# Chapter 5

# Demystifying Twin Paradox

## 5.1 Introduction

As discussed earlier (in Sec.(1.2)) one of the most interesting as well as perplexing paradox of relativity is the twin paradox. Einstein in his 1905 paper[1] on special relativity (SR) first predicted the time dilation effect of moving clocks. From Lorentz transformation (LT) between any two inertial frames in the relativistic world he concluded that a clock $B$ initially synchronized with another stationary clock $A$ at one spatial point, after making a round-trip should fall out of step with the stationary one. He termed the effect as a "peculiar consequence" [2]. Replacing the clocks by human observers, Langevin[3] posed a thought experiment in a problematic form in which a twin $B$ (say) sets off for a space voyage to a distant star with a speed almost equal to the speed of light, and after a brief period, returns to earth at the same speed to meet her stay-at-home brother $A$ (say) to discover that at the end of the trip she is younger than the earth-bound sibling. This counter-intuitive or peculiar result was given the name "twin paradox". This observation is peculiar from the point of view of absolute time since for a common man who believes that time is absolute, any difference between two clocks (initially synchronized) is itself a paradox in a sense that it is unexpected. This definition of the twin paradox focus on the counter-intuitive feature (differential aging) of the problem. Although counter-intuitive both Einstein and Langevin did not see any paradox in this "peculiar consequence" of SR, they recognized early that the situations for the rocket-bound and the stay-at-home twins were not symmetrical and any expectation or claim of symmetrical outcome regarding their ages itself was erroneous. Hence there was no paradox. In fact authors of repute are often found to dismiss the paradox by pointing out that with respect to the inertial frame of the stay-at-home twin, the world lines of the twins in the Minskowski diagram are different and hence the asymmetry in the aging can be attributed to the fact that proper time is not integrable[4].

The other aspect of the twin paradox deals with the idea that the biased result of the twin problem (the stay-at-home twin aged more than the traveller) as if violates the principle of "relativity of motion". For example Dingle[5] in 1957 stated that Einstein made a "regrettable error" and he argued that "According to the postulates of SR, if two identical clocks separate and reunite, there is no observable phenomenon that will show in absolute sense that one rather than other moved. If the postulates of relativity is true, the clocks must be retarded equally or not at all, their readings will agree on reunion if they agreed at separation...". It is thus believed that the principle of relativity demands the ages of the twins to match at the reunion. This aspect of the paradox is a trivial one since the principle of relativity states equivalence of inertial frames of reference only. The principle should not be construed as a statement that all frames of reference are equivalent. In the twin problem the traveller experiences some acceleration at turn-around by virtue of which he lies in a non-inertial frame and hence the principle of relativity of (inertial) motion is not applicable to his frame, thus removing the paradoxical element from the problem.

The above mentioned aspects of the twin problem (first one dealing with the counter-intuitiveness and the second regarding the violation of the principle of relativity of motion) are based on the prediction of the differential aging from the perspective of the stay-at-home twin only. But according to SR one can näively argue that kinematically $A$ is also making a round-trip from the perspective of $B$ and since Lorentz transformation (LT) predicts reciprocal time dilation of moving clocks, clock of $A$ should run slower than that of $B$, hence $B$ can also claim that $A$ should be younger at their reunion. This again generates the "twin paradox" since each of the twin cannot be younger than the other. The third aspect of the twin problem concerns this logical contradiction between the predictions of the twins when the differential aging is calculated from the perspectives of both the twins. This is the most perplexing one. In the present paper we shall focus on resolving this facet of the problem concerning the logical contradiction rather than the counter-intuitive feature of the issue.

The logical contradiction as discussed above comes from the reciprocity of time dilation effect predicted by LT, which is again a consequence of Einstein's scheme of synchronizing clocks (commonly known as standard synchrony), according to which the one-way-speed (OWS) of light is *stipulated* to be equal to its round trip speed. Indeed convention other than that suggested by Einstein can also be used in synchronizing the distant clocks. That other synchronization conventions can be adopted resulting in the possibility of a plethora of relativistic transformations is known as the conventionality of distant simultaneity thesis (CS-thesis)[1] of SR.

The CS-thesis asserts that Einstein's convention is just one among the various possible choices of the OWS of light. This particular convention leads to a set of transformation equations in the relativistic world which are known as the Lorentz transformation (LT). Different choices for the values of the OWS yield different sets of transformation equations with varied structural features. In recent years a new approach based on conventionality of simultaneity to understanding paradoxes in relativity has been found fruitful. In context of the twin paradox Debs and Redhead[4] have shown that the CS approach provides a means to put an end to the question as to where and when the differential aging takes place in the problem.

Although there is no inherent logical contradiction in SR, an apparent one comes

[1] See for example[4, 6, 7]. For a more comprehensive review of the thesis see a recent paper by Anderson, Vetharaniam and Stedman[8].

about because of the particular synchronization scheme adopted by Einstein which is responsible for the reciprocity of length contraction and time dilation effects. One can thus claim that at the heart of the twin paradox (or the apparent logical contradiction) lies the problem of synchronization of distant clocks[9].

In relativity theory the true effects of relativity and that of synchronization convention are mixed up. The role of each of these in causing the twin paradox can be understood if they can be separated. The function of the synchronization convention issue in the twin paradox problem can be best exemplified if one can introduce Einstein's convention (playfully) in the classical (Galilean) world and pose a suitable twin paradox there. In this way the synchronization effect will be delinked from the relativistic ones (since the latter does not exist in the classical world). But can such a paradox exist in the classical world too? The initial reaction would be to answer in the negative since nothing seems to be mysterious or enigmatic in this world and as is well-known, counter-intuitive problems by contrast exist in the relativity theory probably because of its new physical imports. However the answer is in affirmative. One of the so-called new philosophical imports of SR is the notion of relativity of simultaneity. It can be shown that this notion can also be introduced in the Galilean world. In the next section we will provide the transformation equations obtained by incorporating the ingredients of Einstein's synchrony in the classical world. In Sec.5.3 it will be shown that by doing so the paradox can be artificially created even in this world where normally one would not expect it to exist. The perspective of the paradox will hopefully provide deeper understanding of the twin problem and other related issues.

## 5.2 Einstein's Synchronization in the Classical World and Zahar Transformation

Classical world is a kinematical world endowed with a preferred frame of ether $\Sigma_0$ where light propagation is isotropic and the relativistic effects like length contraction of moving rods and time dilation effect of moving clocks are absent. However in the frames other than the preferred frame the two-way-speed (TWS) of light is different in different directions as is expected in the classical world. The transformation equation generally used in this world is the Galilean transformation (GT):indexGalilean Transformation

$$x = x_0 - \beta t_0, y = y_0, t = t_0. \tag{5.1}$$

Using GT one can show that the TWS of light $\overleftrightarrow{c}$ in any other frame $\Sigma$ along any direction $\theta$ with respect to the $x$-axis (direction of relative velocity between $\Sigma_0$ and $\Sigma$) is given as

$$\overleftrightarrow{c}\,(\theta) = \frac{c(1-\beta^2)}{(1-\beta \sin^2\theta)^{1/2}}. \tag{5.2}$$

Hence according to GT, the TWS of light depends upon the direction and it is not equal to the OWS. For example, along the $x$-axis the OWS is $c - v$ and $c + v$ respectively along the positive and negative $x$-directions, where as the TWS i. e. the average round-trip speed of light along the $x$-direction is $c(1 - v^2/c^2)$. However, in a playful spirit one may choose to synchronize the clocks in $\Sigma$ such that the one way speeds, to and fro, along a given direction $\theta$ are the same as $\overleftrightarrow{c(\theta)}$. One may thus mimick the Einstein synchronization scheme to describe the kinematics in this world. The synchrony is somewhat an awkward one in this world but none can prevent one in adopting such a method. For this synchrony GT changes to the following transformations

$$x = (x_0 - vt_0),$$
$$t = {\gamma_v}^2(t_0 - vx_0/c^2). \tag{5.3}$$

where

$$\gamma_v = (1 - v^2/c^2)^{-1/2}. \tag{5.4}$$

is the usual Lorentz factor.

These equations were first obtained by E. Zahar and are therefore known as the Zahar transformations (ZT)[6, 7, 10, 11]. The transformations have been successful in elucidating some recently posed counter-intuitive problems in SR [12, 13]. The presence of the phase term and $\gamma_v^2$ in Eq.(5.3) distinguishes the ZT from GT. The appearance of these terms is thus obviously just an artifact of the adopted synchrony. The corresponding inverse can be calculated as

$$x_0 = {\gamma_v}^2 x + vt,$$
$$t_0 = {\gamma_v}^2 vx/c^2 + t. \tag{5.5}$$

Note that with respect to $\Sigma_0$, as expected, the moving rods do not contract and clocks in motion do not run slow, but both the effects apparently exist relative to any other frame $\Sigma$. The effect of Einstein synchrony is thus manifested through the existence of apparent time dilation and length contraction effects as observed from an arbitrary reference frame $\Sigma$.

From ZT (Eq.(5.3)) between the preferred frame $\Sigma_0$ and an arbitrary frame $\Sigma$, one can obtain the general ZT connecting any two inertial frames $\Sigma_i$ and $\Sigma_k$ as

$$x_i = {\gamma_k}^2(1 - \frac{v_i v_k}{c^2})x_k - (v_i - v_k)t_k, \tag{5.6}$$

$$t_i = {\gamma_i}^2[(1 - \frac{v_i v_k}{c^2})t_k - \frac{\gamma_k^2}{c^2}(v_i - v_k)x_k], \tag{5.7}$$

where the suffixes $i$ and $k$ of coordinates $x$, $t$ and $v$ refer to the coordinates in $\Sigma_i$ and $\Sigma_k$ and velocities of the concerned frames with respect to $\Sigma_0$ respectively. Also $\gamma_i = (1 - \frac{v_i^2}{c^2})^{-1/2}$ and $\gamma_k = (1 - \frac{v_k^2}{c^2})^{-1/2}$.

Clearly a clock stationary with respect to $\Sigma_k$ will suffer a time "dilation" according to

$$\Delta t_i = \frac{1 - v_i v_k/c^2}{1 - v_i^2/c^2} \Delta t_k, \tag{5.8}$$

where $\Delta t_k$ refers to the proper time between two events at the same point of $\Sigma_k$ and $\Delta t_i$ is the corresponding time measured by observers in $\Sigma_i$.

Similarly the space transformation (Eq.(5.6)) gives the length contraction formula (LCF) with respect to $\Sigma_k$ , which is

$$\Delta x_k = \frac{1 - v_k^2/c^2}{1 - v_i v_k/c^2} \Delta x_i. \tag{5.9}$$

Hence any Einstein synchronized reference frames exhibit both time dilation and length contraction effects even in the classical world. So one can say that these effects are just the outcomes of the adopted synchronization convention.

ZT thus represents the Galilean world with Einstein (standard) synchrony. Having obtained the transformation laws for this Einstein synchronized classical world, we are now in a position to pose the paradox.

## 5.3 The Paradox of the Twins

As mentioned in the previous section, if in the classical world the distant clocks in a frame $\Sigma$ are synchronized according to Einstein's method, then GT goes over to ZT which predicts both time dilation and length contraction from the arbitrary frame of reference $\Sigma$. (With respect to the preferred frame however there are no such effects). Now if the earth-bound twin ($A$) of the twin parable is considered to be at rest in the preferred frame $\Sigma_0$ and the rocket-bound one ($B$) in a frame $\Sigma$ then $A$ does not detect any time dilation or length contraction effects. However, $B$ observes the both for a clock or a rod stationary in $\Sigma_0$ (frame of $A$). This leads to an apparently paradoxical situation—no time dilation with respect to the observer $A$ brings him to the conclusion that after the round-trip is over there should not be any differential aging, whereas $B$, with respect to whom $A$ is making the round-trip, contradicts the conclusion and claims that due to time dilation, $A$ should be younger at their reunion. This disagreement between the predictions of the twins gives rise to a twin paradox (in the sense of logical contradiction) in the classical world! Note that the second aspect of the twin paradox is not relevant here since we have already started with a preferred (ether) frame.

One needs to note here that this paradox is different from the well-known twin paradox of the relativistic world in a sense that the predictions of the twins are different in the two cases. In the relativistic world both the twins, due to reciprocal time dilation, argue that the other is the younger one. On the other hand in the classical world only the rocket-bound sibling predicts that her brother is younger, whereas the stay-at-home one observes no differential aging.

The resolution of the ordinary twin paradox should involve demonstration of equal differential aging (zero differential aging in the classical world) from the perspectives of both the twins $A$ and $B$. In posing the paradox twins are allowed to compute the ages of their counterparts by taking into account the time dilation effect only. The time dilation formula is used freely from the perspectives of both the twins and the question of applicability of the formulas with respect to the traveller twin who changes inertial frames (because of her turn around) is ignored. Indeed while the time dilation formula is correct within one inertial frame of the stay-at-home twin $A$, the same formula does not hold with respect to the non-inertial frame attached to $B$. In the abrupt turn-around scenario however, the formula is valid separately in the inertial frames of $B$ in its outward and return journeys. But change of inertial frames by $B$ produces another effect linked with the "relativity of simultaneity" of

SR, in which the key to the resolution of the paradox lies. What happens when a change of reference frame takes place is best exemplified in an article " Case of identically accelerated twins" by S. P. Boughn[9]. In the paper the author has presented a variation of the twin paradox where twins undergo equal accelerations for the same length of time, yet they age differently. He has posed the paradox in the context of SR. In the next section we will recast Boughn's paradox in classical world. It will be shown in Sec.5.5 how Boughn's paradox can be used to resolve the twin paradox posed in the classical world.

## 5.4 Boughn's Paradox in the Classical World

In the above mentioned article[9], a variation on the twin paradox is presented where two twins $P$ and $Q$, initially at rest, at a distance $L$ apart, in an inertial frame $\Sigma_0$, on board two identical rockets (with equal amount of fuel), get equal accelerations for some time in the direction $\overrightarrow{PQ}$ ($x$-direction say) and eventually come to rest (when all their fuels had expended) in a new inertial frame $\Sigma$ moving with a constant velocity $v$ with respect to the former along the positive $x$-direction. From the simple application of LT Boughn obtained an unexpected result that in the new abode the ages of the twins differed—the age of $Q$ turned out to be more than that of $P$. This result is really surprising owing to the fact that the twins throughout had identical local experiences, yet their presynchronized (biological) clocks went out of synchrony! Quantitatively this time-offset or desynchronization is given as

$$\delta t_{desync} = -\gamma_v v L/c^2. \tag{5.10}$$

Hence the two synchronized clocks (twins) separated by a distance $L$ get unsynchronized by an amount $\delta t_{desync}$ when they settle stationary in their new frame.

This desynchronization is real in the sense that it refers to desynchronization in relation to the Einstein convention of synchronization [2]. Let us call this departure from Einstein synchrony as Boughn effect (BE).

The apparently paradoxical result that the spatially separated twins, in spite of their identical history of acceleration age differently is readily explained if one notes that the difference between spatially separated (biological) clocks or the change of relative synchronization cannot have any unequivocal meaning; the clocks can be compared unambiguously only when they are in spatial coincidence. For instance in $\Sigma$, one of the observers can slowly walk towards the other (or both of them can do the walking) and compare their ages (or their clock readings) when they meet. One may therefore define the "reality" of the temporal offset effect due to Boughn provided the clocks are finally compared when they are brought together. Since in the relativistic world the so called "slow transport synchronization" is equivalent to the Einstein synchronization[15], the calculated differential aging or time-offset between their clocks when they were in spatial separation would continue to hold even when the twins meet after their slow walk. BE is thus a real effect (according to the definition) in the relativistic world. However when the twins meet after the slow walk it can easily be seen[16] that they do not have symmetrical experiences, and hence the paradox gets resolved.

In the classical world the situation is different. Below we calculate this temporal offset effect in the classical world by applying ZT between the preferred frame $\Sigma_0$

[2] Incidentally Selleri[14] has noted in different words in this context that after identical acceleration, the two clocks readings define a natural (absolute) synchronization, which is different from Einstein's synchrony; the latter can only be established by resynchronizing them artificially (see also[13]).

and an arbitrary frame $\Sigma$:

$$x_k = (x_{k0} - vt_{k0}),$$
$$t_k = {\gamma_v}^2(t_{k0} - vx_{k0}/c^2). \tag{5.11}$$

where $t_{k0}$ and $x_{k0}$ denote the time and space coordinates of the observer $k$ ($k$ stands for $P$ or $Q$) with respect to $\Sigma_0$ and $t_k$ and $x_k$ refers to the corresponding coordinates in the new frame $\Sigma$.

From the time transformation of Eq.(5.11), we obtain the time difference for the observers $P$ and $Q$ in their new frame as,

$$t_Q - t_P = \gamma_v^2[(t_{Q0} - t_{P0}) - v(x_{Q0} - x_{P0})/c^2]. \tag{5.12}$$

Noting that the distance between the twins i.e $x_{Q0} - x_{P0} = x_0$ remained constant throughout their journey and also assuming that their clocks were initially synchronized i.e assuming $t_{Q0} - t_{P0} = 0$ (since the relative clock readings of $P$ and $Q$ should not change with respect to $\Sigma_0$ as they get identical acceleration for equal amount of time) the time offset $\delta t_{desync} = t_Q - t_P$ between the clocks of the siblings can be calculated as

$$\delta t_{desync} = t_Q - t_P = -\gamma_v^2 v x_0/c^2. \tag{5.13}$$

In the relativistic world if one proceeds in the same manner using LT, then one can easily obtain the desynchronization effect given by Eq.(5.10).

In the relativistic world if one clock is taken towards the other, the time dilation effect is found to depend upon the speed of clock transport and in particular the value goes to zero when the speed is vanishingly small. On the other hand in the classical world, from the calculation of clock transport[3], it can be shown that the integrated effect of time dilation is independent of the speed at which it is transported and the most interesting part is the result is exactly the same as the

[3]These calculations were done in detail in the previous chapter

Boughn's time offset effect between the two clocks with a negative sign. Hence if the observer $P$ walk towards $Q$ (no matter whether slow or fast), the above two effects nullify each other or the net offset between them drop to zero when they meet. In other words, BE vanishes when the clocks are compared at one spatial point. The observation thus demonstrates that although Boughn's paradox can be recast in the classical world, this time offset effect is not real but just an artifact of Einstein synchrony.

Eq.(5.13) gives the Boughn effect between the preferred frame $\Sigma_0$ and an Einstein synchronized frame $\Sigma$ moving with a velocity $v$ with respect to $\Sigma_0$. For future use we will need to obtain the temporal offset between any two arbitrary Einstein synchronized reference frames. This may be calculated as follows:

Consider two such frames $\Sigma$ and $\Sigma'$ moving with velocities $v$ and $u$ respectively with respect to $\Sigma_0$. Writing the transformation equations connecting the space-time coordinates of $\Sigma_0$ and $\Sigma'$ as

$$\begin{aligned} x' &= (x_0 - ut_0), \\ t' &= {\gamma_u}^2(t_0 - ux_0/c^2). \end{aligned} \tag{5.14}$$

one readily obtains transformation equations between $\Sigma$ and $\Sigma'$ as

$$\begin{aligned} x' &= {\gamma_v}^2(1 - uv/c^2)x + (v - u)t, \\ t' &= {\gamma_u}^2[(1 - uv/c^2)t + (v - u){\gamma_v}^2 x/c^2]. \end{aligned} \tag{5.15}$$

Hence, as discussed earlier the twins $P$ and $Q$ (separated by a length $L$ in $\Sigma$) arrives in their new frame $\Sigma'$ producing a temporal offset (desynchronization) between their clocks which is given by (obtained from transformation for time in Eq.(5.15))

$$\delta t'_{desync} = t'_Q - t'_P = {\gamma_u}^2{\gamma_v}^2(v - u)x/c^2, \tag{5.16}$$

Eq.(5.16) thus gives the synchronization gap between any two Einstein synchronized reference frames in the classical world.

In the classical world as we have seen this effect is not real, yet BE can be used successfully to provide an accurate account of differential aging from the perspective of the traveller twin.

## 5.5 Resolution

To analyze the situation let us discuss in detail the whole chain of events from the perspectives of both the twins. For convenience we remove the inconsequential initial and final accelerations from the problem, thus assuming that $B$ makes a flying start and also after the return trip she flies past $A$. So, the only acceleration in our problem is the turn-around acceleration of $B$ which is essential for the final comparison of the clocks (or ages) of the twins at one spatial point after the trip. We show below step by step how the twins make unequivocal predictions regarding their ages.

*Step 1:*

The time dilation formulas from the perspectives of both the twins can be symbolically written as

$$TDF1: \qquad \Delta\tau_B(A) = \Delta t_A(A), \tag{5.17}$$

$$TDF2: \qquad \Delta\tau_A(B) = \gamma_v^{-2}\Delta t_B(B). \tag{5.18}$$

While writing the above equations we have adopted a notation scheme, where $\Delta\tau_B(A)$ [$\Delta\tau_A(B)$] denotes the clock reading of the twin $B$ [$A$] for a time interval between two events which occurred at its position as inferred by the other twin $A$ [$B$] drawn from its own coordinate clocks' records for the interval, $\Delta t_A(A)$ [$\Delta t_B(B)$] and its knowledge of the relevant time dilation effect. Indeed the time intervals $\Delta\tau_B(A)$

or $\Delta\tau_A(B)$ are based on one clock measurements and hence they refer to proper times of $B$ and $A$ respectively. On the other hand $\Delta t_B(B)$ refers to in general, the observed difference in readings (for the same two events) recorded in two spatially separated (synchronized) clocks stationary with respect to the frame of reference attached to $B$. However for the round trip of an object or a clock ($A$ say), the time $\Delta t_B(B)$ is also measured by a single clock ($B$). Although $\tau$-symbol would have been more appropriate in the later case but we shall continue to use the symbol '$t$' to emphasize that the corresponding time is supposed to be the coordinate time.

The length contraction formulas (LCF) are similarly obtained as

$$LCF1: \qquad L_B(A) = L_B(B), \tag{5.19}$$

$$LCF2: \qquad L_A(B) = \gamma_v^{-2} L_A(A). \tag{5.20}$$

Where $L_A(A)$ and $L_B(B)$ are the rest lengths of rods in $\Sigma_0$ and $\Sigma$ respectively and $L_B(A)$ and $L_A(B)$ are the corresponding observed lengths from the other frames ($A$ and $B$ respectively).

Now, in our problem the only distance of interest is that of the distant star from $A$ which is clearly a rest length in $A$ i.e. $L_A(A) = L_0$(say). So the relevant LCF is

$$LCF: \qquad L = \gamma_v^{-2} L_0, \tag{5.21}$$

where $L = L_A(B)$ is the distance of the star measured in $\Sigma$-frame.

*Perspective of A:*

*Step 2:*

Time recorded in $A$-clock for $B$'s journey of distance $2L_0$ is

$$\Delta t_A(A) = 2L_0/v, \tag{5.22}$$

The corresponding time in $B$-clock taking into account TDF1 (Eq.(5.17)) is

$$\Delta\tau_B(A) = \Delta t_A(A) = 2L_0/v. \tag{5.23}$$

*Step 3:*

Differential aging with respect to $A$ is therefore given by

$$\delta t(A) = \Delta t_A(A) - \Delta\tau_B(A) = 0. \tag{5.24}$$

So, $A$ concludes that *both the siblings age the same at their reunion.*

*Perspective of B:*

*Step 4:*

From $B$'s point of view, $A$ makes the round trip with a velocity

$$u = -\gamma_v^{-2} v \tag{5.25}$$

which is obtained from the velocity transformation using ZT.

*Step 5:*

The time elapsed in $B$-clock for this round trip is therefore

$$\Delta t_B(B) = 2L/\left|u\right| = 2\gamma_v^{-2} L_0/\gamma_v^{-2} v = 2L_0/v. \tag{5.26}$$

This is nothing but the $B$-clock time as calculated by $A$ i.e, $\Delta\tau_B(A)$.

*Step 6:*

The same time interval in $A$-clock as calculated by $B$ by the *näive* application of TDF2 (Eq.(5.18) alone on $\Delta t_B(B)$) is obtained as,

$$\Delta\bar{\tau}_A(B) = \gamma_v^{-2} 2L_0/v. \tag{5.27}$$

We have put a bar sign on $\tau$ since the time computed above is erroneous as the direct application of TDF2 on $\Delta t_B(B)$ for the interpretation of $A$-clock time is incorrect because there is a question of desynchronization of distant clocks due to BE which has to be taken into account. Let us clarify this point.

To calculate the time offset due to BE, we first split the frame of reference ($K$) attached to $B$ into two inertial frames $\Sigma$ and $\Sigma'$ which move with velocities $v$ and $-v$ respectively with respect to $\Sigma_0$. Now, just before $B$ decelerates let there be another observer $\bar{A}$ of the same age as that of $B$ (i.e it is assumed that $\bar{A}$'s clock is synchronized with $B$'s in $\Sigma$) and at the same location of $A$ (at a distance $L$ from $B$ with respect to $\Sigma$), comoving with respect to $B$ such that like in Boughn's scenario, $\bar{A}$ and $B$ both undergo the same but arbitrarily large negative acceleration relative to $\Sigma$. From $\Sigma$-frame $\bar{A}$ and $B$ may be considered as Boughn's twins accelerating from rest along the negative $x$-direction (i.e now $\bar{A}$ is forwardly placed with respect to $B$) and settling down finally in the inertial frame $\Sigma'$. BE therefore tells us that with respect to Einstein synchronized clocks in $\Sigma'$, there is a desynchronization effect between the clocks (or ages) of $\bar{A}$ and $B$ which is given by Eq.(5.16) by replacing $u$ by $-v$ as

$$t'_B - t'_{\bar{A}} = \delta t'_{desync} = {\gamma_v}^4 2vL/c^2 = {\gamma_v}^2 2vL_0/c^2, \tag{5.28}$$

The last equality follows from Eq.(5.21).

The above desynchronization due to BE gives the synchronization gap between the two Einstein synchronized reference frames ($\Sigma$ and $\Sigma'$) of the traveller $B$. The twin $B$ during the instantaneous turn-around jumps from one inertial frame $\Sigma$ to the other $\Sigma'$ and because of the desynchronization, the clocks of $\bar{A}$ and $B$ no longer represent the Einstein synchronized coordinate clocks of $\Sigma'$. The presence of the synchronization gap between the instantaneously co moving inertial frames for an accelerated observer restricts one from considering $B$ to remain in a single frame. One can consider $B$ to remain in the single frame $\Sigma$ if the problem is posed such that $B$ instead of turning around would continue to move forward covering the same length of journey with uniform speed as she would do after the turn-around. In that case the coordinate clocks (Einstein synchronized) of $\Sigma$ frame of $B$ could be used to measure the coordinate time and connect the same with the proper time of $A$ through TDF for the entire trip. However in the second leg of $B$'s trip if someone playfully tamper with her synchronization then the coordinate time recorded by $B$ would be erroneous and hence a calculation to obtain the proper time $\tau_A$ of $A$ from this measurement (by applying TDF on it) will give wrong result. The only way to get the correct answer is to first undo the mischief by getting back to the Einstein synchronization that was adopted before and then one is free to use TDF in order to obtain proper time from the coordinate time. Let us now study the scenario when $B$ turns around and immediately jumps to the new inertial frame $\Sigma'$. The synchronization of clocks in the new frame $\Sigma'$ according to Einstein synchrony can be equated with the deliberate alteration of synchronization just discussed in connection with the uniform motion scenario of $B$, since the standard of simultaneity in $\Sigma'$ is thus made different from that in $\Sigma$. It is thus clear that TDF can be used to obtain proper time from the coordinate time provided the latter refers to a *uniform* synchronization. We then ask if there is any way so that one can continue with the standard of simultaneity (synchrony) of $\Sigma$ in $\Sigma'$. The answer is in the affirmative and is provided by Boughn's thought experiment. From the symmetry of the problem it is evident that clocks of $\bar{A}$ and $B$ initially synchronized in $\Sigma$ continue to remain synchronized with respect to $\Sigma$ even when they arrive stationary in $\Sigma'$ after the turn-around acceleration. While calculating from the perspective of $B$, for the round trip of $A$, the time in $B$-clock, $\Delta t_B(B)$ can be easily obtained but one cannot use this to obtain the proper time of $A$, as this does not represent the coordinate time of $B$ in $\Sigma$. Clearly a correction term $\delta t'_{desync}$. is to be added to $\Delta t_B(B)$ in order to obtain the said coordinate time. This correction is equivalent to the process of restoration of the synchronization mentioned in $B$'s non turn-around example. Hence by adding $\delta t'_{desync}$ to $\Delta t_B(B)$ we can carry over the standard of synchronization of coordinate clocks in $\Sigma$ to the new frame $\Sigma'$ and then TDF2 can be correctly used to determine the proper time of $A$.

Therefore the true coordinate time is obtained as

$$\Delta t_B^{coord}(B) = \Delta t_B(B) + \delta t'_{desync} = 2L_0/v + \gamma_v^2 2vL_0/c^2. \tag{5.29}$$

Now applying TDF2 on the true coordinate time $\Delta t_B^{coord}(B)$, $B$ calculates the round-trip time (proper) measured in $A$-clock as

$$\Delta\tau_A(B) = \gamma_v^{-2}\Delta t_B^{coord}(B) = \gamma_v^{-2}(2L_0/v + \gamma_v^2 2vL_0/c^2) = 2L_0/v. \tag{5.30}$$

*Step 7:*

Thus the differential aging from the perspective of $B$ turns out to be,

$$\delta t(B) = \Delta\tau_A(B) - \Delta t_B(B) = 0, \tag{5.31}$$

which agrees with Eq.(5.28), thus resolving the paradox.

## 5.6 Discussion

We have observed that a twin paradox in the classical world can be posed and resolved by using Boughn's paradox which can also be recast in this world. In the classical world when the clocks are synchronized using absolute synchrony, the paradox of the twins does not exist whereas in the same world if the synchronization is performed following Einstein synchrony, the paradox can be posed as in SR. The present treatment proves that the root of the twin problem lies in the choice of Einstein synchronization convention. Indeed the role of Einstein synchrony in this problem is revealed in a rather direct way as the "true" relativistic effects (non-conventional ingredients of SR) are here deliberately suppressed by the consideration of the classical world. The present analysis of the twin paradox clearly explains how the traveller twin makes an error in assessing the coordinate time of the earth bound twin because of the former's turn around acceleration.

Clearly the reader may use the present analysis as a template for resolving the ordinary twin paradox with much confidence fully in the context of special relativity.

It may be further noted that there has been a long standing debate as to whether twin paradox can be explained fully in the context of special relativity or not. Many authors believe that introduction of general relativity and a gravitational field at the point of acceleration is required to deal with the paradox[4, 17]. One may now wonder where does the general relativity fit in while dealing with the twin paradox in the classical world!

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