ON SOME CONCEPTUAL AND COUNTER-INTUITIVE ISSUES IN RELATIVITY THEORY

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I lovingly dedicate this Thesis to my

Dadu (Grandfather)

Preface

This thesis "On Some Conceptual and Counter-intuitive Issues in Relativity Theory" is a culmination of work and learning on the different aspects of Twin Paradox and its ramifications in the theory of Relativity. Though this century old paradox has been a matter of considerable interest in the world of Physics for more than hundred years, articles on the issue still appear in different journals, yet it continues to illuminate our ideas in Relativity theory. In this thesis an effort has been made to take a step ahead to understand the counter-intuitive issue with more clarity. The work was conducted by me in the Department of Physics under the University of North Bengal. During the entire process of the research work there have been several people who provided me with valuable inputs to my work, guided me through and proved to be a very positive influence towards completion of the work. I sincerely acknowledge their efforts and take this opportunity to express my gratitude to them.

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Contents

Notations and Abbreviations

ALT	Approximate Lorentz Transformation
AS	Acoustic Signal
AZT	Approximate Zahar Transformation
BE	Boughn Effect
CMBR	Cosmic Microwave Background Radiation
CS-thesis	Conventionality of Simultaneity thesis
CVL	Constancy of Velocity of Light
DT	Dolphin Transformation
EP	Equivalence Principle
EW	Einstein World
GPS	Global Positioning System
GR	General Relativity
GSDC	Gravitational Slowing Down of Clocks
GW	Galilean World
IAT	Identically Accelerated Twins
LCF	Length Contraction Formula
LT	Lorentz Transformation

MSRP	Modified Second Relativity Postulate
OWS	One Way Speed
RTD	Relativistic Time Dilation
SR	Special Relativity
TDF	Time Dilation Formula
TE	Transformation Equation(s)
TT	Tangherlini Transformation
TWS	Two Way Speed
VSL	Variable Speed of Light
ZT	Zahar Transformation

Chapter 1

General Introduction

1.1 Introduction

The present thesis, as is evident from the title, deals with some conceptual and counter-intuitive issues of the theory of Relativity. One of the greatest scientific achievement at the turn of the century was the introduction of the theory of Relativity by Albert Einstein in 1905. With the introduction of this theory Einstein revolutionized the scientist's way of understanding physics. This theory had to face considerable resistance as it shocked common sense. The basic postulates of this theory played a decisive role in the development of the fundamental laws of mechanics. For almost two centuries Newtonian mechanics was successfully satisfying the scientific and technical demand of the human society, but the introduction of the theory of relativity brought about a revolutionary change in the physical description of the world and the nature of science. Indeed, Einstein's theory of relativity caused a paradigm shift in our understanding of space and time. Since the concept of space and time in the relativistic world differs considerably from that of the everyday life so various difficulties arose in the analysis of certain situations in this world. The altered nature of space and time in this world gave birth to many counter-intuitive predictions and certain apparent logical contradictions which shocked common sense. These are commonly known as "paradoxes". The word "paradox" originates from Latin *paradoxum* or Greek *paradoxos*, meaning a self-contradictory statement or one that appears self-contradictory, while it is actually true. The common feature of these special kind of problems is that a particular physical situation on simple observation from the points of view of different frames of reference give results which apparently contradict each other but if analyzed carefully the contradictions get resolved. The literature is replete with descriptions of quite a number of such paradoxes and their resolutions.

These counter-intuitive problems or the paradoxes have been found to be connected with various conceptual issues of relativity theory. The most fascinating and puzzling among the consequences of relativity is the so-called "twin paradox", a proposed thought experiment analyzing the effects of consequences of the theory on a pair of twins, one of whom travels at or near the speed of light while the other remains "stationary" on Earth. Generations of physicists have applied their knowledge of relativity in resolving this issue, but somehow a debate on this topic still continues. It seems humankind is yet to comprehend the full implications of the twin paradox and its ramifications. That there are more than three hundred articles already written on this subject, articles are still pouring in, editors of pedagogical journals receive about four articles in one issue and the internet is inundated with discussion forums and unpublished articles bring testimony to the above fact.

While dealing with these problems, it is often claimed in literature that one needs to modify the theory in order to get a proper answer to the problem. But to think of any possible violation of a well established theory (for one reason or the other) it is essential first to refine our understanding of the theory. In the twin problem, the question of how the identical twins age differently raise deep questions about the structure of space time and hence provides deeper understanding of the theory of relativity. In particular a new approach to studying the twin paradox employing the conventionality of simultaneity (CS) thesis of Special Relativity (SR) is found to provide deeper understanding regarding the subtleties of the problem enriching our understanding of the subject.

The present investigation attempts to providing better pedagogical and methodological clarity on this famous counter-intuitive problem and its variations. The main text concerning the present study comprises of chapters 3-7 where I have reported the observations and results obtained by me (along with my collaborators) in the last few years. Some of these observations have been published and some have been reported in the national and international meetings and others are under publications. The topic wise summary of these investigations have been provided in Sec.(1.3) Before going into the detail of the investigations, it will be worthwhile to give an outline of the twin problem so that one becomes well acquainted with it from the starting point of our discussion. In the next section of this chapter therefore I will give a brief review of the earlier works and ideas concerning the problem. Chapter 2 will be devoted to studying the CS-thesis, which as mentioned earlier has been found to be an essential tool for the investigations. The different transformations following the CS-thesis has been found to provide a platform for studying the twin problem in different worlds which helps to clarify the issue furthermore. Although this has been reported earlier (by one my collaborator), but for the sake of completeness I will give a detailed report of this in my thesis (Vide. Ref[1] for details). The second chapter will give reader additional background in these issues.

1.2 Twin Paradox

The twin paradox is perhaps the most interesting paradox of Relativity, and turned out to be one of the most discussed subject since the introduction of special relativity. Einstein in his pioneering paper[2] of 1905 first predicted the time dilation of a moving clock or the concept of slowing down of a moving clock as a physical interpretation of relative time. He notes that simultaneity fails for two clocks one of which is at rest and the other in motion–the moving clock when back to its original position will lag behind the stationary one. Replacing the clocks by human observers, Langevin[3] developed the notion of "twin paradox". In 1911 Langevin picturesquely formulated the problem by building a situation where a traveller goes to a distant star with almost the speed of light and returns back to earth in the same manner. At the end of the trip the traveller's age increases by only two years while the time elapsed on earth is almost two centuries[4]. Considering a twin of the traveller remains at rest on earth, the ages of the twins will differ after the space trip of the traveller. This is the standard form of the paradox suggested by Langevin where the twin siblings meet to discover that they age differently. This as such is not a paradox but the "peculiar consequence" of relativistic time dilation (RTD) suffered by the (biological) clock of the traveller twin due to which she returns younger with respect to the earth-bound sibling. The phrase "peculiar consequence" had originally been used by Einstein in 1905[2, 5] to describe the temporal offset effect, which later came to be known as the clock paradox[1] or the twin paradox. In 1911 Einstein, like Langevin gave a dramatical presentation of this peculiar outcome as "If we placed a living organism in a box ... one could arrange that the organism, after any arbitrary lengthy flight, could be returned to its original spot in a scarcely altered condition, while corresponding organisms which had remained in their original positions had already long since given way to new generations. For the moving organism the lengthy time of the journey was mere instant, provided the motion took place with approximately the speed of light" [6]. This peculiar result of twins aging differently was given the name of "twin paradox". Note

[1]Technically the clock paradox is defined otherwise. Debs and Redhead mentioned that "Wesley Salmon refers to this symmetrical time dilatation as the"clock paradox" as opposed to the asymmetrical dilatation which takes place in the twin paradox. Perhaps confusingly "clock paradox"can also refer to the attempt "To avoid ... the (biological) issue of whether a traveller's aging is in accord with the standard clock that he carries. Because proper times are path dependent quantities, the time dilatation which produces the "clock paradox" fails to produce a truely paradoxical version of the twins' story." but here as others have done we will use twin paradox and clock paradox interchangeably.

that the conditions experienced by the earth-bound sibling is visibly different from that of the traveller twin, hence one cannot expect symmetrical result regarding their ages. In fact both Einstein and Langevin recognized early that this "peculiar consequence" may appear paradoxical, but actually it is not. The result becomes paradoxical on consideration of the fact that the time dilation effect depends only on the relative velocities of the two twins (clocks) and is perfectly reciprocal with respect to them. each of the twin can claim that the other's clock is slowing down by the same amount. Further the asymmetric outcome regarding the ages of the twins appear to go against the principle of relativity according to which "the laws of physics are same in all frames of reference". For example Dingle[7] in 1957 stated that Einstein made a "regrettable error" and he argued that "According to the postulates of SR, if two identical clocks separate and reunite, there is no observable phenomenon that will show in absolute sense that one rather than the other moved. If the postulates of relativity is true, the clocks must be retarded equally or not at all, their readings will agree on reunion if they agreed at separation...".

To sum up there are three facets of the problem[2]. First, the counter-intuitive result that the rocket-bound sibling will age less than the earth-bound one is against common sense. For a common man who believes time is absolute gets puzzled to accept any difference between the ages of the twins. Even some authors of relativity claim that the relativistic effects of time dilation and length contraction are not real but a consequence of distant clock synchronization alone[9]; they cannot accept true time dilation of a moving clock when it is brought back to its original position. Second comes the logical contradiction aspect where each of the twin can claim

[2]Scott[8] has noted that "It is paradox, in the dictionary meaning of the word from the view points of (i) absolute time (ii) the special theory of relativity, and (iii) the general theory of relativity."

that since the time dilation effect depends solely on the relative velocity and since Lorentz transformation predicts reciprocal time dilation of moving clocks, the clock of the other twin must run slow, thus generating a conflict between the predictions of the twins. Thirdly, the biased result of the twin problem as if violates the principle of relativity of motion. Considering principle of relativity one can erroneously claim that there is no possible means to distinguish the twin in motion from the twin at rest and hence there cannot be any asymmetry regarding their ages.

The first aspect i.e the counter-intuitive aspect, as mentioned earlier is not paradoxical.Although counter-intuitive the fact has been experimentally proved (see for example Hafale-Keating experiment[10]). It should be noted that neither Einstein nor Langevin did find any paradox in it. Einstein called it as a "peculiar consequence" of Special relativity and Langevin explained the different aging rates as due to the fact that "..only the traveller has undergone an acceleration that changed the direction of his velocity..."[3]. Again, the third aspect is the most trivial one. The principle of relativiy claims equivalence of inertial frames of reference only, but in the twin problem the traveller twin lies in an accelerating frame. The second aspect i.e the logical contradiction aspect is the one which is *most puzzling*, most discussed but still awaits a proper clear cut explanation. The present investigation therefore deals mainly with this aspect of the twin problem.

1.2.1 Standard Resolutions:

Since its inception the paradox have baffled a lot of physicists. They have applied various ideas in understanding this strange phenomenon. The richness of this phenomenon is evident from the fact that the literature is replete with discussions regarding this issue. Relativity in its either forms (special or general) have been used in resolving this problem. The enormous research on this topic can be classified into various categories. In this and the coming sections we shall provide a brief account of these various works. Starting with Langevin's account of the paradox, consider a pair of twins A and B (say), one of whom (A) stays on earth, while the other (B) on board a fast rocket leaves earth with a speed v fractionally less than that of light for a voyage to a distant star at P (at a distance L from A) and subsequently turns around and returns in the same manner to meet the stay-at-home sibling on earth. The duration of the acceleration phase can be assumed to be very small compared to the time it takes during its forward and return journeys, the time for the round trip of B as measured by A may thus be calculated as $2L/v$. The same time in B's clock on account of the relativistic time dilation will be $2\gamma^{-1}L/v$. Since $\gamma^{-1} = (1 - v^2/c^2)^1/2 < 1$, where c is speed of light, at the end of the trip B should be younger than A.

As mentioned in the previous section Langevin and Einstein did not consider this result to be literally paradoxical. Both men argued that the time offset in the twin parable is a natural phenomenon and can be explained considering the time dilation effect of SR. Indeed, with respect to the inertial frame of the earth-bound sibling, the world lines of the twins in the Minkowski diagram are different, and hence the asymmetry in the aging can be attributed to the fact that proper time is a path dependent quantity in SR[4]. This can be clarified as follows. Let us calculate the proper time τ along the two trajectories of the Minkowski space-time as shown in Fig.(1.1) below. It is assumed that the to and fro coordinate-speeds are the same. These trajectories are labeled as path (1) from the origin O of the earth's frame to time $t = 2T$ (say) along the vertical axis (y-axis) and path (2) from the same origin to the turning point P along the oblique line and back again to O', corresponding to the paths of the earth-bound and travelling twin respectively. The outward and the inward speeds being constant the proper times can be obtained by integrating along each trajectory,

$$d\tau = \gamma^{-1} dt. \tag{1.1}$$

The non-integrability of $d\tau$ is evident from the fact that the results are different for the above two paths. Since $\gamma^{-1} < 1$ the proper time along path (2) is always less than that along path (1). This also follows from the elementary notion of geometry: The sum of two sides of a triangle is always different from the third side. Also in SR the space-time is pseudo euclidean, hence $OP + PO' < OO'$. This implies that the traveller will age less than the earth-bound sibling.

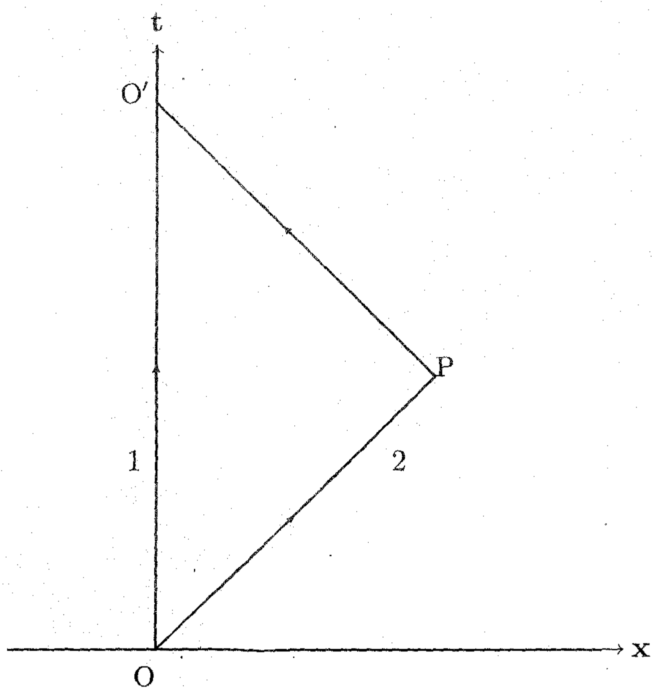

Figure 1.1: **Minkowski diagram**

This simple calculation suggests that the non-integrability of proper time is as if the answer to the paradox and people are often found to express surprise at the suggestion of the paradox itself. Note that this calculation provides an answer to the counter-intuitive aspect of the twin problem i.e. it ascertains that the traveller twin will be younger when the twins meet. This class of resolutions however,is unable to deal with the logical contradiction aspect as it cannot explain the scenario from the frame of the accelerated traveller. As noted by C. S. Unnikrishnan[11] "The calculation is avoided in the frame of B because such a calculation is deemed beyond the scope of special relativity....". The world line approach however allows one to pose the problem in geometrical terms and provides a smart and uniform basis for discussions of the paradox as well as different variations of the clock paradox. In SR the time τ for the clock B and the A-clock time t are related as

$$d\tau^2 = dt^2 - (dx^2 + dy^2 + dz^2)/c^2. \tag{1.2}$$

where (x, y, z) is the displacement of B as measured by A.

From the above equation it is clear that $d\tau$ is always less than dt as B departs from A. For a return journey along any path the time interval on clock B (obtained by integrating the above expression), will thus be less than that of A. Various authors describe and analyze the problem by drawing the world line of the twins (or the clocks) and sometimes for the photons they send. Below we draw[3] world line diagrams for some variations of the twin paradox problem including the standard one.

[3]We owe to G. D. Scott's paper[8] for these diagrams.

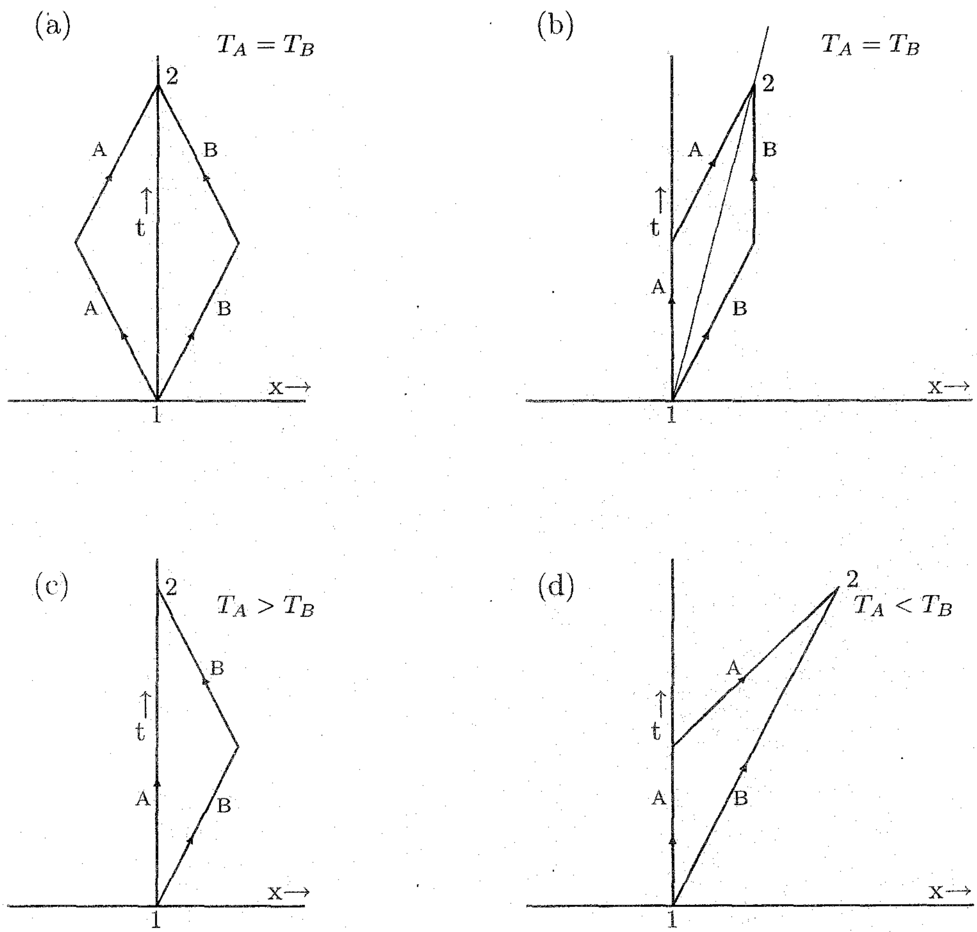

Figure 1.2: **World line diagram for variations on the clock paradox problem.**

The first two diagrams represents the scenario where the twins age the same when they meet at point 2, whereas in the last two diagrams the twins' ages differ. Authors of repute are often found to use the world line diagrams to reconstruct and study variations of the twin paradoxes. For example Debs and Redhead in their most cited paper[4] have extensively used the pedagogical power of these diagrams to analyze the twin problem considering the conventionality of simultaneity thesis. Also there are other articles[11] where the world line diagrams are used to explain the twin problem taking into account the change of line of simultaneity (discussed later) during the turn around of B. This shows the power of these diagrams although the approach is unable to address the subtleties of the problem.

Another type of resolution given to the twin problem involves a change of line of simultaneity during the turn around of the traveller. This type of resolutions came into play when the twin paradox was posed without involving any acceleration, but merely involving change of inertial frames, by including a third observer C. Here thus not two but three inertial frames are involved—The earth frame or Σ_0-frame (the rest frame of A), the frame Σ moving with velocity v relative to A (frame of B) and Σ'-frame moving with velocity $-v$ relative to A (frame of C). In this problem the twin B, at the point of turnaround, transfers the information to the twin C so that A and C can compare their readings as they cross each other, and hence B is not required to physically turn around to compare the ages. An example of such a resolution is the Lord Halsbury's "three brothers thought experiment" [4] in which three brothers (clocks) A, B and C move uniformly with relative velocities along a straight line. It is assumed that each of the observer is equipped with radar, radio and optical equipment for measuring distances, relative velocity and synchronizing clocks[4]. As all the three observers are in inertial frames, each of them, stationary, departing and returning has lines of simultaneity parallel to the lines MP and NP as shown in Fig.(1.3) below. The resolution thus points out that during the transfer of clock information from frame B to frame C the lines of simultaneity changes which causes a discrepancy or a kind of "synchronization jump" MN of the earth clock. This time MN is the reason for the differential aging.

[4]For the present purpose however it is hardly necessary to explain the detailed procedure for such measurements

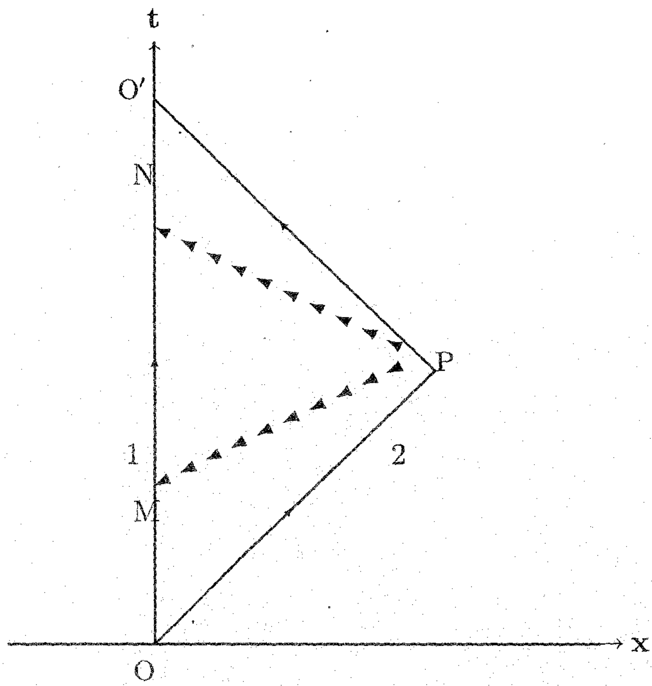

Figure 1.3: **The dotted lines represent the lines of simultaneity from the point of view of the traveller**

For better understanding one may consider that A observes twin B passes him at the time $t_A = 0 = t_B$. He synchronizes his clock on passing. The twin B travels a distance L at a time $t_A = L/v$ and simultaneously passes the third observer C. The sibling C synchronizes his clock to that of B at the position of passing. Finally C crosses the position A at time $t_A = 2L/v$ and A and C compare their clocks. In this case one can use the time dilation formula of SR without any ambiguity. The observer A, on account of time dilation notės the clocks of both B and C run slower by the Lorentz factor γ^{-1}. Thus A observes the reading of C-clock as $2\gamma^{-1}L/v$ at the position of passing when his clock reads $t_A = 2L/v$. Since A, B and C are all assumed to be in *unaccelerated frames of reference*, according to SR then there is no abstract reason to prefer A than B or C. From the view-point of B, the clock A runs slow by the same Lorentz factor γ^{-1} as A observes for B-clock. Similarly

B observes C-clock to be even slower than that of A since B measures the velocity of C to be $2v/(1 + v^2/c^2)$ which is greater than v. Hence B predicts that C-clock reads less than that of A when C passes A. Similarly according to C's perspective, A-clock runs slower by the same Lorentz factor γ^{-1} but C observes B-clock is even slower. This clearly accounts for the fact that C-clock (which has been synchronized to that of B) reads less time than A-clock when the latter passes C, so that the twins finally arrive to the same conclusion.

The explanations of the twin problem based on relativity of simultaneity includes several different but related approaches. One is the above mentioned Lord Halsbury's "three brother" approach, another example is Langevin's radio signal approach. Langevin in his 1911 paper[3] pointed out two grounds based on which the asymmetrical aging of the twins can be explained. Since then these have been the basis of most of the standard explanations of the differential aging[4]. One among them is the radio signal approach. David Bohm[12] described in detail the radio signal approach where the twins stay connected by sending radio signals back and forth. Using the relativistic Doppler shift equations of SR, Bohm observed that the earthbound twin A first received a set of slow pulses and after a time T (which is the time of reception of the first signal after the turn-around of B), received another set of faster ones. For the travelling twin Bohm notes that "If the rocket observer were watching the fixed observer he would then see the life of the latter slowed down at first and later speeded up...." Bohm concludes that for the traveller "...the effect of the speeding up more than balanced that of the slowing down. He would not therefore be surprised to find on meeting with his twin that the latter had experienced more of life than he had...", thus dissolving the paradox.

This treatment of the paradox based on the phenomenon of Doppler shift was also given by Darwin[13] who described a scenario where the twins kept record of each others time by sending out regular time signals (radio or light signals). The analysis can be quantitatively described as follows: Using the relativistic Doppler factor the frequency of a receding source is $[(c-v)/(c+v)]^{1/2}$ while that of an approaching source is $[(c+v)/(c-v)]^{1/2}$. For the outward trip of B, A will thus receive B's signals at $[(c+v)/(c-v)]^{1/2}$ intervals corresponding to a red Doppler shift, whereas when B is approaching, A will receive B's signals at $[(c-v)/(c+v)]^{1/2}$ intervals corresponding to a violet Doppler shift.

The observer A will thus receive slow or $[(c+v)/(c-v)]^{1/2}$ signals from B for the duration of the outward trip or for the time light takes to travel from P to A. For remaining time A will receive fast or $[(c-v)/(c+v)]^{1/2}$ signals from B. Hence A will record time $[(c+v)/(c-v)]^{1/2}L/(c+v) + [(c-v)/(c+v)]^{1/2}L/(c-v)$ or $2\gamma^{-1}L/v$ worth of B's signal.

Similarly for B's record of A's signal, if t is the the total time of the trip as measured by B, then for time $t/2$, B will receive slow signals from A until he reaches P and then upon reversing his motion, he will receive another set of fast signals again for time $t/2$. Since A sends out $2L/v$ worth of signals during the trip,

$$[(c-v)/(c+v)]^{1/2}t/2 + [(c+v)/(c-v)]^{1/2}t/2 = 2L/v \tag{1.3}$$

hence solving one obtains $t = 2\gamma^{-1}L/v$ as described before.

The analysis of the twin problem in terms of Doppler shifts gives a mere description of what is taking place but it does not explain the fault of the standard analysis. Besides it does not provide the reason why there should be an abrupt change of the earth-bound clock when B turns around. This treatment thus evades the paradox but gives no explanations as to where the contradictions lie in the problem.

The second fact that Langevin suggested to explain the lack of symmetry between the paths of the twins was that of the acceleration that the traveller has to face to return to earth. Many authors claim that to obtain a complete explanation of the twin paradox one needs to invoke the effects of the direction-reversing acceleration. The traveller B on reaching the distant star P (at a distance L from A) reverses the direction in a time negligible with respect to L/v and returns back to meet A. A lying in an inertial frame can correctly apply TDF of SR to conclude that B should be younger on their meeting after the trip. On the other hand B on account of the direction reversing acceleration lies in a non-inertial frame, so the observations of B can be shown to be confusing and apparently internally inconsistent[14]. Also as B lies in a non-inertial frame, it is often argued that the postulates of SR are not applicable to B and therefore the claim of reciprocity of RTD between the frames of reference of the twins falls through. Indeed Einstein found this sort of argument preferable in dismissing the paradoxical element in the twin problem[5]. However, although correct, this qualitative argument explaining away the paradoxical element of the twin problem may lead to a misconception that the turn-around acceleration (or the force causing the acceleration) is the direct cause of the differential aging. Many articles have tried to study the role of the acceleration in the twin problem. Whether acceleration is the direct cause of the differential aging has been one of the most debated queries in the context of the twin problem. In section 1.2.3 therefore we shall elaborate on this matter.

Before going to the next section let us discuss a few other trivial approaches adopted in explaining the differential aging. As for example the length contraction effect of SR has been used in literature to explain the asymmetrical aging of the twins. This type of resolution has been discussed by Fremlin[15]. Here it is argued that since B is moving with a constant velocity v relative to A, he will measure the distance AP as $\gamma^{-1}L$ and not L taking into account the relativistic length contraction effect. Hence the time calculated by B for his round trip will be $2\gamma^{-1}L/v$. The asymmetry in this approach can be thought of as due to the fact that the distance point P is fixed relative to A and not to B. Although one can correctly calculate the travel time of A's round-trip on B's clock from the perspective of B in this way, the drawback of this approach is that it remains silent about the reading of A's clock as measured by B. Will it not be dilated?

As another example Bondi[16] presented an article where the aging process of a human being is compared with the mileage of a car and the journey of the traveller twin with that of a vehicle along a curvy road. Referring to the aging of the traveller Bondi has been found to remark "It will *not* therefore come as a surprise to him on his return to earth to find out that he has aged less than the people there, just as the traveller who took the curvy road cannot have been surprised that he covered a longer mileage than the traveller who followed the straight line. Hence there is no clock"paradox", since it is not paradoxical for two persons with different experiences to find that the consequences of their experiences differ....". These trivializing often cavalier statements[5] on the issue are made when one is concerned with the third aspect of the paradox only.

The literature is flooded with various other resolutions of the paradox. However in this vast literature there is surprisingly very little discussions about Einstein's arguments regarding the paradox. Albert Einstein, the father of the "twin paradox" was the first to resolve it. In the next section we present a brief report of Einstein's work on the paradox.

[5]Often these statements are found as passing remarks by some authors who in their scholarly discourse also deal with the deeper aspects of the paradox.

1.2.2 Einstein and the Paradox:

In his 1905 paper Einstein presented the first calculations of time dilation of a moving clock relative to a stationary one. This relativistic effect was shown to predict what he called a "peculiar consequence" which Einstein stated as follows "If there are two synchronous clocks at A, and one of them is moved along a closed curve with constant velocity (v) until it has returned to A, which takes, say t seconds, then this clock will lag on its arrival at A by $\frac{1}{2}t(v/c)^2$ seconds behind the clock that has not been moved"[5, 17] or in other words two synchronized clocks fail to remain simultaneous whether they move in a straight line, a polygonal line or in a closed curve. From this statement it is clear that Einstein believed that the time lag between the two twin's clock is a problem of simultaneity and not of the direction-reversing acceleration. In contrast, in the year 1911 Langevin[3], while resolving the problem stated that "acceleration....which only the voyager has undergone" is the reason for the time difference between the clocks without even mentioning Einstein's argument about simultaneity. This statement of Langevin generated a huge controversy since it achieved a wide circulation among the scientific community. Till 1914 Einstein maintained his view that acceleration was irrelevant for the time gap between the two clocks. In his 1914 letter to Joseph Petzoldt who rejected the conclusions of Einstein's application of closed polygons in a Minkowski space as "a reversion to an absolutistic way of thinking"[18] he wrote"we know nothing though about how (the "moving" clock) U'proceeds relative to (the "stationary system") K while $U'$is in *accelerated* motion. But the travelling speed of U'relative to K can only be influenced *finitely* by a finite acceleration. Thus, if we allow U' to describe a closed path relative to K in such a way that U''s acceleration times disappear against U''s times while moving in a straight line (all seen relative to K), we can then disregard the influence of acceleration times on the angles travelled by the hands of the clock U'. Then we *must* conclude that the hands of U' advance slower while travelling along a closed polygon(al path) than the hands of an identically designed clock that was constantly at rest relative to K....". This confirms that till 1914 Einstein's statement relied mainly on simultaneity and he considered arguments based on acceleration to be unimportant.

In 1918, three years after the introduction of the General theory of relativity, Einstein produced arguments related to acceleration. The theory of relativity faced opposition from a group of anti relativist especially Ernst Gehreke and Phillipp Lenard who not only questioned the theory but accused Einstein of plagiarism and publicity seeking[5, 19]. To defend his theory Einstein presented a paper in the journal *Die Naturwissenschaften* entitled "Dialogue about objections to the theory of relativity"[20] which is written in the form of dialogue between a critic and a relativist. Here Einstein does not repeat his simultaneity argument, rather he argues that postulates of special relativity are not applicable to the moving clock who on account of the acceleration faced lies in a non-inertial frame and so "no contradictions in the foundations of the theory can be construed." This resolution presented by Einstein points out that the physical cause of the asymmetry is the pseudo-gravitational field and the gravitational time dilation of GR[11].

Einstein asserted that the stationary twin, lying in an inertial frame can perform the calculation using SR only and hence considering the time dilation effect he concludes that the travelling twin will age less on their second meeting. From the perspective of the traveller the situation is symmetrical except at the point of acceleration. To analyze the situation from the traveller's frame, Einstein splits the the total time dilation into two parts–first from SR and the second due to the gravitational time dilation according to GR and the equivalence principle. According to Einstein the acceleration experienced by the traveller generates equivalent gravitational fields and since "according to the general theory of relativity, a clock works faster the higher the gravitational potential at the place where it is situated", the traveller should consider this contribution while estimating the ages. Einstein states that if these points are taken into proper consideration then "calculation shows that the consequent advancement amounts to exactly twice as much as retardation during stages of inertial motion. This completely clears up the paradox...."

1.2.3 The Direction-Reversing Acceleration and the Paradox:

Langevin, in his 1911 discussion[3] of the twin problem resolved it in terms of the "acceleration .... which only the voyager has undergone"[5]. Since then the role of acceleration in causing the differential aging has been a matter of great controversy. Numerous articles have struggled with the misconception that the direction-reversing acceleration is the direct cause of the asymmetric aging of the twins. Gruber and Price, in an interesting article[21] dispel the idea of any direct connection between the acceleration and the asymmetric aging by presenting a variation of the paradox where although one of the twins is subjected to undergo an arbitrary large acceleration, no differential aging occurs. In their variation of the problem the rocket-bound sibling undergoes a periodic motion as shown in Fig (1.4) such that

$$x = (V_{max}/\omega) sin\omega t, \tag{1.4}$$

where x and t are the coordinates of a fixed frame on earth and V_{max} as can be seen is the maximum speed achieved by the rocket-bound twin relative to the earth. The acceleration of the rocket (the rocket's 4-acceleration) has a maximum magnitude of $V_{max}\omega$, which occurs at times $\omega t = \pm\pi/2, \pm 3\pi/2....$ The maximum acceleration thus occurs when the particle has zero velocity relative to the fixed frame on earth.

The relativistic results therefore agree with Newtonian (non-relativistic) answers.

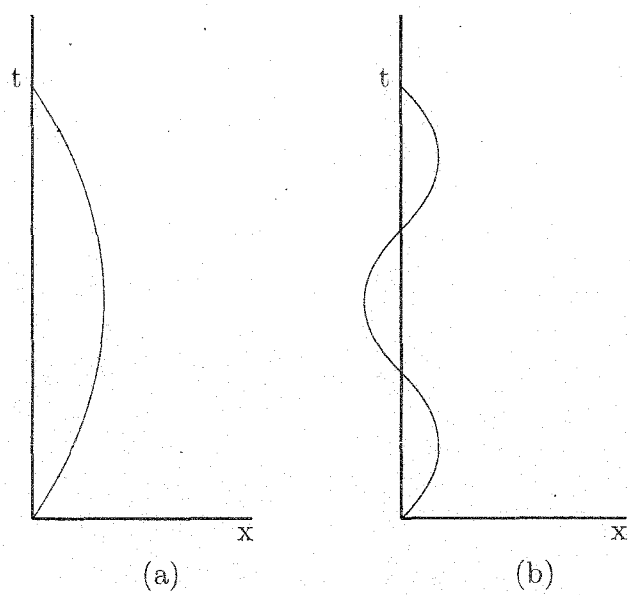

Figure 1.4: **Rocket-bound twin World lines. The world line in (b) has maximum acceleration three times that for the world line in (a).**

The proper time τ of the rocket-bound twin is related to the earth time t by

$$d\tau = dt[1 - (V_{max}/c)^2 cos^2 \omega t]^{1/2}. \tag{1.5}$$

A rocket trip starting and ending at the earth will take an integer number i.e, starting at $t = 0$ and lasting until $\Delta t = n\pi/\omega$. For such a trip the proper time will be

$$\Delta\tau = n \int_0^{\pi/\omega} [1 - (V_{max}/c)^2 cos^2 \omega t]^{1/2} dt, \tag{1.6}$$

so that the ratio of elapsed rocket time to elapsed earth time is given by

$$\Delta\tau/\Delta t = (2/\pi) \int_0^{\pi/2} [1 - (V_{max}/c)^2 cos^2 \theta]^{1/2} d\theta. \tag{1.7}$$

This integral can be evaluated numerically. The important fact evident from this equation is that the time dilation effect is independent of ω. The maximum acceleration, on the other hand, is $V_{max}\omega$ and any other measure of acceleration will be proportional to $V_{max}\omega$. It is thus clear that the time dilation effect is independent of the acceleration . These considerations eventually prove that acceleration *per se* cannot be the root of differential aging. Indeed one can show that one can have arbitrary large acceleration without any significant differential aging!

In another article by Boughn[22] a converse situation is discussed in connection with an interesting variation of the twin problem. Here a scenario is presented where two spatially separated twins age differently although their history of acceleration remains the same. This problem will be discussed in detail in Sec. 1.2.7

The above two examples thus makes it clear that the acceleration is not the direct cause of the asymmetric aging although in order to have twice intersecting trajectories of the twins (this is necessary since the clocks or ages of twins have to be compared at the same space-time events) one cannot normally avoid acceleration[6]. That the acceleration *per se* cannot play a direct role in causing the differential aging is also evident from the usual calculation of the age difference from the perspective of the stay-at-home twin's inertial frame if one notes that the duration of the turn-around process of the rocket can be made arbitrarily small in comparison to that for the rest of the journey and hence the final age difference between the twins can then be attributed to usual time dilation of the traveller during the unaccelerated segment of her journey. In such a calculation the time dilation can also be calculated during the acceleration phase (assuming the clock hypothesis to be true[4]) and it is seen to contribute arbitrarily small value in the age offset if the duration of the

[6]The acceleration can even be eliminated by posing the problem in curved space-time. Examples of this type are discussed in Sec. where some novel versions of the paradox are presented where the traveller although "unaccelerated" eventually meet the stay-at-home one[23, 24, 25, 26, 27, 28, 29, 30].

acceleration phase is assumed to tend to zero.

The role of acceleration in the twin problem has also been studied by Nikolic[31] in his paper entitled "The role of acceleration and locality in the twin paradox". This paper clarifies the quantitative role of acceleration (from the point of view of the accelerated observer) in causing the asymmetry. It is shown therein that the differential lapse of time depends not only on the value of the acceleration itself but also on the relative distance between the observers. The procedure predicts that acceleration has no influence if two observers are at the same position. However if travelling twin moves with constant velocity and suddenly reverses the direction of motion, at this time, it will appear to him that the time of the inertial clock instantaneously jumps forward but there is no such jump of the accelerated clock from the point of view of inertial twin. Hence although acceleration has a secondary role from the point of view of the stationary twin, from the stand point of the traveller the effect of acceleration is far from being trivial.

1.2.4 Resolutions of the Paradox in the General Theory of Relativity:

The question of whether GR is required to be introduced for the explanation of the twin problem arose when Langevin in 1911 proposed the resolution of the paradox in terms of the acceleration faced by the traveller. Many arguments have advanced since then which claim that the introduction of GR and a gravitational field at the point of acceleration of the travelling twin is the right way to explain the asymmetrical aging. Although the clock paradox can be satisfactorily resolved using special relativistic results only, there are authors[6, 12, 20, 32, 33, 34, 35, 36, 37] who seem to feel that for the complete explanation of the twin paradox one needs to take the aid of GR. In this context Bohm notes that "two clocks running at places of different gravitational potential will have different rates." This consideration of the gravitational red shift effect was made by Bohm and also Frisch at the time when the effect was tested for the first time by Pound and Rebka[38]. Even Einstein, the originator of the paradox gave resolutions based on general relativistic results which has been discussed in Sec.(1.2.2). One needs to note here that this was the second argument given by Einstein in the year 1918, the first one being the simultaneity argument based on SR.

GR was introduced in the twin problem in order to explain how the travelling twin perceives the situation during the acceleration phase at turn around. While calculating the ages of the twins one must take into account the effect of the acceleration on the aging. Now since accelerated frames lie outside the domain of SR, it is often claimed that GR needs to be considered. Although several authors[39, 40, 34, 38, 43] have explained the influence of acceleration on the aging within the framework of SR by considering that during the acceleration phase the traveller changes his inertial frames and the difference between these two frames supplies the extra aging of the stay-at-home sibling, but still a group of scientists believe that these explanations are not complete. In this context Harpaz[43] comments "these attempts leave the students with the feeling that something has been kept from them. Although this approach enables one to calculate correctly the age difference between the twins, it does not manifest the 'physical agent' responsible for the creation of such a difference...." Harpaz presents an article[43] where he has shown that the equivalence principle (EP) provides such an agent and that is gravity. The author of this pedagogical article introduces the concept of gravitational slowing down of clocks (GSDC) through EP in calculating the age difference from the perspective of the traveller.

He uses the gravitational red-shift formula, which can be obtained heuristically as

$$\Delta\nu = \nu_0(1 + gh/c^2), \tag{1.8}$$

where g is the acceleration due to (pseudo) gravity and $\Delta\nu$ represents the change of frequency of light observed from a distance h from the source where the frequency of the same light is seen to be ν_0. Interpreting this red-shift effect in terms of GSDC, the formula can be written as

$$t_1 = t_2(1 + \Delta\phi/c^2), \tag{1.9}$$

where t_1 and t_2 are times measured by clocks located at two points P_1 and P_2 (say) and $\Delta\phi = gh$, is the potential difference between these points. As pointed out by Einstein in his second resolution to the paradox, Harpaz too concludes that because of GSDC during the (arbitrary) short duration of B's acceleration a time difference is generated between the clocks of A and B. It has thus been shown that with respect to the traveller the acceleration plays a role by providing an extra time difference between the siblings. This time difference more than offsets the age difference calculated by B solely assuming the reciprocal time dilation so much so that finally B ages less by the correct amount.

In a textbook on GR by Moller[34] another type of general relativistic solution of the twin problem is available, where the problem is resolved by transforming the Lorentz metric into a form that is valid in an arbitrary accelerated reference frame. Moller's treatment has some drawbacks which has been taken care of in an interesting article by Perrin[37]. Perin has set up a specific round-trip situation which has been studied from the point of view of each twin by using the gravitational field equation as its starting point. In order to determine the time elapsed on earth during the periods of acceleration he solved the gravitational field equations and the geodesic equations of motion in the traveller's frame of reference. He has pointed that "The equality of the result obtained by each twin is explicitly exhibited".

In this article the outward trip of the traveller twin B is divided into three phases. In phase 1 B leaves the earth with an acceleration g until she attains a velocity v relative to earth, she then covers a distance L with this constant velocity v in the second phase and in the third phase she decelerates at the same rate and finally stops before reversing her direction of motion. The reverse journey is similarly divided into three parts. First the twin decelerates from rest to velocity $-v$, she then retraces the distance L with the constant velocity and to end with comes to rest on earth after receiving the same amount of acceleration. From special relativistic considerations he calculated the time elapsed on earth to be

$$T = 4\frac{\gamma v}{g} + 2\frac{L}{v}. \tag{1.10}$$

To compute the time recorded in the traveller's clock, as observed on earth he uses the standard time dilation result following LT. This time comes out to be

$$\overline{T} = \frac{2c}{g} ln(\frac{1 + u/c}{1 - u/c}) + 2\gamma^{-1}\frac{L_0}{u}. \tag{1.11}$$

From the perspective of the traveller also the relative motion of A is divided into six phases — four acceleration and two uniform velocity phases. The author interprets the acceleration phases as the turning on of a gravitational field along the direction of motion of the traveller (along $\bar{x}$ say). The author used the S-frame as the frame of the earth-bound twin and the $\bar{S}$-frame as that of the traveller. He claims that the calculation for computing the A-clock time from the perspective of B during acceleration phases lies outside the domain of SR, so GR has to be introduced at this point. "To compute the time elapsed in S, the travelling twin can only use the results of special relativity during phase II. During phases I and III, he must make use of the general theory." Finally after doing the detailed calculations, it is shown that for the paradox to be resolved GR must predict

$$t_I + t_{III} = (1 - v^2/c^2)^{-1/2} 2v/g + Lv/c^2, \tag{1.12}$$

where t_I and t_{III} are the time elapsed in A-clock during the acceleration phases I and III respectively of her forward journey as observed by B.

As mentioned the author assumed the acceleration phases as the turning on of a gravitational field along the $\bar{x}$ direction. The classical gravitational potential corresponding to this field is $\phi(\bar{x}) = g\bar{x}$. With this field Perrin obtained a metric in $\bar{S}$ of the form

$$ds^2 = e^{\pm 2g\bar{x}/c^2}(c^2 d\bar{t}^2 - d\bar{x}^2) - d\bar{y}^2 - d\bar{z}^2 \tag{1.13}$$

The Christoffel symbols corresponding to the metric are

$$\begin{aligned} \left\{ {0 \atop \mu\nu} \right\} &= \pm \tfrac{g}{c^2}(\delta_{\mu 0}\delta_{\nu 1} + \delta_{\nu 0}\delta_{\mu 1}), \\ \left\{ {1 \atop \mu\nu} \right\} &= \pm \tfrac{g}{c^2}(\delta_{\mu 0}\delta_{\nu 0} + \delta_{\nu 1}\delta_{\mu 1}), \\ \left\{ {2 \atop \mu\nu} \right\} &= \left\{ {3 \atop \mu\nu} \right\} = 0. \end{aligned} \tag{1.14}$$

It has been shown in the paper that during phases I and III, the earth twin moves along a geodesic in $\bar{S}$ and hence the time elapsed in S corresponds to the proper time along the geodesic which is obtained by solving the equations of motion

$$\frac{d^2\bar{x}^\mu}{ds^2} + \left\{ {\mu \atop \alpha\beta} \right\} \frac{d\bar{x}^\alpha}{ds}\frac{d\bar{x}^\beta}{ds} = 0. \tag{1.15}$$

With the use of Eq.(1.14) these become

$$\begin{aligned} &\tfrac{d^2\bar{t}}{ds^2} \pm \tfrac{2g}{c^2}\left(\tfrac{d\bar{t}}{ds}\right)\left(\tfrac{d\bar{x}}{ds}\right) = 0, \\ &\tfrac{d^2\bar{x}}{ds^2} \pm g\left(\tfrac{d\bar{t}}{ds}\right)^2 \pm \tfrac{g}{c^2}\left(\tfrac{d\bar{x}}{ds}\right)^2 = 0, \\ &\tfrac{d^2\bar{y}}{ds^2} = \tfrac{d^2\bar{z}}{ds^2} = 0. \end{aligned} \tag{1.16}$$

Solving the above metric equation (Eq.(1.13)) and the geodesic equations of motion (Eq.(1.16)) for $\bar{x}$ and $\bar{t}$ and using the boundary conditions for the phases I and III the time elapsed in S is calculated to be

$$\begin{aligned} t_I &= \tfrac{v}{g}, \\ t_{III} &= 2\tfrac{\gamma v}{g} + \tfrac{Lv}{c^2} - \tfrac{v}{g}. \end{aligned} \tag{1.17}$$

Hence the total of these times is

$$t_I + t_{III} = 2\frac{\gamma v}{g} + \frac{Lv}{c^2}, \tag{1.18}$$

With this value for the earth-clock time, the perspectives of both the twins match, thus resolving the paradox. Hence from his GR analysis Perrin obtained the time elapsed on earth (as calculated by the traveller) during the acceleration phases. In Chapter (7) we shall show that this time elapsed on the earth clock can also be obtained from special relativistic considerations alone by solving some simple equations.

Further in another interesting article by Gron[32] "...it is pointed out that a complete resolution of the twin paradox demands that the travelling twin takes into account the gravitational effect upon the rate of time when he predicts the ageing of his brother...." Two alternative ways to predict this have been presented here.

One makes use of the Lagrangian dynamics in an uniformly accelerated reference frame. In the other method the author assumes that the travelling twin is at rest in an uniformly accelerated reference frame where the line element has the form

$$ds^2 = -(1 + gx/c^2)^2 c^2 dt^2 + dx^2 + dy^2 + dz^2. \tag{1.19}$$

Comparing this with the line element (in the case of time like interval) representing the proper time interval $d\tau$, as measured by a clock following a world line connecting two events with coordinate separation (dt, dx, dy, dz)

$$ds^2 = -c^2 d\tau^2, \tag{1.20}$$

he obtains for a clock stationary at $(x = h)$

$$\Delta\tau = (1 + gh/c^2)\Delta t. \tag{1.21}$$

The author chooses the coordinate system such that the traveller is at the origin each time she experiences a gravitational field. He has shown that at the starting and arrival points the proper time vanishes, but the travelling twin can predict the aging of the earth-bound twin when she experiences a gravitational field at the turning point. The traveller experiences the gravitational field for a time $\Delta t = 2v/g$. Hence using the above equation it is shown that the proper time in the earth clock as observed by the traveller during the acceleration phase turns out to be $2hv/c^2$ which is the same as obtained special relativistically considering the synchronization gap treatment.

In a book of special relativity by Resnick[6] it has been remarked that "Although there is no need to invoke general relativity theory in explaining the twin paradox, the student may wonder what the outcome of the analysis would be if we knew how to deal with accelerated reference frames. We could then use (the) space ship as our reference frame.... We would find that we must have a gravitational field in this frame to account for the accelerations... If, as required in general relativity, we then compute the frequency shifts of light in this gravitational field we come to the same conclusion as in special relativity."

The essence of any general relativistic solution of the twin problem thus lies in introducing an equivalent pseudo-gravitational potential to be experienced by the traveller twin at the time of her direction reversing acceleration. A consequent gravitational time offset effect then provides the extra aging of the stay-at-home twin A required to make the correct prediction by the traveller twin B. Thus acceleration is absolutely essential for the GR analysis. It may be noted here that

SR can resolve the paradox even without involving any sort of acceleration, but GR is incapable to do so, it always states acceleration and hence the equivalent homogeneous gravitational field as the physical cause behind the asymmetrical aging of the twins. But twin problem can be posed without acceleration (by invoking a third inertial observer or by posing the paradox in closed universe setting which we will discuss in the next section). thus a drawback of the GR analysis is that this cannot resolve those variations of the paradox which do not involve any acceleration. In this context Unnikrishnan[11] notes that "all standard resolutions of the twin paradox invoking acceleration or an equivalent pseudo-gravity as a physical effect responsible for asymmetric time dilation are flawed...". In addition, as pointed out by Debs and Redhead[4] and also others[8, 44, 45, 46], that since in the twin problem one deals with the flat space-time (Riemann tensor $R^{\alpha}_{\beta\mu\nu} = 0$), any reference to GR in this context is quite confusing.

We would like to mention here that in chapter (4) we too have explained the canonical twin paradox in terms of GSDC making use of EP, but there we have shown that the gravitational time offset effect of GR (in the case of uniform gravity) follow from EP provided one uses the full machinery of SR and is therefore essentially special relativistic in origin. Apart from this there are also other reasons which reveals the fact that the use of GR in the explanation of the twin problem is essentially trivial. Firstly as Builder[45] pointed out, it is indeed strange to first deny by some authors the applicability of SR in the resolution of the twin paradox and then use the conclusions derived from SR itself by means of the principle of equivalence of GR. Builder notes that ' This tortuous procedure succeeded in evading the paradox rather than resolving it". Builder further wrote that" The equivalence principle states that the description of events in terms of the coordinates of an accelerated reference system is indistinguishable from the description of identical events in terms of coordinates of reference system at rest in an equivalent gravitational field". It is thus evident that through EP one can predict the course of events in a gravitational field using SR, as those events can be described in terms of the coordinates of an equivalent accelerated reference system.

Secondly few authors like Harpaz[43] while searching for a "physical agent" responsible for the extra aging rely on some approximate formulas including that of the gravitational red-shift because of his assumption $v^2/c^2 << 1$ inherent in the analysis, and therefore, the pseudo-gravitational effect has the ability to resolve the paradox only approximately. Clearly there is no valid reason to make such small velocity approximation for the problem.

Thirdly, as discussed in section 1.2.1 the explanation of the paradox based on SR relies on the fact that during the direction reversing acceleration, the travelling twin switches over his inertial frame and the lack of simultaneity of these frames provides the "missing time" which constitutes the reason for the differential aging. Now the lack of agreement in simultaneity is a special relativistic concept without any classical analogue, on the other hand in many standard heuristic derivations of the gravitational red-shift formula (see for example [43, 47, 48, 49]), one finds that no reference to SR is made. Indeed the well-known formula for the red-shift parameter $z = gh/c^2$ is only approximate and is derived by making use of the *classical* Doppler effect for light between the source of light and a detector placed at a distance h along the direction of acceleration g of an Einstein elevator. Thus the equivalence of gravity and acceleration in terms of gravitational red-shift or GSDC therefore turns out to be as if a purely classical (Newtonian) concept in this approximation. The question therefore arises how will then GSDC account for an effect i.e lack of simultaneity which is essentially a standard relativistic phenomenon.

In chapter (4) of this thesis we have tried to clarify these issues.

1.2.5 The Paradox in Compact Space (Unaccelerated Twin Paradox):

In the standard twin problem the acceleration faced by the traveller is the main point of asymmetry between the two twins. As stated earlier this asymmetry can be removed by posing the problem in the framework of a closed space (due to curvature or topology) where it is possible for the rocket twin to return to its starting point without changing direction. One can thus eliminate the acceleration faced by the travelling twin which restrains him from using the special relativistic results correctly. As in compact space no acceleration is involved, the traveller too remains in an inertial frame and hence is entitled to conclude that the other twin is younger, thus generating a paradox which is much more challenging. There are various articles which have dealt with this version of the paradox, in fact it has been shown there that the paradox can be resolved with the rocket sibling aging less after a complete transit around the compact space.

One such variation of the paradox is given by Dray in his article "The twin paradox revisited" [23]. Here the paradox is reformulated in a closed two-dimensional universe which has the shape of an infinitely long cylinder, in which the time t runs *up* the cylinder and the space x runs *around* it. The time axis thus falls parallel to axis of rotation of the cylinder and hence the stationary twin who is at rest in space (x =const) moves straight up the cylinder. The traveller twin departs and returns by going around the cylinder with constant velocity. The traveller as can be seen can come back to the starting point travelling in a straight line, without turning around i.e without suffering any acceleration.

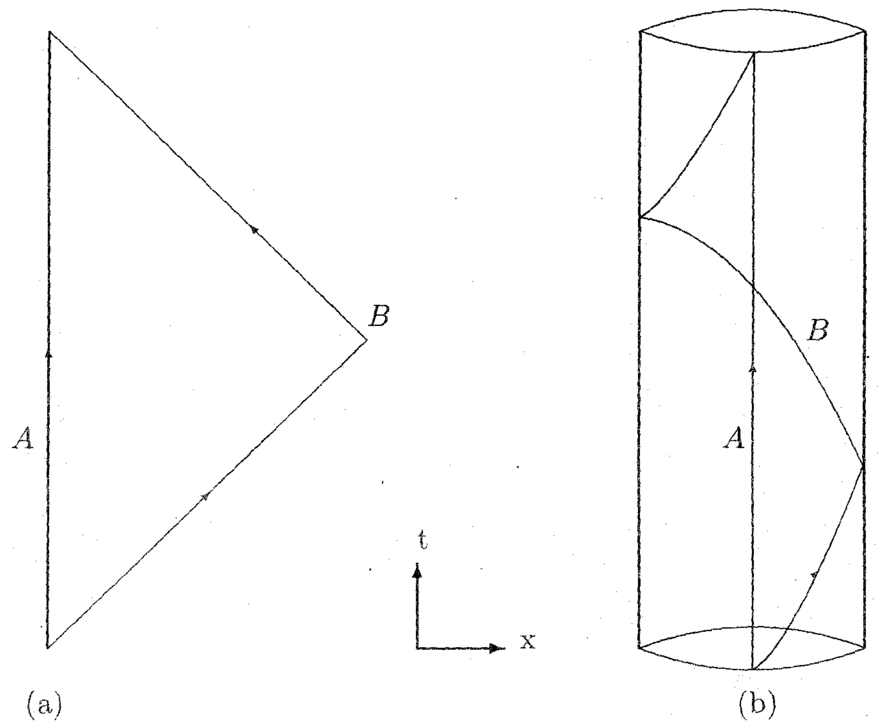

Figure 1.5: **The space-time diagram of the travelling twin B and stationary twin A (a) in a usual twin paradox (b) in a cylindrical universe.**

The author has resolved the issue by showing that although the acceleration can be eliminated from the problem, but still then there is an asymmetry between the twins. The problem has been treated both algebraically and geometrically. In the context of obtaining a geometrical resolution it is believed that one should formulate the paradox in terms of "invariant concepts as opposed to observer dependent concepts". Although we do not agree with this observation in the context of the paradox, for the sake of completeness let us briefly reproduce the arguments.

Dray pointed out that the necessary invariant concept is that of proper time which is just the Lorentzian length of the path and not the Euclidean distance. The triangle inequality in Lorentzian geometry tells that one side of the triangle is greater than the sum of the other two sides. This as if solves the ordinary twin paradox which we have discussed (and criticized)in Sec. For this modified version of the paradox, the author said that "things are just as simple". As a line parallel to the sides of the cylinder is clearly shorter than one that spirals around it.The travelling twin moves along the shorter path and hence is found to be older.

The author further discussed that even though the cylinder is Lorentz invariant, there exists a *preferred* family of observers who should always be oldest in twin paradox calculation between different stationary observers. "Such observers may be singled out by noting that they are the only stationary observers who cannot distinguish between "forward" and "backward"; e.g., by sending light rays in both directions around the cylinder and seeing which returns first".

In relation to this observation we mention here the Hafale-Keating experiment[10], performed by Hafale et.al in order to study the requirement of an asymmetry to get the differential aging. In this experiment, differential aging was observed on two atomic clocks travelling on jets at the same speed around the earth in opposite directions. The rotation of earth provided the required asymmetry to produce the difference in proper times. These two paths without the rotation of the earth can be compared schematically to the two paths going around in the opposite directions on the cylindrical space-time.

An acceleration free version of the paradox has also been discussed by Low[24]. Here the acceleration is avoided by considering a *non-simply-connected* space time where the twins can meet without accelerating as the paths are not homotopic. Two loops (closed curves) are said to be homotopic if they can be continuously deformed into one another. Low observed that " even when no curvature is present the large-scale structure of space time must be taken into account in order to avoid fallacious reasoning." In compact space the asymmetry between the twins is claimed to be not due the acceleration but due to multiply connected topology. This topology has been shown to introduce preferred class of inertial frames[25, 26, 27, 28, 29]. As pointed out by Brans and Stewart[30] who constructed an unaccelerated twin paradox in flat space time "a global analysis leads to the conclusion that the description of the topology of this universe has imposed a preferred state of rest so that the principle of special relativity, although locally valid, is not globally applicable." Similar arguments have been given by Barrow nd Levin who states that "the resolution hinges on the existence of *preferred* frame introduced by the topology, one consequence of which is the inability of the twin in the rocket to synchronize her clock"[29]. They have shown that there can be only one reference frame at rest and all other inertial observers in relative motion live in the universe where both space and time coordinates are known. Thus the solution of twin paradox identifies a preferred place and preferred time at the center of the universe so that observer can be able to synchronize their clocks and observe the smallest volume for the universe. The resolution of the twin problem in closed place thus relies on the selection of a *preferred* frame singled out by the topology of the space.

1.2.6 Twin Problem and the Preferred Frame:

The claim that a preferred frame is required for the complete analysis of the twin problem have been found in various articles. The significant ones among them are the discussions given by Unnikrishnan[11, 50] and Kak[51]. Unnikrishnan, in his paper "On Einstein's resolution of the twin clock paradox", while discussing Einstein's treatment of the twin problem stated that "Einstein's resolution as well as most standard resolutions suffer from a logical fallacy". The fallacy is as follows: the author said that two initially synchronized clocks can register different times only if the rate of the clocks changes differently during motion. Now, in the twin problem the acceleration suffered by the traveller cannot change the rate of the clocks (as rate of clocks are not affected by acceleration), then what is the physical cause behind this changed rates. According to the author this question should be unambiguously answered, but the standard resolution including Einstein's treatment does not show any "logical possibility" which can cause the change in the rate of the clocks required for the explanation of the paradox. As a logically consistent possibility Unnikrishnan then suggests that one has to accept that the "...rate of a clock is modified according to the standard Lorentz factor with the velocity always relative to average rest frame of the universe or the frame in which the cosmic microwave background radiation (CMBR) is isotropic,...". In this case, it is obvious that the twin paradox does not exist. Thus the author concludes that the entire voluminous writings on the twin clock problem can be replaced by this one-sentence resolution which is "...the clocks age with Lorentz factors corresponding to their velocity relative to the preferred frame of the matter-filled universe...." The universe as a preferred frame thus provides an unambiguous solution to the twin problem. This point has been discussed in detail in another paper[50] by the same author.

In the second paper Unnikrishnan has engaged himself for reassessing SR. He notes that the novel theory of relativity should be consistent with the "existence of the massive universe and with the effects of its gravitational interaction on local physics." Also the kinematical effects of SR which depends on the relative motion in flat space-time should be viewed as due to "gravitational effects of the nearly homogeneous and isotropic universe." The correct theory of relativity according to the author is thus the one "with a preferred cosmic rest frame". However the author agrees that the new theory preserves Lorentz invariance. He thus claims that an unambiguous and correct resolution of the twin problem can be obtained from this new theory which he termed as cosmic relativity (which is consistent with the presence of the matter around us in the universe) rather than the old theory i.e special relativity.

As mentioned the author admits that cosmic relativity should preserve Lorentz invariance i.e the theory continues to be Lorentzian like SR, the difference may be structural.

Like Unnikrishnan, Kak presented an article[51] which share the belief that SR is not sufficient to deal with the twin problem as it is said that "the failure of the accepted views and resolutions can be traced to the fact that the special relativity principle formulated originally for physics in empty space is not valid in the matter-filled universe." Kak proposes a new principle for identification of inertial frames in a matter filled universe. He suggested that a frame will be inertial if the universe has spatial isotropy with respect to it. Hence isotropy plays a significant role in this new setup. The author notes that "This provides a means to identify inertial frames. yielding a simple resolution to the twins paradox of relativity theory in such a universe." Further according to the author physical laws can be assumed to be the consequence of large scale structure of the universe such that one can think that two observers in relative motion will have different experiences if isotropy of the universe is not maintained by them equally. It is also claimed that time dilation can be viewed as a consequence of the experience of anisotropy (of the universe) by the moving observer, in which case the resolutions of the twin problem and its variations becomes a trivial exercise. In Kak's treatment the motion of objects are determined against the background of distant stars. This analysis is thus much similar to that of Unnikrishnan's cosmic relativity.

The basic idea behind these explanations is that in order to obtain an unambiguous answer to the twin problem one needs to modify the Special Theory of Relativity to the one involving preferred frame. At this point we would like to mention that we do not agree with this proposal, as to our understanding, the theory of relativity being a self-consistent theory must be able to deal with the consequences of this theory or in other words staying within the theory one should be able to obtain resolution of the twin problem as well as its different variations. This point will be clarified in the coming chapters where we deal with the problem applying only the results of the theory of Relativity. This is best revealed in chapter where we reformulated the twin problem in the classical world using the conventionality of simultaneity thesis of SR. The C-S thesis has been discussed in detail in the next chapter.

1.2.7 Modifications of the Twin Problem:

In an effort to understand clearly the different aspects of the twin problem authors have framed the problem under a variety of circumstances. The literature contains several variations of the clock paradox which are indeed useful additions to the pedagogy of SR in general and twin paradox in particular. Below I will give a brief account of two such variation which have been used extensively in my investigations. The first among them is a scenario given by Boughn[22] where two spatially separated twins age differently although their history of acceleration remains the same. This version of the paradox can be seen to bring out clearly the role of the clock synchronization issue in the twin problem. In the second variant the twin paradox is posed such that each twin lives on one ring of a counter rotating pair of infinitesimally separated rings so that the twins travel on the same circular path but in opposite directions. The resolution of the paradox, as we shall see, focuses attention on the relation between time dilation and clock synchronization.

Boughn's paradox:

An interesting variation of the twin's parable was presented by S. P. Boughn[22] where he discussed *the case of identically accelerated twins* who nevertheless aged differently even after undergoing through the same chain of events. Boughn's scenario involves twins Dick(D) and Jane(J) who were initially at rest at a distance x_0 apart in an inertial frame Σ_0 (the rest frame of their mom and dad) and experienced same amount of acceleration along the direction $\overrightarrow{DJ}$ (x-direction say). After some time, when all their fuels had expended, they came to rest in another inertial frame Σ, moving with non-zero velocity v relative to Σ_0. From the simple application of Lorentz transformation Boughn obtained a surprising result that at the end of the trip, when the twins settle in their final frame Σ, the forwardly placed (with respect to the direction of acceleration) twin J turned out to be more aged than her brother D! In other words if the twins are considered as two synchronized clocks, the outcome would be a net time-offset effect between these clocks in Σ.

The result is surprising since in the problem it has been assumed that the twins D and J throughout had identical experiences, yet their presynchronized (biological) clocks went out of synchrony.

The amount of this desynchronization or the age difference can be quantified from LT as follows:

The time transformation of LT is

$$t_i = \gamma(t_{0i} - vx_{0i}/c^2), \tag{1.22}$$

where t_{0i} and x_{0i} refer to the time and space coordinates of the observer i (i stands for D or J) with respect to the frame of their mom and dad i.e Σ_0 and t_i denotes the corresponding time in their final frame Σ. $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor and c is the speed of light in free space. From the above equation it follows that

$$t_J - t_D = \gamma[(t_{0J} - t_{0D}) - v(x_{0J} - x_{0D})/c^2]. \tag{1.23}$$

Since both the twins had identical experiences the ages of the twins with respect to the frame of their mom and dad, Σ_0 must be the same throughout their journey, even after the completion of their acceleration phase. In other words the clocks of the twins Dick and Jane which were initially synchronized in Σ_0 continue to remain so in Σ_0 during their acceleration phases and thereafter. But what is simultaneous to mom and dad is not simultaneous with respect to their new frame Σ. As pointed out by Boughn, the birthdays of the twins which occur at the same time in Σ_0 (i.e $t_{0J} = t_{0D}$), does not occur at the same time in Σ. This is evident from the above Eq.(1.23) which shows that for $t_{0J} = t_{0D}$, the time difference $t_J - t_D \neq 0$ as the twins are separated by a constant distance $x_{0J} - x_{0D} = x_0$, the difference is rather

$$\Delta t_{desync} = t_J - t_D = -\gamma v x_0/c^2, \tag{1.24}$$

implying in Σ, the birthday of Jane occurs before that of Dick and hence Jane is found to be older than her brother Dick in their new abode by the above precise amount.

This paradox put forth by Boughn concerning the case of identically accelerated twins can however be solved as soon as one recognizes that age difference of two observers at different locations or time offset between two spatially separated clocks does not have any unequivocal meaning. Two clocks can be compared unambiguously only when they are at the same spatial point. For example in Σ, one of the observers can slowly walk towards the other (or both of them can do the walking) and compare their ages (or their clock readings) when they meet. In the relativistic world the so called "slow transport synchronization" is equivalent to the Einstein synchronization[21], and hence the calculated differential aging or time-offset between the clocks of the spatially separated twins would continue to hold even whe they meet after their slow walk. But when the twins meet it can easily be seen[4 that they do not have symmetrical experiences, and hence the paradox gets resolved. Although the paradoxical element of the problem goes away, the fact remains that the differential aging for the "case of identically accelerated twins" given by Eq.(3) is *correct* and as has been already pointed out this time-offset remains unchanged even if the observers slowly walk towards each other and compare their clocks or ages when they meet[21].

This temporal offset effect of identically accelerated clocks gives an important insight into the behavior of clocks in a uniform gravitational field, for, according to EP "...all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system" [52]. This suggests, as correctly remarked by Boughn that two clocks at rest in a uniform gravitational field are in effect perpetually being accelerated into the new frames and hence the clock at the higher gravitational potential (placed forward along the direction of acceleration) runs faster. we shall see in chapter-3 that the time-offset relation (Eq. (1.20)) of Boughn's paradox can be interpreted as the accumulated time difference between two spatially separated clocks because of the pseudo-gravity experienced by the twins. However the connection between gravity with this temporal offset through EP was first pointed out by Barron and Mazur [53], who derived the approximate formula for the "clock rate difference". In chapters (3) to (7) We shall see the importance of this time-offset relation (Eq. (1.24)) in accounting for the asymmetrical aging of the standard twin paradox from the perspective of the traveller twin. This article establishes the fact that at the heart of the twin problem lies the clock synchronization issue of SR.

Circular Twin Paradox:

Although the paradox first appeared in Lightman et.al[54] in 1975, a more elaborate discussion on the problem has been given by Cranor et.al[55] in the beginning of this century. In this problem as said earlier, each twin lives on a ring of counter rotating pair of rings which are infinitesimally separated such that the twins travel along opposite direction in the same circular path. The observers on the ring of one twin should claim the clock of the other twin slowed down by time dilation and other contradicting the claim. The resolution of the paradox focuses attention in the relation of time dilation to clock synchronization. According to Cranor et.al's story the two rings have been assumed to move with equal and opposite angular velocity ω with respect to the frame of reference of the laboratory. One twin Lisa with a team of observers live stationary at every point on a ring of radius R whereas the other twin Bart lives on the other identical ring. The author assumes that Bart moves at the speed $v = \omega R$ in the counterclockwise direction through the laboratory while Lisa's ring move with the same speed in clockwise direction. The twin will pass each other so that they can easily observe the other's clock. Initially they are at the same place and notice that their clocks both read $t = 0$. According to the velocity addition formula of special relativity (SR) the observers on Lisa's ring detect Bart's clock moves at speed

$$v_{rel} = 2v/(1 + v^2/c^2). \tag{1.25}$$

Due to time dilation formula of SR, Lisa's team observes Bart's clock ticking more slowly than the proper time of their clocks by a relative Lorentz factor

$$\gamma_{rel} = (1 - v_{rel}^2/c^2)^{-1/2} = (1 + v^2/c^2)/(1 - v^2/c^2), \tag{1.26}$$

which shows that his clock will lag behind the clock of next of Lisa's observers that he passes. His clock should be seen to lag more and more as he passes successive members of Lisa's team. Finally after the half rotation he ultimately passes Lisa again and notice that their clock will disagree– Bart's clock lags behind Lisa's clock.

Using the notion of SR, Bart and his observers can argue in the same way that Lisa's clock will lag behind his clock at future meeting indicating a paradox.

The resolution of the paradox hinges on the fact that the *time dilation formula of SR holds good provided the coordinate clocks of any frame of reference (inertial) are synchronized according to Einstein's method of synchronization.* The authors correctly point out the difficulty in synchronizing coordinate clocks on rotating frame in Einstein's way. Indeed they point out how the synchronization gap appears when one tries to synchronize clocks along the rim of a rotating ring. They were able to explain the paradox in terms of this synchronization gap or desynchronization of clocks on a rotating platform but some remarks by the authors are rather unfortunate. Let us clarify this below.

Note that Einstein described his synchrony only for inertial frames of reference since a rotating ring represents a set of non-inertial frames, the three other schemes of synchronization described by authors are worth studying. According to method 1 the ring is initially non rotating and any two infinitesimally separated coordinate clocks of the ring are synchronized according to Einstein's method by using the light signal in the usual way by stipulating the one-way speed (OWS) of light is the same as its two-way speed (TWS). The ring can be set into rotation uniformly such that all points of the ring are "treated identically". The method 2 uses a light flash from a big laboratory clock stationed at the center of rotation. The ring is at rest and the observers of the ring upon receiving the light flash can all set their clocks to read $t = 0$. As before the rings are again uniformly put into the motion after the completion of synchronization process. The method 3 on th other hand is almost same as method 2 in every respect except the fact that th coordinate clocks on the ring are synchronized when the ring is already in motio It is clear that the method 1, 2 and 3 synchronization schemes all suggest absol synchrony, according to which two spatially separated events that are simultaneous with respect to the rotating ring are also simultaneous with the laboratory frame. In this connection the authors make the following remarks "If methods 1, 2 or 3 are used for synchronization of ring clocks, then events that are simultaneous to Lisa and Milhouse will also be simultaneous to the Lab observers. It follows that Lisa and Milhouse, and more generally the entire set of observers on Lisa's ring, are not correctly synchronized to constitute special relativity reference frames. This explains what we already know must be true: There will be no lagging of Bart's clock observed as it passes each of Lisa's observers. For the relativistically inappropriately synchronized clocks of Lisa's observers, there is no time dilation of Bart's clock".(In the quotation Milhouse is the closest neighbour of Lisa on the ring in the counter clockwise direction.)

One may conclude from the above discussions that time dilation of SR is the result of the relativity of simultaneity alone. Although in special case it is true that Bart's clock will not be dilated with respect to Lisa's observers, it is generally not true that non-relativity of simultaneity implies no time dilation. It can be shown that time dilation can also be observed even if the co-ordinate clocks are inappropriately synchronized. For example in the relativistic world with absolute synchronization one can observe both time dilation and "time contraction" (for the inverse transformation).

In chapter(6) of this thesis a detailed discussion regarding this variation of the paradox will be given.

1.3 Topic Wise Summary

1.3.1 The Case of Identically Accelerated Twins and the Ordinary Twin Paradox

In chapter (3) the usual twin paradox is explained rigorously in terms of the paradox concerning the differential aging of identically accelerated twins (IAT). Unlike many other resolutions of the paradox, the present approach does away with the prospect of drastic alteration of the remote clock (of the Earth bound twin) from the point of view of the traveller. The case of IAT explains how the rocket bound sibling, in spite of its non inertiality can freely use the special relativistic time dilation formula and yet predict the correct differential aging for the ordinary twin paradox.

1.3.2 Paradox of Identically Accelerated Twins in Different Worlds and the Ordinary Twin Paradox

In chapter (4) the desynchronization effect in the clock readings of the *identically accelerated twins* has been obtained in the relativistic as well as in the classical world with different synchronizations. The conventionality of simultaneity thesis of special relativity is introduced to study the effect in these different worlds. The role of the synchronization convention in producing the desynchronization effect has been clarified. In addition to this the twin paradox is explained using the principle of equivalence. A variation of the twin paradox is used to connect gravity with the desynchronization effect. Through the principle of equivalence gravity is introduced into the twin problem as a physical agent which supplies the required extra aging to the traveller during the acceleration phase due to the gravitational time-offset effect. A brief discussion of the work by Price and Gruber [Am. J. Phys.

64(8),1996] is also presented and some additional examples are provided to further emphasize their point that the question of where the differential aging occur in the twin problem has no definite meaning.

1.3.3 Demystifying Twin Paradox

In chapter (5) the conventionality of simultaneity thesis of special relativity due to Reichenbach and Grünbaum is introduced in the classical world and hence a variation of the twin paradox is posed in this world. A desynchronisation effect between two identically accelerated twins (clocks) in this world is then used to resolve the paradox. The absence of "true" time dilation and length contraction effects in the classical world allows one to see more clearly the role of desynchronisation of coordinate clocks in the frame of reference of the traveller twin in the abrupt turn-around scenario. The reader may use the present analysis as a template for resolving the ordinary twin paradox fully in the context of special relativity.

1.3.4 Circular Twin paradox Revisited

In chapter (6) an interesting variation of the classical twin paradox called circular twin paradox (CTP) has been critically examined. In the CTP discussed in the literature, the twins live on a pair of counter-rotating rings with identical size so that the twins trace the same circular trajectories in opposite directions. To observers of one twin the clocks of the other twin suffer time dilation of special relativity. But when the twins meet during their rotation their clocks must agree on which clock is lagging. In the particular case when the angular velocities of the twins are equal and opposite, the symmetry of the problem demands that the clocks of the observers should have identical readings during their meetings. This is the paradox.

The paradox has been resolved by clearly showing the role of synchronization in the time dilation of clocks in the classical world. The drawbacks of an earlier paper on this issue has also been discussed and clarified by choosing to synchronize the team of observers on both the rings in absolute way.

1.3.5 Conventionality of Simultaneity, Absolute Synchronization and Twin Paradox in the Non-abrupt turn-around Scenario

In the concluding chapter (chapter(7)) a complete treatment of the twin paradox has been presented in the non-abrupt turn-around scenario. In this picture the rocket-bound twin is finitely accelerated with some proper acceleration during its take off, then uniformly move for some time and then stops after finite deceleration when the turn-around takes place. The twin then proceeds towards the stay at home twin with the same kind of acceleration,uniform velocity and deceleration phases and then meets her sibling. Although it is generally believed that this kind of problem requires general relativity to deal with, the present article refutes the claim by showing explicitly how the problem can be treated fully in the context of *special relativity.* It has been shown that with respect to the rocket-bound observer the differential aging can be calculated in this case using the conventionality of simultaneity thesis of SR to show that it matches with the differential aging predicted by the earth twin, thus showing the GR treatment to be redundant. This is the first time where twin paradox is solved fully with the ingredients of SR in the non-abrupt turn-around scenario.

1.4 List of Papers (Published/under Publication)

(1) **The Principle of Equivalence and the Twin Paradox**– S. K. Ghosal, Saroj Nepal and ***Debarchana Das***, *Found. Phys. Lett.* **18**(7), 603-619 (2005). (extended version of the paper is included in the thesis)

(2) **A Twin Paradox for 'Clever' Students**– S. K. Ghosal, Saroj Nepal and ***Debarchana Das***, *arXiv:gr-qc/0511126v1*, (2005). (not included in the thesis)

(3) **Relativity in 'Cosmic Substratum' and the UHECR Paradox**– S. K. Ghosal, Saroj Nepal and ***Debarchana Das***,– *Proc. of Physical Interpretation of Relativity Theory*, held at Moscow, July 2-5, 2007. (not included in the thesis)

(4) **Twin Paradox: A Classic Case of 'Like Cures Like'**– S. K. Ghosal, Saroj Nepal and ***Debarchana Das***, appear in "Mathematics, Physics and Philosophy in the interpretations of Relativity theory" held in Budapest, September 7–9, 2007. [**WWW.phil-inst.hu/~szekely/PIRT_BUDAPEST/ft/Ghosal-ft.pdf**] (not included in the thesis)

(5) *"Paradoxical Twins in Classical and Relativistic Worlds"*–S. K. Ghosal, ***Saroj Nepal***, Debarchana Das, Biplab Raychaudhuri and Minakshi Sarkar, *Proc. of Seminar on Hundred Years of Three Seminal Papers of Albert Einstein and Contemporary Ideas"*, held in the Physics Department of North Bengal University, 3–5 January, 2005 (Abstracted).

(6) **The Case of Identically Accelerated Twins and the Ordinary Twin Paradox**– S. K. Ghosal, Saroj Nepal and ***Debarchana Das***,(under publication).

(7) **Paradox of Identically Accelerated Twins in Different Worlds and the Ordinary Twin Paradox**– S. K. Ghosal and ***Debarchana Das***, (ready for communication).

(8) **Demystifying Twin Paradox**– S. K. Ghosal and ***Debarchana Das***, (ready

for communication).

(9) **Circular Twin paradox Revisited**– S. K. Ghosal and ***Debarchana Das***, (ready for communication).

(10) **Conventionality of Simultaneity,Absolute Synchronization and Twin Paradox in the Non-abrupt turn-around Scenario**– S. K. Ghosal, ***Debarchana Das*** and Saroj Nepal, (ready for communication).

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Chapter 2

Conventionality of Simultaneity (CS) Thesis

2.1 Conventionality of Simultaneity Thesis

The procedure for distant clock synchronization in the theory of relativity has an element of convention. This is known as the conventionality of distant simultaneity (CS) thesis. In literature the CS-thesis has been successfully used to deal with some interesting (apparent) paradoxes of relativity such as the twin paradox, tippitop paradox[1], Selleri paradox and the likes. As regards to the twin paradox, the thesis facilitates one to study the paradox and its variants in different worlds (other than the special relativistic world). In one of the most cited paper on the twin paradox[2] a novel approach to understanding the twin problem based on the conventionality of simultaneity has been presented providing a clearer way to settle the often discussed issue of twins relative aging. As stated earlier the CS-approach has also been fruitfully used to examine some other paradoxes of relativity[1, 3]. In the coming chapters the CS-thesis will be seen to play a significant role in dealing with the counter-intuitive problem, enlightening our understanding of certain subtle issues regarding the problem, a brief review of the CS-thesis of SR therefore may be in order.

The role of conventionality in defining simultaneity of distant events or in synchronizing spatially separated clocks in a given inertial frame is one of the most debated issue in literature[3, 4, 5]. In relativity theory the spatially separated clocks in a given inertial frame are synchronized by light signal whose one-way-speed (OWS) must be known beforehand. But to measure the OWS of the signal from one point to another one requires to have two presynchronized clocks and hence the whole process ends up in a logical circularity. To get out of this problem a convention is adopted in assuming the OWS of the signal within certain bound. (The round-trip speed or the two-way-speed (TWS) of the signal is however a convention-free entity since only one clock is employed for measuring it and hence the problem of synchronization of spatially separated clocks does not enter.) To break the circularity Einstein *stipulated* that the OWS of light is the same as its TWS. This stipulation of equality of to and fro speeds of light (along any given direction) in an inertial frame is known as *standard synchronization* (or Einstein synchronization) convention in the literature. The CS-thesis asserts that Einstein's convention is just one among the various possible alternative conventions (termed as non-standard synchronization) to synchronize clocks at different locations. This particular convention leads to a set of transformation equations in the relativistic world which are known as the Lorentz transformation (LT). Many of the results (or formulae) obtained by Einstein thus depended on his special choice of synchronization. Even the fact that two inertial observers in relative motion come to different conclusions regarding the simultaneity of two spatially distant events is also a matter of such a simultaneity convention.

The possibility of using a synchronization convention other than that suggested by Einstein was first discussed by Reichenbach[6] in 1928 and later by Grünbaum[7]. They claimed that the simultaneity between events in an inertial frame is a matter of convention and the conventionality lies in the assumption regarding the OWS of light. As according to Einstein's proposition for synchronizing distant clocks the time of arrival and the consequent reflection by a mirror at one clock position is determined by considering that the latter is halfway in time between the departure of the light signal and its arrival at the position of the other clock from where the light signal is sent out for synchronization. By specifying a value for the OWS of light one can obtain an unique light signal procedure for the synchronization of distant clocks, hence any prescription for OWS value(s) is equivalent to adoption of a convention for clock synchronization[8]. Different choices for the values of the OWS yield different synchronization conventions. Einstein himself referred to his choice of OWS as a free stipulation for giving an empirical meaning of distant simultaneity[9]. According to the CS-thesis a range of choices are possible, all fully equivalent with respect to experimental outcome. One needs to note here that the thesis allows any synchronization convention with the requirement that it is consistent with the round-trip principle, according to which the average speed of a light ray over any closed path has a constant value. It may not be out of place in this context to mention that the second relativity postulate should be restated by replacing the phrase "velocity of light" by "round trip speed of light" or "TWS of light". The round-trip principle should be satisfied by the conventions in SR as this principle is a consequence of the theory prior to adoption of any criterion for distant simultaneity and may in principle be tested with a single clock.

Although first eminently put forward by Reichenbach and Grünbaum, the thesis after formulation attracted a considerable amount of discussions in the literature. Possibility of using synchronization convention other than that adopted by Einstein has also been much discussed. John Winnie[10] was the first to study the consequences of this approach to establishing simultaneity where he made no assumption regarding the OWS of light. He developed the so called ϵ-Lorentz transformations (using Reichenbach's notation) adopting non-Einstein one-way velocity assumption or non-standard synchronization convention in general. ϵ is called the *synchronization* or *Reichenbach parameter* which is given by the equation

$$t_2 = t_1 + \epsilon(t_3 - t_1), \tag{2.1}$$

where t_1, t_2 and t_3 gives the time measured in two spatially separated clocks which are synchronized using light signals. t_1 and t_3 are the times of transmission and reception of the signal respectively by the first clock and t_3 is the time recorded by the second clock when the signal reaches it. For causality reason the value of ϵ is restricted as

$$0 < \epsilon < 1. \tag{2.2}$$

Note that Einstein's convention is equivalent to the assumption $\epsilon = 1/2$. While developing the ϵ-Lorentz transformation Winnie assumed a principle called "principle equal passage time" which he used in addition to the "Linearity principle" and the "Round-trip light principle". These principles were in fact shown to be independent of one-way velocity assumption and hence may form the basis of SR without distant simultaneity assumptions. Winnie's approach was extended by Ungar[11] who considered a generalized Lorentz transformation group that does not embody Einstein's isotropy convention. This approach suggested by Ungar suited well for establishing the results of Winnie as well as some new results. However these discussions were confined to only single dimension. It was noted later by some authors that at least a two-dimensional analysis was necessary as one-dimensional analysis restricts one from using the isotropy of one-way speed of light which follows from the modified second relativity postulate and therefore some subtleties and richness of the relativistic physics[12] will have to be sacrificed.

Mansouri and Sexl[13] in a series of important papers developed a test theory of SR and investigated the role of convention in various definitions of clock synchronization and simultaneity. They showed that two principal methods of synchronization could be considered: system internal and system external synchronization. Einstein's synchronization scheme (using the light signal) and slow clock transport synchronization (where all the clocks are collected at a given locality and synchronized and then after slowly transported back to their respective space points in a given reference frame) turn out to be equivalent if and only if and only if the time dilation factor is given by Einstein result $(1 - v^2/c^2)^{-1/2}$. The authors formulated an ether theory where simultaneity was absolute and was kinematically equivalent to SR.

Later Sjödin[14] developed the thesis by considering the whole issue more generally and also by assuming the role of synchronization in SR and some related theories. He gave all logically possible linear transformations between inertial frames depending on physical behavior of scales and clocks in motion with respect to the so called "physical vacuum" and then examined LT in the light of true length contraction and time dilation. In an effort to separate the true effects and the effects due to synchronization convention the author considered two special cases: The Newtonian world– without any contraction of moving bodies and slowing down of moving clocks and Lorentzian world– with longitudinal contraction of moving bodies and slowing down of clocks. The author then used *standard synchronization* in the Newtonian world (which was later termed as Pseudo-standard synchrony by Ghosal, Mukhopadhyay and Chakraborty[12]) and obtained the transformations which were already derived by Zahar[15][1]. These transformations show that in the Newtonian world the (apparent) relativistic effects are only due to choice of special synchrony. Using absolute synchronization in the Lorentzian world Sjödin obtained the transformations that were obtained by Tangherlini[?] which showed the "real" effects. In this way Sjödin concluded that the confusion regarding the existence of the ether and the reality of length contraction/time dilation effects was mainly due to the mixing up of the effects arising out of synchronization and the real contraction of moving bodies and retardation of moving clocks.

As said earlier, different choices for the values of the OWS yield different sets

[1]These transformations were seen to provide much clarity to some counter-intuitive issues of SR. In relation to the twin problem the importance of this transformation will be revealed in the coming chapters.

of transformation equations with varied structural features. LT is just one among them which is obtained by using Einstein convention. The other transformation equations although different outwardly, will predict the same kinematical world. These structurally different transformation equations have been found provide much insight into many conceptual issues including some interesting paradoxes in SR. (In the present investigation some of these have been used.) Some of these important transformation equations which explicitly incorporate the CS-thesis are given below. These equations relate coordinates x, y, z and time t in an inertial frame Σ with those (x',y',z',t') in another inertial frame Σ'.

Winnie transformations:

Winnie obtained his ϵ-Lorentz transformations based on three synchrony independent principles "the round trip light principle, the "principle of equal passage times" and the "linearity principle" which is as follows (The interesting derivation of these transformation equations are in Ref[10]).

$$\begin{aligned} x' &= \alpha^{-1}(x - \vec{v_\epsilon} t), \\ t' &= \alpha^{-1} t[2\vec{v_\epsilon} c^{-1}(1 - \epsilon - \epsilon') + 1] - xc^{-2}[2c(\epsilon - \epsilon') + 4\vec{v_\epsilon}(\epsilon)(1 - \epsilon)], \end{aligned} \tag{2.3}$$

where $\vec{v_\epsilon}$ denotes the relative speed of Σ' with respect to Σ,

$$\alpha = [(c - \vec{v_\epsilon}(2\epsilon - 1))^2 - \vec{v_\epsilon}^2]^{1/2}/\epsilon, \tag{2.4}$$

and ϵ (as given by Eq (2.1))and ϵ' are Reichenbach parameters in the two frames which are in relative motions.

Regarding the vector sign over the velocity a clarification is needed. The vector sign does not imply that the transformation equations involve more than one dimension, the arrow sign only emphasizes the non-reciprocity of relative velocity when $\epsilon \neq 1/2$ (i.e for non-standard synchronization). The equation could also have been written in terms of $\overleftarrow{v_{\epsilon'}}$ which denotes the relative speed of Σ to Σ' and in general $\vec{v}_\epsilon \neq \overleftarrow{v}_{\epsilon'}$.

Selleri transformations:

Selleri gave the space-time transformation between frames Σ and Σ' whose general form is given by

$$\begin{aligned} x' &= (x - \beta ct)/R(\beta) \\ y' &= y \\ z' &= z \\ t' &= R(\beta)t + \epsilon(x - \beta ct) + e(y + z), \end{aligned} \tag{2.5}$$

where ϵ and e are two undetermined functions of relative velocity v, $\beta = v/c$ and $R(\beta) = (1 - \beta)^{1/2}$. The demand of rotational invariance around x-axis gives $e = 0$, giving the final form of these transformation equations as

$$\begin{aligned} x' &= (x - \beta ct)/R(\beta) \\ y' &= y \\ z' &= z \\ t' &= R(\beta)t + \epsilon(x - \beta ct). \end{aligned} \tag{2.6}$$

The transformation Eqs.(2.3) and (2.6) represent the relativistic world.

For absolute synchronization ($\epsilon = 0$) the consequent transformation equations are

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma^{-1}t, \end{aligned} \tag{2.7}$$

which are known as Tangherlini transformations[16] or inertial transformations[17, 18, 19]. An interesting point to note here is that although the above equations represent the relativistic world, simultaneity is not relative in character i.e it is absolute. This transformation plays a significant role while dealing with the twin problem. This is reported in Chapter of this thesis.

Zahar Transformation:

The classical or Galilean world is a world where there is no time dilation and hence the clock transport synchronization holds without any ambiguity. The transformation equations generally used in this world are the well known Galilean transformations. However, by (playfully) incorporating Einstein's procedure of light signal synchronization in this world one observes that the Galilean transformations are replaced by the Zahar transformation (named after E. Zahar who obtained these transformation equations originally in 1977[3, 12, 14, 15]).

$$\begin{aligned} x' &= x - vt, \\ t' &= \gamma^2(t - vx/c^2). \end{aligned} \tag{2.8}$$

In addition to the above mentioned transformation equations there are some other interesting transformation equations following CS approach where synchronization is achieved by non-luminal signal (in general) using the standard synchronization procedure. The equations are quite general in nature in the sense that the world (classical or relativistic) is not specified beforehand. These equations are obtained in the next section.

2.2 Non-Luminal Clock Synchronization in Different Worlds

The CS-thesis of SR can be understood more clearly by developing relativity within a medium. In an interesting paper Ghosal et.al [12] dealt with the CS issue in a novel way by considering "Relativity in a substrate". There instead of using light, spatially separated clocks were imagined to be synchronized using "acoustic signal" following Einstein's mode of synchronization. It has been shown there that transformation equations obtained by synchronizing clocks using non-luminal signal following Einstein convention in the relativistic world helped to visualize clearly the conventionality ingredients in the standard formulation of SR.

Before discussing the content of the paper let us start with the following observations. In the standard formulation of SR light has two different roles to play. On the one hand it acts as a synchronizing agent, on the other hand it has invariant two-way-speed (TWS) in vacuum. The second role has a basis in the empirically verifiable property, but the first one is purely prescriptive in origin. In the derivation of the LT, these two roles are mixed up. The inseparability contributes to several misconceptions and prejudices in relativity theory. In an effort to separate these two roles one may introduce non-luminal signal to synchronize clocks and re derive the transformation equations. To achieve this the authors[12] considered reference frames submerged in a substrate. In order to derive the transformation equations, they proposed to synchronize the clocks by some other signal (acoustic signal (AS)) which is a characteristic of the substratum. To start with the authors considered that an acoustic wave is generated at $t = 0$ at the common origin of the frames S_i and S_k. Except for the frame S_0 which is at rest relative to the substratum, in all other frames the velocity of AS in the positive and negative x-directions will not be the same. Using the CS-thesis they defined the synchronization of clocks so that these two velocities are equal in all frames although their values vary from frame to frame. This synchrony is called the *pseudo-standard synchrony* other than Einstein's standard synchrony. According to pseudo-standard synchrony along the x-axis, the one dimensional wave front equation will be

$$x_k^2 = a_{kx}^2 t_k^2, \tag{2.9}$$

where x_k denotes the co-ordinates of a frame S_k which is moving with respect to S_0 frame which is fixed in the substrate and a_{kx} is the TWS of the AS in the x-

direction.

Two-way-speed (TWS) of AS will not be the same in all directions, for example along the y-direction the wave front equation will be

$$y_k^2 = a_{ky}^2 t_k^2 \tag{2.10}$$

where a_{ky} is the TWS of AS along y-direction and may have different value from a_{kx}. Hence the acoustic wave front will not be spherical in frames other than in S_0 frame.

The Derivation of Transformation Equations:

To derive the transformation equations (TE) between two arbitrary inertial frame S_i and S_k moving with relative velocity v_{ik} the authors used TE in the linear form as,

$$\begin{aligned} x_k &= \alpha_{ik}(x_i - v_{ik}t_i), \\ y_k &= y_i, \\ t_k &= \xi_{ik}x_i + \beta_{ik}t_{ik}, \end{aligned} \tag{2.11}$$

where α_{ik}, ξ_{ik} and β_{ik} are constant that were determined using pseudo-standard synchrony. By virtue of the chosen synchrony, one can set the condition

$$x_k^2 - a_{kx}^2 t_k^2 = \lambda_{ik}^2(x_i^2 - a_{ix}^2 t_i^2), \tag{2.12}$$

where λ_{ik} is a scale factor that is independent of the space and time coordinates. Using Eqs.(2.11) and (2.12) the transformation coefficients were obtained as

$$\alpha ik = \lambda_{ik}\gamma_{ik} \tag{2.13}$$

$$\beta_{ik} = \alpha ik/\rho_{ik} \tag{2.14}$$

$$\xi_{ik} = -\frac{\alpha_{ik}/\rho_{ik}}{v_{ik}/a_{ix}^2} \tag{2.15}$$

with

$$\gamma_{ik} = (1 - v_{ik}^2/a_{ix}^2)^{-1/2}. \tag{2.16}$$

and

$$\rho_{ik} = a_{kx}/a_{ix} \tag{2.17}$$

Incorporating these the transformation Eqs.(2.11) was written as,

$$\begin{aligned} x_k &= \lambda_{ik}\gamma_{ik}(x_i - v_{ik}t_i), \\ t_k &= (\lambda_{ik}/\rho_{ik})\gamma_{ik}(t_i - v_{ik}x_i/a_{ix}^2). \end{aligned} \tag{2.18}$$

As in the preferred frame S_0 according to adopted synchronization scheme, the TWS of AS is isotropic for

$$a_x^2 + a_y^2 = a_0^2, \tag{2.19}$$

where a_x and a_y are the x and y components of the velocity of the wavefront and a_0 is the isotropic signal speed.

The TWS in an arbitrary frame S_k along x- direction was obtained as

$$a_{kx} = \frac{\alpha_{0k}a_0(1 - v_{0k}^2/a_0^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.20}$$

Similarly the TWS in y- direction in S_k

$$a_{ky} = \frac{a_0(1 - v_{0k}^2/a_0^2)}{\beta_{0k} + \xi_{0k}v_{0k}}. \tag{2.21}$$

For any other signal whose isotropic TWS (equal to its OWS) in S_0 is a_0' (which may differ from a_0) the TWS along x- direction was

$$a'_{kx} = \frac{\alpha_{0k}a'_0(1 - v_{0k}^2/a_0'^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.22}$$

and along the y-direction

$$a'_{ky} = \frac{a'_0(1 - v_{0k}^2/a_0'^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.23}$$

where a'_{kx} and a'_{ky} are the TWS of the signal as measured from S_k in the longitudinal and in the transverse directions respectively. One needs to note here that in order to obtain these relations it has been assumed that with respect to S_0 under the chosen synchrony, the OWS of "other" signal is isotropic and is equal to its TWS. It has thus been tacitly assumed that in the preferred frame S_0 the pseudo-standard synchronization with AS and with the "other" signal are equivalent.

Making use of Eqs.(2.13), (2.20) and (2.21) the scale factors were obtained as

$$\lambda_{0k} = a_{kx}/a_{ky} \tag{2.24}$$

Also

$$\lambda_{ik} = \frac{\lambda_{0k}}{\lambda_{0i}} = \frac{a_{kx}}{a_{ky}}\frac{a_{iy}}{a_{ix}}. \tag{2.25}$$

Using this value for λ_{ik} Eq.(2.18) becomes

$$\begin{aligned} x_k &= (a_{kx}/a_{ky})(a_{iy}/a_{ix})[(x_i - v_{ik}t_i)/(1 - v_{ik}^2/a_{ix}^2)^{1/2}], \\ t_k &= (a_{iy}/a_{ky})[(t_i - (v_{ik}/a_{ix}^2)x_i)/(1 - v_{ik}^2/a_{ix}^2)^{1/2}]. \end{aligned} \tag{2.26}$$

With respect to preferred frame S_0 (where $a_{0x} = a_{0y} = a_0$) the TE from S_0 to any other inertial frame S_k was thus obtained as

$$\begin{aligned} x_k &= (a_{kx}/a_{ky})[(x_0 - v_{0k}t_0)/(1 - v_{0k}^2/a_0^2)^{1/2}], \\ t_k &= (a_0/a_{ky})[(t_0 - (v_{0k}/a_0^2)x_0)/(1 - v_{0k}^2/a_{0x}^2)^{1/2}]. \end{aligned} \tag{2.27}$$

In a lighter vein the authors termed this set of transformation equations as dolphin transformations (DT) as if these transformations are perceived by intelligent dolphins equipped with standard rods and clocks under water. As can be seen the Dolphin transform in the present form can be used as a space-time relation between two frames provided one knows the TWS of AS in these two frames. If instead of AS light signal was used for synchronization of clocks, then by virtue of CVL postulate in SR

$$a_{ix} = a_{iy} = a_{kx} = a_{ky} = c, \tag{2.28}$$

which gives the familiar LT. However in absence of any communication with the outside world, *apparently* c does not play any role in DT even though the dolphins live in the relativistic world where we know c plays a fundamental role! As said in the article, in DT, c will appear as a *physical constant* through a_{kx} and a_{ky}. In order to make DT usable, the dolphins will have to measure the TWS of AS in S_k as a function of velocity v_{0k} and one can anticipate that they will eventually find that

$$\begin{aligned} a_{kx} &= a_{kx}(v_{0k}, c), \\ a_{ky} &= a_{ky}(v_{0k}, c), \end{aligned} \tag{2.29}$$

where c would appear not as the speed of light but as some physical constant. Putting $a'_{kx} = a'_{ky} = a'_0 = c$ in the two-way velocity transformation given by Eqs.(2.22) and (2.23 and using Eqs(2.13-2.16) it was demonstrated that

$$\rho_{0k} = \frac{(1 - v_{0k}^2/a_0^2)}{(1 - v_{0k}^2/c^2)}, \tag{2.30}$$

and

$$\lambda_{0k} = \frac{(1 - v_{0k}^2/a_0^2)^{1/2}}{(1 - v_{0k}^2/c^2)^{1/2}}. \tag{2.31}$$

or by Eqs.(2.17) and (2.24)

$$a_{kx} = a_0 \frac{(1 - v_{0k}^2/a_0^2)}{(1 - v_{0k}^2/c^2)}, \tag{2.32}$$

and

$$a_{ky} = a_0 \frac{(1 - v_{0k}^2/a_0^2)^{1/2}}{(1 - v_{0k}^2/c^2)^{1/2}}. \tag{2.33}$$

By inserting Eqs.(2.31) and (2.32) in Eq.(2.27) the DT for the relativistic world was obtained as

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0)/(1 - v_{0k}^2/a_0^2)^{1/2}, \\ t_k &= (1 - v_{0k}^2/c^2)^{1/2}(1 - v_{0k}^2/a_0^2)^{-1}[t_0 - (v_{0k}/a_0^2)x_0]. \end{aligned} \tag{2.34}$$

The authors note down certain important consequences of DT which are the following:

1. The transformation equations contain TWS of synchronizing signal. The simultaneity is relative. Under this synchrony relative speeds are not symmetric in general.

2. The speed of AS a_0 is conventional. c appears as a physical constant - the TWS of light - and is not based on any convention. The factor $(1 - v_{0k}^2/c^2)^{1/2}$ is due to real effects. The other factor, $(1 - v_{0k}^2/a_0^2)$ arises from the synchronization procedure which is evident from the presence of the term a_0. Thus this clarifies that different synchronization procedure may not have relativity of simultaneity but they can predict length contraction and time dilation effects. From the DT, the length contraction factor (LCF) and time dilation factor (TDF) were calculated to be

$$\begin{aligned} LCF &= (1 - v_{0k}^2/a_0^2)/(1 - v_{0k}^2/c^2)^{1/2}, \\ TDF &= (1 - v_{0k}^2/c^2)^{1/2}/(1 - v_{0k}^2/a_0^2). \end{aligned} \tag{2.35}$$

3. In the derivation of DT the two roles of the light signal which are mixed up in standard SR (as discussed before) are clearly separated.

An important point regarding DT is that it can be well used to obtain some important transformation equations in relativistic and classical worlds by making use of the properties of the synchronization signal:

Lorentz transformation *(relativistic world with Einstein synchronization)*:

In the standard synchrony the synchronization agent is light. Putting $a_0 = c$ in DT one obtains the Lorentz transformation.

Tangherlini transformation *(relativistic world with absolute synchronization)*:

If in the *preferred frame* the speed of synchronization signals $a_0 \rightarrow \infty$ then we obtains (for $S_0 \rightarrow S_k$) the Tangherlini transformation

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma^{-1}t, \end{aligned} \tag{2.36}$$

Zahar transformation *(Einstein synchronization and classical world)*:
In the classical world the velocity addition law is the Galilean one. Then the TWS of AS is obtained to be

$$\begin{aligned} a_{kx} &= a_0(1 - v_{0k}^2/a_0^2), \\ a_{ky} &= a_0(1 - v_{0k}^2/a_0^2)^{1/2} \end{aligned} \tag{2.37}$$

Inserting these expressions for a_{kx} and a_{ky} in the DT (and in particular in Eq.(2.41) we obtain DT in in classical world

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0), \\ t_k &= [t_0 - (v_{0k}/a_0^2)x_0]/(1 - v_{0k}^2/a_0^2). \end{aligned} \tag{2.38}$$

In the standard synchrony, ($a_0 = c$) DT then becomes Zahar transformation (ZT) as we have discussed earlier

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0), \\ t_k &= [t_0 - (v_{0k}/c^2)x_0]/(1 - v_{0k}^2/c^2). \end{aligned} \tag{2.39}$$

Galilean transformation *(absolute synchrony and classical world)*:
It has been shown that in the classical world if the synchronizing signal's speed is assumed to be arbitrarily large (hypothetically) so that one may put $a_0 \rightarrow \infty$ in Eq.(2.38), one is then seen to retrieve the familiar form of GT.

DT thus is shown to act as an originator of these various important transformation equation with varied structural features. This has been seen to throw light on various counter-intuitive issues of Relativity.

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Chapter 3

The Case Of Identically Accelerated Twins and the Ordinary Twin Paradox

3.1 Introduction

In a pedagogically interesting and thought provoking article, S. P. Boughn[1] discussed a variation of the twin paradox parable where twins (P and Q say) on board two identical rockets (with equal amount of fuel), initially at rest a distance L apart in an inertial frame Σ, underwent identical accelerations for some time in the direction $\overrightarrow{PQ}$, and eventually came to rest (when all their fuels had expended) with another inertial frame Σ' moving with non-zero relative velocity v with respect to Σ. From the simple application of Lorentz transformation Boughn obtained a rather surprising result that in the new abode (Σ') the age of P became less than that of Q! Viewed differently, if the twins would carry presynchronized clocks, the outcome would have been a net time-offset effect between these clocks in Σ'.

The result is counter-intuitive by virtue of the fact that the twins of the parable throughout had identical local experiences yet their presynchronized clocks (also their own biological clocks) went out of synchrony!

Quantitatively this time-offset or desynchronization turns out to be

$$\delta t'_{desync} = -\gamma_v vL/c^2, \tag{3.1}$$

where the Lorentz factor

$$\gamma_v = (1 - v^2/c^2)^{-1/2} \tag{3.2}$$

and c is the speed of light in free space.

The result can be seen to follow from the simple application of LT:

$$\begin{aligned} x_k{}' &= \gamma_v(x_k - vt_k) \\ t_k{}' &= \gamma_v(t_k - vx_k/c^2), \end{aligned} \tag{3.3}$$

where in the current context, t_k and x_k denote the time and space coordinates of the observer k (k stands for P or Q) with respect to Σ and the prime refers to the corresponding coordinates of the observers in Σ' where they arrive and settle stationary after their acceleration phases are over.

From the time transformation of Eq.(3.3) one obtains

$$t_Q' - t_P' = \gamma_v[(t_Q - t_P) - v(x_Q - x_P)/c^2]. \tag{3.4}$$

The clocks of the observers P and Q are initially synchronized in Σ and continue (as the symmetry of the situation demands) to remain synchronized with respect to the frame during their acceleration phases and thereafter. For a pair of events simultaneous with respect to Σ, say for instance birthdays of P and Q, $t_Q - t_P = 0$. Using this and noting that the twins are separated by a distance $L = x_Q - x_P$, the last equation gives Eq.(3.1) where the desynchronization between the clocks P and Q is denoted by $\delta t'_{desync} = t'_Q - t'_P$. Obviously the above desynchronization corresponds to a differential aging of the twins in their new abode.

The apparently paradoxical result that the twins age differently in spite of their identical history of acceleration is readily explained if one notes that for spatially separated (biological) clocks the change of relative synchronization cannot have any unequivocal meaning. They can only be compared unambiguously when they are in spatial coincidence. For instance in Σ', one of the observers can slowly walk towards the other (or both of them can do the walking) and compare their ages (or their clock readings) when they meet. Since in the relativistic world the so called "slow transport synchronization" is equivalent to the Einstein synchronization[2], the calculated differential aging or time-offset between their clocks when they were spatially separated would continue to hold even when the twins meet after their slow walk. However in that case it can easily be seen[3] that they do not have symmetrical experiences, and hence the paradox gets resolved.

While the paradoxical element of the counter-intuitive outcome melts away, the fact remains that the differential aging for the "case of identically accelerated twins" given by Eq.(3.1) is *correct* and the time-offset can be verified at one spatial point as has already been pointed out. Boughn in his paper claimed that the ordinary twin paradox could be explained in terms of this effect (which hereafter will be referred to as the Boughn effect (BE))[1]. According to the parable of the ordinary twin paradox, Adam (A) stays at home on earth in a frame of reference Σ_0, while his traveller twin sister Beatrice (B) on board a fast rocket leaves earth with velocity v for a voyage to a distant star and subsequently turns around and then returns with the same speed v to meet her stay-at-home sibling to discover that they age differently. By applying time dilation formula (TDF) of special relativity (SR) on B's (biological) clock, A predicts that B should be younger on her return. The apparent paradox arises if B tries to apply the special relativistic TDF on A's clock (pretending that A is doing all the moving) and makes the contradictory claim that it is B who should be younger after the round-trip.

In this context Boughn observed that according to twin B, twin A would age less rapidly by a factor $1/\gamma$ during the entire trip. However, with obvious reference to the time-offset effect discussed earlier, Boughn further argued that because of acceleration at turn around, there would be a change in synchronization between the two twins' clocks. This change would overcompensate for the apparent slowdown in twin A's aging and finally twin A would be the older of the two. This was how both the twins could finally agree on their predictions.

But can one really explain the ordinary twin paradox in terms of Boughn effect?

[1]Although the problem of "identically accelerated twins" perhaps was well known to many in one form or the other, many authors[1, 2, 3, 4] have chosen to refer to Boughn's paper to describe this effect (paradox). The present paper also continues with this tradition in calling this effect after Boughn.

Commenting on Boughn's paper, Desloge and Philpott[3] noted that using Boughn's scenario "to analyse the twin paradox can be misleading since twins A and B do not start and end together whereas the twins in the conventional twin problem do." In spite of the criticism we feel that the pedagogical power of Boughn's paradox *can be used* to explain the usual twin paradox. However as outlined in the preceding paragraph, the brief account given by Boughn himself to this end is only a qualitative one. Besides, Boughn's paradox refers to the time-offset between two twins whose spatial separation has been maintained constant throughout with respect to Σ. One may therefore wonder how this can be related to the desynchronization of twins (of the ordinary twin paradox), one of whom remains stationary while the other makes the round-trip. In a recent paper, referring to the desynchronization of clocks in the case of identically accelerated twins (clocks), Styer[5] attempted to provide a resolution of the ordinary twin paradox in terms of it. However, in doing so the author in his otherwise interesting paper treated the Earth clock and the star clock as the identically accelerated ones whereas indeed the travelling twin suffered the turn-around acceleration thus rendering his arguments untenable. Besides, the resolution relied on an arbitrarily rapid jumping ahead of the Earth (remote) clock during the brief turn-around of the traveller. However, this drastic alteration in the reading of the remote clock is often difficult to accept[6]. Perhaps this is the reason why this resolution (as Styer himself had put it) "leaves most students with a gnawing pit in their guts".

The purpose of the present paper among other things is to remove this gnawing pit in the guts, and show how Boughn's paradox can be fruitfully used to resolve the usual twin paradox *quantitatively*. To our knowledge this has not been done before. Indeed the actual demonstration of unequivocal prediction for differential aging from both the twins' perspectives by employing BE will be found to be a non-trivial exercise. A careful analysis of the problem will provide a lot of additional insight into the century old counter-intuitive puzzle. It will be shown how the travelling twin in spite of her non-inertiality can make use of the special relativistic time dilation formula to obtain the proper time of the stay-at-home twin and yet predict the correct differential aging from her perspective. Interestingly, as the paper attempts to explain one paradox in terms of another, the present endeavour may be looked upon, in a lighter vein, as to justify the proverb "like cures like".

3.2 Coordinate Time, Time Dilation and Desynchronization

The relativistic time dilation effect relates times of two different nature. One concerns the rate of ticking of a moving clock at its position and the corresponding time is known as the proper time (often denoted by τ) of the clock. The other refers to readings of spatially separated coordinate clocks (at rest with respect to some inertial frame of reference), as the concerned clock moves past these coordinate clocks. Time recorded by the coordinate clocks are therefore known as coordinate time which may be denoted by t. Note that, since the coordinate clocks are spatially separated, the coordinate time for a given pair of events depends on the synchronization convention (or the standard of simultaneity) adopted to synchronize these coordinate clocks. In SR we adopt the standard synchronization or Einstein synchronization according to which the one-way-speed of light is *stipulated* to be equal to its round trip speed. The proper time τ of a clock however is independent of any synchronization convention.

The standard relativistic TDF which connects τ and t is therefore valid provided the coordinate clocks are synchronized following Einstein's convention. According to the conventionality of simultaneity thesis[2] however, other quite equally valid synchronization schemes can be adopted but in that case the relativistic TDF will not be valid. Since in the twin paradox thought experiment, τ of one twin (clock) is "calculated" from the "knowledge" of the coordinate time elapsed in the other twin's frame of reference, one must ascertain the latter with great caution.

The genesis of the twin paradox lies in the failure to do so in the frame of reference attached to the traveller twin. Let us now clarify this. Consider the abrupt turn around scenario of the standard twin parable. Assume that the turn around of B takes place when the distance between the twins (with respect to Σ) measures L say. Now, just before the deceleration phase starts, one may consider another observer Alfred ($\bar{A}$) of the same age as that of Beatrice (i.e it is assumed that $\bar{A}$'s clock is synchronized with B's in Σ) and at the same location of A comoving with respect to B such that, like in Boughn's scenario, $\bar{A}$ and B both undergo the same but arbitrarily large negative acceleration with respect to Σ, which moves with constant velocity v with respect to Σ_0. From Σ frame, B and $\bar{A}$ may be considered as Boughn's twins accelerating from rest along the negative x-direction (i.e now $\bar{A}$ is forwardly placed with respect to B) and settles in some inertial frame Σ' moving with velocity $-w$ (say) with respect to Σ (and $-v$ with respect to Σ_0).

BE therefore tells us that with respect to Einstein synchronized clocks in Σ', there is a desynchronization effect between the clocks (or ages) of Alfred and Beatrice,

$$t'_B - t'_{\bar{A}} = \delta t'_{desync} = \gamma_w w L/c^2, \tag{3.5}$$

which has been obtained from Eq.(3.1) replacing γ_v and v by γ_w and $-w$ respec-

[2]See for example[4, 7, 8]. For a more comprehensive review of the thesis see a recent paper by Anderson, Vetharaniam and Stedman[9].

tively. Note that here

$$\gamma_w = (1 - w^2/c^2)^{-1/2} = (1 + v^2/c^2)/(1 - v^2/c^2), \tag{3.6}$$

where we have used

$$w = 2v/(1 + v^2/c^2). \tag{3.7}$$

The last relation of course follows from the relevant relativistic velocity addition law. Using the last two expressions in Eq.(3.5) one obtains

$$\delta t'_{desync} = 2v\gamma_v^2 L/c^2. \tag{3.8}$$

The above desynchronization also corresponds to a synchronization gap between the Einstein synchronized reference frames Σ and Σ'. The presence of this synchronization gap between instantaneously comoving inertial frames for an accelerated observer is the reason why such frames cannot be meshed together.

Owing to the (instantaneous) turn-around, Beatrice switches her inertial frame and because of desynchronization, the clocks of Alfred and Beatrice no longer represent the Einstein synchronized coordinate clocks of Σ'. Now consider the following scenario: Instead of not turning around if Beatrice would continue to move forward covering the same length of journey with uniform speed as she would do after the turn-around, coordinate clocks (Einstein synchronized) of Σ frame of Beatrice could be used to measure the lapse of coordinate time for A's trip and connect the same with the proper time of Adam through TDF for A's entire one-way trip. However if during the second phase of the trip someone would playfully tamper with the synchronization, any coordinate time measurement following it would then be erroneous and hence a calculation to obtain the proper time τ_A of A from this measurement (by applying TDF on it) would give wrong result. Clearly, in order to get the correct answer the remedy is to first undo the playful tampering of synchronization by getting back to the Einstein synchronization that was adopted before and then one would be free to use TDF in order to obtain the proper time from the coordinate time. Let us now see what the corresponding situation is if we consider Beatrice's turn-around. In this case the second leg of Beatrice's journey corresponds to the inertial frame Σ'. The adoption of Einstein synchronization in this frame can be equated with the deliberate alteration of synchronization just discussed in connection with the uniform motion scenario of Beatrice, since the standard of simultaneity in Σ' is thus made different from that in Σ which corresponds to the earlier leg of Beatrice's trip.

It is clear that the proper time and coordinate time of a clock are connected by TDF of special relativity provided the coordinate clocks are standardly synchronized and they remain so during measurements. We then ask if there is any way so that one can continue with the standard of simultaneity (synchrony) of Σ in Σ'. The answer is in the affirmative and is provided by Boughn's thought experiment. From the symmetry of the problem it is evident that clocks of Alfred and Beatrice initially synchronized in Σ continue to remain synchronized with respect to Σ even when they arrive stationary in Σ' after the turn-around acceleration[3]. From B's perspective one can easily obtain the round-trip time $\Delta t_B(B)$ recorded in B's clock for A's journey (see later), but this does not correspond to the coordinate time for the same in Σ. Clearly a correction term $\delta t'_{desync}$, is to be added to $\Delta t_B(B)$ (see below) in order to obtain the said coordinate time. This correction is equivalent to the process of restoration of the synchronization mentioned in Beatrice's non

[3]In other words as if the clocks $\bar{A}$ and B behave in an obstinate manner and refuse to be synchronized in the new frame according to the standard synchrony *automatically*. The clock readings are to be tampered with in order to resynchronize them in Σ' according to the Einstein synchrony. If instead the clocks are left alone, these coordinate clocks then define absolute synchronization which Selleri refers to as "nature's choice"[10, 11].

turn-around example.

3.3 Resolution

Before we proceed to provide the quantitative resolution of the twin paradox using BE, let us for convenience, remove the inconsequential initial and final accelerations from the problem. We thus assume that B makes a flying start and also after the return trip she flies past A. The only unavoidable acceleration that we keep is the one associated with B's turn-around without which A and B cannot compare their clocks (or ages) at one spatial point after the latter's round-trip. The resolution can now be laid down in the following steps:

Perspective of A:

Step 1:

The reciprocity of the relativistic TDF from the perspective of A and B can symbolically be expressed as,

$$TDF1: \qquad \Delta\tau_B(A) = \gamma_v^{-1}\Delta t_A(A), \tag{3.9}$$

$$TDF2: \qquad \Delta\tau_A(B) = \gamma_v^{-1}\Delta t_B(B). \tag{3.10}$$

In the above we follow a notation scheme, where $\Delta\tau_B(A)$ $[\Delta\tau_A(B)]$ denotes the B $[A]$-clock reading for a time interval between two events occurred at its position as inferred by the observer A $[B]$. The inference of course is drawn from its own coordinate clocks' records for the interval, $\Delta t_A(A)$ $[\Delta t_B(B)]$ and its knowledge of the relevant time dilation effect. Indeed the time intervals $\Delta\tau_B(A)$ or $\Delta\tau_A(B)$ are based on one clock measurements and hence they refer to proper times of B and A respectively.

Regarding the notations $\Delta t_B(B)$ or $\Delta t_A(A)$, a clarification is needed. While, for example $\Delta\tau_A(B)$ refers to the difference between one clock (A) reading for two events, $\Delta t_B(B)$ refers to in general, the observed difference in readings (for the same pair of events) recorded in two spatially separated (synchronized) clocks stationary with respect to the frame of reference attached to B. However when $\Delta t_B(B)$ concerns measurement of the round trip time of an object or a clock (A say), it also refers to a single clock (B) measurement. Although τ-symbol would have been more appropriate in the later case but we shall continue to use the symbol 't' to emphasize that the corresponding time is supposed to be the coordinate time.

We now quote the relevant length contraction formula (LCF),

$$LCF: \qquad L = \gamma_v^{-1} L_0, \tag{3.11}$$

where L_0 is the distance of the distant star from the earth (measured in Σ_0) and L is the corresponding distance measured in Σ.

Step 2:

A-clock time for B's up and down travel of distance $2L_0$ is

$$\Delta t_A(A) = 2L_0/v, \tag{3.12}$$

and using the above result, the B-clock time for the same as calculated by A using TDF 1 (Eq.(3.9)) is

$$\Delta\tau_B(A) = \gamma_v^{-1} 2L_0/v. \tag{3.13}$$

Step 3:

Differential aging with respect to A is therefore given by

$$\delta t(A) = \Delta t_A(A) - \Delta\tau_B(A) = (1 - \gamma_v^{-1}) 2L_0/v. \tag{3.14}$$

Perspective of B:

Step 4:

From B's point of view, A makes the round trip and B measures the time for this trip as $\Delta t_B(B)$. This is nothing but the B-clock time as calculated by A, $\Delta\tau_B(A)$ which is given by Eq.(3.13). Hence

$$\Delta t_B(B) = \gamma_v^{-1} 2L_0/v \tag{3.15}$$

This can also be seen in the following way. According to B, A travels a distance $\gamma_v^{-1}2L_0$ (using LCF Eq.(3.11)) for the round trip . The speed of A with respect to B is also v as LT honours the reciprocity of relative velocity. Hence the travel time $\Delta t_B(B)$ is again calculated as $\gamma_v^{-1}2L_0/v$.

Step 5:

The same time interval in A-clock as calculated by B by the *näive* application of TDF2 (Eq.(10)) on $\Delta t_B(B)$) is obtained as,

$$\Delta\bar{\tau}_A(B) = \gamma_v^{-2} 2L_0/v. \tag{3.16}$$

This is however incorrect since desynchronization of distant clocks due to BE has not been taken into account and hence we have put a bar sign on τ, to be removed later after correction.

Step 6:

The above expression must be corrected by taking into account the BE. To calculate this effect we first split the frame of reference (K) attached to B into two inertial frames Σ and Σ' which move with velocities v and $-v$ respectively with respect to Σ_0. As discussed in Sec.3.2, $\bar{A}$ and B separated by a length L in Σ after deceleration arrives in the final frame of reference Σ' producing a temporal offset (desynchronization) between their clocks which is given by Eq.(3.8)

$$\delta t'_{desync} = 2v\gamma_v^2 L/c^2 = 2v\gamma_v L_0/c^2, \tag{3.17}$$

where for the last equality we have made use of Eq.(3.11). Going back to Eq.(3.15), leading to Eq.(3.16) one now discovers that the application of Eq.(3.10) on $\Delta t_B(B)$ to obtain $\Delta\tau_A(B)$ is a mistake since, as has been explained in Sec.2, the former does not represent the coordinate time as the coordinate clocks in K fail to remain synchronized according to the standard synchronization scheme as B changes her inertial frame from Σ to Σ' for her turn around acceleration. This is the lesson we learn from BE. One therefore needs to add[4] this desynchronization effect (Eq.(3.17)) to $\Delta t_B(B)$ before the application of TDF2 (See Appendix for clarification).

Adding $\delta t'_{desync}$ to $\Delta t_B(B)$ will undo the "resynchronization" of clocks (see Sec.3.2 and footnote(4)) in Σ'[5] and hence the standard synchronization of coordinate clocks in Σ will be carried over in Σ' as well. This is indeed the precondition that ensures the applicability of the relativistic TDF.

Therefore the true coordinate time is obtained as

$$\Delta t_B^{coord}(B) = \Delta t_B(B) + \delta t'_{desync} = 2\gamma_v^{-1} L_0/v + 2v\gamma_v L_0/c^2. \tag{3.18}$$

Now applying TDF2 on the true coordinate time $\Delta t_B^{coord}(B)$, B calculates the round-trip time (proper) measured in A-clock as

$$\Delta\tau_A(B) = \gamma_v^{-1}(2\gamma_v^{-1} L_0/v + 2v\gamma_v L_0/c^2). \tag{3.19}$$

Step 7:

Thus the differential aging from the perspective of B turns out to be,

$$\delta t(B) = \Delta\tau_A(B) - \Delta t_B(B) = (1-\gamma_v^{-1})2L_0/v, \tag{3.20}$$

[4]Whether one should add or subtract this desynchronization effect depends on its definition, as $\delta t'_{desync}$ could have been defined as $t'_{\bar{A}} - t'_B$, in which case one would need to subtract the effect for undoing the resynchronization.

[5]Resynchronization has been tacitly assumed in calculating $\Delta t_B(B)$ (see arguments following Eq.(3.15)) when reciprocity of relative velocity and LCF has been assumed to be valid in the frame of reference of B in her return journey.

which agrees with Eq.(3.14).

3.4 Summary

Boughn has shown that two identically accelerated twins initially at rest with some inertial frame ages differently when they arrived stationary in another inertial frame (after their acceleration phases are over). Although the outcome is counter-intuitive (since in spite of the twins' accelerations being symmetric in every respect, they age differently), the effect is an undeniable fact. It has been remarked in the literature that ordinary twin paradox can be explained in terms of the paradox of the identically accelerated twins. In this paper we have taken up the issue and solved the usual twin paradox *quantitatively* using the Boughn paradox. We have considered the abrupt turn around scenario and the essence of the present approach to resolve the issue lies in recognizing the fact that the coordinate clocks (that of Alfred and Beatrice say) of Σ no longer represent the Einstein-synchronized coordinate clocks in Σ' after the turn around. Indeed, these coordinate clocks of Σ carry over their synchronization convention in Σ', a lesson we learn from Boughn paradox. With these clocks the standard of simultaneity in Σ, according to Einstein convention is preserved in spite of their acceleration; in Σ' though, these clocks are not Einstein-synchronized. This departure from Einstein synchronization of the clocks is reflected in the Boughn effect.

Relativistic TDF can be used to calculate the proper time of A-clock from the coordinate time in the frame of reference attached to B provided the coordinate clocks represent *uniform* synchronization according to Einstein's scheme. It has been shown that the round-trip time $\Delta t_B(B)$ of Adam as recorded by Beatrice's clock cannot represent the readings of the coordinate clocks of Beatrice's frame of reference having constant synchronization (because of the frame's turn-around) and it has been explained how this can be corrected using the Boughn effect. Thus the genesis of the paradox lies in the mistake in the reasoning by Beatrice who näively use the TDF on $\Delta t_B(B)$ to draw inference regarding the proper time of Adam's clock. Once this is recognized the problem gets dissolved fully in the context of SR.

3.5 Appendix

Correction to coordinate time:

A careful analysis is required to understand the precise role of BE quantitatively in estimating the (coordinate) time of A's trip with respect to B.

Rewriting Eq.(3.5) as

$$t'_{\bar{A}} = t'_B - \delta t'_{desync}, \tag{3.21}$$

where $\delta t'_{desync}$ is given by Eq.(3.17),

one can interpret the equation by saying that with respect to standardly synchronised clocks in Σ', $\bar{A}$-clock as if, has been put behind the B-clock by the amount $\delta t'_{desync}$ when these clocks arrive stationary in Σ'. In reality nothing happens to these clocks during the brief period of acceleration. Indeed if one wishes to treat these clocks as Einstein-synchronized coordinate clocks in Σ' one requires to resynchronize them by hand.

During the return trip, A leaves the clock $\bar{A}$ at some time and meets B at a later time. The elapsed coordinate time for this trip is the difference between these times. If for any pair of events the time difference (B-clock time minus A-clock time) with respect to spatially separated Einstein-synchronized clocks is X', the same with respect to these clocks before resynchronization (that is if one continues with the synchronization in Σ) will clearly be more as $\bar{A}$-clock carries an offset $-\delta t'_{desync}$.

This is given by

$$X = X' + \delta t'_{desync}. \tag{3.22}$$

For the outward trip of A the coordinate time of travel with respect to Σ is

$$\Delta t_1 = L/v, \tag{3.23}$$

for the return trip the same in Σ' (with respect to resynchronized clocks) is again given by

$$\Delta t'_2 = L/v, \tag{3.24}$$

but this is not the lapse of coordinate time with respect to Σ.

Clearly after undoing the resynchronization, i.e, when $\bar{A}$ and B clocks are left alone and these untampered clocks are used as coordinate clocks, the coordinate time for A's return trip then should read, following Eq.(3.22) as $\Delta t_2 = \Delta t'_2 + \delta t'_{desync}$, and the same for the entire trip of A is given by

$$\Delta t_B^{coord}(B) = \Delta t_1 + \Delta t_2 = \Delta t_1 + \Delta t'_2 + \delta t'_{desync} = \Delta t_B(B) + \delta t'_{desync}, \tag{3.18}$$

where we have put $\Delta t_B(B) = \Delta t_1 + \Delta t_2$.

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Chapter 4

Paradox of Identically Accelerated Twins in Different Worlds and the Ordinary Twin Paradox

4.1 Introduction

As is evident from the title, in this chapter we shall discuss the desynchronization effect of the case of identically accelerated twins undergoing through the same chain of events in the relativistic and classical world. This is an extension of an earlier paper "Principle of Equivalence and the Twin Paradox" where a variation of the twin paradox is used to connect gravity with the the desynchronization effect. The paradox has been explained there using EP whose role is to provide a physical agent i.e. gravity which can supply the required extra aging to the traveller during the acceleration phase through a gravitational time offset effect. I should mention here that the paper "Principle of Equivalence and the Twin Paradox" has been reported by one of my collaborators, but still for the sake of completeness I reproduce the whole article as a background.

As discussed earlier in the canonical version of the twin paradox, of the two twins initially living on earth (assumed to be an inertial frame), one leaves the earth by a fast rocket to a distant star and then returns to meet her stay-at-home brother to discover that they age differently. This as such is not a paradox since the rocket-bound sibling, on account of her high velocity will suffer relativistic time dilation of her (biological) clock throughout her journey and will therefore return younger with respect to her brother. Indeed with respect to the inertial frame of the stay-at-home twin, the world lines of the twins in the Minkowski diagram are different (although from the description of the problem the end points of these lines i.e the time and the place of departure and that of their reunion, meet) and hence the asymmetry in the aging can be attributed to the fact that proper time is not integrable[1]. The paradox arises if one näively treats the perspectives of the twins symmetrically. For example if the traveller twin considers herself to remain stationary and relate the motion to her brother, she would (erroneously) expect her brother to stay younger by believing that the Lorentz transformation (LT) predicts reciprocal time dilation of moving clocks. Qualitatively the resolution lies in the observation that one of the twins is in an accelerated (non-inertial) frame of reference and hence the postulates of Special Relativity (SR) are not applicable to it and therefore the claim of reciprocity of time dilation between the frames of reference of the twins falls through. Indeed Einstein himself found this sort of argument preferable in dismissing the paradoxical element in the twin problem[2]. However this suggestion should not be construed as a statement that the resolution of the paradox falls outside the purview of SR . On the contrary much of the expositions found in the literature on the subject deal with the problem in the frame work of SR alone[1], although many tend to believe that the introduction of GR and a gravitational field at the point of acceleration is the right way to understand the asymmetry in the perspectives of the twins. Bohm notes in the context that " two clocks running at places of different gravitational potential will have different rates"[5]. This suggests that EP can directly be used to explain the asymmetry (difference between the experiences of the rocket-bound and the stay-at-home twin). However, as pointed out by Debs and Redhead[1] and also others[6], that since in the twin problems one deals with flat space-time, any reference of GR in this context is quite confusing.

Coming back to the issue of acceleration, one finds often that the direct role of acceleration of the rocket-bound twin in causing the differential aging has been much criticized although it is quite clear that in order to have twice intersecting trajectories of the twins (this is necessary since the clocks or ages of the twins have to be compared at the same space-time events) one cannot avoid acceleration.

[1]Very extensive treatment is available in Special Relativity Theory-Selected Reprints[3], (see also Ref.[4]). For newer expositions see for example Ref.[1] and references therein.

In an interesting article Gruber and Price[7] dispel the idea of any direct connection between acceleration and asymmetric aging by presenting a variation of the paradox where although one twin is subjected to undergo an arbitrarily large acceleration, no differential aging occurs. That the acceleration per se cannot play a role is also evident from the usual calculation of the age difference from the perspective of the inertial frame of the stay-at-home twin if one notes that the duration of the turn-around process of the rocket can be made arbitrarily small in comparison to that for the rest of the journey and hence the final age difference between the twins can then be understood in terms of the usual relativistic time dilation of the traveller twin during essentially the unaccelerated segment of her journey [2]. One is thus caught in an ambivalent situation that, on the one hand the acceleration does not play any role, on the other hand the paradox is not well posed unless there is a turn-around (acceleration) of the traveller twin[3].

In order to get out of this dichotomy it is enough to note that from the point of view of the traveller twin, the acceleration (or the change of reference frame in the abrupt turn-around scenario) is important. The consideration of this acceleration only has the ability to explain that the expectation of symmetrical time dilation of the stationary twin from the point of view of the rocket-bound twin is incorrect.

In an interesting paper A.Harpaz[10] tries to explain the twin paradox by calculating the age difference from the perspective of the traveller twin directly by applying EP i.e by introducing GSDC. From the previous discussions it may seem unnecessary (or even confusing) to invoke gravity in the essentially special rela-

[2]In such a calculation the time dilation is also calculated during the acceleration phase (assuming the clock hypothesis to be true[1]) and is shown to contribute arbitrarily small value in the age offset if the duration of the acceleration phase is assumed to tend to zero.

[3]Here we are considering the standard version of the paradox and the variation where the twins live in a cylindrical universe[8, 9] has been kept out of the present scope.

tivistic problem. However the fact is, Harpaz's approach apparently provides an alternate explanation for the differential aging from the traveller's perspective.

The author of the pedagogical article observes that although the special relativistic approach can correctly account for the age difference between the twins, " it does not manifest the 'physical agent' responsible for the creation of such a difference" [10]. It is held that EP provides such an agent and that is gravity. But how does gravity find way into the problem? Gravity enters through EP and its connection with the resolution of the paradox can symbolically be written as

$$Acceleration \overset{EP}{\rightarrow} Gravity \rightarrow Gravitational\ red\text{-}shift \rightarrow GSDC \rightarrow Extra\ aging$$

where the last item of the flow diagram indicates that with respect to the rocket-bound twin, GSDC provides the extra aging of the stay-at-home one, explaining the asymmetrical aging of the problem.

However while there is as such no harm in understanding the twin problem from a different perspective (here, this is in terms of GSDC), Harpaz's approach suffer from two fold conceptual difficulties which we will elaborate in the next section. These difficulties include the fact that the calculations are only approximate. The other difficulty will be seen to be of more fundamental in nature. The aim of the present study (reported in this chapter) is to remove these difficulties and give an *accurate* account of the asymmetric aging from the perspective of the rocket-bound twin directly in terms of a time-offset between the siblings which is introduced due to the pseudo-gravity experienced by the traveller twin.

4.2 GSDC and Extra Aging

In the standard version of the twin paradox the differential aging from the perspective of the stay-at-home (inertial) observer A can easily be calculated assuming that for the most parts of the journey of the traveller twin B, the motion remains uniform except that there is a turn-around acceleration of the rocket so that finally the siblings are able to meet and compare their ages. In the Minkowski diagram the whole scenario is characterized primarily by three events: (1) Meeting of the world lines of A and B when the voyage starts taking place. (2) the turn around of B and (3) meeting of the world lines when A and B reunite. For the paradox it is not necessary that at events (1) and (2), the relative velocity between A and B has to be zero, since ages or clocks can be compared at a point even if the observers are in relative motion, therefore the analysis of the problem can be done by considering the acceleration only during the turn-around. The duration of the acceleration phase can be considered to be arbitrarily small compared to the time it takes during its forward and return journeys and hence the age difference occurs due to the usual relativistic time dilation of a clock for its uniform motion. This is clearly given by

$$Age\ \ difference = 2t_A(1 - \gamma_v^{-1}) \approx 2t_A \frac{v^2}{c^2}, \tag{4.1}$$

where $2t_A$ is the time the rocket takes for its entire journey (up and down) in uniform speed v and $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$ is the usual Lorentz factor.

The paradox is resolved if one can show that B also predicts the same difference in spite of the fact that the time dilation effect is reciprocal. Clearly some new considerations (that were absent in arriving at Eq.(4.1)) must offset this reciprocal time dilation and also this must provide some extra aging to A from the point of view of B so that the age difference remains independent of the two perspectives. One of these new considerations, as has already been pointed out, is the one of a synchronization gap that B discovers due to her change of inertial frame during her entire voyage. This has been clearly demonstrated by Bondi[11] in the context of

Lord Hulsbery's three brother approach[1] to understanding the twin paradox.

The other way of understanding the same thing is the consideration of pseudo-gravity experienced by B because of its turn-around. In order to demonstrate how EP plays the role in the analysis, Harpaz uses the gravitational red-shift formula, which can be obtained heuristically (using the EP) as

$$\Delta\nu = \nu_0(1 + \frac{gh}{c^2}), \tag{4.2}$$

where g is the acceleration due to (pseudo) gravity and $\Delta\nu$ represents the change of frequency of light observed from a distance h from the source where the frequency of the same light is seen to be ν_0. Interpreting this red-shift effect in terms of GSDC, the formula can be written as

$$t_1 = t_2(1 + \frac{\Delta\Phi}{c^2}), \tag{4.3}$$

where t_1 and t_2 are times measured by clocks located at two points P_1 and P_2 (say) and $\Delta\Phi = gh$, is the potential difference between these points. It has been shown that with respect to B the acceleration plays a role by providing an extra time difference between B and A, because of the integrated effect of GSDC during the (arbitrarily) short duration of B's acceleration. This time difference more than offsets the age difference calculated by B solely assuming the reciprocal time dilation so much so that finally B ages less by the correct amount. As pointed out earlier there are two conceptual difficulties in understanding the treatment. First, in an effort to find a "physical agent" responsible for the extra aging, Harpaz relies on some approximate formulae including that of the gravitational red-shift because of his assumption, $v^2/c^2 \ll 1$ inherent in the analysis, and therefore, the pseudo-gravitational effect has the ability to resolve the paradox only approximately. Clearly there is no valid reason to make any such small velocity approximation for the problem. One might of course argue that for the author's stated purpose it would be enough to show that the " physical agent" i.e. gravity is at work when B's point of view is considered. However, it will be shown that such an argument would also not hold good and the reason for it concerns the second difficulty. The explanations based on SR relies on the fact that during the direction reversing acceleration, the travelling twin changes from one reference frame to another and the lack of simultaneity of one reference frame with respect to the other provides the "missing time" which constitutes the reason for the differential aging[1]. Now the lack of agreement in simultaneity is a special relativistic concept without any classical analogue, on the other hand in many standard heuristic derivations of the gravitational red-shift formula (see for example[12, 13, 14]) which is also followed by the author of Ref.[10], one finds that no reference to SR is made. Indeed the well-known formula for the red-shift parameter $Z = gh/c^2$ is only approximate and is derived by making use of the classical Doppler effect for light between the source of light and a detector placed at a distance h along the direction of acceleration g of an Einstein elevator[10]. According to EP an observer within the elevator will " attribute his observations in the elevator, to the existence of a uniform gravitational field in a rest system of reference"[10]. Thus the equivalence of gravity and acceleration in terms of gravitational red-shift or GSDC therefore turns out to be as if a purely classical (Newtonian) concept in this approximation! How then is GSDC able to account for an effect, viz. the lack of simultaneity which is essentially a standard relativistic phenomenon?

In the next section we will show that indeed the EP can explain the twin paradox exactly provided the connection of EP and GSDC is obtained using the full machinery of SR.

4.3 EP and the Gravitational Time Offset

S. P. Boughn [15] in an interesting article presented a paradox in SR with a great pedagogical power. This paradox put forth by Boughn involves twins who age differently although they experience equal amount of acceleration for the same time. According to Boughn's parable twins P and Q on board two identical rockets (with equal amount of fuel), initially at rest a distance x_0 apart in an inertial frame Σ_0, underwent identical accelerations for some time in the direction $\overrightarrow{PQ}$ (x-direction say) and eventually came to rest (when all their fuel had expended) in another inertial frame Σ moving with velocity v relative to Σ_0. Using Lorentz transformation (LT) Boughn obtained a counter-intuitive result that after the completion of the acceleration phase, the age of the forwardly placed (with respect to the direction of acceleration) twin Q turned out to be more than that of her brother P!

The result is counter-intuitive owing to the fact that the twins of the parable throughout had identical local experiences yet their presynchronized (biological) clocks went out of synchrony. The amount of this desynchronization or the age difference comes out to be

$$\delta t_{desync} = -\frac{\gamma_v v x_0}{c^2}. \tag{4.4}$$

where the Lorentz factor

$$\gamma_v = (1 - v^2/c^2)^{-1/2} \tag{4.5}$$

and c is the speed of light in free space.

The above result can be obtained from the simple application of LT:

$$t_k = \gamma_v(t_{0k} - \frac{v x_{0k}}{c^2}), \tag{4.6}$$

where t_{0k} and x_{0k} denote the time and space coordinates of the observer k (k stands for P or Q) with respect to their first inertial frame Σ_0 and t_k is the time in their final frame Σ where they settle stationary after the acceleration phase is over.

From Eq.(4.6) it follows that

$$t_Q - t_P = \gamma_v[(t_{0Q} - t_{0P}) - v(x_{0Q} - x_{0P})/c^2]. \tag{4.7}$$

Since both the twins had identical experiences the ages of the twins with respect to the frame of their mom and dad, Σ_0 must be the same throughout their journey, even after the completion of their acceleration phase. In other words the clocks of the twins P and Q which were initially synchronized in Σ_0 continue to remain so in Σ_0-frame during their acceleration phases and thereafter. But what is simultaneous to mom and dad is not simultaneous with respect to their new frame Σ. For a pair of events simultaneous with respect to Σ_0, say for instance birthdays of the twins which occur at the same time in Σ_0 (i.e $t_{0Q} = t_{0P}$), does not occur at the same time in Σ. This is evident from the above Eq.(4.7) which shows that for $t_{0Q} = t_{0P}$ the time difference $t_Q - t_P \neq 0$ as the twins are separated by a constant distance $x_{0Q} - x_{0P} = x_0$, the difference is rather given by Eq.(4.4) where $\delta t_{desync} = t_Q - t_P$.

This temporal offset effect of identically accelerated clocks gives an important insight into the behaviour of clocks in a uniform gravitational field, for, according to EP "...all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system"[13]. This suggests, as correctly remarked by Boughn that two clocks at rest in a uniform gravitational field are in effect perpetually being accelerated into the new frames and hence the clock at the higher gravitational potential (placed forward along the direction of acceleration) runs faster. With this insight we write Eq.(4.4) as

$$t - t_0 = -\frac{\gamma_v(t)v(t)x_0}{c^2} = -f(t), \tag{4.8}$$

where now t and t_0 are the readings of two clocks at higher and lower potentials respectively and also $f(t)$ stands for the right hand side of Eq.(4.4) without the minus sign

$$f(t) = \gamma_v(t)v(t)x_0/c^2. \tag{4.9}$$

In terms of differentials one may write Eq.(4.8) as

$$\delta t - \delta t_0 = -\dot{f}(t)\delta t, \tag{4.10}$$

where the time derivative $\dot{f}(t) = \frac{gx_0}{c^2}$, with $g = \frac{d}{dt}(\gamma_v v)$ is the *proper acceleration*. We may now replace δt and δt_0 by n and n_0, where the later quantities corresponds to the number of ticks (second) of the clocks at their two positions. We therefore have,

$$\frac{n - n_0}{n_0} = -\dot{f}(t), \tag{4.11}$$

or in terms of frequency of the clocks

$$-\frac{\delta\nu}{\nu_0} = \dot{f}(t), \tag{4.12}$$

where $\delta\nu$ refers to the frequency shift of an oscillator of frequency ν_0. The slowing down parameter for clocks, $-\delta\nu/\nu_0$ in Eq.(4.12) is nothing but the so called red-shift parameter Z for which we obtain the well-known formula[4]

$$Z = \frac{gx_0}{c^2}. \tag{4.13}$$

One thus observes that the time-offset relation (4.8) of Boughn's paradox can be interpreted as the accumulated time difference between two spatially separated clocks because of the pseudo-gravity experienced by the twins.[5] We shall see the importance of the time-offset relation (4.8) in accounting for the assymetrical aging of the standard twin paradox from the perspective of the traveller twin.

[4]In terms of *ordinary* acceleration $\overline{g} = \frac{dv}{dt}$, measured with respect to S the formula comes out to be $Z = \frac{\overline{g}\gamma x_0}{c^2}(1 - \frac{v^2\gamma^2}{c^2})$ which for small velocities can also be written as $Z = \frac{\overline{g}x_0}{c^2}$.

[5]The connection between gravity with this temporal offset through EP was first pointed out by Barron and Mazur[16], who derived the approximate formula for the "clock rate difference" mentioned in the previous foot-note.

Also, the above time offset effect between the identically accelerated twins calculated by Boughn can however be explained by noting that for spatially separated clocks the change of relative synchronization cannot be unequivocally determined. The clocks or the ages of the twins can be compared unambiguously only when they are at the same spatial point. For example in Σ either of the observers can slowly walk towards the other or both the observers can walk symmetrically towards each other and compare their clocks (ages) when they meet.

One may therefore define without much ado the reality of the temporal offset effect, provided the clocks are finally compared when they are brought together. We shall hereafter refer to the desynchronization effect between the identically accelerated twins when they are in spatial separation as the apparent Boughn effect (ABE) and that which persists (if any) when those clocks are brought together by slow transport as real Boughn effect (RBE). To check the reality of the time offset effect in the relativistic world, we consider any two arbitrary reference frames Σ_1 and Σ_2 moving with relative velocity w. LT between these two frames is

$$\begin{aligned} x_1 &= \gamma_w(x_2 + wt_2) \\ t_1 &= \gamma_w(t_2 + wx_2/c^2). \end{aligned} \tag{4.14}$$

The above time transformation gives the time dilation suffered by a clock stationary in Σ_2 as

$$\Delta t_1 = \gamma_w \Delta t_2, \tag{4.15}$$

where Δt_2 denotes the proper time between two events at the same point in Σ_2 and Δt_1 is the corresponding time measured by the observers in Σ_1.

Now if two synchronized clocks are spatially separated by a distance x in Σ_1 and a third clock attached to Σ_2 covers this distance slowly, then the time taken by this clock to cover the distance in Σ_1 is

$$\Delta t_1 = \frac{x}{w}. \tag{4.16}$$

The corresponding time measured in the Σ_2-clock (using Eq.(4.15)) is

$$\Delta t_2 = \gamma_w{}^{-1}\frac{x}{w}. \tag{4.17}$$

The difference of these times is therefore

$$\delta t = \Delta t_2 - \Delta t_1 = (\gamma_w{}^{-1} - 1)\frac{x}{w}. \tag{4.18}$$

This δt represents the integrated effect of time dilation suffered by the clock when it is being transported. Clearly this effect is dependent on the velocity of clock transport. Indeed, from the above equation one can see that for very small velocity i. e for $w \longrightarrow 0$ the above time reduces to zero

$$w \overset{Lt}{\longrightarrow} 0\ \delta t = w \overset{Lt}{\longrightarrow} 0\ \frac{(\gamma_w{}^{-1} - 1)x}{w} = 0. \tag{4.19}$$

In other words one can say that the contribution of the time dilation effect to the time offset can be reduced to zero by transporting the clocks very slowly. In the relativistic world therefore, if two spatially separated clocks are desynchronized by some amount then on their reunion (after slow transport) they will carry the same amount of desynchronization. The time offset due to slow transport of the clocks is thus zero, viewed differently, the effect of slow transport is null i.e slow transport effect, $STE = 0$.

If the two clocks stationary in Σ_1 are considered as two Boughn observers then the desynchronization between these clocks (as in Boughn's thought experiment) will continue to be there when the clocks are brought together slowly for comparison. In other words, the apparent Boughn effect (as defined earlier) is equivalent to the real Boughn effect. In the above example involving the twins P and Q one can therefore see that the age of the forwardly placed twin Q turns out to be more than that of P even after they are reunited (after slow walk) at the same location, showing that the time offset effect suggested by Boughn has an absolute meaning[17], or the effect is a real one (according to our definition) in the relativistic world. The conclusion for the relativistic world can therefore be symbolically represented as $ABE \equiv RBE$ and $STE = 0$.

In order to understand the issue more clearly we will provide some counter examples. As the first example we consider Boughn's scenario in the classical (Galilean) world. In this world, it is well-known that mysteries and paradoxes do not generally exist. However quite surprisingly we shall see that the counter-intuitive effect discussed by Boughn can be reinvented in the Galilean world too. In the next section (Sec.(4.4)) we study the question of reality of Boughn's time offset effect in such a world. We shall indeed see that there can be ABE but not RBE in the Galilean world.

In another example we consider the relativistic world with non-standard synchronization of coordinate clocks in the moving frame. We shall see that although there will not be any desynchronization of clocks, the RBE will pop up during their slow walk reunion.

4.4 Galilean World: Reality of Boughn Effect

Classical or Galilean world is a kinematical world endowed with a preferred frame (of ether) Σ_0 with respect to which the speed of light c is isotropic and moving rods and clocks do not show any length contraction and time dilation effects. However in any arbitrary frame Σ (other than the preferred frame) the speed of light changes and depends on direction. As the clocks do not experience any time dilation effect so they can be transported freely and hence all clocks can be synchronized at one spatial point and then may be transported with arbitrary speed to different locations[6]. In the classical world one generally uses the Galilean transformation (GT) to compare events in different inertial frames, which is

$$\begin{aligned} x &= x_0 - vt_0, \\ t &= t_0, \end{aligned} \tag{4.20}$$

From GT one can obtain the two way speed (TWS) of light $\overleftrightarrow{c}(\theta)$ in Σ along any direction θ with respect to the x-axis (direction of relative velocity between Σ_0 and Σ) as

$$\overleftrightarrow{c}(\theta) = \frac{c(1-\beta^2)}{(1-\beta\sin^2\theta)^{1/2}}. \tag{4.21}$$

This TWS of light is not the same as the one-way speed (OWS) since, for example, along the x-axis the TWS is $c(1 - v^2/c^2)$, while the OWS is $c - v$ and $c + v$ in the positive and negative x-directions respectively. However, one may playfully choose to synchronize the clocks in Σ such that the OWS (to and fro) along a given direction θ are the same as $\overleftrightarrow{c}(\theta)$, this is nothing but Einstein's stipulation in SR, commonly known as the standard synchrony. This synchrony is somewhat awkward in the Galilean world but there is nothing wrong in adopting such a method. In the Galilean world for this synchrony the transformation equations are given by

$$\begin{aligned} x &= (x_0 - vt_0), \\ t &= \gamma_v^2(t_0 - vx_0/c^2). \end{aligned} \tag{4.22}$$

These equations were first obtained by E. Zahar and are therefore known as the Zahar transformations (ZT)[18, 19, 20, 21]. The phase term and γ_v^2 in the time

[6]In the relativistic world due to the time dilation effect this process is generally forbidden.

transformation distinguishes ZT from GT. ZT has been successfully used in clarifying some recently posed paradoxes of SR[22, 23]. An interesting feature of ZT is the existence of apparent time dilation and length contraction effects as observed from an arbitrary reference frame Σ whereas with respect to the preferred frame Σ_0 there are no such effects.

The notion of relativity of simultaneity can thus be imported to the classical world, as can be seen from the time transformation of Eq.(4.22) that two events that are simultaneous in Σ_0 frame are not necessarily so in Σ. By adopting Einstein's mode of convention one should therefore be able to recast the effect linked with relativity of simultaneity or the Boughn effect (BE) in the Galilean world too. Using the above transformation and extending the arguments leading to Eq.(4.4), here too one can obtain a time offset or desynchronization between identically accelerated twins,

$$\delta t_{desync} = -\frac{\gamma_v^2 v x_0}{c^2}. \tag{4.23}$$

By simple alteration of the mode of synchronizing distant clocks, one is thus able to recast the Boughn's paradox even in the Galilean world. The above time offset effect or the differential aging between two spatially separated twins is therefore an artifact of the adopted synchronization scheme.

Although BE can be artificially created in the Galilean world the question here arises whether the effect persists even when the spatially separated clocks are brought together for comparison (as it does in the relativistic world). As mentioned earlier one may define the reality of BE provided the clocks are finally compared when they are brought together. Below we check the reality of BE in the classical world by calculating the effect of clock transport from ZT.

From Eq.(4.22) one can readily obtain the transformation equation of coordinates between any two arbitrary inertial frames Σ_1 and Σ_2 as,

$$x_1 = {\gamma_2}^2(1 - \frac{v_1 v_2}{c^2})x_2 - (v_1 - v_2)t_2, \tag{4.24}$$

$$t_1 = {\gamma_1}^2[(1 - \frac{v_1 v_2}{c^2})t_2 - \frac{\gamma_2^2}{c^2}(v_1 - v_2)x_2], \tag{4.25}$$

where v_1 and v_2 refer to the velocities of the concerned frames, i.e. Σ_1 and Σ_2 respectively with respect to the preferred frame Σ_0 . Also $\gamma_n = (1 - \frac{v_n^2}{c^2})^{-1/2}$ with $n = 1, 2$.

From the above time transformation the time dilation suffered by a clock stationary with respect to Σ_2 is

$$\Delta t_1 = \frac{1 - v_1 v_2/c^2}{1 - {v_1}^2/c^2}\Delta t_2, \tag{4.26}$$

where Δt_2 refers to the proper time between two events at the same point of Σ_2 and Δt_1 is the corresponding time measured by observers in Σ_1.

Now suppose two synchronized clocks are spatially separated by a distance x in Σ_1 and a third clock attached to Σ_2 slowly covers the distance, then the time taken by this clock to cover this distance in Σ_1 is given by

$$\Delta t_1 = \frac{x}{w}, \tag{4.27}$$

where

$$w = \frac{(1 - \frac{v_1^2}{c^2})(v_2 - v_1)}{1 - \frac{v_1 v_2}{c^2}}. \tag{4.28}$$

is the relative velocity of Σ_2 with respect to Σ_1.

The corresponding time measured by the third clock (Σ_2 - clock) may be obtained from Eq.(4.26) as

$$\Delta t_2 = \frac{1 - v_1^2/c^2}{1 - v_1 v_2/c^2}.\frac{x}{w}, \tag{4.29}$$

The difference of these two times

$$\delta t = \Delta t_2 - \Delta t_1 = \frac{v_1 x}{c^2}\gamma_1^2. \tag{4.30}$$

The above equation shows that the integrated effect of time dilation in the classical world due to clock transport is independent of the speed (v_2) at which the clock is transported. The effect is thus non-vanishing even for very slow speed of clock transport. This is just the contrary to the scenario in the relativistic world where we have seen that the result for clock transport is very much dependent on the speed of the clock and in particular vanishes for very small speed.

If as before the two clocks (stationary in Σ_1) are assumed to represent the two Boughn's observers, they have precisely the same amount (δt given by Eq.(4.30)) of desynchronization with a negative sign. The two effects thus nullify each other, or in other words, on comparison of these clocks at one spatial point (i.e. if one of the observer walk towards the other, no matter whether slow or fast) the result will be zero time difference (differential aging). This observation thus demonstrates that although the time-offset effect can be recast in the Galilean world, it is just an artifact and not real according to our definition of "reality" of the effect. That the effect has no absolute meaning is also evident from the fact that it is dependent on the synchronization convention, as, if instead of ZT one uses GT (Eq.(4.20)) to synchronize distant clocks then clearly one cannot find any ABE or RBE. However with ZT one can create ABE in the classical world but cannot obtain RBE since ABE is just the negative of STE. As in the relativistic world here too one can symbolically summarize the conclusion as $ABE \neq RBE$ and $ABE = -STE$.

Thus GSDC cannot be obtained from this Boughn's effect in the classical world via EP. Conversely Boughn's temporal offset may be regarded as an integrated effect of GSDC while in the classical world if it exists is just an artifact of the synchrony. This proves that the connection of the time-offset and GSDC is purely relativistic in nature.

4.5 Absence of BE in the Relativistic World!

In order to further clarify the matter let us consider the other extreme where in the pure relativistic world, apparently there may not be any time offset effect but there will still be differential aging in "real" terms. In order to understand this we will have to incorporate the conventionality of simultaneity (CS) thesis[7] in the relativistic world (we have already considered this thesis in the context of the classical world, although we did not mention it, while we used ZT to describe the world).

In relativity theory the spatially separated clocks in a given inertial frame are synchronized by light signal whose OWS must be known beforehand. But to measure the OWS of the signal from one point to another one requires to have two presynchronized clocks and hence the whole process ends up in a logical circularity. To get out of this problem a convention is adopted in assuming the OWS of the signal within certain bounds. To break the circularity Einstein stipulated the OWS of light to be equal to c which is the same as its TWS. This convention is known as Einstein synchrony or the standard synchrony. Later Reichenbach and Grünbaum[25, 26] first suggested that other synchronization conventions can as well be adopted in synchronizing the clocks. They claimed that the simultaneity between events in an inertial frame is a matter of convention and the conventionality lies in the assumption regarding the OWS of light. This fact that the synchronization procedure in SR has an element of convention is known as conventionality of distant simultaneity or the CS-thesis.

The CS-thesis asserts that Einstein's convention is just one among the various possible choices of the OWS of light. This particular convention leads to a set of

[7] A comprehensive review of the thesis is available in a recent paper by Anderson, Vetharaniam and Stedman[24]. See also Ref.[1, 19, 21].

transformation equations in the relativistic world which are known as the Lorentz transformation (LT). Different choices for the values of the OWS yield different sets of transformation equations with varied structural features. For example it is known that the relativistic world can also be described by the so called Tangherlini transformations (TT) by adopting absolute synchrony[19, 21, 20, 27, 28, 29]

$$\begin{aligned} x &= \gamma_v(x_0 - vt_0), \\ t &= \gamma_v^{-1}t_0.. \end{aligned} \tag{4.31}$$

Note that the absence of spatial coordinate in the time transformation above, shows that the distant simultaneity is absolute. In the light of CS thesis, "relativity of simultaneity" (often considered to be one of the fundamental imports of SR), loses its meaning in this absolute synchrony set-up since there is no lack of synchrony between spatially separated events as observed from different inertial frames. It is quite evident therefore that in the relativistic world with absolute synchrony the time offset effect suggested by Boughn does not exist, but still then, as we will see later, one can obtain a temporal offset effect.

We repeat the calculations done in Sec.(4.4) using TT in the place of ZT. As before from Eq.(4.31), one may obtain the transformation equation between any two arbitrary frames Σ_1 and Σ_2 moving with velocities v_1 and v_2 respectively relative to Σ_0 as,

$$x_1 = \gamma_1[{\gamma_2}^{-1}x_2 + \gamma_2(v_2 - v_1)t_2], \tag{4.32}$$

$$t_1 = {\gamma_1}^{-1}\gamma_2 t_2, \tag{4.33}$$

where $\gamma_1 = (1 - \frac{v_1^2}{c^2})^{-1/2}$ and $\gamma_2 = (1 - \frac{v_2^2}{c^2})^{-1/2}$.

The above time transformation relation gives the time dilation formula with respect to Σ_1-frame, which is

$$\Delta t_1 = {\gamma_1}^{-1}\gamma_2\Delta t_2, \tag{4.34}$$

where Δt_2 refers to the proper time between two events at the same point in Σ_2-frame and Δt_1 is the corresponding time (coordinate time) measured in Σ_1.

As discussed earlier (in the context of ZT), let us here too consider two synchronized clocks (or twins) in Σ_1, spatially separated by a distance x and another clock attached to Σ_2-frame slowly covers this distance with speed w which is given as

$$w = {\gamma_1}^2(v_2 - v_1). \tag{4.35}$$

The time taken by the Σ_2-clock to cover this distance in Σ_1 is given by

$$\Delta t_1 = \frac{x}{w}. \tag{4.36}$$

The corresponding time in Σ_2, taking into account the time dilation effect is

$$\Delta t_2 = \gamma_1 {\gamma_2}^{-1} \frac{x}{w}. \tag{4.37}$$

The difference between these times measured in the two frames is thus

$$\delta t = \Delta t_2 - \Delta t_1 = (\gamma_1 {\gamma_2}^{-1} - 1)\frac{x}{w} = \frac{x(\gamma_1 {\gamma_2}^{-1} - 1)}{{\gamma_1}^2(v_2 - v_1)}. \tag{4.38}$$

Clearly δtdepends not only on v_1 but also on v_2. The result is thus qualitatively different from that one obtains in the Galilean world(with Einstein synchrony). However, if now the Σ_2-clock is transported very slowly such that $w \longrightarrow 0$ or in other words $v_2 \longrightarrow v_1$ then the above time difference reduces to

$$w. \overset{Lt}{\longrightarrow} 0\ (\delta t) = -x v_1/c^2. \tag{4.39}$$

As in the classical world, here too we can see that the above value does not vanish even if the clock is transported very slowly. Thus when the spatially separated synchronized clocks (or twins) are brought together by slow transport, this non-vanishing integrated effect of time dilation gives rise to a time offset or a non-null differential aging between the twin's (biological) clocks. Hence even in this absolute synchrony set-up where there is no question of relativity of simultaneity and therefore of BE, the differential aging or the temporal offset pop up as a time dilation effect when the clocks are brought together by slow transport. In the notation form one can write here $ABE = 0$ and $RBE = STE$.

4.6 Resolution of the Ordinary Twin Paradox Using BE

Let us now move on to the details of the arguments leading to Eq.(4.1). The outward trip of the traveler twin B from the point of view of the earth twin is composed of two phases. In the first phase, the rocket moves a distance L_A in time t_{A1} with uniform velocity v which is given by

$$t_{A1} = \frac{L_A}{v}, \tag{4.40}$$

and in the second phase, which corresponds to the deceleration phase of the rocket which finally stops before it takes the turn-around, the time t_{A2} taken by B is given by

$$t_{A2} = \frac{\gamma v}{g}, \tag{4.41}$$

where the proper acceleration g has been assumed to be uniform with respect to the earth frame. In the present analysis this term does not contribute since we consider the abrupt turn-around scenario where t_{A2} tends to zero as $g \to \infty$; however for the time being we keep it. Therefore the total time elapsed in S for the entire journey is given by

$$T_A = \frac{2L_A}{v} + 2t_{A2}. \tag{4.42}$$

Now we compute this time as measured in B's clock by taking the time dilation effect from the point of view of A. For phase 1 this time t_{B1} may be computed as

$$t_{B1} = \gamma^{-1} t_{A1} = \frac{\gamma^{-1} L_A}{v}, \tag{4.43}$$

where we have applied the simple time dilation formula. For phase 2 however this time-dilation formula is differentially true as the speed is not a constant i. e one may write

$$dt_{B2} = (1 - \frac{v^2}{c^2})^{1/2} dt_{A2} = (1 - \frac{v^2}{c^2})^{1/2} \frac{1}{g} d(\gamma v). \tag{4.44}$$

Hence after integration one obtains[30]

$$t_{B2} = \frac{c}{2g} ln(\frac{1 + v/c}{1 - v/c}). \tag{4.45}$$

However once again this tends to zero as $g \to \infty$. In any case we shall however not need this expression any more. Therefore the total elapsed time measured in B's clock for the complete journey is given by

$$T_B = \frac{2\gamma^{-1} L_A}{v} + 2t_{B2}. \tag{4.46}$$

The differential aging from the point of view of A is thus

$$\delta T_A = T_A - T_B = \frac{2L_A}{v}(1 - \gamma^{-1}) + 2(t_{A2} - t_{B2}). \tag{4.47}$$

From the point of view of B the stay-at-home observer A is moving in the opposite direction and as before one may divide the relative motion of A into two phases, phase I and phase II, where the later corresponds to the acceleration phase. The phase II may be interpreted as turning on of a gravitational field. When this field is switched off (marking the end of the acceleration phase), the phase I starts where the stay-at-home observer A moves with a velocity $-v$ up to a distance L_B which on account of the Lorentz contraction of L_A is given by,

$$L_B = L_A(1 - \frac{v^2}{c^2})^{\frac{1}{2}}, \tag{4.48}$$

and the corresponding elapsed time t_{B1} is given by,

$$t_{BI} = \frac{L_B}{v} = \frac{\gamma^{-1} L_A}{v}. \tag{4.49}$$

This obviously comes out to be the same as t_{B1} since the result is obtained from considerations with respect to the inertial observer A. Similarly t_{BII} i.e. B-clock's time during phase II should be the same as t_{B2} during which the gravitational field is turned on, i.e

$$t_{BII} = t_{B2}, \tag{4.50}$$

and hence the total time

$$\tau_B = 2t_{BI} + 2t_{BII} = \frac{2\gamma^{-1} L_A}{v} + 2t_{BII} = T_B. \tag{4.51}$$

The corresponding time of A's clock by taking into account the time dilation effect is

$$t_{AI} = \gamma^{-1} t_{BI} = \frac{\gamma^{-2} L_A}{v}. \tag{4.52}$$

Writing A-clock's time during phase II from B's perspective as t_{AII}, one may write for A's clock time for the entire journey as

$$\tau_A = 2t_{AI} + 2t_{AII} = \frac{2\gamma^{-2} L_A}{v} + 2t_{AII}. \tag{4.53}$$

The difference of these times of clocks A and B as interpreted by the observer B, is given by,

$$\delta T_B = \tau_A - \tau_B = \frac{2\gamma^{-1} L_A}{v}(\gamma^{-1} - 1) + 2(t_{AII} - t_{BII}). \tag{4.54}$$

Note that at the moment we do not know the value of t_{AII}, since it refers to the time measured by A as interpreted by B when it is in its acceleration phase. The paradox is resolved if

$$\delta T_A = \delta T_B. \tag{4.55}$$

In other words using Eqs.(4.47) and (4.54) one is required to have,

$$t_{AII} = \frac{L_A}{v}(1 - \gamma^{-2}) + t_{A2} = \frac{L_A v}{c^2} + t_{A2}. \tag{4.56}$$

In the abrupt turn-around scenario, as we have already observed $t_{A2} = 0$, one therefore must have

$$t_{AII} = \frac{L_A v}{c^2} = \frac{\gamma L_B v}{c^2}. \tag{4.57}$$

The resolution of the twin paradox therefore lies in accounting for this term. It is interesting to note that the term is independent of the acceleration in phase II. This is possibly the implicit reason why the role of acceleration in the explanation of the twin paradox is often criticized in the literature. However we shall now see how, we can interpret this term as an effect of the direction reversing acceleration (or the pseudo-gravity) experienced by the traveller twin.

Now recall the Boughn-effect of temporal offset between two identically accelerated observers. To be specific, consider an inertial frame of reference S attached to the observer B when it is in the uniform motion phase (phase I). Suppose now there is another observer B' at rest in S at a distance L_B behind B and both of them get identical deceleration and eventually come to rest with respect to A in the frame of reference S', which is moving with velocity $-v$ in the x-direction with respect to S. According to Boughn-effect then the clocks of these two observers get desynchronized and the amount of this desynchronization is given by the expression (4.4) only with the sign changed, that means

$$desync = \frac{\gamma v L_B}{c^2}, \tag{4.58}$$

which is nothing but t_{AII}. It has already been pointed out that this Boughn-effect may be interpreted as the effect of pseudo-gravity (in this case as experienced by the observer B) according to EP. In terms of the pseudo acceleration due to gravity the above expression can also be obtained as

$$desync = \frac{g\Delta t_B L_B}{c^2}. \tag{4.59}$$

Note that $g\Delta t_B$ is finite (equal to γv) even if $g \rightarrow \infty$.

The observer B' which is L_B distance away from B is spatially coincident with A, hence, in calculating the clock time of A from B's perspective this time-offset due to Boughn-effect must be taken into account. This effect is ignored when the twin paradox is posed by näively asserting the reciprocal time-dilation effect for the stay-at-home and the rocket-bound observers. Clearly the paradox is resolved if the Boughn-effect or the pseudo gravitational effect is taken into consideration.

4.7 Test of Boughn-Effect

We have seen that the Boughn-effect can be interpreted as the integrated effect of GSDC. The experimental test of GSDC or the gravitational red-shift is therefore a test of a differential Boughn-effect in a way. On the contrary one may directly measure the integrated effect by the following means:

First two atomic clocks may be compared (synchronized) at the sea level, then one of the clocks may be slowly transported to a hill station of altitude h and then kept there for some time T. In this time these two atomic clocks according to Boughn scenario are perpetually accelerated from a rest frame S to a hypothetical inertial frame S' moving with velocity v, with proper acceleration g so that $\gamma v = gT$. Boughn-effect therefore predicts a temporal offset (see Eqs.(4.58) and (4.59)),

$$\Delta t_{offset} = \frac{ghT}{c^2}. \tag{4.60}$$

This offset can be checked by bringing the hill station clock down and then comparing its time with the sea level one. Any error introduced in the measurement due to transport of clocks can be made arbitrarily small compared to Δt_{offset} by increasing T. As a realistic example for h =7000ft (altitude of a typical hill station in India), and T =1 year and taking the average g to be about $9.8m/sec^2$, the Boughn-effect comes out to be in the micro-second order:

$$\Delta t_{offset} = 7.3\mu s, \tag{4.61}$$

which is easily measurable without requiring sophisticated equipments, such as those used in Pound-Rebka type experiments.

It is interesting to note that from the empirical point of view the effect is not entirely unknown. For example Rindler[12], in seeking to cite an evidence for the GSDC effect, remarks: "Indeed, owing to this effect, the US standard atomic clock kept since 1969 at the National Bureau of standards at Boulder, Colorado, at an altitude of 5400ft. gains about five microseconds each year relative to a similar clock kept at the Royal Greenwich Observatory, England, ...". However one can consciously undertake the project with all seriousness, for the accurate determination of the time-offset (with the error bars and all that), not merely to prove GSDC but to verify the Boughn-effect of SR. It is worth while to note that the empirical verification of this time-offset as a function of T would not only test the Boughn-effect and the integral effect of GSDC but it would also provide empirical support for the relativity of simultaneity of SR. So far no experimental test has been claimed to be the one verifying the relativity of simultaneity. Indeed SR is applicable in the weak gravity condition of the earth so that gravity can be thought of as a field operating in the flat (Minkowskian) background of the space-time[31]. Clearly because of EP, the earth with its weak gravity has the ability to provide a convenient Laboratory to test some special relativistic effects like the relativity of simultaneity or the Boughn-effect.

4.8 Concluding Remarks

In a paper by Price and Gruber[17] an extension of the Boughn's twin paradox is presented. It is shown there that there is no meaning to the question as to where does the differential aging occur in the twin problem. To prove the above statement, the Boughn's scenario is studied from the point of view of a new observer, the uncle of the twins who got separated from the family before the birth of the twins and settled in the final frame of the twins after experiencing some acceleration. The uncle saw the twins born at the same place and then drift apart very slowly to start their rocket trip. The uncle being in a frame different from that of mom and dad did not not see the twins start their journey simultaneously, he rather saw the forwardly placed twin to start the trip first and hence he concluded that at the end of the trip the twins aged differently. Although the result seems to be straight forward apparently, but as pointed out in this paper this result inferred by the uncle is quite strange, as the uncle concludes that the aging has taken place before the twins enter the rocket. The question here is, to the uncle the twins were born at the same place at the same time, then what had happened to the twins during separating when they did not even suffer any acceleration or even move fast. The answer to the question, as provided in this paper is 'the twins "...lose simultaneity during the *arbitrary slow* motion of their separation!..." Although strange, the result can be obtained from the simple application of LT between the frame of mom and dad and that of the uncle.

The time difference between the clocks of the twin during separating from each other as seen by the uncle ($\Delta t'$ say) can be connected to that measured in the frame of mom and dad (Δt say) through the time transformation of LT as

$$\Delta t' = \gamma_v(\Delta t - \frac{v\Delta x}{c^2}), \tag{4.62}$$

Let us consider that each of the twin move symmetrically in the opposite directions with a very small speed v to cover a distance $L/2$. In this case with respect to the frame of mom and dad the time measured in the clocks of both the twins will be the same owing to their symmetric motion, or $\Delta t = 0$. Using this and also noting that $\Delta x = L$, the above equation gives

$$\Delta t' = \gamma_v vL/c^2. \tag{4.63}$$

Thus the uncle observes the twins age differently before the initiation of the rocket-trip. In Sec.1, however, we have seen that the differential aging of the twins occurred during the rocket-trip. One therefore cannot find a definite answer as to where the differential aging occurs, showing that the question is meaning less. In an effort to further justify this point we try to study the paradox of the uncle in the absolute synchrony set-up, i.e with TT (Eq.(4.31)).

It is evident from the time transformation of Eq.(4.31) that if as in the above example the twins separate symmetrically in the frame of their mom and dad then, even in the frame of the uncle no differential aging occurs. Hence with respect to the uncle too the twins age the same when they on board the rocket. When the twins settle at rest in their final frame (i.e the frame of the uncle) their clocks are seen to be synchronized with respect to the uncle. But the scenario changes when the twins walk to meet each other. As we have seen in the previous section during the slow walk meeting of the twins a differential aging crops up between them due to the time dilation effect. In short with absolute synchrony set-up one concludes that the differential aging occurs during the slow walk reunion of the twins.

The above example shows that answer to *where* the differential aging occurs depend not only on the perspective of the observer but also on the adopted mode of synchronization.

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Chapter 5

Demystifying Twin Paradox

5.1 Introduction

As discussed earlier (in Sec.(1.2)) one of the most interesting as well as perplexing paradox of relativity is the twin paradox. Einstein in his 1905 paper[1] on special relativity (SR) first predicted the time dilation effect of moving clocks. From Lorentz transformation (LT) between any two inertial frames in the relativistic world he concluded that a clock B initially synchronized with another stationary clock A at one spatial point, after making a round-trip should fall out of step with the stationary one. He termed the effect as a "peculiar consequence" [2]. Replacing the clocks by human observers, Langevin[3] posed a thought experiment in a problematic form in which a twin B (say) sets off for a space voyage to a distant star with a speed almost equal to the speed of light, and after a brief period, returns to earth at the same speed to meet her stay-at-home brother A (say) to discover that at the end of the trip she is younger than the earth-bound sibling. This counter-intuitive or peculiar result was given the name "twin paradox". This observation is peculiar from the point of view of absolute time since for a common man who believes that time is absolute, any difference between two clocks (initially synchronized) is itself a paradox in a sense that it is unexpected. This definition of the twin paradox focus on the counter-intuitive feature (differential aging) of the problem. Although counter-intuitive both Einstein and Langevin did not see any paradox in this "peculiar consequence" of SR, they recognized early that the situations for the rocket-bound and the stay-at-home twins were not symmetrical and any expectation or claim of symmetrical outcome regarding their ages itself was erroneous. Hence there was no paradox. In fact authors of repute are often found to dismiss the paradox by pointing out that with respect to the inertial frame of the stay-at-home twin, the world lines of the twins in the Minskowski diagram are different and hence the asymmetry in the aging can be attributed to the fact that proper time is not integrable[4].

The other aspect of the twin paradox deals with the idea that the biased result of the twin problem (the stay-at-home twin aged more than the traveller) as if violates the principle of "relativity of motion". For example Dingle[5] in 1957 stated that Einstein made a "regrettable error" and he argued that "According to the postulates of SR, if two identical clocks separate and reunite, there is no observable phenomenon that will show in absolute sense that one rather than other moved. If the postulates of relativity is true, the clocks must be retarded equally or not at all, their readings will agree on reunion if they agreed at separation...". It is thus believed that the principle of relativity demands the ages of the twins to match at the reunion. This aspect of the paradox is a trivial one since the principle of relativity states equivalence of inertial frames of reference only. The principle should not be construed as a statement that all frames of reference are equivalent. In the twin problem the traveller experiences some acceleration at turn-around by virtue of which he lies in a non-inertial frame and hence the principle of relativity of (inertial) motion is not applicable to his frame, thus removing the paradoxical element from the problem.

The above mentioned aspects of the twin problem (first one dealing with the counter-intuitiveness and the second regarding the violation of the principle of relativity of motion) are based on the prediction of the differential aging from the perspective of the stay-at-home twin only. But according to SR one can näively argue that kinematically A is also making a round-trip from the perspective of B and since Lorentz transformation (LT) predicts reciprocal time dilation of moving clocks, clock of A should run slower than that of B, hence B can also claim that A should be younger at their reunion. This again generates the "twin paradox" since each of the twin cannot be younger than the other. The third aspect of the twin problem concerns this logical contradiction between the predictions of the twins when the differential aging is calculated from the perspectives of both the twins. This is the most perplexing one. In the present paper we shall focus on resolving this facet of the problem concerning the logical contradiction rather than the counter-intuitive feature of the issue.

The logical contradiction as discussed above comes from the reciprocity of time dilation effect predicted by LT, which is again a consequence of Einstein's scheme of synchronizing clocks (commonly known as standard synchrony), according to which the one-way-speed (OWS) of light is *stipulated* to be equal to its round trip speed. Indeed convention other than that suggested by Einstein can also be used in synchronizing the distant clocks. That other synchronization conventions can be adopted resulting in the possibility of a plethora of relativistic transformations is known as the conventionality of distant simultaneity thesis (CS-thesis)[1] of SR.

The CS-thesis asserts that Einstein's convention is just one among the various possible choices of the OWS of light. This particular convention leads to a set of transformation equations in the relativistic world which are known as the Lorentz transformation (LT). Different choices for the values of the OWS yield different sets of transformation equations with varied structural features. In recent years a new approach based on conventionality of simultaneity to understanding paradoxes in relativity has been found fruitful. In context of the twin paradox Debs and Redhead[4] have shown that the CS approach provides a means to put an end to the question as to where and when the differential aging takes place in the problem.

Although there is no inherent logical contradiction in SR, an apparent one comes

[1]See for example[4, 6, 7]. For a more comprehensive review of the thesis see a recent paper by Anderson, Vetharaniam and Stedman[8].

about because of the particular synchronization scheme adopted by Einstein which is responsible for the reciprocity of length contraction and time dilation effects. One can thus claim that at the heart of the twin paradox (or the apparent logical contradiction) lies the problem of synchronization of distant clocks[9].

In relativity theory the true effects of relativity and that of synchronization convention are mixed up. The role of each of these in causing the twin paradox can be understood if they can be separated. The function of the synchronization convention issue in the twin paradox problem can be best exemplified if one can introduce Einstein's convention (playfully) in the classical (Galilean) world and pose a suitable twin paradox there. In this way the synchronization effect will be delinked from the relativistic ones (since the latter does not exist in the classical world). But can such a paradox exist in the classical world too? The initial reaction would be to answer in the negative since nothing seems to be mysterious or enigmatic in this world and as is well-known, counter-intuitive problems by contrast exist in the relativity theory probably because of its new physical imports. However the answer is in affirmative. One of the so-called new philosophical imports of SR is the notion of relativity of simultaneity. It can be shown that this notion can also be introduced in the Galilean world. In the next section we will provide the transformation equations obtained by incorporating the ingredients of Einstein's synchrony in the classical world. In Sec.5.3 it will be shown that by doing so the paradox can be artificially created even in this world where normally one would not expect it to exist. The perspective of the paradox will hopefully provide deeper understanding of the twin problem and other related issues.

5.2 Einstein's Synchronization in the Classical World and Zahar Transformation

Classical world is a kinematical world endowed with a preferred frame of ether Σ_0 where light propagation is isotropic and the relativistic effects like length contraction of moving rods and time dilation effect of moving clocks are absent. However in the frames other than the preferred frame the two-way-speed (TWS) of light is different in different directions as is expected in the classical world. The transformation equation generally used in this world is the Galilean transformation (GT):indexGalilean Transformation

$$x = x_0 - \beta t_0, y = y_0, t = t_0. \tag{5.1}$$

Using GT one can show that the TWS of light $\overleftrightarrow{c}$ in any other frame Σ along any direction θ with respect to the x-axis (direction of relative velocity between Σ_0 and Σ) is given as

$$\overleftrightarrow{c}\ (\theta) = \frac{c(1-\beta^2)}{(1-\beta\sin^2\theta)^{1/2}}. \tag{5.2}$$

Hence according to GT, the TWS of light depends upon the direction and it is not equal to the OWS. For example, along the x-axis the OWS is $c - v$ and $c + v$ respectively along the positive and negative x-directions, where as the TWS i. e. the average round-trip speed of light along the x-direction is $c(1 - v^2/c^2)$. However, in a playful spirit one may choose to synchronize the clocks in Σ such that the one way speeds, to and fro, along a given direction θ are the same as $\overleftrightarrow{c}(\theta)$. One may thus mimick the Einstein synchronization scheme to describe the kinematics in this world. The synchrony is somewhat an awkward one in this world but none can prevent one in adopting such a method. For this synchrony GT changes to the following transformations

$$x = (x_0 - vt_0),$$
$$t = {\gamma_v}^2(t_0 - vx_0/c^2). \tag{5.3}$$

where

$$\gamma_v = (1 - v^2/c^2)^{-1/2}. \tag{5.4}$$

is the usual Lorentz factor.

These equations were first obtained by E. Zahar and are therefore known as the Zahar transformations (ZT)[6, 7, 10, 11]. The transformations have been successful in elucidating some recently posed counter-intuitive problems in SR [12, 13]. The presence of the phase term and γ_v^2 in Eq.(5.3) distinguishes the ZT from GT. The appearance of these terms is thus obviously just an artifact of the adopted synchrony. The corresponding inverse can be calculated as

$$x_0 = {\gamma_v}^2 x + vt,$$
$$t_0 = {\gamma_v}^2 vx/c^2 + t. \tag{5.5}$$

Note that with respect to Σ_0, as expected, the moving rods do not contract and clocks in motion do not run slow, but both the effects apparently exist relative to any other frame Σ. The effect of Einstein synchrony is thus manifested through the existence of apparent time dilation and length contraction effects as observed from an arbitrary reference frame Σ.

From ZT (Eq.(5.3)) between the preferred frame Σ_0 and an arbitrary frame Σ, one can obtain the general ZT connecting any two inertial frames Σ_i and Σ_k as

$$x_i = {\gamma_k}^2(1 - \frac{v_i v_k}{c^2})x_k - (v_i - v_k)t_k, \tag{5.6}$$

$$t_i = {\gamma_i}^2[(1 - \frac{v_i v_k}{c^2})t_k - \frac{\gamma_k^2}{c^2}(v_i - v_k)x_k], \tag{5.7}$$

where the suffixes i and k of coordinates x, t and v refer to the coordinates in Σ_i and Σ_k and velocities of the concerned frames with respect to Σ_0 respectively. Also $\gamma_i = (1 - \frac{v_i^2}{c^2})^{-1/2}$ and $\gamma_k = (1 - \frac{v_k^2}{c^2})^{-1/2}$.

Clearly a clock stationary with respect to Σ_k will suffer a time "dilation" according to

$$\Delta t_i = \frac{1 - v_i v_k/c^2}{1 - v_i^2/c^2} \Delta t_k, \tag{5.8}$$

where Δt_k refers to the proper time between two events at the same point of Σ_k and Δt_i is the corresponding time measured by observers in Σ_i.

Similarly the space transformation (Eq.(5.6)) gives the length contraction formula (LCF) with respect to Σ_k , which is

$$\Delta x_k = \frac{1 - v_k^2/c^2}{1 - v_i v_k/c^2} \Delta x_i. \tag{5.9}$$

Hence any Einstein synchronized reference frames exhibit both time dilation and length contraction effects even in the classical world. So one can say that these effects are just the outcomes of the adopted synchronization convention.

ZT thus represents the Galilean world with Einstein (standard) synchrony. Having obtained the transformation laws for this Einstein synchronized classical world, we are now in a position to pose the paradox.

5.3 The Paradox of the Twins

As mentioned in the previous section, if in the classical world the distant clocks in a frame Σ are synchronized according to Einstein's method, then GT goes over to ZT which predicts both time dilation and length contraction from the arbitrary frame of reference Σ. (With respect to the preferred frame however there are no such effects). Now if the earth-bound twin (A) of the twin parable is considered to be at rest in the preferred frame Σ_0 and the rocket-bound one (B) in a frame Σ then A does not detect any time dilation or length contraction effects. However, B observes the both for a clock or a rod stationary in Σ_0 (frame of A). This leads to an apparently paradoxical situation—no time dilation with respect to the observer A brings him to the conclusion that after the round-trip is over there should not be any differential aging, whereas B, with respect to whom A is making the round-trip, contradicts the conclusion and claims that due to time dilation, A should be younger at their reunion. This disagreement between the predictions of the twins gives rise to a twin paradox (in the sense of logical contradiction) in the classical world! Note that the second aspect of the twin paradox is not relevant here since we have already started with a preferred (ether) frame.

One needs to note here that this paradox is different from the well-known twin paradox of the relativistic world in a sense that the predictions of the twins are different in the two cases. In the relativistic world both the twins, due to reciprocal time dilation, argue that the other is the younger one. On the other hand in the classical world only the rocket-bound sibling predicts that her brother is younger, whereas the stay-at-home one observes no differential aging.

The resolution of the ordinary twin paradox should involve demonstration of equal differential aging (zero differential aging in the classical world) from the perspectives of both the twins A and B. In posing the paradox twins are allowed to compute the ages of their counterparts by taking into account the time dilation effect only. The time dilation formula is used freely from the perspectives of both the twins and the question of applicability of the formulas with respect to the traveller twin who changes inertial frames (because of her turn around) is ignored. Indeed while the time dilation formula is correct within one inertial frame of the stay-at-home twin A, the same formula does not hold with respect to the non-inertial frame attached to B. In the abrupt turn-around scenario however, the formula is valid separately in the inertial frames of B in its outward and return journeys. But change of inertial frames by B produces another effect linked with the "relativity of simultaneity" of

SR, in which the key to the resolution of the paradox lies. What happens when a change of reference frame takes place is best exemplified in an article " Case of identically accelerated twins" by S. P. Boughn[9]. In the paper the author has presented a variation of the twin paradox where twins undergo equal accelerations for the same length of time, yet they age differently. He has posed the paradox in the context of SR. In the next section we will recast Boughn's paradox in classical world. It will be shown in Sec.5.5 how Boughn's paradox can be used to resolve the twin paradox posed in the classical world.

5.4 Boughn's Paradox in the Classical World

In the above mentioned article[9], a variation on the twin paradox is presented where two twins P and Q, initially at rest, at a distance L apart, in an inertial frame Σ_0, on board two identical rockets (with equal amount of fuel), get equal accelerations for some time in the direction $\overrightarrow{PQ}$ (x-direction say) and eventually come to rest (when all their fuels had expended) in a new inertial frame Σ moving with a constant velocity v with respect to the former along the positive x-direction. From the simple application of LT Boughn obtained an unexpected result that in the new abode the ages of the twins differed—the age of Q turned out to be more than that of P. This result is really surprising owing to the fact that the twins throughout had identical local experiences, yet their presynchronized (biological) clocks went out of synchrony! Quantitatively this time-offset or desynchronization is given as

$$\delta t_{desync} = -\gamma_v v L/c^2. \tag{5.10}$$

Hence the two synchronized clocks (twins) separated by a distance L get unsynchronized by an amount δt_{desync} when they settle stationary in their new frame.

This desynchronization is real in the sense that it refers to desynchronization in relation to the Einstein convention of synchronization [2]. Let us call this departure from Einstein synchrony as Boughn effect (BE).

The apparently paradoxical result that the spatially separated twins, in spite of their identical history of acceleration age differently is readily explained if one notes that the difference between spatially separated (biological) clocks or the change of relative synchronization cannot have any unequivocal meaning; the clocks can be compared unambiguously only when they are in spatial coincidence. For instance in Σ, one of the observers can slowly walk towards the other (or both of them can do the walking) and compare their ages (or their clock readings) when they meet. One may therefore define the "reality" of the temporal offset effect due to Boughn provided the clocks are finally compared when they are brought together. Since in the relativistic world the so called "slow transport synchronization" is equivalent to the Einstein synchronization[15], the calculated differential aging or time-offset between their clocks when they were in spatial separation would continue to hold even when the twins meet after their slow walk. BE is thus a real effect (according to the definition) in the relativistic world. However when the twins meet after the slow walk it can easily be seen[16] that they do not have symmetrical experiences, and hence the paradox gets resolved.

In the classical world the situation is different. Below we calculate this temporal offset effect in the classical world by applying ZT between the preferred frame Σ_0

[2] Incidentally Selleri[14] has noted in different words in this context that after identical acceleration, the two clocks readings define a natural (absolute) synchronization, which is different from Einstein's synchrony; the latter can only be established by resynchronizing them artificially (see also[13]).

and an arbitrary frame Σ:

$$
\begin{aligned}
x_k &= (x_{k0} - vt_{k0}), \\
t_k &= {\gamma_v}^2(t_{k0} - vx_{k0}/c^2).
\end{aligned}
\tag{5.11}
$$

where t_{k0} and x_{k0} denote the time and space coordinates of the observer k (k stands for P or Q) with respect to Σ_0 and t_k and x_k refers to the corresponding coordinates in the new frame Σ.

From the time transformation of Eq.(5.11), we obtain the time difference for the observers P and Q in their new frame as,

$$t_Q - t_P = \gamma_v^2[(t_{Q0} - t_{P0}) - v(x_{Q0} - x_{P0})/c^2]. \tag{5.12}$$

Noting that the distance between the twins i.e $x_{Q0} - x_{P0} = x_0$ remained constant throughout their journey and also assuming that their clocks were initially synchronized i.e assuming $t_{Q0} - t_{P0} = 0$ (since the relative clock readings of P and Q should not change with respect to Σ_0 as they get identical acceleration for equal amount of time) the time offset $\delta t_{desync} = t_Q - t_P$ between the clocks of the siblings can be calculated as

$$\delta t_{desync} = t_Q - t_P = -\gamma_v^2 v x_0/c^2. \tag{5.13}$$

In the relativistic world if one proceeds in the same manner using LT, then one can easily obtain the desynchronization effect given by Eq.(5.10).

In the relativistic world if one clock is taken towards the other, the time dilation effect is found to depend upon the speed of clock transport and in particular the value goes to zero when the speed is vanishingly small. On the other hand in the classical world, from the calculation of clock transport[3], it can be shown that the integrated effect of time dilation is independent of the speed at which it is transported and the most interesting part is the result is exactly the same as the

[3]These calculations were done in detail in the previous chapter

Boughn's time offset effect between the two clocks with a negative sign. Hence if the observer P walk towards Q (no matter whether slow or fast), the above two effects nullify each other or the net offset between them drop to zero when they meet. In other words, BE vanishes when the clocks are compared at one spatial point. The observation thus demonstrates that although Boughn's paradox can be recast in the classical world, this time offset effect is not real but just an artifact of Einstein synchrony.

Eq.(5.13) gives the Boughn effect between the preferred frame Σ_0 and an Einstein synchronized frame Σ moving with a velocity v with respect to Σ_0. For future use we will need to obtain the temporal offset between any two arbitrary Einstein synchronized reference frames. This may be calculated as follows:

Consider two such frames Σ and Σ' moving with velocities v and u respectively with respect to Σ_0. Writing the transformation equations connecting the space-time coordinates of Σ_0 and Σ' as

$$\begin{aligned} x' &= (x_0 - ut_0), \\ t' &= {\gamma_u}^2(t_0 - ux_0/c^2). \end{aligned} \tag{5.14}$$

one readily obtains transformation equations between Σ and Σ' as

$$\begin{aligned} x' &= {\gamma_v}^2(1 - uv/c^2)x + (v-u)t, \\ t' &= {\gamma_u}^2[(1 - uv/c^2)t + (v-u){\gamma_v}^2 x/c^2]. \end{aligned} \tag{5.15}$$

Hence, as discussed earlier the twins P and Q (separated by a length L in Σ) arrives in their new frame Σ' producing a temporal offset (desynchronization) between their clocks which is given by (obtained from transformation for time in Eq.(5.15))

$$\delta t'_{desync} = t'_Q - t'_P = {\gamma_u}^2{\gamma_v}^2(v-u)x/c^2, \tag{5.16}$$

Eq.(5.16) thus gives the synchronization gap between any two Einstein synchronized reference frames in the classical world.

In the classical world as we have seen this effect is not real, yet BE can be used successfully to provide an accurate account of differential aging from the perspective of the traveller twin.

5.5 Resolution

To analyze the situation let us discuss in detail the whole chain of events from the perspectives of both the twins. For convenience we remove the inconsequential initial and final accelerations from the problem, thus assuming that B makes a flying start and also after the return trip she flies past A. So, the only acceleration in our problem is the turn-around acceleration of B which is essential for the final comparison of the clocks (or ages) of the twins at one spatial point after the trip. We show below step by step how the twins make unequivocal predictions regarding their ages.

Step 1:

The time dilation formulas from the perspectives of both the twins can be symbolically written as

$$TDF1: \qquad \Delta\tau_B(A) = \Delta t_A(A), \tag{5.17}$$

$$TDF2: \qquad \Delta\tau_A(B) = \gamma_v^{-2}\Delta t_B(B). \tag{5.18}$$

While writing the above equations we have adopted a notation scheme, where $\Delta\tau_B(A)$ [$\Delta\tau_A(B)$] denotes the clock reading of the twin B [A] for a time interval between two events which occurred at its position as inferred by the other twin A [B] drawn from its own coordinate clocks' records for the interval, $\Delta t_A(A)$ [$\Delta t_B(B)$] and its knowledge of the relevant time dilation effect. Indeed the time intervals $\Delta\tau_B(A)$

or $\Delta\tau_A(B)$ are based on one clock measurements and hence they refer to proper times of B and A respectively. On the other hand $\Delta t_B(B)$ refers to in general, the observed difference in readings (for the same two events) recorded in two spatially separated (synchronized) clocks stationary with respect to the frame of reference attached to B. However for the round trip of an object or a clock (A say), the time $\Delta t_B(B)$ is also measured by a single clock (B). Although τ-symbol would have been more appropriate in the later case but we shall continue to use the symbol 't' to emphasize that the corresponding time is supposed to be the coordinate time.

The length contraction formulas (LCF) are similarly obtained as

$$LCF1: \qquad L_B(A) = L_B(B), \tag{5.19}$$

$$LCF2: \qquad L_A(B) = \gamma_v^{-2} L_A(A). \tag{5.20}$$

Where $L_A(A)$ and $L_B(B)$ are the rest lengths of rods in Σ_0 and Σ respectively and $L_B(A)$ and $L_A(B)$ are the corresponding observed lengths from the other frames (A and B respectively).

Now, in our problem the only distance of interest is that of the distant star from A which is clearly a rest length in A i.e. $L_A(A) = L_0$(say). So the relevant LCF is

$$LCF: \qquad L = \gamma_v^{-2} L_0, \tag{5.21}$$

where $L = L_A(B)$ is the distance of the star measured in Σ-frame.

Perspective of A:

Step 2:

Time recorded in A-clock for B's journey of distance $2L_0$ is

$$\Delta t_A(A) = 2L_0/v, \tag{5.22}$$

The corresponding time in B-clock taking into account TDF1 (Eq.(5.17)) is

$$\Delta\tau_B(A) = \Delta t_A(A) = 2L_0/v. \tag{5.23}$$

Step 3:

Differential aging with respect to A is therefore given by

$$\delta t(A) = \Delta t_A(A) - \Delta\tau_B(A) = 0. \tag{5.24}$$

So, A concludes that *both the siblings age the same at their reunion.*

Perspective of B:

Step 4:

From B's point of view, A makes the round trip with a velocity

$$u = -\gamma_v^{-2}v \tag{5.25}$$

which is obtained from the velocity transformation using ZT.

Step 5:

The time elapsed in B-clock for this round trip is therefore

$$\Delta t_B(B) = 2L/|u| = 2\gamma_v^{-2}L_0/\gamma_v^{-2}v = 2L_0/v. \tag{5.26}$$

This is nothing but the B-clock time as calculated by A i.e, $\Delta\tau_B(A)$.

Step 6:

The same time interval in A-clock as calculated by B by the *näive* application of TDF2 (Eq.(5.18) alone on $\Delta t_B(B)$) is obtained as,

$$\Delta\bar{\tau}_A(B) = \gamma_v^{-2}2L_0/v. \tag{5.27}$$

We have put a bar sign on τ since the time computed above is erroneous as the direct application of TDF2 on $\Delta t_B(B)$ for the interpretation of A-clock time is incorrect because there is a question of desynchronization of distant clocks due to BE which has to be taken into account. Let us clarify this point.

To calculate the time offset due to BE, we first split the frame of reference (K) attached to B into two inertial frames Σ and Σ' which move with velocities v and $-v$ respectively with respect to Σ_0. Now, just before B decelerates let there be another observer $\bar{A}$ of the same age as that of B (i.e it is assumed that $\bar{A}$'s clock is synchronized with B's in Σ) and at the same location of A (at a distance L from B with respect to Σ), comoving with respect to B such that like in Boughn's scenario, $\bar{A}$ and B both undergo the same but arbitrarily large negative acceleration relative to Σ. From Σ-frame $\bar{A}$ and B may be considered as Boughn's twins accelerating from rest along the negative x-direction (i.e now $\bar{A}$ is forwardly placed with respect to B) and settling down finally in the inertial frame Σ'. BE therefore tells us that with respect to Einstein synchronized clocks in Σ', there is a desynchronization effect between the clocks (or ages) of $\bar{A}$ and B which is given by Eq.(5.16) by replacing u by $-v$ as

$$t'_B - t'_{\bar{A}} = \delta t'_{desync} = {\gamma_v}^4 2vL/c^2 = {\gamma_v}^2 2vL_0/c^2, \tag{5.28}$$

The last equality follows from Eq.(5.21).

The above desynchronization due to BE gives the synchronization gap between the two Einstein synchronized reference frames (Σ and Σ') of the traveller B. The twin B during the instantaneous turn-around jumps from one inertial frame Σ to the other Σ' and because of the desynchronization, the clocks of $\bar{A}$ and B no longer represent the Einstein synchronized coordinate clocks of Σ'. The presence of the synchronization gap between the instantaneously co moving inertial frames for an accelerated observer restricts one from considering B to remain in a single frame. One can consider B to remain in the single frame Σ if the problem is posed such that B instead of turning around would continue to move forward covering the same length of journey with uniform speed as she would do after the turn-around. In that case the coordinate clocks (Einstein synchronized) of Σ frame of B could be used to measure the coordinate time and connect the same with the proper time of A through TDF for the entire trip. However in the second leg of B's trip if someone playfully tamper with her synchronization then the coordinate time recorded by B would be erroneous and hence a calculation to obtain the proper time τ_A of A from this measurement (by applying TDF on it) will give wrong result. The only way to get the correct answer is to first undo the mischief by getting back to the Einstein synchronization that was adopted before and then one is free to use TDF in order to obtain proper time from the coordinate time. Let us now study the scenario when B turns around and immediately jumps to the new inertial frame Σ'. The synchronization of clocks in the new frame Σ' according to Einstein synchrony can be equated with the deliberate alteration of synchronization just discussed in connection with the uniform motion scenario of B, since the standard of simultaneity in Σ' is thus made different from that in Σ. It is thus clear that TDF can be used to obtain proper time from the coordinate time provided the latter refers to a *uniform* synchronization. We then ask if there is any way so that one can continue with the standard of simultaneity (synchrony) of Σ in Σ'. The answer is in the affirmative and is provided by Boughn's thought experiment. From the symmetry of the problem it is evident that clocks of $\bar{A}$ and B initially synchronized in Σ continue to remain synchronized with respect to Σ even when they arrive stationary in Σ' after the turn-around acceleration. While calculating from the perspective of B, for the round trip of A, the time in B-clock, $\Delta t_B(B)$ can be easily obtained but one cannot use this to obtain the proper time of A, as this does not represent the coordinate time of B in Σ. Clearly a correction term $\delta t'_{desync}$. is to be added to $\Delta t_B(B)$ in order to obtain the said coordinate time. This correction is equivalent to the process of restoration of the synchronization mentioned in B's non turn-around example. Hence by adding $\delta t'_{desync}$ to $\Delta t_B(B)$ we can carry over the standard of synchronization of coordinate clocks in Σ to the new frame Σ' and then TDF2 can be correctly used to determine the proper time of A.

Therefore the true coordinate time is obtained as

$$\Delta t_B^{coord}(B) = \Delta t_B(B) + \delta t'_{desync} = 2L_0/v + \gamma_v^2 2vL_0/c^2. \tag{5.29}$$

Now applying TDF2 on the true coordinate time $\Delta t_B^{coord}(B)$, B calculates the round-trip time (proper) measured in A-clock as

$$\Delta\tau_A(B) = \gamma_v^{-2}\Delta t_B^{coord}(B) = \gamma_v^{-2}(2L_0/v + \gamma_v^2 2vL_0/c^2) = 2L_0/v. \tag{5.30}$$

Step 7:

Thus the differential aging from the perspective of B turns out to be,

$$\delta t(B) = \Delta\tau_A(B) - \Delta t_B(B) = 0, \tag{5.31}$$

which agrees with Eq.(5.28), thus resolving the paradox.

5.6 Discussion

We have observed that a twin paradox in the classical world can be posed and resolved by using Boughn's paradox which can also be recast in this world. In the classical world when the clocks are synchronized using absolute synchrony, the paradox of the twins does not exist whereas in the same world if the synchronization is performed following Einstein synchrony, the paradox can be posed as in SR. The present treatment proves that the root of the twin problem lies in the choice of Einstein synchronization convention. Indeed the role of Einstein synchrony in this problem is revealed in a rather direct way as the "true" relativistic effects (non-conventional ingredients of SR) are here deliberately suppressed by the consideration of the classical world. The present analysis of the twin paradox clearly explains how the traveller twin makes an error in assessing the coordinate time of the earth bound twin because of the former's turn around acceleration.

Clearly the reader may use the present analysis as a template for resolving the ordinary twin paradox with much confidence fully in the context of special relativity.

It may be further noted that there has been a long standing debate as to whether twin paradox can be explained fully in the context of special relativity or not. Many authors believe that introduction of general relativity and a gravitational field at the point of acceleration is required to deal with the paradox[4, 17]. One may now wonder where does the general relativity fit in while dealing with the twin paradox in the classical world!

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Chapter 6

Circular Twin Paradox Revisited

6.1 Introduction

In an interesting paper [1] Cranor et.al present a variation of the twin paradox where each twin leaves on one ring of a counter rotating pair of infinitesimally separated rings so that the siblings trace approximately the same circular path but travel in opposite directions. (The observers on the ring of one twin should claim the clock of the other twin slowed down by time dilation and other contradicting the claim. The resolution of the paradox as has been claimed focuses attention to the relation of time dilation to clock synchronization.) According to Cranor et.al's story the two rings have been assumed to move with equal and opposite angular velocity $\omega = vR$ with respect to the frame of reference of the laboratory where v and R denote the linear speed and radii of the rings respectively. Specifically one twin Lisa is assumed to reside stationary on the ring which is moving in the clockwise direction, whereas the other twin Bart lives on the other identical ring. Both Lisa and Bart are assumed to have their teams of observers stationary at every point of their respective rings. The twins pass each other periodically when they can easily compare their clocks. Initially they are at the same place and observe that their clocks both read $t = 0$ (say). According to the velocity addition formula of special relativity (SR) the observers on Lisa's ring detect Bart's clock moves with relative speed

$$v_{rel} = 2v/(1 + v^2/c^2). \tag{6.1}$$

Due to relativistic time dilation formula in comparison to the coordinate clocks of Lisa's team of observers Bart's clock appears to tick more slowly by a relative Lorentz factor

$$\gamma_{rel} = (1 - v_{rel}^2/c^2)^{-1/2} = (1 + v^2/c^2)/(1 - v^2/c^2), \tag{6.2}$$

which shows that Bart's clock will lag behind the clock of next of Lisa's observer (Millhouse) that he passes. It may be argued that the time lag of Bart's clock builds up as he passes the successive members of Lisa's team and finally after the half rotation he ultimately passes Lisa again and therefore Lisa should observe that Bart's clock lags behind Lisa's at the second meeting. This conclusion can soon be dismissed by two counter-reasonings: (1) Bart and his team of observers can also argue in the same way that Lisa's clock instead will lag behind Bart's in future meetings contradicting the claim by Lisa just mentioned. (2) With respect to laboratory observers the motions of Lisa and Bart are identical, differing only in their sense of rotations. Since handedness of motion cannot have any effect on time dilation there should be no reason why the clocks of Lisa and Bart should disagree in future meetings if they were synchronized in their first meeting.

The resolution of the paradox hinges on the fact that the time dilation formula of special relativity (SR) holds good provided the coordinate clocks of any frame of reference (inertial) are synchronized according to Einstein's method of synchronization. The authors correctly point out the difficulty in synchronizing coordinate clocks on rotating frame in Einstein's way. Indeed they have shown how a discontinuity in synchronization appears when one tries to synchronize clocks along the rim of a rotating ring. The author explain the paradox in terms of this synchronization gap or desynchronization of clocks on a rotating platform[1]. In order to emphasize the intimate connection between time dilation and clock synchronization (i.e. the definition of simultaneity) in a reference frame with respect to which time dilation of clocks are to be considered, the authors of reference (1) discussed in passing some non-standard synchronization schemes that one can adopt on a rotating ring. How-

[1]In fact the twin paradox and Sagnac effect are thus connected since the origin of the Sagnac effect can be traced back to this desynchronization [2, 3, 4]

ever the relation between time dilation and clock synchronization is a tricky issue. Misconceptions may easily arise if one is not sufficiently cautious.Indeed we feel that some of the statements made in the article on the issue are either outcome or lacks sufficient clarity so that the beginning relativity students might misinterpret the remarks and make wrong conclusions from them.

The purpose of the present paper is to point out and remove these shortcomings of the otherwise excellent paper and provide some additional clarifications concerning the relationship between clock synchronization and time dilation in SR. The paper under scrutiny has described three possible schemes for synchronization of clocks on a rotating ring in addition to the standard one. According to these methods any two spatially separated events on a rotating ring which are simultaneous with respect to the observers on the ring are also simultaneous with respect to laboratory. In order to facilitate clarifying our points we briefly reproduce only one of these synchronization schemes. Indeed it is not necessary to describe all the three methods since they all provide the same sort of synchronization. The synchronization method is as follows: One considers the ring to be already rotating. At the center of the rotating ring one has a light flash. Upon receiving the flash the ring observers set all their clocks to read the same time say $t = 0$, thus completing the procedure. It has truly been argued that the flash at the center "favours no particular observer,..." hence "a moment of simultaneity on a ring will also be a moment of simultaneity in the lab...." In the literature this type of synchronization is often known as absolute synchronization. In the following sections we consider some of the remarks made by Cranor et.al concerning the absolute synchronization and provide our clarification on the related issues.

6.2 Does Absolute Synchronization Imply Absence of Time Dilation?

Referring to the non-standard synchronizations on a rotating ring the authors note (or the authors' statements may be so construed) that it does not produce time dilation. To the students it may appear that since this particular synchronization imply the absolute nature of simultaneity it cannot produce time dilation. Indeed referring to the fact that two events simultaneous with respect to laboratory will also be simultaneous with respect to Lisa's observers, the authors observe "...this explains what we already know must be true. There will be no lagging of Bart's clock observed as it passes each of Lisa's observers. *For the relativistically inappropriately synchronized clocks of Lisa's observers, there is no time dilation of Bart's clock....*"

It is true that partly owing to this particular synchronization, there will be no lagging of Bart's clock with respect to the frame of reference of Lisa but the fact does not follow from the reason that simultaneity is absolute in Lisa's frame with respect to laboratory. In fact the absolute simultaneity argument does not have the ability to distinguish between Bart and any other observer who is rotating with some *arbitrary* angular velocity, since the agreement of simultaneity is an issue between the laboratory observers and Lisa's observers and therefore it has nothing to do with Bart's clock. Hence anyone is entitled to assume that the reference of "Bart's clock" in the quoted statement of the last paragraph is only incidental and one may incorrectly interpret the remark to mean that with respect to a reference frame where clocks are absolutely synchronized with reference to laboratory will not observe any time dilation! However such an interpretation is correct specifically for Bart's clock. One can prove that it not generally valid. The point however is the absence of time dilation of Bart's clock with respect to the inappropriately synchronized clocks of Lisa's observers is not obvious from the absolute simultaneity reasoning used by the authors. This may be seen as follows:

Cranor et.al considers a novel method (method 3 according to the reference) of Synchronization in Lisa's frame of reference when it is already in uniform (angular) motion[2]

The team of observers on Lisa's frame receive a light flash sent from the from the centre of rotations and on the basis of prior instructions set their clocks to $t = 0$ at the moment they see the flash. The authors correctly argued that since light flash at the centre does not favour any particular observer, the "moment of simultaneity on the rotating ring frame will also be a "moment of simultaneity" in the laboratory frame.

Now the transformation between the laboratory frame and a moving inertial frame (which may be instantaneously comoving with the rotating frame) can be written as

$$\begin{aligned} x' &= \gamma(x - vt), \\ t' &= \gamma^{-1}t, \end{aligned} \tag{6.3}$$

where the primed quantities refer space and time in the frame moving with relative velocity v with respect to the laboratory frame.

Indeed one can verify that the above transformations represent a relativistic world but the clocks of moving frames are so synchronized that simultaneity is absolute (i.e. if $\Delta t = 0$, $\Delta t'$ is also zero). The world is relativistic in the sense that moving clocks dilate and moving rods contract with respect to the singled-out frame (laboratory frame) as they would do according to Lorentz transformation; but since different synchronization scheme have been adopted in moving frames, the time di-

[2]In the present article we shall not consider method 1 and 2 as they will attract the problem of Ehrenfest paradox[5, 6] which will complicate synchronization scenario.

lation and length contraction results from those frames in general differ from those obtained in the standard formulation of SR. However that does not mean that the world is non-relativistic. Time dilation and length contraction are partly synchronization convention dependent effects. One may verify that in the moving frames two way speed (TWS) of light is always c. It must be so as TWS is a convention independent quantity.

Now specifically let Σ be the laboratory frame, Σ_1 and Σ_2 be the frames of Lisa and Bart moving with velocities v_1 and v_2 respectively relative Σ. The transformation equations for space-time between the instantaneously co moving frames of Lisa and laboratory using absolute synchronization (Tangherlini Transformation[7, 8, 9, 10, 11]) are given as

$$\begin{aligned} x_1 &= \gamma_1(x - v_1 t), \\ t_1 &= \gamma_1^{-1} t, \end{aligned} \tag{6.4}$$

where $\gamma_1 = (1 - v_1^2/c^2)^{-1/2}$ is the usual Lorentz factor.

With the inverse transformation

$$\begin{aligned} x &= \gamma_1^{-1} x_1 + \gamma_1 v_1 t_1, \\ t &= \gamma_1 t_1. \end{aligned} \tag{6.5}$$

Similarly the transformations between Σ and Σ_2 are

$$\begin{aligned} x_2 &= \gamma_2(x + v_2 t), \\ t_2 &= \gamma_2^{-1} t. \end{aligned} \tag{6.6}$$

The inverse is

$$\begin{aligned} x &= \gamma_2^{-1} x_2 - \gamma_2 v_2 t_2, \\ t &= \gamma_2 t_2. \end{aligned} \tag{6.7}$$

From Eqs.(6.5) and (6.7), we obtain the transformation equations connecting Σ_1 and Σ_2 which are given as

$$\begin{aligned} x_2 &= \gamma_1^{-1} \gamma_2 x_1 + \gamma_1 \gamma_2 (v_1 + v_2) t_1, \\ t_2 &= \gamma_1 \gamma_2^{-1} t_1. \end{aligned} \tag{6.8}$$

Similarly

$$x_1 = \gamma_1\gamma_2^{-1}x_2 + \gamma_1\gamma_2(v_1 + v_2)t_2,$$
$$t_1 = \gamma_1^{-1}\gamma_2 t_2. \tag{6.9}$$

The corresponding time dilation formulas (TDF) are

TDF1 (representing time dilation of Lisa's clock with respect to Bart's observers):

$$\Delta t_2 = \gamma_1\gamma_2^{-1}\Delta t_1. \tag{6.10}$$

TDF2 (representing time dilation of Bart's clock with respect to Lisa's observers):

$$\Delta t_1 = \gamma_1^{-1}\gamma_2\Delta t_2. \tag{6.11}$$

As is evident from the above equations only for $v_1 = v_2$ i.e. $\gamma_1 = \gamma_2$, the synchronization do not produce any time dilation, but otherwise $(\gamma_1 \neq \gamma_2)$ $\Delta t_1 \neq \Delta t_2$, or in other words there is some time dilation (contraction) effect. Thus proving that although in the special case it is true that Bart's clock will not be dilated relative to Lisa's observers. it is generally not true that "non-relativity of simultaneity implies no time dilation...."

In addition to what we have already discussed we have learnt an important lesson. There are many relativists who do not like to believe in the CS-thesis of the relativity theory. But there is no question of believing or not believing since in the present problem we can see how the clocks in different moving (rotating) frames can be absolutely synchronized. Indeed no body can prevent one from doing so. In one of the most cited article Debs and Redhead [12] have been found to be a bit apologetic about their using of the CS-thesis in providing clarifications on the twin paradox. To our mind, as we have clarified there aught not to be any reason to be hesitant at all.

6.3 Einstein Synchronization on Lisa's ring

Having understood the role of absolute synchronization in *evading* the paradox let us now consider the standard (Einstein) synchronization on the rotating ring. In addition in order to understand the role of synchronization we would like to eliminate relativistic effects from the problem. We have already seen in the previous chapter that the origin of the clock paradox lies essentially in the Einstein's synchronization convention. We shall therefore consider a classical world where except in a preferred frame (Laboratory frame) two way speeds of light will not be isotropic. Einstein synchronization in this frame would mean that along any line the one way speeds in one (forward) direction (whatever may be its value) will be the same in the other (reverse) direction. Transformation equations that represent this synchronization is represented by the so called Zahar transformation (ZT)[7, 8, 10, 13]

$$\begin{aligned} x &= (x_0 - vt_0), \\ t &= \gamma^2(t_0 - vx_0/c^2), \end{aligned} \tag{6.12}$$

where x' and t' denotes the space and time coordinates of an event in the instantaneous frame co moving with Lisa (say) and x_0 and t_0 are the same in the laboratory frame. The inverse of ZT is readily obtained as

$$\begin{aligned} x_0 &= \gamma^2(x + v\gamma^{-2}t), \\ t_0 &= \gamma^2 vx/c^2 + t. \end{aligned} \tag{6.13}$$

The above transformation indicates that Lisa's clock will not experience any time dilation as $\Delta t_0 = \Delta t$ (since x the position of Lisa's clock remains constant in Lisa's frame). But from the forward transformation (Eq.(6.12))it is evident that a clock at rest on a ring fixed in the laboratory frame (which apparently rotates with respect to Lisa) will experience time dilation through the relation $\Delta t = \gamma^2 \Delta t_0$.

The general ZT between two frames Σ_i and Σ_k instantaneously co moving with two rotating rings can be obtained as

$$x_i = {\gamma_k}^2(1 - \frac{v_i v_k}{c^2})x_k - (v_i - v_k)t_k, \tag{6.14}$$

$$t_i = {\gamma_i}^2[(1 - \frac{v_i v_k}{c^2})t_k - \frac{\gamma_k^2}{c^2}(v_i - v_k)x_k], \tag{6.15}$$

where terms with suffixes i and k represents respective quantities in Σ_i and Σ_k respectively, v_i and v_k refer to velocity with respect to the laboratory frame Σ_0.

If specifically the frames of Lisa and Bart are Σ_i and Σ_k we have $v_i = v = -v_k$ and the above equation can be written as

$$x_i = \gamma^2(1 + \frac{v^2}{c^2})x_k - 2vt_k, \tag{6.16}$$

$$t_i = \gamma^2[(1 + \frac{v^2}{c^2})t_k - \frac{2\gamma^2}{c^2}(v)x_k], \tag{6.17}$$

where $\gamma^2 = {\gamma_i}^2 = {\gamma_k}^2 = (1 - v^2/c^2)^{-1/2}$.

The circular twin paradox can now be posed in the classical world also. It is evident from the time transformation of Eq.(6.17) that Bart's clock will experience time dilation with respect to Lisa's team of observers and in the same way Lisa's clock will experience *same* time dilation with respect to Bart's team of observers! If this happens we can argue as Cranor et.al explains "Since Bart's clock agrees with Lisa's as he passes her at $t = 0$, time dilation means that his clock will lag behind the clock of the next of Lisa's observers that he passes. As he passes successive members of Lisa's observing team, his clock should be seen to lag further and further behind. One-half rotation later, the observer he passes will be Lisa...." One therefore concludes that when Bart passes her, their clocks will not read the same.

Now this cannot happen, on the ground of symmetry of the problem as Bart's and Lisa's motions differ only in handedness. This handedness of motion cannot have any effect on the rate of ticking of clocks. This is the paradox. The difference with that posed by Cranor et.al lies in the fact that ours is a classical world. There is no "real" time dilation and length contraction yet these can be apparently created by not choosing absolute synchronization. Clearly the apparent relativistic effects that we have seen in this world is an artifact of synchronization process.

The prompt remedy of the paradox is to go back to absolute synchronization and immediately ZT will be replaced by GT and Lisa's and Bart's clocks will always agree. Only in this case this agreement will always be true irrespective of their speeds. In the relativistic case however they will not agree if Bart's angular velocity differs from that of Lisa's.

However choosing absolute synchronization merely evades the paradox. One needs to resolve the contradictory claims of the two observers. This can be done in the following way: As stated, Milhouse on Lisa's ring is the next observer in the anticlockwise direction. Bart's clock will lag Milhouse's when they pass each other. Assume now that Einstein synchronization is used starting with Lisa, and proceeds in the anticlockwise direction to Milhouse and then proceeding around the ring until the clock of the last observer Selma gets synchronized (with light signal). In this case the angular position of Lisa (angle 0) and that of Selma (angle 2π) coincide but their clocks will not agree. Thus the synchronization produces a discontinuity (synchronization gap). The amount of discontinuity in the relativistic case has been shown by Cranor et.al to be

$$Disc(1) = 2\pi R v \gamma / c^2. \tag{6.18}$$

When calculated in detail, the time dilation of Bart's clock will produce a time lag with Selma's clock precisely the above $Disc(1)$. At the very same location as Selma lies Lisa whose clock lags Selma's by $Disc(1)$ and therefore agrees precisely with Bart's clocks.

In the present case the discontinuity in the synchronization (time difference between Lisa's and Selma's clock) turns out to be

$$Disc(2) = 2\pi R v\gamma^2/c^2, \tag{6.19}$$

and in the same way the time lag of Bart's clock with Selma's will be compensated by $Disc(2)$ when compared with Lisa's.

Incidentally $Disc(1)$ and $Disc(2)$ represent the phase shifts between counter rotating light beams on a rotating platform. This phase shift is known as the Sagnac effect.

The above analysis clarifies that in an accelerated frame one has to live with these discontinuities if Einstein synchronization is adopted. This discontinuity is the root of all troubles in standard twin paradox scenario also. Absolute synchronization has the ability to free ourselves from apparent contradictions. The next chapter will be devoted to discussing this issue in detail.

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Chapter 7

Conventionality of Simultaneity, Absolute Synchronization and Twin Paradox in the Non-abrupt Turn-around Scenario

7.1 Introduction

This chapter also deals with the ordinary twin paradox but with a difference. Since the advent of SR hundreds of papers on twin paradox have appeared in the literature. Barring a few, most of the authors have asserted that there is no "paradox". Indeed once "paradox" gets resolved the term "paradox" should never be used in the concerned context. Yet almost all the authors dealing with the topic concerning twins continue to refer to it as the twin paradox. There is no point however to give excuses for using the phrase "twin paradox" or raising this sementic issue at the fag end of this thesis. But there is a reason. If the counter-intuitive issue had really long been resolved it would not remain still one of the most enduring puzzles of physics.

As for example in 1918, Einstein came up with the resolution of the paradox, in terms of his own general theory of relativity; on the other hand say about eighty years after Einstein's 1905 paper an article appears in a reputed journal with the provocative title "Twin paradox: A complete treatment from the point of view of each twin"[1] indicating as if so far complete treatment was not available! Some relatively recent papers[2, 3, 4] even challenge SR in the context of the paradox. Unnikrishnan[2] claims to have shown "Logical and physical flaws" in Einstein's analysis and further remarked in some context that "special relativity principle formulated originally for physics in empty space is not valid in the matter filled universe." Some of Unnikrishnan's arguments however was promptly challenged by Gron[5] as it was rightly shown that there was serious error in the former's analysis.

A few years ago (2006) there had been a considerable hype in the media stating for example "...scientist solves Einstein's twin paradox" referring to a paper that appeared in J.of theo.phys. Students of Physics would be puzzled since it would mean that twin paradox was being solved for the first time! What we would mean to say is that in spite of various authors having remarked that clock paradox or twin paradox have long been solved the debate on the issue still continues. Pesic[6] had rightly remarked "... numerous papers continue to illuminate the problem. Alternative situations have emerged that surely would have delighted Einstein".

One of such alternative situations concerns the so-called non-abrupt turn-around scenario where the rocket bound twin B during turn-around gets finitely decelerated. Let us consider this now.

In the twin problem, as said earlier, the traveller Barbara (B) faces some acceleration during turn-around by virtue of which she lies in an accelerated frame, whereas Alex (A), the stay at home one remains in an inertial frame. Since Lorentz transformation (LT) holds only between two inertial frames, one cannot use LT to compute the ages from the perspective of the accelerated observer B. In case of abrupt turn-around however, one can work with LT by assuming that the accelerated motion of B can be decomposed into two uniform motions for her forward and return journeys. But a correction is to be introduced in the calculation of time due to change of simultaneity when B jumps from one inertial frame to another. The main problem arises when one considers a more realistic situation where the acceleration faced by the traveller is finite. Here the question arises can one deal with the situation in the same way when the turn-around is non-abrupt (finite acceleration).

It is often believed that one needs general relativity (GR) to deal with the situation[1, 7, 8, 9]. For example Perrin in his paper "Twin paradox: A complete treatment from the point of view of each twin"[1] set up a round-trip situation assuming finite (proper) acceleration of B and worked through from the perspectives of each twin. In particular the author obtained the time elapsed on the clock of the stay-at-home twin during the periods of acceleration by solving the Einstein's field equation and the geodesic equation in the frame of reference of the traveller twin. The calculation is quite involved and one may have the feeling that the result thus obtained cannot be obtained purely from special relativistic considerations. However we will show below (Sec.7.3) that the A-clock time can be calculated fully in the context of SR provided one synchronizes the coordinate clocks suitably in the frame of reference of the traveller twin.

To understand this synchronization issue it is worthwhile to first discuss briefly the conventionality of simultaneity thesis (CS-thesis)[10, 11, 12, 13, 14, 15, 16, 17] of SR. According to CS-thesis[1], in relativity theory various conventions can be adopted in synchronizing distant clocks in a given inertial frame. Einstein's synchronization convention which asserts that the one-way-speed (OWS) of light is isotropic and is equal to its two-way-speed (TWS) 'c' is just one among these possible modes of synchronization. Now if we can obtain a synchronization scheme where there is no change of simultaneity when one shifts from one inertial frame to another, then in the twin problem the effect due to the change of synchronization of the coordinate clocks of B can be eliminated. In such a mode of synchronization, time should be absolute. The proposed synchronization scheme is known as absolute synchronization. Indeed in a previous chapter we have seen that correction of time due to the change of simultaneity in the traveller's frame during turn-around suggests how naturally absolute synchronization emerges as the best synchronization in the context of acceleration. In the relativistic world if the distant clocks in an inertial frame Σ are synchronized according to absolute synchronization, then the

[1]A comprehensive review of the thesis is presented in a paper by Anderson,Vetharaniam and Stedman[18]Also one can study the papers of Ref[12, 13, 19]

transformation equation thus generated is given as

$$x = \gamma(x_0 - ut_0),$$
$$t = \gamma^{-1}t_0, \tag{7.1}$$

where u is the relative velocity of Σ with respect to the preferred frame Σ_0 and $\gamma = (1 - u^2/c^2)^{-1/2}$ is the usual Lorentz factor. The transformation Eq.(7.1) is known as the Tangherlini transformation (TT)[12, 13, 14, 20, 21] or often inertial transformation[15].

Coming back to the context of the twin paradox, when the traveller B faces some finite acceleration, she continuously changes her inertial frames. In case of abrupt turn-around however, since as we have already mentioned, one can easily find the correction term that is to be added for the change of simultaneity between these two inertial frames. On the other hand, for finite acceleration one will need to find ways to incorporate the effects due to change of simultaneity in a continuous manner. However any endeavour towards this end is not witnessed in the literature.

We propose to deal with the situation by choosing the absolute synchronization from the onset so that one does away with the need to incorporate corrections due to change of simultaneity that we discussed earlier. Thus, using TT we will be able to recover the expression for the A-clock time (that have been obtained by Perrin[1] in a complicated manner by solving Einstein's field equations and the geodesic equations) from B's perspective.simply from the consideration of SR alone[2]. To our knowledge no such *"complete treatment"* of the paradox involving finite acceleration in the context of SR is available in literature. In this paper we shall present such an analysis. We hope that this work will provide a deeper understanding regarding the subtleties of the problem. Perhaps this will also put a stop to the long controversy

[2]It may be noted that the context of SR does not change for merely choosing a different (coordinate) clock synchronization scheme.

regarding the necessity of the involvement of GR in the resolution of the twin problem. Before presenting our main calculations we shall give in the next section a brief review of Perrin's[1] analysis. Sec.7.4 will be devoted to summarizing our results and conclusions.

7.2 A Brief Review of Perrin's Paper

Robert Perrin in his paper "Twin paradox: A complete treatment from the point of view of each twin" [1] divided the round-trip of the traveller into acceleration phases and uniform velocity phases. He divided the round-trip into six phases as follows:
Phase 1 is the acceleration phase where the traveller leaves the earth with a proper acceleration g until her velocity increases from 0 to u relative to the earth twin.
Phase 2 is the uniform velocity phase where the traveller covers a distance L_0 with the constant velocity u.
Phase 3 is the deceleration phase where the traveller moves with an acceleration '$-g$' and turns around.
Phase 4 is the moment after turn-around where the traveller suffers the same amount of acceleration and travels toward earth with velocity $-u$.
Phase 5 is again the constant velocity phase where the traveller moves toward earth with velocity $-u$.
Phase 6 is the final acceleration phase where the traveller suffers the acceleration g and ends up at rest on the earth:

From special relativistic consideration it has been calculated that the time elapsed on earth for the entire round-trip of the traveller is

$$T = 4\frac{\gamma u}{g} + 2\frac{L_0}{u}. \tag{7.2}$$

The above result is obtained in detail in the next section.

To compute the time recorded in the traveller's clock, the stay-at-home sibling uses the standard time dilation result following LT. This time comes out to be

$$\overline{T} = \frac{2c}{g} ln(\frac{1 + u/c}{1 - u/c}) + 2\gamma^{-1}\frac{L_0}{u}. \tag{7.3}$$

From the point of view of the traveller twin the author similarly divided the trip into six phases and interpreted the acceleration phases as the turning on of a gravitational field along the x-direction. From the perspective of the traveller the time elapsed on her clock is the same as that calculated by the earth-bound observer and that elapsed on earth during the uniform velocity phases are obtained using the length contraction formula. But in order to calculate the time elapsed on earth *during the accelerated segments* of the journey Perrin solved the gravitational field equations and the geodesic equations of motion. He claimed that these calculations cannot be completed without involving GR.

From his quite involved GR analysis he showed that the time elapsed on earth (as calculated by the traveller) during the acceleration phases comes out to be

$$t_I + t_{III} = 2\frac{\gamma u}{g} + \frac{L_0 u}{c^2}, \tag{7.4}$$

where t_I and t_{III} are the time elapsed on earth during the acceleration phases I and III respectively.

With this value for the earth-clock time, perspective of both the twins matches, thus resolving the paradox. In the next section we shall show that the time elapsed on the earth clock can be obtained from special relativistic considerations alone by solving some simple equations.

7.3 Tangherlini Transformation and the Twin Paradox

As mentioned in Sec.7.1 clock synchronization is a matter of convention and different conventions yield different sets of transformation equations. Selleri suggested that in general the space-time transformation between the preferred frame Σ_0 and an arbitrary frame Σ with relative velocity u is given as

$$\begin{aligned} x &= \gamma(x_0 - ut_0), \\ t &= \gamma^{-1}t_0 + \epsilon(x_0 - ut_0). \end{aligned} \tag{7.5}$$

which represents a set of theories equivalent to SR. The free parameter ϵ depends on the simultaneity convention adopted in the frame Σ and is a function of the relative velocity u. For different values of ϵ we get different sets of transformation equations. For example, for standard synchrony

$$\epsilon = -\beta\gamma/c = -u\gamma/c^2. \tag{7.6}$$

For this value of ϵ Eq.(7.5) reduces to LT. From Eq.(7.5) one can also obtain the OWS' of light along the positive (c'_+) and negative (c'_-) x-directions in the frame Σ as

$$\begin{aligned} \tfrac{1}{c'_+} &= \tfrac{1}{c} - \left[\tfrac{u}{c^2} + \epsilon\gamma^{-1}\right], \\ \tfrac{1}{c'_-} &= \tfrac{1}{c} + \left[\tfrac{u}{c^2} + \epsilon\gamma^{-1}\right] \end{aligned} \tag{7.7}$$

Putting the value of ϵ given by Eq.(7.6) one can easily check that the OWS' of light in Σ comes out to be the same as its TWS c, as is demanded by the standard synchrony.

For $\epsilon = 0$ Eq.(7.5) reduces to

$$\begin{aligned} x &= \gamma(x_0 - ut_0), \\ t &= \gamma^{-1}t_0, \end{aligned} \tag{7.8}$$

which is the same as Eq.(7.1). So for $\epsilon = 0$ we get the Tangherlini transformation (TT) representing the relativistic world with absolute synchrony.

The inverse of TT can be calculated as

$$\begin{aligned} x_0 &= \gamma^{-1}(x + \gamma^2 ut) = \gamma^{-1}(x - vt), \\ t_0 &= \gamma t, \end{aligned} \tag{7.9}$$

where v is the relative velocity of Σ_0 with respect to Σ which is given as

$$|v| = \left|-\gamma^2 u\right| \tag{7.10}$$

To obtain the A-clock time from the perspective of B we consider that the stay-at-home twin (A) is attached to the frame Σ_0 and the traveller (B) to Σ. We follow closely the approach suggested by Perrin[1] and hence divide the round-trip similarly into six phases—two uniform velocity and four acceleration phases.

Below we shall compute the ages from the perspective of both the twins A and B step-wise.

Step 1:

Before calculating the ages from the individual perspectives let us first write down the time dilation formulas and the length contraction formulas.

Time Dilation Formulas:

The transformation equations (7.1) and (7.9) suggests that the twin A attached to the preferred frame observes time dilation whereas the other twin B observes the time to be contracted. So from the perspective of A we get a time dilation formula (TDF) but from the perspective of B we obtain a time contraction formula (TCF), which are

$$TDF: \qquad \Delta\tau_B(A) = \gamma^{-1}\Delta t_A(A), \tag{7.11}$$

$$TCF: \qquad \Delta\tau_A(B) = \gamma\Delta t_B(B). \tag{7.12}$$

In the above $\Delta\tau_X(Y)$, (where X and Y stands for both A or B) denotes the time recorded in X-clock for two events which occurred at the position of X, as inferred by another observer Y, drawn from Y's coordinate clock record $\Delta t_Y(Y)$ and the relevant TDF (in case Y represents A) or TCF (in case Y represents B). Also the interval $\Delta\tau_X(Y)$ is based on a single clock measurement and hence it refers to the proper time of X.. So the τ-symbol stands for the proper time. The interval $\Delta t_Y(Y)$, as mentioned earlier, is the coordinate time of Y and it gives the difference in clock readings for the same set of events (which occurred in X-frame) measured by two spatially separated (synchronized) coordinate clocks stationary with respect to the frame of reference attached to Y. However for the round trip of the observer X, only a single clock (of Y) is required to measure the interval $\Delta t_Y(Y)$, but even then it still represents the coordinate time, so t-symbol is used instead of τ.

Length contraction formulas:

Similarly from the set of equations (7.1) and (7.9) we obtain two length contraction formulas, more correctly contraction from the perspective of A and dilation from that of B which are

$$LCF1: \qquad L_B(A) = \gamma^{-1} L_B(B), \tag{7.13}$$

$$LCF2: \qquad L_A(B) = \gamma L_A(A). \tag{7.14}$$

Where $L_A(A)$ and $L_B(B)$ are the rest lengths of rods in the frames of A (i.e Σ_0) and B (i.e Σ) respectively. The lengths $L_B(A)$ and $L_A(B)$ are the corresponding lengths noted by the observers in the other frames (A and B respectively). In the twin problem the only distance of interest is that of the point of turn-around from A which is clearly a rest length in A i.e. $L_A(A) = L_0$ (say). Hence the relevant LCF is that given by Eq.(7.14), which is

$$L = \gamma L_0, \tag{7.15}$$

with $L = L_A(B)$.

Perspective of A:

Step 2:

Phase 1, as said before, is the acceleration phase where the acceleration of Σ-frame is

$$g_u = \frac{du}{dt_0} = (1 - \frac{u^2}{c^2})^{3/2} g, \tag{7.16}$$

or

$$dt_A(A) = dt_0 = \frac{du}{(1 - \frac{u^2}{c^2})^{3/2} g}. \tag{7.17}$$

The phase 1 ends when B attains a velocity u with respect to A, hence on integrating the right hand side from velocity 0 to u we can get the total time elapsed in A-clock during phase 1

$$\Delta t_A(A)_1 = \frac{1}{g} \int_0^u \frac{du}{(1 - \frac{u^2}{c^2})^{3/2}} = \frac{\gamma u}{g} \tag{7.18}$$

Note that in the notation $\Delta t_A(A)_1$, the subscript 1 denotes the phase 1.

Now the entire journey is composed of four such acceleration phases, i.e, phases 1,3,4 and 6. Hence considering all these phases the total time elapsed is

$$4\Delta t_A(A)_1 = 4\frac{\gamma u}{g}. \tag{7.19}$$

Step 3:

The remaining two phases i.e 2 and 5 are the uniform velocity phases where the twin B traverses a distance L_0 with the constant velocity u. Hence the total time elapsed during these phases is

$$2\Delta t_A(A)_2 = 2\frac{L_0}{u}. \tag{7.20}$$

Step 4:

Considering all the six phases the total time measured in A-clock comes out to be

$$\Delta t_A(A) = 4\frac{\gamma u}{g} + 2\frac{L_0}{u}. \tag{7.21}$$

Step 5:

The time recorded in B-clock as observed by A can be obtained by applying the TDF given by Eq.(7.11). For the uniform velocity phases this formula can be used directly but for the acceleration phases the speed is not constant and hence the TDF is differentially true. For the acceleration phases therefore we have

$$dt_B(A) = \gamma^{-1} dt_A(A), \tag{7.22}$$

On integrating the above equation we obtain the time recorded in B-clock during each acceleration phase as

$$\Delta t_B(A)_1 = \int_0^u \gamma^{-1} dt_A(A) = \int_0^u \frac{du}{(1-\frac{u^2}{c^2})g} = \frac{c}{2g} ln(\frac{1+u/c}{1-u/c}). \tag{7.23}$$

Step 6:

For the uniform velocity phases the calculation is simple and the time recorded in B-clock as computed by A is

$$\Delta t_B(A)_2 = \gamma^{-1}\Delta t_A(A)_2 = \gamma^{-1}\frac{L_0}{u}. \tag{7.24}$$

Step 7:

The total time recorded in B -clock as observed by A thus comes out to be

$$\Delta t_B(A) = 4\Delta t_B(A)_1 + 2\Delta t_B(A)_2 = \frac{2c}{g} ln(\frac{1+u/c}{1-u/c}) + 2\gamma^{-1}\frac{L_0}{u}. \tag{7.25}$$

Step 8:

From Eqs. (7.21) and (7.25) the differential aging from the point of view of A can be calculated as

$$\delta t(A) = 2\frac{L_0}{u}(1-\gamma^{-1}) + 4\frac{\gamma u}{g} - \frac{2c}{g} ln(\frac{1+u/c}{1-u/c}). \tag{7.26}$$

Perspective of B:

Step 9:

From the point of view of the traveller B the journey can be similarly divided into six phases. The phase I is as before the acceleration phase. From the principle of equivalence one may interpret this as the turning on of a gravitational field in the $-x$ direction until the earth-bound twin attains a velocity $-v$. During this phase the acceleration experienced by the earth-bound twin is

$$g_v = \frac{dv}{dt} = (1 + \frac{4v^2}{c^2})^{1/2} g, \tag{7.27}$$

or

$$dt_B(B) = dt = \frac{dv}{(1 + \frac{4v^2}{c^2})^{1/2} g}. \tag{7.28}$$

Hence the time elapsed in the B-clock $\Delta t_B(B)_I$ can be obtained by integrating the right hand side between speeds 0 and v, thus giving

$$\Delta t_B(B)_I = \frac{1}{g} \int_0^v \frac{dv}{(1 + \frac{4v^2}{c^2})^{1/2}} = \frac{c}{2g} \ln \left[\frac{2v}{c} + \left(1 + \frac{4v^2}{c^2} \right)^{1/2} \right]. \tag{7.29}$$

Replacing v by $\gamma^2 u$ in the above equation gives

$$\Delta t_B(B)_I = \frac{c}{2g} ln(\frac{1 + u/c}{1 - u/c}). \tag{7.30}$$

which is the same as $\Delta t_B(A)_1$ given by Eq.(7.23).

Step 10:

On the completion of phase I, phase II starts where A moves with a constant velocity $-v$ and covers a distance $L = \gamma L_0$. The time in B-clock during this phase is thus

$$\Delta t_B(B)_{II} = \frac{L}{|-v|} = \frac{\gamma L_0}{\gamma^2 u} = \frac{\gamma^{-1} L_0}{u}, \tag{7.31}$$

where we have made use of Eq.(7.10).

Step 11:

During phase III, the gravitational field is again turned on as in phase I but now in the opposite sense unless A comes to rest and reverses his direction. The return trip of A is same as the outward trip with two acceleration and one uniform velocity phases. The total time elapsed in B-clock (considering all these phases) thus comes out to be

$$\Delta t_B(B) = 4\frac{c}{2g}ln(\frac{1+u/c}{1-u/c}) + 2\frac{\gamma^{-1}L_0}{u} = \Delta t_B(A). \tag{7.32}$$

Step 12:

In this step we shall compute the time elapsed in A-clock as observed by B during the acceleration phase I (or during each acceleration phase). Note that during these phases, as said earlier, a gravitational field is turned on, so Perrin[1] claimed that GR is required for the calculations to be performed. In contrast we shall do the calculation using SR alone.

To obtain this time assuming absolute synchronization we use the TCF (Eq.(7.12)). During phase I this time is

$$\Delta t_A(B)_I = \int_0^v \gamma dt_B(B) = \frac{1}{g}\int_0^v \frac{dv}{\sqrt{1-u^2/c^2}\sqrt{1+4v^2/c^2}} \tag{7.33}$$

Solving the equation and writing v in terms of u, we get

$$\Delta t_A(B)_I = \frac{\gamma u}{g}. \tag{7.34}$$

Step 13:

The time elapsed in A-clock during the uniform velocity phase is

$$\Delta t_A(B)_{II} = \gamma \Delta t_B(B)_{II} = \frac{L_0}{u}. \tag{7.35}$$

Step 14:

The total time elapsed in A-clock as calculated by B is thus

$$\Delta t_A(B) = 4\Delta t_A(B)_I + 2\Delta t_A(B)_{III} = 4\frac{\gamma u}{g} + 2\frac{L_0}{u} = \Delta t_A(A). \tag{7.36}$$

On comparison of Eqs.(7.32)and (7.36) we can see that the differential aging from the perspective of B comes out to be the same as that from the perspective of A

$$\delta t(B) = \delta t(A). \tag{7.37}$$

The perspectives of both the twins match, thus dissolving the paradox completely.

7.4 Summary

The present analysis shows that the whole calculation can be performed considering SR alone. Even for finite acceleration, during the acceleration phases one can easily obtain the A-clock time fully in the context of SR if the clocks are synchronized following absolute synchronization convention. To our knowledge the proposed analysis is the first one to treat the twin problem involving finite acceleration lying in the realm of SR. This treatment thus makes it evident that for the *complete treatment* of the twin problem whose origin lies in SR only special relativistic effects are sufficient, any reference to GR in the context of the ordinary twin paradox is redundant. The above treatment also shows that the twin problem does not arise at all if the clocks are synchronized according to absolute synchrony. Hence it reveals clearly the fact that at the heart of the standard twin problem lies the adopted synchronization convention, i.e the Einstein synchrony since, as we have seen, in the relativistic world if the clocks are synchronized according the standard synchrony then there develops a contradiction between the predictions of the twins, but if in the same relativistic world the clocks are synchronized following absolute synchrony the contradiction vanishes. As said earlier, in the ordinary twin problem, the rocket-bound sibling is subjected to a turn-around acceleration due to which she shifts from one inertial frame to another. For the clocks synchronized according to Einstein synchrony, the simultaneity being relative, a change in synchrony occurs at this point of turn around, and hence while computing the ages the twin B has to take proper care of this effect, in addition to the time dilation effect[3]. In our earlier works we have shown that the paradox gets resolved if the effect linked with relativity of simultaneity is taken care of. It has been shown there that the reason behind this desynchronization is that the clocks in the first frame of B refuse to be synchronized in her second frame *automatically*. The clocks if left to themselves define absolute synchrony; any other synchrony (with $\epsilon \neq 0$ in Eq.(7.5)) can be achieved only through human intervention i.e artificially. Selleri therefore called absolute synchrony as *"nature's choice"*. The twin paradox thus arises when B misses out this desynchronization effect while calculating the ages. But for the clocks synchronized according to absolute synchrony the desynchronization and hence the paradox does not arise. So this work of ours makes it clear that the twin problem is an artifact of Einstein's mode of synchronization.

[3]The stay-at-home twin A being in an inertial frame can correctly calculate the ages considering only the time dilation effect.

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THE PRINCIPLE OF EQUIVALENCE AND THE TWIN PARADOX

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The canonical twin paradox is explained by making a correct use of the principle of equivalence. The role of the principle of equivalence is to provide a physical agent i.e gravity which can supply the required extra aging to the rocket-bound sibling during its acceleration phase through a gravitational time-offset effect. We follow an approach where a novel variation on the twin paradox is used to connect gravity with the desynchronization in the clocks of two spatially distant, identically accelerated observers. It is shown that this approach removes certain drawbacks of an earlier effort which claims to exploit the equivalence principle in explaining the differential aging in the paradox.

Key words: special relativity, general relativity, twin paradox, equivalence principle, gravitational slowing down of clocks, conventionality of simultaneity, Zahar transformation.

1. INTRODUCTION

The principle of equivalence between acceleration and gravity is considered as a *cornerstone* of Einstein's theory of gravitation or that of general relativity (GR). According to Einstein, the principle states that: "A system in a uniform acceleration is equivalent to a system at rest immersed in a uniform gravitational field" [1]. Text books often introduce GR by first demonstrating that the equivalence principle (EP) predicts gravitational red-shift, which Einstein viewed as a test of general relativity. However, we now regard it as a more basic test

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of EP and the existence of curved space-time [2]. The phenomenon of gravitational red-shift, which has been tested by precision experiments by Pound-Rebka and Snider in the sixties [3, 4] is also interpreted as that of gravitational slowing down of clocks (GSDC). The GSDC has now been tested with much accuracy by using a hydrogen maser clock with extraordinary frequency stability flown on a rocket to an altitude of about 10,000 km [2]. In the literature GSDC phenomenon has been found to play an important role in resolving the notorious twin paradox [5].

In the canonical version of the twin paradox, of the two twins initially living on earth (assumed to be an inertial frame), one leaves the earth by a fast rocket to a distant star and then returns to meet her stay-at-home brother to discover that they age differently. This as such is not a paradox since the rocket-bound sibling, on account of her high velocity will suffer relativistic time dilation of her (biological) clock throughout her journey and will therefore return younger with respect to her brother. Indeed, with respect to the inertial frame of the stay-at-home twin, the world lines of the twins in the Minkowski diagram are different (although from the description of the problem, the end points of these lines, i.e the time and the place of departure and that of their reunion, meet) and hence the asymmetry in the aging can be attributed to the fact that proper time is not integrable [6]. The paradox arises if one näively treats the perspectives of the twins symmetrically. For example, if the traveller twin considers herself to remain stationary and relate the motion to her brother, she would (erroneously) expect her brother to stay younger by believing that the Lorentz transformation (LT) predicts reciprocal time dilation of moving clocks. Qualitatively the resolution lies in the observation that one of the twins is in an accelerated (non-inertial) frame of reference and hence the postulates of special relativity (SR) are not applicable to it and therefore the claim of reciprocity of time dilation between the frames of reference of the twins falls through. Indeed, Einstein himself found this sort of argument preferable in dismissing the paradoxical element in the twin problem [7]. However, this suggestion should not be construed as a statement that the resolution of the paradox falls outside the purview of SR . On the contrary much of the expositions found in the literature on the subject deal with the problem in the frame work of SR alone[1], although many tend to believe that the introduction of GR and a gravitational field at the point of acceleration is the right way to understand the asymmetry in the perspectives of the twins. Bohm notes in the context that " two clocks running at places of different gravitational potential will have different rates" [10]. This suggests that EP can directly be used to explain the asymmetry (difference between the experiences of the

[1]Very extensive treatment is available in Special Relativity Theory-Selected Reprints [8], (see also Ref. [9]). For newer expositions see for example Ref. [6] and references therein.

rocket-bound and the stay-at-home twin). However, as pointed out by Debs and Redhead [6] and also others [11], that since in the twin problems one deals with flat space-time, any reference of GR in this context is quite confusing.

Coming back to the issue of acceleration, one finds often that the direct role of acceleration of the rocket-bound twin in causing the differential aging has been much criticized although it is quite clear that in order to have twice intersecting trajectories of the twins (this is necessary since the clocks or ages of the twins have to be compared at the same space-time events) one cannot avoid acceleration.

In an interesting article Gruber and Price [12] dispel the idea of any direct connection between acceleration and asymmetric aging by presenting a variation of the paradox where although one twin is subjected to undergo an arbitrarily large acceleration, no differential aging occurs. That the acceleration per se cannot play a role is also evident from the usual calculation of the age difference from the perspective of the inertial frame of the stay-at-home twin if one notes that the duration of the turn-around process of the rocket can be made arbitrarily small in comparison to that for the rest of the journey and hence the final age difference between the twins can then be understood in terms of the usual relativistic time dilation of the traveller twin during essentially the unaccelerated segment of her journey[2].

One is thus caught in an ambivalent situation that, on the one hand the acceleration does not play any role, on the other hand the paradox is not well posed unless there is a turn-around (acceleration) of the traveller twin[3].

In order to get out of this dichotomy it is enough to note that from the point of view of the traveller twin, the acceleration (or the change of reference frame in the abrupt turn-around scenario) is important. The consideration of this acceleration only has the ability to explain that the expectation of symmetrical time dilation of the stationary twin from the point of view of the rocket-bound twin is incorrect.

In an interesting paper A.Harpaz [5] tries to explain the twin paradox by calculating the age difference from the perspective of the traveller twin directly by applying EP i.e by introducing GSDC. From the previous discussions it may seem unnecessary (or even confusing) to invoke gravity in the essentially special relativistic problem. However the fact is, Harpaz's approach apparently provides an alternate

[2]In such a calculation the time dilation is also calculated during the acceleration phase (assuming the clock hypothesis to be true [6]) and is shown to contribute arbitrarily small value in the age offset if the duration of the acceleration phase is assumed to tend to zero.

[3]Here we are considering the standard version of the paradox and the variation where the twins live in a cylindrical universe [13, 14] has been kept out of the present scope.

explanation for the differential aging from the traveller's perspective.

The author of the pedagogical article observes that although the special relativistic approach can correctly account for the age difference between the twins, " it does not manifest the 'physical agent' responsible for the creation of such a difference" [5]. It is held that EP provides such an agent and that is gravity. But how does gravity find way into the problem? Gravity enters through EP and its connection with the resolution of the paradox can symbolically be written as

$$Acceleration \xrightarrow{EP} Gravity \rightarrow Gravitational\ red\text{-}shift \rightarrow GSDC \rightarrow Extra\ aging,$$

where the last item of the flow diagram indicates that with respect to the rocket-bound twin, GSDC provides the extra aging of the stay-at-home one, explaining the asymmetrical aging of the problem.

However while there is as such no harm in understanding the twin problem from a different perspective (here, this is in terms of GSDC), Harpaz's approach suffer from two fold conceptual difficulties which we will elaborate in the next section. These difficulties include the fact that the calculations are only approximate. The other difficulty will be seen to be of more fundamental in nature. The aim of the present paper is to remove these difficulties and give an *accurate* account of the asymmetric aging from the perspective of the rocket-bound twin directly in terms of a time-offset between the siblings which is introduced due to the pseudo-gravity experienced by the traveller twin.

2. GSDC AND EXTRA AGING

In the standard version of the twin paradox the differential aging from the perspective of the stay-at-home (inertial) observer A can easily be calculated assuming that for the most parts of the journey of the traveller twin B, the motion remains uniform except that there is a turn-around acceleration of the rocket so that finally the siblings are able to meet and compare their ages. In the Minkowski diagram the whole scenario is characterized primarily by three events: (1) Meeting of the world lines of A and B when the voyage starts taking place, (2) the turn around of B and (3) meeting of the world lines when A and B reunite. For the paradox it is not necessary that at events (1) and (2), the relative velocity between A and B has to be zero, since ages or clocks can be compared at a point even if the observers are in relative motion, therefore the analysis of the problem can be done by considering the acceleration only during the turn-around. The duration of the acceleration phase can be considered to be arbitrarily small compared to the time it takes during its forward and return journeys and hence the age difference occurs due to the usual relativistic time dilation of a clock for its uniform motion. This is clearly given by

$$\text{Age difference} = 2t_A(1-\gamma^{-1}) \approx 2t_A v^2/c^2, \tag{1}$$

where $2t_A$ is the time the rocket takes for its entire journey (up and down) in uniform speed v and $\gamma = (1-\frac{v^2}{c^2})^{-1/2}$ is the usual Lorentz factor.

The paradox is resolved if one can show that B also predicts the same difference in spite of the fact that the time dilation effect is reciprocal. Clearly some new considerations (that were absent in arriving at Eq.(1)) must offset this reciprocal time dilation and also this must provide some extra aging to A from the point of view of B so that the age difference remains independent of the two perspectives. One of these new considerations, as has already been pointed out, is the one of a synchronization gap that B discovers due to her change of inertial frame during her entire voyage. This has been clearly demonstrated by Bondi [15] in the context of Lord Hulsbery's three brother approach [6] to understanding the twin paradox.

The other way of understanding the same thing is the consideration of pseudo-gravity experienced by B because of its turn-around. In order to demonstrate how EP plays the role in the analysis, Harpaz uses the gravitational red-shift formula, which can be obtained heuristically (using the EP) as

$$\Delta\nu = \nu_0(1 + gh/c^2), \tag{2}$$

where g is the acceleration due to (pseudo) gravity and $\Delta\nu$ represents the change of frequency of light observed at a distance h from the source where the frequency of the same light is seen to be ν_0. Interpreting this red-shift effect in terms of GSDC, the formula can be written as

$$t_1 = t_2(1 + \Delta\Phi/c^2), \tag{3}$$

where t_1 and t_2 are times measured by clocks located at two points P_1 and P_2 (say) and $\Delta\Phi = gh$, is the potential difference between these p oints. It has been shown that with respect to B the acceleration plays a role by providing an extra time difference between B and A, because of the integrated effect of GSDC during the (arbitrarily) short duration of B's acceleration. This time difference more than offsets the age difference calculated by B solely assuming the reciprocal time dilation so much so that finally B ages less by the correct amount. As pointed out earlier there are two conceptual difficulties in understanding the treatment. First, in an effort to find a "physical agent" responsible for the extra aging, Harpaz relies on some approximate formulae including that of the gravitational red-shift because of his assumption, $v^2/c^2 \ll 1$ inherent in the analysis, and therefore, the pseudo-gravitational effect has the ability to resolve the paradox only approximately. Clearly there is no valid reason to make any such small velocity approximation for the problem. One might of course argue that for the author's stated purpose it would be enough to show that the " physical agent" i.e. gravity is at work when B's point of view is considered. However, it will be shown that such an argument would also not hold good and the reason for it concerns the second difficulty. The explanations based on SR relies on the fact that during the direction reversing acceleration, the travelling twin changes from one reference frame to another and the lack of simultaneity of one reference frame with respect to the other provides the "missing time" which constitutes the reason for the differential aging [6]. Now the lack of agreement in simultaneity is a special relativistic concept without any classical analogue, on the other hand in many standard heuristic derivations of the gravitational red-shift formula (see for example [16–18]) which is also followed by the author of Ref. [5], one finds that no reference to SR is made. Indeed the well-known formula for the red-shift parameter $Z = gh/c^2$ is only approximate and is derived by making use of the classical Doppler effect for light between the source of light and a detector placed at a distance h along the direction of acceleration g of an Einstein elevator [5]. According to EP an observer within the elevator will " attribute his observations in the elevator, to the existence of a uniform gravitational field in a rest system of reference" [5]. Thus the equivalence of gravity and acceleration in terms of gravitational red-shift or GSDC therefore turns out to be as if a purely classical (Newtonian) concept in this approximation! How then is GSDC able to account for an effect, viz. the lack of simultaneity which is essentially a standard relativistic phenomenon?

In the next section we will show that indeed the EP can explain the twin paradox exactly provided the connection of EP and GSDC is obtained using the full machinery of SR.

3. EP AND THE GRAVITATIONAL TIME OFFSET

In an interesting paper Boughn [19] presents a variation of the twin paradox where two twins A and B on board two identical rockets (with equal amount of fuel), initially at rest a distance x_0 apart in an inertial frame S, get identical accelerations for some time in the direction AB (x-direction say), and eventually come to rest (when all their fuel has been expended) with respect to another inertial frame S' moving with velocity v along the positive x-direction with respect to S. From the simple application of Lorentz transformation Boughn obtains a very surprising result that after the acceleration phase is over, the age of A becomes less than that of B.

The result is counter-intuitive by virtue of the fact that the twins throughout have identical local experiences but their presynchronised (biological) clocks go out of synchrony. The amount of this time offset turns out to be

$$\Delta t' = -\gamma v x_0/c^2. \tag{4}$$

The result follows from the simple application of LT which one may write for time as

$$t_k' = \gamma(t_k - v x_k/c^2), \tag{5}$$

where t_k and x_k denote the time and space coordinates of the observer k (k stands for A or B) with respect to S and the prime refers to the corresponding coordinates in S'.
From Eq.(5) it follows that

$$t_B' - t_A' = \gamma[(t_B - t_A) - v(x_B - x_A)/c^2]. \tag{6}$$

Assuming the clocks of the observers A and B are initially synchronized in S, i.e assuming $t_B - t_A = 0$ and also noting that $x_B - x_A = x_0$ remains constant throughout their journeys, the time offset between these clocks is given by the expression (4) provided $\Delta t'$ is substituted for $t_B' - t_A'$.

The paradox however can be explained by noting that for spatially separated clocks the change of relative synchronization cannot be unequivocally determined. The clocks can only be compared when they are in spatial coincidence. For example, when in S' either of the observers can slowly walk towards the other or both the observers can walk symmetrically (with respect to S') towards the other and compare their clocks (ages) when they meet. However in that case one can show [20] that they do not have identical local experiences— thus providing the resolution of the paradox.

While the paradoxical element of the problem goes away, the fact remains that the result (4) is correct and this time offset remains unchanged even if they slowly walk towards each other and compare their clocks (ages) when they meet [21].

This temporal offset effect of identically accelerated clocks gives an important insight into the behaviour of clocks in a uniform gravitational field, for, according to EP "...all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system" [17]. This suggests, as correctly remarked by Boughn that two clocks at rest in a uniform gravitational field are in effect perpetually being accelerated into the new frames and hence the clock at the higher gravitational potential (placed forward along the direction of acceleration) runs faster. With this insight we write Eq.(4) as

$$t - t_0 = -\gamma(t)v(t)x_0/c^2 = -f(t), \tag{7}$$

where now t and t_0 are the readings of two clocks at higher and lower potentials respectively and also $f(t)$ stands for the right hand side of Eq.(4) without the minus sign

$$f(t) = \gamma(t)v(t)x_0/c^2. \tag{8}$$

In terms of differentials one may write Eq.(7) as

$$\delta t - \delta t_0 = -\dot{f}(t)\delta t, \tag{9}$$

where the time derivative $\dot{f}(t) = \frac{gx_0}{c^2}$, with $g = \frac{d}{dt}(\gamma v)$ is the *proper acceleration.*
We may now replace δt and δt_0 by n and n_0, where the later quantities corresponds to the number of ticks (second) of the clocks at their two positions. We therefore have

$$(n - n_0)/n_0 = -\dot{f}(t), \tag{10}$$

or, in terms of frequency of the clocks,

$$-\delta\nu/\nu_0 = \dot{f}(t), \tag{11}$$

where $\delta\nu$ refers to the frequency shift of an oscillator of frequency ν_0. The slowing down parameter for clocks, $-\delta\nu/\nu_0$ in Eq.(11) is nothing but the so called red-shift parameter Z for which we obtain the well-known formula[4]

$$Z = gx_0/c^2. \tag{12}$$

One thus observes that the time-offset relation (7) of Boughn's paradox can be interpreted as the accumulated time difference between two spatially separated clocks because of the pseudo-gravity experienced by the twins.[5] We shall see the importance of the time-offset relation (7) in accounting for the asymmetrical aging of the standard twin paradox from the perspective of the traveller twin. However before that, in the next section we show that the connection of the time-offset and GSDC is purely relativistic in nature.

4. BOUGHN'S PARADOX IN THE CLASSICAL WORLD

The origin of Boughn's paradox can be traced to the space dependent part in the time transformation of LT. The existence of this term is indeed the cause of relativity of simultaneity in SR.

The notion of relativity of simultaneity however can also be imported to the classical world. By classical or Galilean world we mean a kinematical world endowed with a preferred frame (of ether) S with

[4]In terms of *ordinary* acceleration $\overline{g} = dv/dt$, measured with respect to S the formula comes out to be $Z = (\overline{g}\gamma x_0/c^2)(1 - v^2\gamma^2/c^2)$ which for small velocities can also be written as $Z = \overline{g}x_0/c^2$.

[5]The connection between gravity with this temporal offset through EP was first pointed out by Barron and Mazur [22], who derived the approximate formula for the "clock rate difference" mentioned in the previous foot-note.

respect to which the speed of light c is isotropic and moving rods and clocks do not show any length contraction and time dilation effects. However the speed of light measured in any other inertial frame S' moving with velocity v with respect to S will change and will depend on direction. The synchronization of spatially separated clocks is generally not an issue in this world as clocks can be transported freely without having to worry about time dilation, therefore all clocks can be synchronized at one spatial point and then may be transported with arbitrary speed to different locations. (The process is generally forbidden in SR). Clearly one uses the Galilean transformation (GT) to compare events in different inertial frames. Using GT one can show that the two way speed (TWS) of light $\overleftrightarrow{c}$ in S' along any direction θ with respect to the x-axis (direction of relative velocity between S and S') is given by

$$\overleftrightarrow{c}\,(\theta) = c(1-\beta^2)/(1-\beta\sin^2\theta)^{1/2}. \tag{13}$$

According to GT this TWS is not the same as the one-way speed (OWS) of light, for example, along the x-axis it is $c-v$ and $c+v$ in the positive and negative x-directions respectively, while the two way speed, i.e the average round-trip speed of light along the x-direction is given by $c(1-v^2/c^2)$. However, in a playful spirit one may choose to synchronize the clocks in S' such that the one way speeds, to and fro are, the same as $\overleftrightarrow{c}$. This is similar to Einstein's stipulation in SR which is commonly known as the standard synchrony. In the Galilean world the synchrony is somewhat an awkward one but none can prevent one in adopting such a method. For this synchrony GT changes to the following transformations

$$x' = (x-vt), \quad t' = \gamma^2(t - vx/c^2), \tag{14}$$

which was first obtained by E. Zahar and is therefore known as the Zahar transformation (ZT) [23–26]. The transformations have been successfully used to clarify some recently posed counter-intuitive problems in SR [27, 28]. The presence of the phase term and γ^2 in Eq.(14) distinguishes the ZT from GT. Clearly the appearance of these terms is just an artifact of this synchrony.

One is thus able to recast Boughn's paradox using the above transformations and extending the arguments leading to the Eq.(4), one obtains, for the differential aging,

$$\Delta t' = -\gamma^2 v x_0/c^2. \tag{15}$$

The above expression for the differential aging between two spatially separated twins is also therefore an artifact of the synchrony.

Let us note that ZT has many interesting features which include the existence of apparent time dilation and length contraction effects as observed from an arbitrary reference frame S'. (With respect to the preferred frame however there are no such effects). We have already pointed out that the temporal offset between clocks cannot have any unequivocal meaning unless it corresponds to measurement at one spatial point.

One may therefore define without much ado the reality of the temporal offset effect due to Boughn (hereafter referred to as Boughn-effect), provided the clocks are finally compared when they are brought together. In the relativistic world a clock is slowly transported towards the other in order to minimize the time dilation effect in the process. In this world if one of the pre-synchronized spatially separated clocks is brought to the other in an arbitrarily slow motion, it can be seen that when they are compared at the position of the second clock, they remain synchronized. In other words if two clocks have an initial temporal offset between them (due to Boughn-effect or otherwise) when separated, the value for this offset will remain unchanged when they are brought together for comparison. Boughn-effect is thus a real effect (according to the definition) in the relativistic world. In the classical world the situation is different. Below we calculate the effect of clock transport from ZT.

From ZT between a preferred frame S_0 and an arbitrary frame S, one may write the transformation equation between any inertial frames S_i and S_k as

$$x_i = {\gamma_k}^2(1 - v_i v_k/c^2)x_k - (v_i - v_k)t_k, \tag{16}$$

$$t_i = {\gamma_i}^2[(1 - v_i v_k/c^2)t_k - (\gamma_k^2/c^2)(v_i - v_k)x_k], \tag{17}$$

where the suffixes i and k of coordinates x, t, and v refer to the coordinates in S_i and S_k and velocities of the concerned frames with respect to S_0, respectively; also $\gamma_i = (1 - v_i^2/c^2)^{-1/2}$ and $\gamma_k = (1 - v_k^2/c^2)^{-1/2}$.

Clearly a clock stationary with respect to S_k will suffer a time "dilation" according to

$$\Delta t_i = [(1 - v_i v_k/c^2)/(1 - v_i^2/c^2)]\Delta t_k, \tag{18}$$

where Δt_k refers to the proper time between two events at the same point of S_k and Δt_i is the corresponding time measured by observers in S_i.

Consider now two synchronized clocks are spatially separated by a distance x in S_i and a third clock attached to S_k slowly covers the distance. The time taken by the clock to cover this distance in S_i is given by

$$\Delta t_i = x/w, \tag{19}$$

where w is the relative velocity of S_k with respect to S_i. The corresponding time measured by the third clock (S_k - clock) may be obtained from Eq.(18).

From ZT the relative velocity formula is obtained as

$$w = (1 - v_i^2/c^2)(v_k - v_i)/(1 - v_i v_k/c^2). \tag{20}$$

Using Eqs. (18), (19), and (20), one obtains for the difference of these two times

$$\delta t' = \Delta t_k - \Delta t_i = (v_i x/c^2)\gamma_i^2. \tag{21}$$

This non-vanishing integrated effect of the time dilation in the classical world due to clock transport is independent of the speed (v_k) at which the clock is transported. In contrast, in the relativistic world one finds different values for the effect for different velocities and in particular the value is zero when the speed is vanishingly small.

If now the two stationary (with respect to S_i) clocks refer to two Boughn's observers A and B, they have precisely this amount (Eq.(21)) of temporal offset with a negative sign and hence if the observer A walks towards B no matter whether slow or fast, the result will be the zero time difference between the clocks when compared at one spatial point. This observation demonstrates that although Boughn's paradox can be recast in the Galilean world the time-offset effect is just an artifact and not real according to our definition of "reality" of the effect. Thus GSDC cannot be obtained from this Boughn's effect in the classical world via EP. Conversely Boughn's temporal offset may be regarded as an integrated effect of GSDC while in the classical world if it exists is just an artifact of the synchrony.

5. RESOLUTION

Let us now move on to the details of the arguments leading to Eq.(1). The outward trip of the traveller twin B from the point of view of the earth twin A is composed of two phases. In the first phase, the rocket moves a distance L_A in time t_{A1} with uniform velocity v which is given by

$$t_{A1} = L_A/v, \tag{22}$$

and in the second phase, which corresponds to the deceleration phase of the rocket which finally stops before it takes the turn-around, the time t_{A2} taken by B is given by

$$t_{A2} = \gamma v/g, \tag{23}$$

where the proper acceleration g has been assumed to be uniform with respect to the earth frame. In the present analysis this term does not contribute since we consider the abrupt turn-around scenario where t_{A2} tends to zero as $g \to \infty$; however for the time being we keep it. Therefore the total time elapsed in S for the entire journey is given by

$$T_A = 2L_A/v + 2t_{A2}. \tag{24}$$

Now we compute this time as measured in B's clock by taking the time dilation effect from the point of view of A. For phase 1 this time t_{B1} may be computed as

$$t_{B1} = \gamma^{-1} t_{A1} = \gamma^{-1} L_A/v, \tag{25}$$

where we have applied the simple time dilation formula. For phase 2 however this time dilation formula is differentially true as the speed is not a constant i.e., one may write

$$dt_{B2} = (1 - v^2/c^2)^{1/2} dt_{A2} = (1 - v^2/c^2)^{1/2} d(\gamma v)/g. \tag{26}$$

Hence, after integration, one obtains [29]

$$t_{B2} = \frac{c}{2g} ln \left(\frac{1 + v/c}{1 - v/c} \right). \tag{27}$$

However once again this tends to zero as $g \to \infty$. In any case we shall however not need this expression any more. Therefore the total elapsed time measured in B's clock for the complete journey is given by

$$T_B = 2\gamma^{-1} L_A/v + 2t_{B2}. \tag{28}$$

The differential aging from the point of view of A is thus

$$\delta T_A = T_A - T_B = (2L_A/v)(1 - \gamma^{-1}) + 2(t_{A2} - t_{B2}). \tag{29}$$

From the point of view of B the stay-at-home observer A is moving in the opposite direction and as before one may divide the relative motion of A into two phases, phase I and phase II, where the later corresponds to the acceleration phase. The phase II may be interpreted as turning on of a gravitational field. When this field is switched off (marking the end of the acceleration phase), the phase I starts where the stay-at-home observer A moves with a velocity $-v$ up to a distance L_B which on account of the Lorentz contraction of L_A is given by,

$$L_B = L_A(1 - v^2/c^2)^{\frac{1}{2}}, \tag{30}$$

and the corresponding elapsed time t_{BI} is given by,

$$t_{BI} = \frac{L_B}{v} = \gamma^{-1} L_A/v. \tag{31}$$

This obviously comes out to be the same as t_{B1} since the result is obtained from considerations with respect to the inertial observer A.

Similarly t_{BII} i.e. B-clock's time during phase II should be the same as t_{B2} during which the gravitational field is turned on, i.e.,

$$t_{BII} = t_{B2}, \tag{32}$$

and hence the total time

$$\tau_B = 2t_{BI} + 2t_{BII} = 2\gamma^{-1}L_A/v + 2t_{BII} = T_B. \tag{33}$$

The corresponding time of A's clock by taking into account the time dilation effect is

$$t_{AI} = \gamma^{-1}t_{BI} = \gamma^{-2}L_A/v. \tag{34}$$

Writing A-clock's time during phase II from B's perspective as t_{AII}, one may write for A's clock time for the entire journey as

$$\tau_A = 2t_{AI} + 2t_{AII} = 2\gamma^{-2}L_A/v + 2t_{AII}. \tag{35}$$

The difference of these times of clocks A and B as interpreted by the observer B, is given by

$$\delta T_B = \tau_A - \tau_B = (2\gamma^{-1}L_A/v)(\gamma^{-1} - 1) + 2(t_{AII} - t_{BII}). \tag{36}$$

Note that at the moment we do not know the value of t_{AII}, since it refers to the time measured by A as interpreted by B when it is in its acceleration phase. The paradox is resolved if

$$\delta T_A = \delta T_B. \tag{37}$$

In other words, using Eqs. (29) and (36), one is required to have,

$$t_{AII} = (L_A/v)(1 - \gamma^{-2}) + t_{A2} = L_A v/c^2 + t_{A2}. \tag{38}$$

In the abrupt turn-around scenario, as we have already observed $t_{A2} = 0$, one therefore must have

$$t_{AII} = L_A v/c^2 = \gamma L_B v/c^2. \tag{39}$$

The resolution of the twin paradox therefore lies in accounting for this term. It is interesting to note that the term is independent of the acceleration in phase II. This is possibly the implicit reason why the role of acceleration in the explanation of the twin paradox is often criticized in the literature. However we shall now see how, we can interpret this term as an effect of the direction reversing acceleration (or the pseudo-gravity) experienced by the traveller twin.

Now recall the Boughn-effect of temporal offset between two identically accelerated observers. To be specific, consider an inertial frame of reference S attached to the observer B when it is in the uniform motion phase (phase I). Suppose now there is another observer B' at rest in S at a distance L_B behind B and both of them get identical deceleration and eventually come to rest with respect to A in the frame of reference S', which is moving with velocity $-v$ in the x-direction with respect to S. According to Boughn-effect then the clocks of these two observers get desynchronized and the amount of this desynchronization is given by the expression (4) only with the sign changed, that means

$$desync = \gamma v L_B/c^2, \tag{40}$$

which is nothing but t_{AII}. It has already been pointed out that this Boughn-effect may be interpreted as the effect of pseudo-gravity (in this case as experienced by the observer B) according to EP. In terms of the pseudo acceleration due to gravity the above expression can also be obtained as

$$desync = g\Delta t_B L_B/c^2. \tag{41}$$

Note that $g\Delta t_B$ is finite (equal to γv) even if $g \to \infty$.

The observer B' which is L_B distance away from B is spatially coincident with A, hence, in calculating the clock time of A from B's perspective this time-offset due to Boughn-effect must be taken into account. This effect is ignored when the twin paradox is posed by näively asserting the reciprocal time dilation effect for the stay-at-home and the rocket-bound observers. Clearly the paradox is resolved if the Boughn-effect or the pseudo gravitational effect is taken into consideration.

6. CONCLUDING REMARKS: TEST OF BOUGHN-EFFECT

We have seen that the Boughn-effect can be interpreted as the integrated effect of GSDC. The experimental test of GSDC or the gravitational red-shift is therefore a test of a differential Boughn-effect in a way. On the contrary one may directly measure the integrated effect by the following means:

First two atomic clocks may be compared (synchronized) at the sea level, then one of the clocks may be slowly transported to a hill station of altitude h and then kept there for some time T. In this time these two atomic clocks according to Boughn scenario are perpetually accelerated from a rest frame S to a hypothetical inertial frame S' moving with velocity v, with proper acceleration g so that $\gamma v = gT$. Boughn-effect therefore predicts a temporal offset (see Eqs.(40) and (41)),

$$\Delta t_{\text{offset}} = ghT/c^2. \tag{42}$$

This offset can be checked by bringing the hill station clock down and then comparing its time with the sea level one. Any error introduced in the measurement due to transport of clocks can be made arbitrarily small compared to Δt_{offset} by increasing T. As a realistic example for h =7000ft (altitude of a typical hill station in India), and T =1 year and taking the average g to be about $9.8m/sec^2$, the Boughn-effect comes out to be in the micro-second order:

$$\Delta t_{\text{offset}} = 7.3\mu s, \tag{43}$$

which is easily measurable without requiring sophisticated equipments, such as those used in Pound-Rebka type experiments.

It is interesting to note that from the empirical point of view the effect is not entirely unknown. For example Rindler [16], in seeking to cite an evidence for the GSDC effect, remarks: "Indeed, owing to this effect, the US standard atomic clock kept since 1969 at the National Bureau of standards at Boulder, Colorado, at an altitude of 5400ft; gains about five microseconds each year relative to a similar clock kept at the Royal Greenwich Observatory, England, ..." However one can consciously undertake the project with all seriousness, for the accurate determination of the time-offset (with the error bars and all that), not merely to prove GSDC but to verify the Boughn-effect of SR. It is worth while to note that the empirical verification of this time-offset as a function of T would not only test the Boughn-effect and the integral effect of GSDC but it would also provide empirical support for the relativity of simultaneity[6] of SR. So far no experimental test has been claimed to be the one verifying the relativity of simultaneity. Indeed SR is applicable in the weak gravity condition of the earth so that gravity can be thought of as a field operating in the flat (Minkowskian) background of the spacetime [30]. Clearly because of EP, the earth with its weak gravity has the ability to provide a convenient Laboratory to test some special relativistic effects like the relativity of simultaneity or the Boughn-effect.

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[6]In the light of the C-S thesis, however, "relativity of simultaneity" loses its absolute meaning, since for example if absolute synchrony is used, there is no lack of synchrony between two spatially separated events as observed from different inertial frames, however, the differential aging or the temporal offset will pop up as a time dilation effect in the absolute synchrony set-up when the clocks are brought together by slow transport. The details of this issue is a subject matter of another paper by the authors in preparation.

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