PART - TWO

TO STUDY THE BOUNDARY LAYER FOR THE FLOW OF VISCOUS STRATIFIED FLUID PAST A POROUS BED

Introduction : The motion of viscous stratified fluid largely depends on the magnitude of stratification factor and porosity factor. So, they must have an effect on the boundary layer also. Boundary layer flow over a flat plate in rotating frame of reference has gained considerable attention due to its varied and wide application in the areas of geophysics and astro-physics. The fluid flow in a porous pipe and porous channel has been investigated by a number of workers in a recent past. Sellers, Yuan, Berman and Wang, Chiu, Soo and Singleton investigated the problem of boundary layer flow of dusty fluid over a semi finite flat plate. Gupta and Pop (1975) analysed the boundary layer growth in a rotating liquid with suspended particles. Kapur and Gupta (1964) considered the steady boundary layer equation for a power law fluid flowing between parallel plates. Mathur and Nandanan (1972) have studied the laminar boundary layer flow of an oldroyed fluid under the influence of pressure gradient with and without suction through a wedge. Srivastava and Maiti (1966) have solved the boundary layer equation for two dimensional flow of a second order fluid. Sharma and Singh (1992) investigated three dimensional laminar boundary layer free convect flow past (i) a porous flat plate (ii) a porous vertical plate when slightly sinusoidal transverse suction velocity distribution is applied at the plate.

The study of the flow of the viscous fluid past a porous medium without stratification has been studied by Beavers and Joseph (1967). Channabassappa and Ranganna (1976) considered the flow of viscous stratified fluid of porous bed with the anticipation that stratification may provide a technique for studying the pore size in a porous medium. In this paper he has shown that 'slip velocity' is proportional to the pressure gradient. The boundary layer just beneath the permeable interface and the friction factor are also obtained.

In this present note we have studied the effect of stratification factor and porosity factor on the boundary layer of the flow of viscous stratified fluid between a rigid impermeable bed and a fixed permeable bed. It is observed that for a fixed n the boundary layer δ decreases with the increase of σ, (Table-1) and for fixed σ,the boundary layer δ increases with the decrease of n (Table-2).

Mathematical Formulation of the Problem And Its Solution :

Here we consider the flow of viscous fluid between two parallel plate channel of height h where the upper one is rigid impermeable wall moving with an exponentially decaying velocity and the lower boundary wall is permeable.

A physical model for this problem considered here have been depicted in Figure-1 which consists of two zones. Zone one is the region occupied by the viscous fluid from moving impermeable rigid plate to the interface where the flow is characterised by free flow and is governed by Navier-Stokes equations. Zone-2 lies below the interface where the flow is governed by modified Darcy Law.

The motion being taken uni-directional we choose u the velocity in the x direction, y axis perpendicular to direction of motion, ρ the density, μ the co-efficient of viscosity and p the pressure at any point the space occupied by the fluid.

IMPERMEABLE PLATE

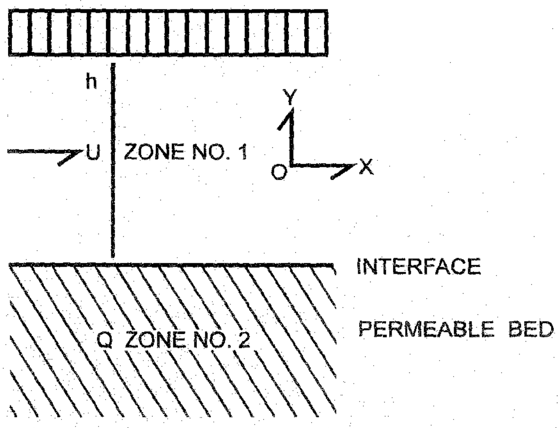

Fig. 1 PHYSICAL MODEL

Now, while studying the stratified viscous flow of variable viscosity between a porous bed and an impermeable plate. It is seen there exists a thin boundary layer just beneath the interface.

As we are interested in studying the effect of σ and n on the boundary δ. We consider the boundary layer type equation.

The basic boundary layer equation is

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\mu}{k}u \quad \ldots\ldots\ldots\ (1)$$

The boundary conditions

$$u = u_B \text{ at } y = 0 \quad \ldots\ldots\ldots\ldots\ (2)$$

$$u = Q_0 e^{-n\delta} = -k/\mu_0 \, e^{-n\delta}\left(\frac{\partial p}{\partial x}\right) \text{at } y = -\delta \; \ldots\ldots\ldots\ldots (3)$$

$$\mu = \mu_0 e^{-\beta y}, \; \rho = \rho_0 \, e^{-\beta y} \; \ldots\ldots\ldots\ldots (4)$$

$$Q = Q_0 \, e^{\beta y} \ldots\ldots\ldots\ldots (5)$$

is the basic equation for zone-2.

Where, $Q_0 = -\,{}^{1}\!/_{\mu_0}\left(\frac{\partial p}{\partial x}\right)$ ……… (5a)

Here μ_0 and ρ_0 are the co-efficients of viscosity and density respectively at the interface y=0. Also β represents the stratification factor, u_B the slip velocity at the nominal surface y=0 and Q_0 is given by (5a), k is the permeability co-efficient (has the dimension of length square).

Let us make equation (1) dimensionless by using the quantities

$$u' = {}^{u}\!/_{u_m}, \; t' = ht/u_m, \; k' = \frac{k}{h}, \; y' = \frac{y}{h}$$

$$p' = \frac{p}{\rho_0 u_m^2}, \; \delta' = \frac{\delta}{h}, \; V_B = \frac{u_B}{u_m}$$

Where u_m is the maximum velocity of the flow. Equation (1) now reduces to

$$\frac{\partial u'}{\partial t'} + \frac{n}{R}\frac{\partial u'}{\partial y'} - \frac{1}{R}\frac{\partial^2 u'}{\partial y'^2} + \frac{\sigma^2}{R}u' = -e^{ny'}\frac{\partial p'}{\partial x'} \; \ldots\ldots\ldots (6)$$

The corresponding boundary conditions are

$$u' = V_B' \text{ at } y' = 0 \; \ldots\ldots\ldots\ldots (7)$$

$$u' = \frac{Q_0}{u_m} e^{-n\delta'} = -\frac{R}{\sigma^2} e^{-n\delta'} \frac{\partial p'}{\partial x'} \text{ at } y' = -\delta' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where $n = \beta h,\ R = \frac{u_m h \rho_0}{\mu_0},\ V_0 = \frac{\mu_0}{\rho_0})$

Dropping dashes equation (6) becomes we get,

$$\frac{\partial u}{\partial t} + \frac{n}{R}\frac{\partial u}{\partial y} - \frac{1}{R}\frac{\partial^2 u}{\partial y^2} + \frac{\sigma^2}{R} u = -e^{ny}\frac{\partial p}{\partial x} \quad \ldots\ldots\ldots\ldots (9)$$

and the corresponding boundary conditions are

$$u = V_B \text{ at } y = 0 \ldots\ldots\ldots\ldots (10)$$

$$u = \frac{Q_0}{u_m} e^{-n\delta} = -\frac{R}{\sigma^2} e^{-n\delta} \frac{\partial p}{\partial x} \text{ at } y = -\delta \quad \ldots\ldots\ldots (11)$$

Here n is the non-dimensional stratification factor, σ is the porosity factor, R is the Reynoldss number, V_B is the dimensionless slip velocity at the nominal surface, V_0 is Kinematic co-efficient of viscosity at the interface y=0.

Let us now we assume that the flow is due to a pressure gradient given by

$$-\frac{\partial p}{\partial x} = Ae^{-\alpha_1 t} \ldots\ldots\ldots\ldots (12)$$

When A & α_1 are known real constants, we assume that the velocity is given by

$$u = ve^{-\alpha_1 t} \quad \ldots\ldots\ldots\ldots\ldots (13)$$

and $V_B = u_* e^{-\alpha_1 t}$ .... (14)

Equation (9) now becomes

$$\frac{d^2v}{dy^2} - n\frac{dv}{dy} - \left(\sigma^2 - \alpha_1 R\right)V = -Ae^{ny} \text{ .... (15)}$$

Solution of equation (15) is

$$v = e^{xy/2}\left\{C \cosh \alpha_2 y + D\ Sinh\ \alpha_2 y\right\} + \frac{A}{\left(\sigma^2 - \alpha_1 R\right)}e^{ny} \text{ .... (16)}$$

Where C &D are functions independent of y and

$$\alpha_2 = \frac{\sqrt{n^2 + 4\left(\sigma^2 - \alpha_1 R\right)}}{2}$$

Boundary conditions for u follows from (10) & (11)

$$V = u_* \text{ at } y = 0 \text{ .... (17)}$$

$$V = \frac{R}{\sigma^2} Ae^{-n\delta} \text{ at } y = -\delta \text{ .... (18)}$$

Determining C, D by means of (17) & (18) and substituting in (13), we obtain

$$u = ve^{\alpha_1 t} = e^{ny/2}\left[\left(v_B - \frac{Ae^{-\alpha_1 tR}}{\sigma^2 - \alpha_1 R}\right)Cos\ h\alpha_2 y + \left(V_B - \frac{A\mathrm{Re}^{-\alpha_1 t}}{\sigma^2 - \alpha_1 R}\right)Cot\ h\alpha_2\ \delta\ Sin\ h\alpha_2 y\right.$$
$$\left. + \frac{\left(\alpha_1 R^2 Ae^{-\alpha_1 t}\right)e^{-n\delta/2}}{\sigma^2\left(\sigma^2 - \alpha_1 R\right)Sin\ h\alpha_2\delta} Sin\ h\alpha_2 y\right] + \frac{Ae^{-\alpha_1 t}\ e^{nyR}}{\left(\sigma^2 - \alpha_1 R\right)} \text{ .... (19)}$$

We know that at the edge of the boundary layer, the shear has to be zero.

In other words

$$\frac{\partial u}{\partial y} = 0 \text{ at } y = -\delta \text{ .... } (20)$$

Then from Equation (19), using (20), we get,

$$\alpha_2\left(V_2 - \frac{Ae^{\beta_1 tR}}{\sigma^2 - \alpha_1 R}\right) + \frac{\alpha_2 A\alpha_1 R^2 e^{-\alpha_1 t}}{\sigma^2\left(\sigma^2 - \alpha_1 R\right)} \operatorname{Cosh} \alpha_2 \delta e^{-n\delta/2}$$

$$+ \frac{nARe^{-\alpha_1 t}}{\sigma^2 - \alpha_1 R}\left(1 - \frac{\alpha_1 R}{2\alpha^2}\right) \operatorname{Sinh} \alpha_2 \delta e^{-n\delta/2} = 0 \text{ .... } (21)$$

This equation for δ is transcendental and it is difficult to obtain an analytical solution. However, we feel that since the boundary layer and stratification factors are very small, we can neglect cubes and higher powers of δ and obtain

$$a\delta^2 + b\delta + c = 0 \text{ .... } (22)$$

$$a = \frac{n^2 c}{8} - \frac{n}{2}\alpha_2\alpha_3, \quad b = \alpha_2\alpha_4 - \frac{nc}{2} + \alpha_2\alpha_3$$

$$c = 2\alpha_2\alpha_3\left(\alpha_5 - \frac{1}{n\sigma^2}\right), \alpha_3 = {}^{n}\!/_{2}\, Ae^{-\alpha_1 tR},$$

$$\alpha_4 = \alpha_2\left(V_B - 2\alpha_2\alpha_5\right),$$

$$\alpha_5 = \left\{nR\left(\sigma^2 - \alpha_1 R\right)\right\}^{-1}$$

Now, we assume, $V_B = 1, R = 1, \alpha_1 = 1, A' = 1$ for all calculations. The boundary layer δ, as a function of porosity factor σ, is calculated numerically for different values of n as shown in table (1).

DISCUSSION :

To study the effect of stratification factor and porosity factor on the boundary layer of the flow of viscous stratified fluid between a rigid impermeable bed and a fixed permeable bed. Zone-1 is the region occupied by the viscous fluid from moving impermeable rigid plate to the interface when flow is characterised by free flow and is governed by Navier-Stokes equations. Zone-2 lies below the interface where the flow is governed by modified Darcy law.

From Table-1 it is clear that for a fixed n the boundary layer δ increases with the increase of σ.

From Table-2 it is clear that for fixed σ, the boundary layer thickness δ increases with increase of n.

Table – 1

n=0.2

$\sigma \rightarrow$	5	10	15	20	25
$\delta \rightarrow$	8.0638	16.4086	24.6369	31.5360	41.0170

n =0.4

$\sigma \rightarrow$	5	10	15	20	25
$\delta \rightarrow$	8.3386	16.6895	24.9137	31.7891	41.2901

n = 0.6

$\sigma \rightarrow$	5	10	15	20	25
$\delta \rightarrow$	8.6546	16.9831	25.1986	32.0475	41.5680

n = 0.8

$\sigma \rightarrow$	5	10	15	20	25
$\delta \rightarrow$	9.0025	17.2885	25.4900	32.3128	41.8494

n = 1

$\sigma \rightarrow$	5	10	15	20	25
$\delta \rightarrow$	9.3868	17.6119	25.7949	32.5812	42.1386

The boundary layer thickness δ, as a function of stratification n, is also calculated numerically for different values of σ as shown in Table-2

Table-2

$\sigma = 5$

$n \rightarrow$	0.2	0.4	0.6	0.8	1
$\delta \rightarrow$	8.0638	8.3386	8.6546	9.0025	9.3868

$\sigma = 10$

$n \rightarrow$	0.2	0.4	0.6	0.8	1
$\delta \rightarrow$	16.4086	16.6895	16.9831	17.2885	17.6119

$\sigma = 15$

$n \rightarrow$	0.2	0.4	0.6	0.8	1
$\delta \rightarrow$	24.6369	24.9137	25.1986	25.4900	25.7949

$\sigma = 20$

$n \rightarrow$	0.2	0.4	0.6	0.8	1
$\delta \rightarrow$	31.5360	31.7891	32.0475	32.3128	32.5812

$\sigma = 25$

$n \rightarrow$	0.2	0.4	0.6	0.8	1
$\delta \rightarrow$	41.0170	41.2901	41.5680	41.8494	42.1386