Chapter 3

INFLATIONARY UNIVERSE WITH TACHYON FIELD

3.1 Introduction

The present astrophysical observations predict that the universe is possibly made up of exotic kind of matters. The believe is due to the fact that the fields of the standard model are not enough to understand the present observations. Consequently a number of cosmological models with exotic kinds of matter are probed recently. Tachyon fields which originates in a class of string theories are considered to be one of the promising candidate for accounting the deficiency in the density parameter in the universe. Sen [77] has shown that tachyon condensate in a class of string theories can be described by an effective scalar field, but Lagrangian for such a scalar field is very different. The evolution of this tachyonic field may have cosmological significance which may be worth exploring in the context of accelerating universe. The field may contribute to dark energy and dark matter at the present epoch. It is not known definitely the shape of the potentials for the tachyon field, but there are attempts to obtain the potentials in the context of accelerating universe [78]. Later Sami [79] obtained inflationary universe solution with tachyon in the brane world model using the above technique. However, in the model one has to consider a potential which supports a slow roll evolution for a successful inflation. Recently, Chervon and his collaborators [51] gave an elegant way to express the field equation for a scalar field cosmology which permits inflation without assuming slow roll condition. We obtain here cosmological solutions with tachyon using the Chervon's method and determine the corresponding effective potential for the field which leads to inflation [80]. The motivation is to explore an exact inflationary solution with tachyon, analogous to slow-roll one for a particular transformation of the field (basically without slow roll approximation) that was adopted by Chervon *et al* [51] for a scalar field. It reveals that though inflation is obtained with scalar field without restriction on its potential, in the case of tachyon field it is different. The tachyonic inflation is permitted for a restricted domain of the values of the field.

3.2 Exact Inflationary Solution without Restrictions on Tachyon field Potential

For a spatially flat homogeneous and isotropic universe, the field equations with tachyon field as the matter are

$$H^2 = \frac{\kappa^2}{3}\left[\frac{V(\Psi)}{\sqrt{1-\dot{\Psi}^2}}\right], \tag{3.1}$$

$$\frac{\ddot{\Psi}}{1-\dot{\Psi}^2} + 3H\dot{\Psi} + \frac{V'(\Psi)}{V(\psi)} = 0 \tag{3.2}$$

where $\kappa^2 = 8\pi G$ and $V(\Psi)$ is a potential for the tachyon field Ψ, $H = \frac{d}{dt}[\ln a(t)]$ is the Hubble parameter and $a(t)$ is the scale factor of the universe. Using the method adopted by Zhuravlev and Chervon [51], eqs. (3.1)-(3.2) can be rewritten as

$$H^2 = \frac{\kappa^2}{3} U(\Psi), \tag{3.3}$$

$$3H\dot{\Psi} = -\frac{d}{d\Psi}(\ln U(\Psi)), \tag{3.4}$$

where the total energy density $U(\Psi)$ is given by

$$U(\Psi) = \frac{V(\Psi)}{\sqrt{1 - W^2(\Psi)}} \tag{3.5}$$

with $W(\Psi) = \dot{\Psi}$. Thus the approach adopted here does not require any restriction on the potential for slow roll approximation or parameters [47,51,81]. Eqs. (3.3) and (3.4) now determine the scale factor, which is given by

$$a(t) = a_o\, exp\left[-\kappa^2 \int \frac{U(\Psi)}{\frac{d}{d\Psi}(\ln U)} d\Psi\right]. \tag{3.6}$$

The number of e-folds is given by

$$N(t) = \int_{t_o}^{t_e} H dt = \ln\frac{a_e}{a_o} = -\kappa^2 \int_{\Psi_o}^{\Psi_e} \frac{U}{\frac{d}{d\Psi}(\ln U)} d\Psi \tag{3.7}$$

where Ψ_o is the magnitude of field when inflation begins at $t = t_o$ and Ψ_e is the field when inflation ends. The functions $U(\Psi)$ and $W(\Psi)$ are related as

$$\sqrt{3\kappa^2 U(\Psi)}\, W(\Psi) = -\frac{d}{d\Psi}[\ln U(\Psi)]. \tag{3.8}$$

It is now clear that only one of the functions $U(\Psi)$ and $W(\Psi)$ is arbitrary. Using fine tunning of the potential method adopted by Chervon [51] we get

$$V(t) = \frac{\sqrt{3}}{\kappa^2}\left(\frac{\dot{a}}{a}\right)\left[2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right]^{1/2}, \tag{3.9}$$

$$\Psi(t) = \pm\sqrt{\frac{2}{3}} \int \sqrt{-\frac{1}{\frac{\dot{a}}{a}} \frac{d}{dt} \ln\left(\frac{d}{dt} \ln a\right)}\, dt. \tag{3.10}$$

The square of the first derivative of the field now becomes

$$\dot{\Psi}^2 = -\frac{2\dot{H}}{3H^2}. \tag{3.11}$$

We now determine the total energy density as

$$U(\Psi) = \frac{3}{\kappa^2} H^2 \tag{3.12}$$

and the corresponding physical potential $V(\Psi)$ reduces to

$$V(\Psi) = \frac{3}{\kappa^2}\sqrt{1 + \frac{2\dot{H}}{3H^2}}\, H^2 \tag{3.13}$$

which is same as the potential $V(t)$ given in (3.9). The field eqs. (3.3) and (3.4) are written in such a way that the conditions of inflation are satisfied automatically since the kinetic term in (3.3) and $\ddot{\Psi}$ term in eq. (3.4) do not appear in the new set of equations.

3.3 Toy Model

In the earlier section it is evident that we have two equations (3.11) and (3.13) and three unknowns, the system of equations may be solved if one of them is assumed. Let us consider an ansatz for the tachyonic field

$$\Psi = \alpha \, \ln t \tag{3.14}$$

where α is a parameter (One can choose $\Psi = \alpha \, \ln(t+1)$ so that $\Psi = 0$ at $t = 0$, but the final conclusion will not be affected). The Hubble parameter is obtained integrating eq. (3.11) which is given by

$$H = \frac{1}{\beta - \frac{3\alpha^2}{2t}} \tag{3.15}$$

where β is a constant. On integrating eq. (3.15) once again one obtains the scale factor which is given by

$$a(t) = a_o e^{\frac{t}{\beta}} \left(2\beta t - 3\alpha^2\right)^{\frac{3\alpha^2}{2\beta^2}}. \tag{3.16}$$

The number of e-folding is

$$N = \left[\frac{t}{\beta} + \frac{3\alpha}{2\beta^2} \ln\left(2\beta t - 3\alpha^2\right)\right]_{t_{initial}}^{t_{final}}. \tag{3.17}$$

In this case the energy density is

$$U(\Psi) = \frac{3}{\kappa^2} \frac{1}{\left(\beta - \frac{3\alpha^2}{2} e^{-\frac{\Psi}{\alpha}}\right)^2}. \tag{3.18}$$

The corresponding potential $V(\Psi)$ is given by

$$V(\Psi) = \frac{3}{\kappa^2} \frac{\sqrt{1 - \alpha^2 e^{-2\frac{\Psi}{\alpha}}}}{\left(\beta - \frac{3\alpha^2}{2} e^{-\frac{\Psi}{\alpha}}\right)^2}. \tag{3.19}$$

The number of e-folding in terms of field becomes

$$N = \frac{1}{\beta}\left(e^{\Psi_e/\alpha} - e^{\Psi_o/\alpha}\right) + \ln\left(\frac{2\beta e^{\frac{\Psi_e}{\alpha}} - 3\alpha^2}{2\beta e^{\frac{\Psi_o}{\alpha}} - 3\alpha^2}\right)^{\frac{3\alpha^2}{2\beta^2}} \tag{3.20}$$

Here Ψ increases slowly (logarithmically) for $t \geq M_P^{-1}$. However $\dot{\Psi}^2$ decreases rapidly leading to $V(\Psi) \rightarrow constant$ when $\beta \neq \frac{3\alpha^2}{2}$. It is evident that an inflationary scenario is permitted for initial value of the tachyon field $\Psi_o \geq \alpha \ \ln\alpha$ and the tachyonic inflation ends at Ψ_e when $\dot{\Psi} \rightarrow 0$. The tachyonic potential $V(\Psi)$ is restricted as it becomes imaginary if $\Psi < \alpha \ \ln\alpha$. Thus the potential is regular for a particular sector of the tachyon field which is different from that of scalar field model where scalar field potential $V(\phi)$ is regular everywhere.

3.4 Discussions

In this chapter cosmological solutions with tachyon field which admits inflation are presented. The relevant potential to obtain inflation is determined. For a logarithmically varying field one obtains inflationary scenario for a tachyon field satisfying the constraint $\Psi_o > \alpha\ (\ln \alpha)$ to begin with. The tachyon field increases as the universe evolves and later transits to an inflationary phase.

From eq. (3.16) it is noted that the scale factor $a(t)$ depends on the multiplication of two functions of time. One of them is an exponential function of t i.e. $e^{t/\beta}$ where $1/\beta$ is the expansion rate, considered in the slow-roll approximation technique and the second function acts as a correction term to get an exact solution from the approximate solution. It is evident from above that singularity exists at $\Psi = \alpha\left(\ln \frac{3\alpha^2}{2\beta}\right)$. To avoid the singularity we choose $\beta > \frac{3\alpha}{2}$. If we compare the number of e-folding N given in (3.20) to that obtained from slow-roll approximation technique, only the first term of (3.20) corresponds to the slow-roll condition with a restriction on tachyon field Ψ. In sec. 3.3 we have taken Ψ as a logarithmic function of time to study the inflationary solution. One can study the same by taking other functionals of Ψ. All the solutions have some general feature, which are as follows : Integrating the equation (3.11), we get

$$\frac{1}{H} = \frac{3}{2} f(\Psi) + \beta \tag{3.21}$$

where $f(\Psi) = \int W(\Psi)\, d\Psi$. Using equations (3.3) and (3.21) we obtain,

$$\frac{U^2(\Psi)}{U'(\Psi)} = -\frac{1}{\kappa^2} \frac{1}{f'^2(\Psi)\left(\frac{3}{2} f(\Psi) + \beta\right)} \tag{3.22}$$

and the e-folding,

$$N = \int_{\Psi_o}^{\Psi_e} \frac{d\Psi}{f'(\Psi)\left(\frac{3}{2}f(\Psi)+\beta\right)}. \tag{3.23}$$

Considering slow-roll approximation technique one determines the number of e-folding which is given by

$$N = \int_{\Psi_o}^{\Psi_e} \frac{d\Psi}{f'(\Psi)} \tag{3.24}$$

Comparing equations (3.23) and (3.24) we conclude that the extra term $\left(\frac{3}{2}f(\Psi)+\beta\right)$ appears as a multiplier with $f'(\Psi)$ in equation (3.23) which may be considered to be a correction term to the slow-roll approximation. It results an exact value of e-folding.

On integrating eq. (3.21) we get the scale factor

$$a(t) = a_o\, exp\left[\int \frac{dt}{\frac{3}{2}f(\Psi)+\beta}\right]. \tag{3.25}$$

It is evident from the above that the value of Ψ, for which $f(\Psi) = -\frac{2}{3}\beta$ is not permitted. Here two cases arise (i) when $f(\Psi) < 0$ and $\beta > 0$ then $|f(\Psi)| > 2\beta/3$ and (ii) when $f(\Psi) > 0$ and $\beta < 0$ then $f(\Psi) > 2|\beta|/3$ for a realistic solution. It is also noted that by considering the slow-roll approximation technique, the scale factor becomes

$$a(t) = a_o\, exp\left(\int \frac{dt}{\beta}\right). \tag{3.26}$$

where $1/\beta$ is the expansion rate of the inflationary universe. Thus compared to the slow role approximation the scale factor is now dependent on $f(\Psi)$ also.