Chapter 1

INTRODUCTION

1.1 Introduction

The last three decades witnessed a spurt in activities world wide in understanding the overlapping fields of particle physics and cosmology. The success of the unified theory of weak and electromagnetic interactions proposed by Salam and Weinberg inspired the development of Grand unified theories (in short, GUT) in which strong interaction is also included in the unification scheme. The elementary particle physics has entered a very interesting stage with the development of unified theories. The successes of superstring theories have raised the hope of unifying gravitational interaction along with the other fundamental interactions in nature, namely electroweak and strong interactions. It has been predicted that such a unification may occur at an energy scale $M_P \sim 10^{19}$ GeV (Planck scale). This energy scale is huge which is far from that is attainable in the man made laboratory. The standard cosmological models suggest that such an energy scale perhaps was available shortly after the creation of the universe. The early universe, therefore, may be considered as a unique physical laboratory for testing the new theories of particle interactions. There has also been considerable progress in the understanding of the cosmological models of the early universe. A cosmological model is a mathematical description of the observed universe which is concerned with the fundamental queries on creation, evolution, large scale structure formation of the universe. The standard cosmological model is based on Einstein's General theory of Relativity (in short, GTR) with perfect fluid assumption. In GTR, the Einstein's field equation determines the evolution of the universe filled with matter or radiation. In 1917, Einstein wanted to build a static model of the universe using his new gravitational theory as those days observational results were limited to our Galaxy only, but failed to obtain a static universe. In order to accommodate a static universe Einstein modified the field equation introducing a repulsive term, known as the cosmological constant (Λ). However, Hubble's discovery made it clear that the universe is not static but expanding. Theoretically, existence of a nonstatic expanding universe solution was obtained by de Sitter, Friedmann and Lemaitre independently before Hubble's discovery without a cosmological constant. Einstein uttered that introduction of a cosmological constant (Λ) in the field equation was a great mistake once he realized that the universe is expanding. However, in the last couple of decades there has been a growing interest to include a cosmological constant term again and again in order to describe cosmological observational evidences. The Hubble's discovery admits an expanding universe which is in favour of the big-bang model of the universe. According to the Big Bang model, the Universe originated in an extremely dense and hot state. Since then, space itself is expanding with the evolution of time. It has been realized that the standard big-bang model is fairly successful in explaining the 2.7^o K cosmic microwave background radiation (in short, CMBR), the expansion of the universe (Hubble redshift), primordial nucleosynthesis and the cosmic abundances of the light elements *e.g.* Deuteron, Helium (^4He) etc.. In spite of successes, the big-bang model cannot be regarded as a complete theory of the universe because it fails to explain the early universe and unable to address some of the observed features of the universe. In standard cosmology with perfect fluid assumption, the following problems, namely *flatness problem, horizon problem, singularity problem, small scale inhomogeneity problem etc.* have been cropped up when Big bang model extrapolated to probe the early universe and the issues remain unsettled for a long time. More details on these issues can be found in several textbooks [1] and in some earlier reviews [2]. The concepts of recent theories of particle physics are also found to be in conflict when implemented in cosmology. It is, therefore, felt essential to look for a cosmological model which can solve basic problems both in particle physics and cosmology. Some of the outstanding problems can be resolved satisfactorily by advocating the idea of inflation.

Gliner [3] was the first to note that inflationary solution may be obtained in GTR for an appropriate stress tensor corresponding to matter with properties of a vacuum. Later, Starobinsky [4] obtained inflation using trace anomaly in GTR. However, the efficacy of the theory is known only after the seminal work of Guth [5]. Guth employed temperature dependent phase transition mechanism in cosmology and showed how the idea of inflation may be effectively used to construct a viable cosmological model. During inflationary epoch, the vacuum energy density behaves as an effective positive cosmological constant allowing the universe to expand exponentially (or quasi-exponentially), $a(t) \sim e^{\int H\, dt}$ where $H = \frac{\dot{a}}{a}$ represents Hubble parameter. Consequently this expansion in the scale factor allows a small causally coherent region to grow bigger then the observable universe when the exponent $\int H\, dt > 65$. Later the vacuum like state decays and its energy transforms into heat, the universe becomes hot and the scale factor $a(t)$ of the radiation dominated universe grows more slowly like $a(t) \sim \sqrt{t}$. This was the prevailing scenario for a long time. But recently a lot has been changed in understanding the late universe because of observations, which will be discussed later in this thesis. The inflationary universe scenario opens up new avenues in the interface of particle physics and cosmology leading to solutions of some of the outstanding problems both in cosmology and in particle physics. The horizon and the flatness problems get resolved in this framework. The origin of the large scale structure formation of the universe emerged in this scenario with satisfactory explanation. Therefore, inflation is an attractive idea which is essential in building cosmological models of the early universe.

In Guth's model, the universe was trapped in a metastable state (supercooled) which thereafter could decay through the process of bubble nucleation via quantum tunneling. Bubbles of true vacuum spontaneously formed in the sea of false vacua and rapidly begins to expand at the speed of light. Guth later realized that in the model the universe fails to reheat properly. In case inflation last long enough it could solve the initial condition problem satisfactorily, but the collision rate between bubbles in this scenario became exceedingly low. It has been understood that inflationary model based on a first order phase transition, could not provide a satisfactory explanation of how to exit from the inflationary phase without disturbing the good properties of the standard cosmological model [5, 6]. Consequently attempt has been made to solve this *'graceful exit'* problem in a new model which is known as *'new inflation'* [7]. In this model, inflation is obtained using a homogeneous scalar field, which is rolling down a potential energy hill instead of tunneling out of a false vacuum state. When the field rolls down the potential very slowly compared to the expansion rate of the universe, it permits inflation. However, if the hill becomes steeper inflation ends and reheating can occur at the minimum because of the oscillations of the scalar field. It was shown that although new inflation does not produce a perfectly symmetric universe, it sets a tiny quantum fluctuations in the inflaton field. It was shown by Mukhanov and Chibisov [8] that the tiny quantum fluctuations of the scalar field originated during the evolution of the early universe could act as the primordial seeds for all structure formation in the late universe. The recent observation, namely, CMBR data agrees precisely with the theoretical prediction by inflation for structure formation.

Cosmological models are also studied with second order phase transition and found that those are plagued with severe fine-tuning problems [9]. The inflationary models of the early universe including the new inflationary model [7] make use of a phase transition which requires some fine-tuning either in the potential $V(\phi)$ of the inflaton field ϕ or in initial conditions of the universe. Linde [10, 11] in 1983 proposed a new approach to obtain an inflationary universe scenario which apparently needs no specific fine tuning. In the new model, a scalar field is still needed but the initial state of the field is non-thermal. The advantage of the model is that it is not essential to restrict to a particular initial configuration $\phi = 0$ as was required in the *new inflation* model. The homogeneous field instead can take any value provided by a random initial distribution satisfying the constraint $V(\phi) \leq M_P^4$. This constraint is necessary so that the quantum behavior of gravity does not dominate and a classical description of space-time remains valid. Since the initial data for the model are randomly distributed, the scenario is known as chaotic inflationary scenario. Chaotic model is based on a random distribution of the values of the vacuum scalar field (also called inflaton field) ϕ at time $t \sim t_p = M_p^{-1}$, where t_p is the Planck time and M_p, the Planck energy. The inflationary solution can be realized in a semiclassical theory, i.e., describing gravity classically but matter by quantum fields. Consequently the equation of state necessary for inflation with a scalar field is achieved when its potential energy dominates over the kinetic energy density of the universe. During this epoch, the vacuum energy behaves like an effective positive cosmological constant resulting in a huge expansion of the universe in a very short epoch. The scenario can be realized even when ϕ is initially highly fluctuating $\left(\frac{1}{2}\dot{\phi}^2 \gg V(\phi)\right)$ [12]. Papantonopoulos *et al.* [13] have shown that the favourable condition for inflation $\left(\frac{1}{2}\dot{\phi}^2 \ll V(\phi)\right)$, emerges quite naturally if one introduces an axion field ξ which has only derivative coupling to ϕ. Linde assumed that subject to the condition $V(\phi) < M_p^4$ all values of ϕ are equally probable as initial values. The problems of Big bang model can be solved satisfactorily if the observed part of the universe emerges from a region having initially $\phi_o(t = M_p^{-1}) > 3M_p$. The lower bound on ϕ ensures that there is sufficient inflation. It was also shown by Paul *et al.* [14] that chaotic model is fairly general and can be extended even to a universe with initial anisotropy. In the model if the field increases or decreases due to quantum fluctuation then in addition to chaotic inflation another inflationary scenario may be realized in the former case because of moving up the potential by the field due to the effect of the quantum fluctuations, which is known as *eternal chaotic inflation* [15].

Another version of inflation is also found in the literature where the universe expands with a less rapid inflation known as *Power law inflation.* The *Power law inflation* is characterized by a period in which $a(t)$ grows as t^A with $A \gg 1$. An example of this type may be obtained by using an exponential type potential $V(\phi) = V_0\, exp(-\lambda\phi)$ with $\lambda > 0$. The exponential potential is found in the Salam-Sezgin model [16] of $N = 2$ supergravity coupled to matter and in some Kaluza-Klain theories [17]. The causal structure of the space-time in inflationary models differs considerably from that in the standard Big-Bang model. The horizon problem gets resolved as the observable universe in the inflationary scenario had a smaller size compared to a causal horizon volume. Inflation also helps us to resolve the primordial monopole problem that was introduced in cosmology by the grand unified theories (in short GUT). At the end of inflation the density of primordial monopole becomes extremely small and fresh production of primordial monopole may not occur if the reheating temperature is settled below the temperature of GUT symmetry breaking ($T < 10^{14} GeV$). Another important feature of inflation is that it provides a mechanism of small scale density inhomogeneities. The quantum fluctuations generated during inflation lead to the observed density perturbations [18]. The scale invariant Harrison-Zeldovich [19] spectrum emerges naturally leading to an initial seed for galaxy formation. However, such a scale invariance spectrum can not be obtained in *power law inflation* [20] for a finite exponent (A). The *exponential expansion* is, therefore, more attractive.

Cosmological models for the early universe have also been investigated with an imperfect cosmic fluid in GTR [21]. The effective energy-momentum tensor of an imperfect fluid changes in such a way that it leads to a negative pressure accommodating both exponential and power law inflation. Viscosity arises due to various dissipative processes which may play an important role in the evolution of the early universe. The various processes that may responsible for viscosity are, the decoupling of neutrinos during the radiation era, the decoupling of matter from radiation during the recombination era, creation of superstrings during the quantum era, particle collisions involving gravitons, cosmological quantum particle creation processes and during the formation of galaxies [22].

A large number of inflationary models in the context of ever changing fundamental theory have been proposed during the last three decades. The attractive idea of inflation was taken up by cosmologists and the following cosmological models have been emerged during the last 28 years:

• (1980-1989): R^2-inflation [4], Old inflation [5], New inflation [7], Chaotic inflation [10-15], Double inflation [23], Power-law inflation [20], SUGRA inflation [24], Extended inflation [25].

• (1990-1999): Hybrid inflation [26], SUSY D-term inflation [27], Assisted inflation [28], Brane inflation [29].

• (2000-2008): Super-natural inflation [30], K-inflation [31], D3-D7 inflation [32], DBI inflation [33], Racetrack inflation [34], Tachyon inflation [35], Hill top inflation [36], Landscape model [37].

The high precision observations of the *cosmic microwave background* (CMB) temperature fluctuations made by the *Wilkinson Microwave Anisotropy Probe* (WMAP) [38, 39] and other observations may help to select a suitable model of the universe in future. It is expected that the observational constraints, when taken into account, might lead to a more acceptable model of the universe.

There has also been a growing interest in building cosmological models involving dimensions more than the usual four. Kaluza and Klein [40] independently introduced an extra dimension with an aim to unify gravity with electromagnetic interaction. But their idea does not work well because of several technical difficulties. Recently there is a paradigm shift in understanding higher dimensional theories. The investigations seem to lead to a general belief that a consistent theory of quantum gravity cannot be obtained within the framework of point field theories. The advent of the string theory [41] has opened up new and interesting possibilities in this context. String theories may be looked upon as describing the interactions of a few massless and an infinite set of massive states, with masses which are multiples of Planck mass. The superstring theories may be regarded as the low energy regimes of supergravity theories. The striking discovery that in ten dimensions a supergravity theory coupled to Yang-Mills fields with gauge group SO(32) or $E_8\, X\, E_8$ is anomaly free [42] has inspired considerable activities in this area.

Cosmological models mentioned above can explain some of the observations of the universe but are not truly acceptable because of their limitations. Out of the many versions of inflationary models of the early universe proposed in the large volume of literature only a few are considered to be realistic. Further these also have their own merits and demerits. The objective of the thesis is to study some of the cosmological models both in the framework of four and higher dimensional gravity taking into account the recent cosmological and astrophysical results.

The observations from COBE (*Cosmic Background Explorer*) predict that the present universe might have emerged from an early inflating phase in the past and then settles down into the matter dominated phase through the intermediate radiation phase. In recent times an interesting prediction from cosmological observation is that the present universe is passing through an accelerating phase of expansion. It has also been estimated from cosmological observation that 4% matter in the universe is observable, 70% of the universe consists of dark energy and 26% matter is in the form of dark matter [39, 43]. To accommodate the present acceleration of the universe various theories in GTR are proposed considering either (i) a modification of the matter sector or (ii) a modification of the gravitational sector with a new physics. A large volume of literature appeared to address the present accelerating phase by modifying the matter sector of the Einstein field equation other than that of a scalar field *e.g.* chaplygin gas, phantom field, tachyon field etc. called exotic matter field. Moreover, it has been realized that modification of the Einstein-Hilbert action with higher order terms in curvature invariants that are become effective in the high curvature region play an important role in cosmological model building of the very early universe satisfactorily. The modified theory which is also called higher derivative gravity permits inflation in the early epoch [44] and a consistent scenario of early universe may be realized. In analogy it may be important to explore modification of the Einstein gravitational action with terms that might be important at extremely low curvature region to describe the present cosmic acceleration. There are various attempts to explain the cosmic speed up with a modification of the Einstein Hilbert action by considering modification of the gravitational part of the action including quadratic (αR^2) and/or cubic (βR^3) terms, along with a new (δ/R) term, where R is the Ricci scalar and α, β, δ are dimensional parameters. These theories will be taken up here to address present accelerating phase in addition exploring to models of the early universe.

At present there seems to be no alternative to inflationary scenario. But in spite of all the attractive features of cosmological inflation, its mechanism of realization still remains to be *ad hoc.* It is not known when and how the universe has entered the inflationary phase. As inflation operates at Planck's scale, it is interesting to explore cosmological models in the context of string theory which allows such a scale. It is, therefore, not surprising that M/String theory inspired models are under active consideration in cosmology at present. Recently, there is a paradigm shift in higher dimensional cosmology. The recent idea is that the universe is a (3+1) dimensional brane embedded in a higher dimensional spacetime [45] and the usual matter field and force except for gravity are confined on the brane [29, 46]. In this picture all the matter fields are confined to the brane whereas gravity can propagate in the bulk. The scenario has interesting cosmological implications, in particular, the prospects of inflation are enhanced on the brane due to the modifications in the Friedmann equation. While discussing the applications of brane-worlds, one often assumes Einstein gravity in the bulk and then projects the dynamics on to the brane. This leads to the high energy corrections to the Friedmann equation which changes the expansion dynamics in the early universe. In the theory, gravity is regarded as a higher dimensional theory which reduces effectively to a four dimensions at lower energies. Such higher dimensional theories not only resolve some of the problems of the big bang model as discussed earlier but also opens up the possibility of solving the hierarchy problem in particle physics by considering large compactified extra dimensions and making use of string scale accessible to future laboratory experiments [46]. To be in the better spirit with string theory, one should include the higher order curvature invariant terms to the Einstein-Hilbert action, as the term occurs naturally in it.

One typically considers an inflationary phase driven by the potential or vacuum energy of a scalar field, the inflation, whose dynamics is determined by the Klein-Gordon equation [47]. More recently, however, motivated by string theory, other non-standard scalar field actions have been used in cosmology. In k-inflation [31] higher-order scalar kinetic terms in the action can, without the help of the potential, drive an inflationary evolution. In this context, models of quintessence such as k-essence may also resolve the coincidence problem [48]. One particular model of k-inflation which has recently attracted a great deal of attention is tachyon inflation [35]. Tachyon fields are also important as at late times it can account for the density necessary for dark matter.

The thesis contains studies on cosmological models of the early universe both in the usual four and higher dimensions taking into account some of the recent theories discussed above. The probability for quantum creation of an inflationary universe with or without a pair of black holes are also explored using gravitational instantons. Gravitational instantons are Euclidean solution of the gravitational action with a finite probability. The observational constraints are also taken into account to determine different parameters of the theory for a consistent cosmological scenario. Both the early universe and present accelerating phase of the universe are explored. The problem of getting an accelerating universe with usual matter field is identified and we propose a scenario which admits late acceleration.

1.2 Methodology

In GTR, the Einstein's field equation in four dimensions is given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu} \tag{1.1}$$

where $\mu, \nu = 0, 1, 2, 3$; $g_{\mu\nu}$ is the 4-dimensional metric, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar and $T_{\mu\nu}$ is the energy momentum tensor. The L.H.S. of eq.(1.1) describes the property of a space-time whereas the R.H.S. is determined by the matter content in the universe. The most general space-time metric consistent with homogeneity and isotropy is the Robertson-Walker(RW) metric which is given by

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)\right] \tag{1.2}$$

where ($k = +1, 0, -1$), $k = +1$ for positively curved spatial sections (closed universe), $k = 0$ for local flatness (flat universe) and $k = -1$ for negatively curved spatial sections (open universe), $a(t)$ represents the scale factor of the universe. Using the energy-momentum tensor for perfect fluid, $T_{\mu\nu} = [\rho, -p, -p, -p]$ where ρ is the energy density and p is the isotropic pressure, the metric eq.(1.2), eq.(1.1) yields the following field equations:

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G\rho}{3} \tag{1.3}$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}=-8\pi Gp \tag{1.4}$$

These equations are known as Friedmann equations. The conservation equation is

$$\frac{d\rho}{dt}+3(\rho+p)H=0$$

The above equation and the set of equations (1.3) and (1.4) are the key equations in cosmology. Equation (1.4) can be obtained using eq.(1.3) and the above equation. Thus any two of the equations are sufficient and may be employed to obtain cosmological solution for a known equation of state $p=p(\rho)$. Using the above equations one obtains Raychaudhuri equation, which is

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p) \tag{1.5}$$

For an accelerated universe, $\ddot{a}>0$ implies $p<-\frac{\rho}{3}$, which is not possible with perfect fluid. In the next section the necessary condition for obtaining an accelerating universe is discussed:

A. CONDITIONS FOR INFLATION WITH DIFFERENT FIELDS:

i) For scalar field :

The Lagrangian for scalar field, $\phi(\mathbf{r},t)$ is given by

$$L=-\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\;-V(\phi), \tag{1.6}$$

where $V(\phi)$ is the scalar field potential. The stress-energy tensor for this field is

$$T_{\mu\nu}=\partial_\mu\phi\,\partial_\nu\phi\;-g_{\mu\nu}L(\phi). \tag{1.7}$$

The total energy density (ρ) and the pressure (p) in terms of scalar field are then obtained from eq. (1.7) which are given by

$$\rho=\frac{1}{2}\dot{\phi}^2+\frac{1}{2}a^{-2}(\nabla\phi)^2+V(\phi), \tag{1.8}$$

$$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}a^{-2}(\nabla\phi)^2 - V(\phi) \tag{1.9}$$

where $a(t)$ is the scale factor of the universe. For a homogeneous scalar field $\phi = \phi(t)$, the energy density and pressure becomes

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{1.10}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{1.11}$$

Using eqs. (1.10) and (1.11), eq.(1.5) reduces to Raychaudhuri equation :

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\dot{\phi}^2 - V(\phi)\right) \tag{1.12}$$

it is evident that for an accelerating universe one has, $\ddot{a} > 0$ which leads to $\dot{\phi}^2 < V(\phi)$.

ii) For tachyonic field :

The Lagrangian for a tachyonic field (ψ) is given by

$$L = V(\psi)\sqrt{1 + g^{\mu\nu}\,\partial_\mu\psi\,\partial_\nu\psi} \tag{1.13}$$

with tachyonic potential $V(\psi)$. The energy density (ρ) and pressure (p) for tachyonic field are given by

$$\rho = \frac{V(\psi)}{\sqrt{1 - \dot{\psi}^2}}, \tag{1.14}$$

$$p = -V(\psi)\sqrt{1 - \dot{\psi}^2} \tag{1.15}$$

In this case pressure remains negative ($p < 0$) for $\dot{\psi}^2 < 1$. It is also interesting to note that as $p \to 0$ the energy density $\rho \to \infty$, which may be attained when $\dot{\psi}^2 \to 1$.

iii) For phantom field :

The Lagrangian for phantom field (Φ) is given by

$$L = \frac{1}{2}\partial_\mu\Phi\,\partial^\mu\Phi\; - V(\Phi) \tag{1.16}$$

it has an unusual sign before the kinetic energy. The energy density and pressure of a homogeneous phantom field are

$$\rho = -\frac{1}{2}\dot{\Phi}^2 + V(\Phi), \tag{1.17}$$

$$p = -\frac{1}{2}\dot{\Phi}^2 - V(\Phi) \tag{1.18}$$

in this case p is always a negative quantity, the field may be considered an important candidate for obtaining present accelerating universe due to unusual sign before the kinetic energy of the field. The equation of state parameter $\omega_\Phi = \frac{\dot{\Phi}^2+2V}{\dot{\Phi}^2-2V}$, where $\omega_\Phi < -1$ can be attained for $\dot{\Phi}^2 < 2V$. In the next section, we discuss realization of inflationary solution with a scalar field.

B. REALIZATION OF INFLATION IN A SCALAR FIELD COSMOLOGY:

For a potential energy dominated region, $\dot{\phi}^2 << V(\phi)$; consequently eqs. (1.10) and (1.11) yields

$$p = -\rho \cong -V(\phi) \tag{1.19}$$

i.e. a negative pressure. The Friedmann equation with a homogeneous scalar field ($\phi = \phi(t)$) as the sole energy source is given by for a flat universe ($k = 0$) :

$$H^2 = \frac{1}{3M_{pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \tag{1.20}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{1.21}$$

where M_P represents the Planck mass and $M_P^2 = \frac{1}{8\pi G}$. Where prime represents derivative with respect to the field ϕ. The standard approximation technique for analyzing inflation is the slow-roll approximation. The condition for inflation in this case is achieved neglecting $\ddot{\phi}$ term in eq.(1.21) and neglecting kinetic energy of ϕ compared to the potential energy in (1.20). Friedmann equation and the scalar field equation of motion then reduces to

$$H^2 \simeq \frac{V(\phi)}{3M_P^2}. \tag{1.22}$$

$$3H\dot{\phi} \simeq -V'(\phi), \tag{1.23}$$

The slow-roll parameters are defined as

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \tag{1.24}$$

$$\eta = M_P^2 \frac{V''}{V} \tag{1.25}$$

Both the slow roll parameters ϵ and η become less than unity during the inflation. For inflation we have,

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \tag{1.26}$$

which leads to $\frac{\dot{H}}{H^2} > -1$. Thus using eq. (1.22) one obtains,

$$-\frac{\dot{H}}{H^2} \simeq \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 = \epsilon \tag{1.27}$$

Consequently, if the slow-roll approximation is valid ($\epsilon \ll 1$), then inflation is guaranteed. Smallness of the other parameter η helps us to ensure that inflation will continue for a sufficient period. The number of e-folds of inflation is given by

$$N(t) = \ln \frac{a(t_{end})}{a(t)} \tag{1.28}$$

where $a(t_{end})$ represents scale factor of the universe at the end of inflation. However inflation ends when $\epsilon(\phi_{end}) = 1$. We can express N in terms of the potential also employing the eqs. (1.22) and (1.23), which is given below

$$N(t) = \int_{t}^{t_{end}} H dt \simeq \frac{1}{M_P^2} \int_{\phi}^{\phi_{end}} \frac{V}{V'} d\phi \tag{1.29}$$

1.3 AIM of the Work

The objective of the investigation is to study some specific issues relevant for cosmological model building, which incorporates the quantum nature of matter or both matter and gravity in the early epochs. The quantum creation of a universe leading to a classical universe which emerges with all features observed today including the results available from *COBE* will be taken up here. We intend to investigate cosmological models taking into account the constraints imposed by the recent cosmological observations from *WMAP*, *Supernovae type I* red shift survey at high redshift etc.. It is now understood that most of the matter in the universe are in the form of either dark matter and/or dark energy. Dark energy in the universe leads to an accelerating present universe. There are various proposals to account the recent cosmic acceleration either by modifying the gravitational sector of the action or by introducing exotic matter in GTR. The aim of the thesis is to investigate cosmological models and some of the conceptual issues in cosmology in the framework of such theories. Gravitational instanton technique is also taken up here to study early universe.

1.4 Summary of the Work

• In Chapter 1, a review of the inflationary models of the early universe that developed in the recent past are presented. The aims and objective of the thesis work is also presented here.

• In Chapter 2, a computation of the probability for quantum creation of an inflationary universe with or without a pair of black holes in a modified gravity is presented using gravitational instantons. The action of the modified theory of gravity contains $\alpha\ R^2$ and $\delta\ R^{-1}$ terms in addition to a cosmological constant (Λ) in the Einstein-Hilbert action. The probabilities for the creation of universe with a pair of black holes have been evaluated considering two different kinds of spatial sections, one which accommodates a pair of black holes and the other without black hole. We adopt a technique prescribed by Bousso and Hawking [49] to evaluate the above creation probability in a semiclassical approximation using Hartle-Hawking boundary condition [50]. We note a class of new and physically interesting gravitational instanton solutions characterized by the parameters in the action. These instantons may play an important role in the creation of the early universe. We also note that the probability of creation of a universe with a pair of black holes is strongly suppressed with a positive cosmological constant when $\delta = \frac{4\Lambda^2}{3}$ for $\alpha > 0$ but it is more probable for $\alpha < -\frac{1}{6\Lambda}$. In GTR, BH obtained gravitational instanton in the presence of Λ. However, it is shown in this chapter that the modified gravity permits instanton solutions, even without a cosmological constant if one begins with a negative δ.

• In Chapter 3, inflationary solution of the early universe is obtained considering a tachyon field. The technique of Zhuravlev and Chervon [51] to obtain inflationary cosmological models without restrictions on a scalar field potential is employed here. We note that like scalar field, the inflationary solution obtained here with tachyon field does not depend on the potential. However, unlike scalar field, inflation with tachyon is obtained for a restricted domains of the field to begin with. The tachyonic potential is determined for which one gets early inflation. Unlike the scalar field potential, the tachyonic potential is not regular at all values of the field. The solution obtained here with a tachyon field is new.

• In Chapter 4, cosmological models with phantom field in an anisotropic Bianchi-I universe with a cosmological constant is presented. It is noted that kinetic energy dominated regime of the phantom field inflates and at a later epoch it transits to an inflationary universe. A class of new cosmological solutions are obtained for anisotropic Bianchi-I universe with an initial anisotropy satisfying an upper bound which is determined by the parameter of the kinetic part of the field. Here anisotropy decreases and it leads to a flat FRW universe. It is found that phantom field admits a nonsingular universe.

• In Chapter 5, inflationary dynamics with a scalar field in an inverse coshyperbolic potential in the brane-world model is explored. The coshyperbolic potential is found to be useful recently as a toy model in cosmology to probe the universe. It is found that a sufficient inflation may be obtained in the early universe with the potential allowing slow-roll approximation in the high energy limit. We determine the minimum values of the initial field required to obtain sufficient inflation and also determine the relevant inflationary parameters. The numerical values of spectral index of the scalar perturbation spectrum are determined by varying the number of e-foldings for different initial values of the inflaton field. The result obtained here is in good agreement with the current observational limits.

• In Chapter 6, cosmological solutions in the Randall- Sundrum type II brane-world model [52] is obtained with or without Gauss-Bonnet term in the presence of an imperfect fluid. Describing imperfect fluid by truncated Israel-Stewart theory [53] cosmological solutions are explored. It is found that it admits accelerating phase of the universe. The field equations can be reduced to a set of autonomous differential equation and the stability of the equilibrium points corresponding to cosmological solutions are studied.

• In Chapter 7, concluding remarks and future work is presented.