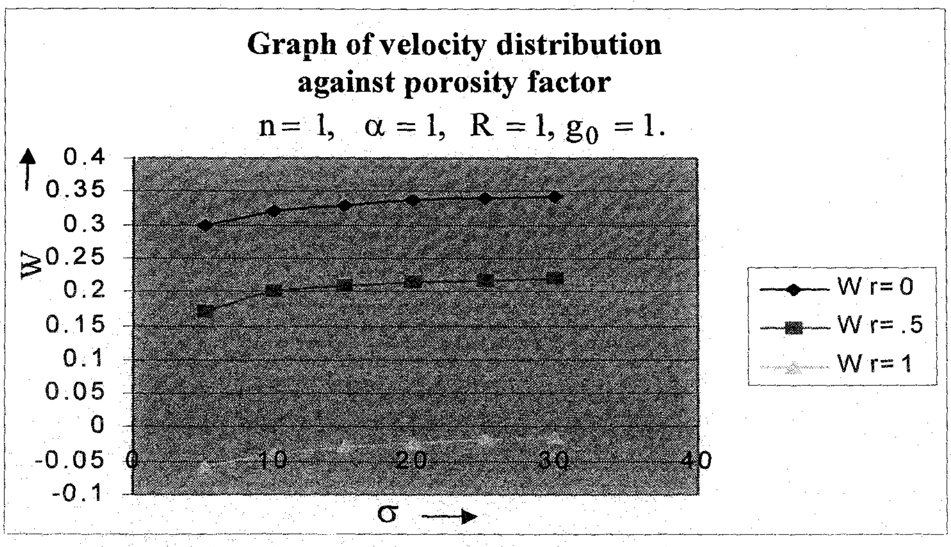


Fig. 3

$\sigma \rightarrow$	5	10	15	20	25	30
$W]_{r=0}$	.299	.319	.330	.336	.341	.343
$W]_{r=0.5}$	.176	.196	.207	.213	.218	.220
$W]_{r=1}$	-.06	-.04	-.029	-.023	-.018	-.016

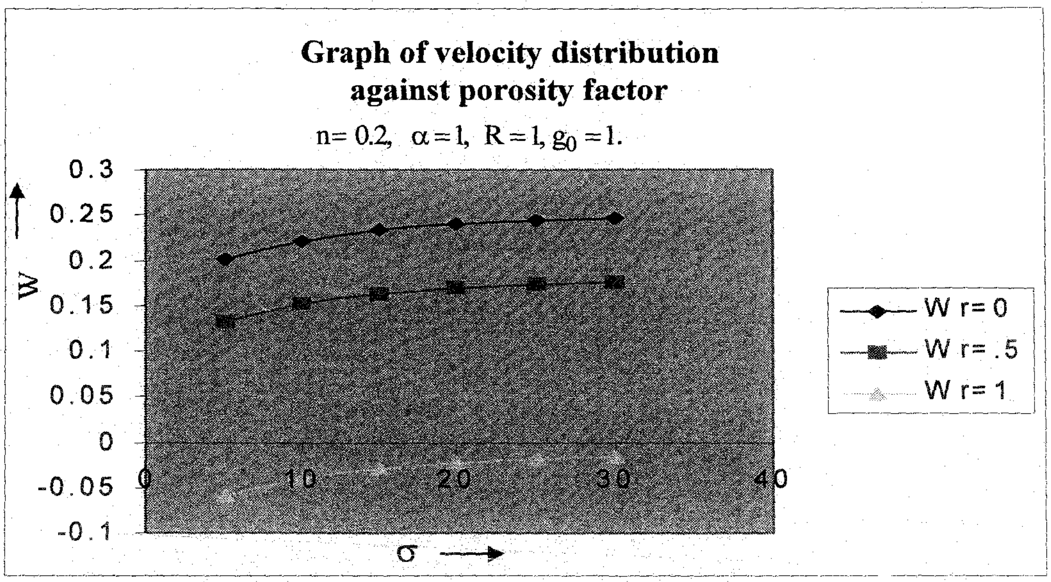


Fig. 4

σ →	5	10	15	20	25	30
$W]_{r=0}$	.203	.223	.234	.240	.245	.247
$W]_{r=0.5}$	.132	.152	.163	.169	.174	.176
$W]_{r=1}$	-.06	-.04	-.029	-.023	-.018	-.016