CHAPTER 4

Viscous Cosmologies with Variable G and Λ in R^2 Gravity

4.1 Introduction

Cosmological models considering perfect fluid as a source of matter in the framework of higher derivative gravity have been studied in the literature [108] to obtain viable cosmological scenario of the early universe. When particle number is conserved in the perfect fluid, the continuity equation is $\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$ i.e., $n^{\alpha}_{;\alpha} = 0$ where n is particle number density. However in the case of the particle production, one get $n^{\alpha}_{;\alpha} = \dot{n} + 3Hn = vn$, where v is the rate of particle production. The entropy is conserved for a perfect fluid in equilibrium $S^{\alpha}_{;\alpha} = 0$. Perfect fluid in equilibrium generates zero entropy and it does not admit frictional heat flow, because the dynamics is reversible and does not admit dissipative process. However in cosmology and in astrophysics a deviation from perfect fluid assumption is necessary to accommodate observational evidences. It is important to consider effect of viscosity in the early universe.

To describe a relativistic theory of viscosity, Eckart [66] made the first attempt. However, the theories of dissipation in Eckart formulation suffers from serious shortcoming, viz., causality and stability [117] regardless of the choice of equation of state. The problem arises due to first order nature of the theory, since it considers only first order deviation from equilibrium. It has been shown that the problems of relativistic imperfect fluid may be resolved by including higher order deviation terms in the transport equation [118]. Israel and Stewart [67], and Pavon [68] developed a fully relativistic formulation of the theory taking into account second order deviation terms in the theory, which is termed as "transient" or "extended" irreversible thermodynamics (in short, *EIT*). The crucial difference between the standard Eckart and the extended Israel-Stewart transport equations is that the latter is a differential evolution equations, while the former is a algebraic relations. Extended irreversible thermodynamics takes its name from the fact that the set needed to describe non-equilibrium states is extended to include the dissipative variables. The price paid for the improvements that *EIT* brings is that new thermodynamic coefficients (e.g., τ, ζ, λ, η) are introduced. The problem of causality of Eckart theory ($\tau = 0$) have been removed in *EIT* by taking a finite value of relaxation time ($\tau > 0$). The extended theory is also known as causal thermodynamics, second-order thermodynamics and transient thermodynamics. The FIS theory leads to a TIS theory with reduced bulk viscosity at equilibrium (i.e., $|\Pi \ll \rho|$). The amount of reduction depends on the size of τ relative to H. If $\tau H \ll 1$, the reduction is insignificant, i.e., the result of TIS and FIS theory may be comparable. But if τH is close to 1, the reduction could be significant i.e. in this case the result of TIS and FIS theory differs dramatically. One expects the coincidence to be better close to the equilibrium case.

In the case of irreversible process, the form of the energy momentum tensor becomes

$$T_{\mu\nu} = \rho u_\alpha u_\beta + (p + \Pi) h_{\alpha\beta} + q_\alpha u_\beta + q_\beta u_\alpha + \pi \alpha\beta,$$

where q is an energy flux , u is the four-velocity and π_{ij} is the anisotropic stress. In irreversible thermodynamics, the entropy is no longer conserved, but grows, according to the second law of thermodynamics (i.e., $S^\alpha_{;\alpha} \geq 0$). The entropy generated in a dissipative process that begins at t_0 and ends at $t_0 + \Delta t$ is given by

$$\Delta\Sigma = -\int_{t_0}^{t_0+\Delta t} \frac{a^3}{T}(3H\Pi + q^\alpha_{;\alpha} + \dot{u}_\alpha q^\alpha + \sigma_{\alpha\beta}\pi^{\alpha\beta})dt.$$

The observed universe has large entropy $\sim 10^{88}$. Inflationary cosmology predicts that nearly all of this entropy may be generated during reheating process at the end of the inflation i.e., all other dissipative processes in the cosmological evolution of the universe make a tiny contribution to entropy production by comparison. If one assumes that the universe is exactly isotropic and homogeneous i.e., an FRW universe - then the symmetries show that only scalar dissipation is possible i.e., $q_\alpha = 0 = \pi_{\alpha\beta}$. Bulk viscosity arises typically in mixtures either of different species or of the species but with different energies.

The solutions of the full causal theory are well behaved at all the times. Therefore, the best currently available theory for analyzing dissipative processes in the universe is the full Israel-Stewart theory (FIS). Using the transport equation obtained from *EIT*, in addition to dynamical equation obtained from either Einstein gravity [116] or modified gravity [121] cosmological solutions are generally obtained. The motivation is to explore cosmological solutions in the modified theory of gravity with variable G and Λ in the presence of imperfect fluid described by FIS theory. It is interesting to explore the behaviour of G and Λ in this case.

The characteristics temperature of the evolving universe will also be determined in the presence of viscosity. However, it may be pointed out here that a number of literature [122] determine the evolution of temperature in the presence of viscosity starting from Gibbs equation, which will be also taken up here.

The chapter is organized as: in sec 4.2, the gravitational action and the relevant field equations in the higher derivative theory of gravity is presented. In sec. 4.3, cosmological solutions are presented. In sec. 4.4, distance modulus curves are presented to compare theoretical results with observation. Finally, in sec. 4.5, a summary of the result is discussed.

4.2 Gravitational Action and Dynamical Equations

Let us consider a gravitational action with higher order term in the scalar curvature (R) containing a variable gravitational constant ($G(t)$), which is

$$I = -\int \left[\frac{1}{16\pi G(t)} f(R) + L_m\right] \sqrt{-g}\; d^4x \tag{4.1}$$

where $f(R)$ is a function of R and its higher power including a variable cosmological constant $\Lambda(t)$, g is the determinant of the four dimensional metric and L_m represents the matter Lagrangian.

Variation of the action (4.1) with respect to $g_{\mu\nu}$ yields

$$f_R(R)\; R_{\mu\nu} - \frac{1}{2}\; f(R)\; g_{\mu\nu} + f_{RR}(R)\,(\nabla_\mu \nabla_\nu R - g_{\mu\nu} \nabla^\mu \nabla^\nu g_{\mu\nu}) +$$

$$f_{RRR}(R)\,(\nabla_\mu R \nabla_\nu R - \nabla^\sigma R \nabla_\sigma R\; g_{\mu\nu}) = -8\pi G(t)\; T_{\mu\nu} \tag{4.2}$$

where ∇_μ is the covariant differential operator, $f_R(R)$ represents the derivative of $f(R)$ with respect to R and $T_{\mu\nu}$ is the effective energy momentum tensor for matter determined by L_m . For a flat Robertson-Walker metric the trace and (0,0) components of eq. (4.2) are given by

$$Rf_R(R) - 2f(R) \; + \; 3f_{RR}(R)\left(\ddot{R} + 3\frac{\dot{a}}{a}\dot{R}\right) + 3f_{RRR}(R)\dot{R} + 8\pi G(t)\, T = 0, \qquad (4.3)$$

$$f_R(R)\, R_{00} + \frac{1}{2}f(R) - 3f_{RR}(R)\frac{\dot{a}}{a}\dot{R} + 8\pi G(t)\, T_{00} = 0. \qquad (4.4)$$

We now consider a higher order gravity, namely, $f(R) = R + \alpha R^2 - 2\Lambda(t)$. The field equation becomes

$$H^2 - 6\alpha\left[2H\ddot{H} - \dot{H}^2 + 6\dot{H}H^2\right] = \frac{8\pi G(t)\rho}{3} + \frac{\Lambda(t)}{3}, \qquad (4.5)$$

and the conservation equation is given by

$$\dot{\rho} + 3(\rho + p)H = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right), \qquad (4.6)$$

where ρ and p are the energy density and pressure of the perfect fluid respectively. Equations (4.5) and (4.6) are the key equations to study cosmological models with a perfect fluid in the presence of time varying G and Λ. To include, the effect of viscosity in the above, the perfect fluid pressure in eq. (4.6) is replaced by an effective pressure p_{eff}, which is given by $p_{eff} = p + \Pi$, where p is isotropic pressure and Π is the bulk viscous stress. In *EIT*, the bulk viscous stress Π satisfies the transport equation

$$\Pi + \tau\dot{\Pi} = -3\zeta H - \frac{\epsilon}{2}\tau\Pi\left[3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\zeta}}{\zeta} - \frac{\dot{T}}{T}\right] \qquad (4.7)$$

where ζ is the coefficient of bulk viscosity, τ is the relaxation coefficient for transient bulk viscous effects and T is the absolute temperature of the universe. The parameter ϵ takes the value 0 or 1. Here $\epsilon = 0$ represents truncated Israel-Stewart theory and $\epsilon = 1$ represents full Israel-Stewart (FIS) causal theory. One recovers the non-causal Eckart theory for $\tau = 0$. The conservation eq. (4.6) including viscous fluid is given by :

$$\dot{\rho} + 3(\rho + p + \Pi)H = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right). \qquad (4.8)$$

For a constant G, Λ and $\Pi = 0$, eq. (4.8) reduces to the usual continuity equation for a barotropic fluid. It is evident from eq. (4.8) that in order to satisfy the energy conservation, a decaying vacuum term, time varying gravitational constant and viscosity necessarily lead to matter production. We consider an equation of state for the isotropic fluid pressure given by

$$p = (\gamma - 1)\rho \tag{4.9}$$

where γ $(1 \leq \gamma \leq 2)$ is a constant. The deceleration parameter (q) is related to H as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{4.10}$$

The deceleration parameter is negative for accelerating phase and positive for decelerating phase of the universe. The temperature of the universe is defined by the Gibbs equation which is given by

$$TdS = d\left(\frac{\rho}{n}\right) + p\, d\left(\frac{1}{n}\right), \tag{4.11}$$

where S is the entropy of the universe. The temperature of the universe in EIT frame work is obtained through Gibbs integrability condition which is given by

$$n\frac{\partial T}{\partial n} + (\rho + p)\frac{\partial T}{\partial \rho} = T\frac{\partial p}{\partial \rho}. \tag{4.12}$$

Suppose the pressure and temperature are barotropic. For perfect fluid case, the form of T is considered as $T \sim \rho^{\frac{\gamma-1}{\gamma}}$, where γ is the EOS parameter. The temperature is also determined from Gibbs integrability condition given by

$$\frac{\dot{T}}{T} = -3H\left[\left(\frac{\partial p}{\partial \rho}\right)_n + \frac{\Pi}{T}\left(\frac{\partial T}{\partial \rho}\right)_n\right]. \tag{4.13}$$

It is not simple to find out temperature variation in FIS theory. To determine the temperature of a viscous universe in this case one may use the eq. (4.7), in addition to eq. (4.13), the Gibbs integrability condition. Both the equations are considered to determine the appropriate behaviour of temperature permitted by the theory. investigate different cosmologies permitted here.

4.3 Cosmological Solutions

The system of Eqs. (4.5), (4.7)-(4.9) is employed to determine cosmological solutions. The system of equations is not closed as it has eight unknowns (ρ, γ, τ, ζ, G Λ, $a(t)$, T)

to be determined from four equations. We assume the following widely accepted *adhoc* relations

$$\zeta = \beta\rho^s, \ \tau = \beta\rho^{s-1} \tag{4.14}$$

where $\zeta \geq 0$, $\tau \geq 0$, $\beta \geq 0$ and $s \geq 0$. Using a known function of Λ in terms of the Hubble parameter (H), (namely, $\Lambda = 3mH^2$ where m is dimensionless constant) the cosmological solutions are explored.

4.3.1 *Power-law model*

In this case we consider a power law expansion of the universe given by

$$a(t) = a_0 t^D \tag{4.15}$$

where a_0 and D are constants which are to be determined from the field equation. The accelerating mode of expansion ($q < 0$) of the universe is obtained for $D > 1$. In the absence of particle creation (i.e., $n^{\alpha}_{;\alpha} = 0$) eq. (4.8) may be decoupled as follows:

$$\dot{\rho} + 3\gamma\rho H + 3\Pi H = 0, \tag{4.16}$$

$$8\pi \, \dot{G} \, \rho + \dot{\Lambda} = 0. \tag{4.17}$$

Using eqs. (4.5), (4.15) and (4.17), we obtain Newton's gravitational constant which varies as

$$G = G_0 \left[t^2 + \rho_2\right]^{\frac{m}{1-m}}, \tag{4.18}$$

where we replaced $\rho_2 = \frac{18\alpha(2D-1)}{1-m}$ and $G_0 = const$. The initial value of $G(t)$ is determined in terms of coupling parameter (α), which however vanishes when $D = \frac{1}{2}$. In this case the gravitational parameter $G(t)$ increases with time for $\Lambda > 0$ and it decreases for $\Lambda < 0$. It is also noted that the theory admits a constant G when Λ vanishes. The evolution of energy density is obtained from eq. (4.17), which is

$$\rho = \rho_0 \left[t^2 + \rho_2\right]^{\frac{1-2m}{1-m}} \ t^{-4}, \tag{4.19}$$

where $\rho_0 = 3(1-m)D^2$ (with $8\pi G_0 = 1$). For a physically realistic solution (i.e., $\rho > 0$) the upper boundary on cosmological constant is $\Lambda < 3H^2$ (i.e., $m < 1$). For $m = \frac{1}{2}$ the variation of energy density is determined by R^2 term only. In this case the energy density decreases as ($\rho \sim t^{-4}$), whereas it is independent of α. The bulk viscous stress obtained from eq. (4.16) is given by

$$\Pi = -\left[\Pi_0 t^2 + \Pi_2\right]\left[t^2 + \rho_2\right]^{\frac{-m}{1-m}} t^{-4}, \tag{4.20}$$

where $\Pi_0 = D(3\gamma D(1-m) - 2)$ and $\Pi_2 = 18\alpha D(2D-1)(3\gamma D - 4)$. For physically realistic solution bulk viscous stress is essentially negative, which demands $\Pi_0 > 0$ i.e., $D > \frac{2}{3\gamma(1-m)}$. We note that the bulk viscous stress decreases as $|\Pi| \sim t^{-\frac{2}{1-m}}$ for the following two cases:

(i) $\alpha = 0$ and $D \neq \frac{1}{2}$ or (ii) $\alpha \neq 0$ and $D = \frac{1}{2}$.

It is evident that for $m < 1$, bulk viscous stress ($|\Pi|$) is found to decrease with time. The advantage of the FIS theory is that the behaviour of temperature can be predicted. Using eq. (4.14) in FIS theory ($\epsilon = 1$), the transport eq. (4.7) reduces to a differential equation given by

$$\frac{\dot{T}}{T} = 3H - \frac{\dot{\rho}}{\rho} + \frac{6H\rho}{\Pi} + \rho^{1-s}\frac{2}{\beta} + \frac{2\dot{\Pi}}{\Pi}. \tag{4.21}$$

Using eqs. (4.19) and (4.20) in eq. (4.21) we get temperature of the universe. On integrating we obtain

$$T = T_0\,\frac{\Pi^2 a^3}{\rho} e^{\int \frac{6\rho H}{\Pi} dt} e^{\frac{2}{\beta}\int \rho^{1-s} dt}, \tag{4.22}$$

where T_0 is a arbitrary constant. The above expression for the temperature of the universe satisfies the Gibbs integrability condition (4.13). We note the following:

(i) For $s = \frac{3}{4}$, $m = \frac{1}{2}$ and $\alpha = 0$, we note power law decrease of the temperature, which is $T = T_0\, t^{-\alpha_1}$, where $\alpha_1 = \frac{6\rho_0 D}{\Pi_0} + 4 - 3D - \frac{2\rho_0^{\frac{1}{4}}}{\beta}$. The decreasing mode of temperature is ensured for $\alpha_1 \geq 0$. Putting the expression of temperature (T) in the presence of viscosity in Gibbs condition (4.13) one obtains, $\alpha_1 = \frac{4(\gamma-1)}{\gamma}$. Here the negative value of bulk viscous stress (Π) demands $3\gamma D > 4$. However, In the absence of viscosity, the variation of temperature can be obtained from eq. (4.13) which yields $T = T_0 t^{-3D(\gamma-1)}$. It is evident from fig. (4.1) that a universe filled with viscous fluid is more hotter than that of universe without viscosity. It is also evident from fig. (4.1) that the temperature of the universe is higher for lower a values of γ at a given instant of time.

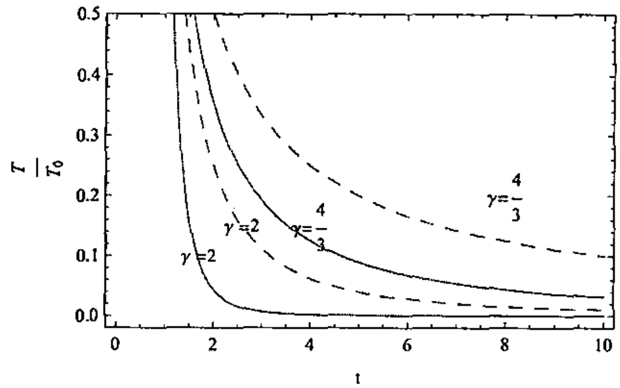

Figure 4.1: shows the plot of $\frac{T}{T_0}$ vs t for a different value of γ with $D = \frac{3}{2}$. Here the solid line represent the variation in the absence of viscosity while the dashing line represent that in the presence of viscosity.

(ii) For $s = \frac{3}{4}$, $m = \frac{5}{6}$, $\alpha \neq 0$ and $D \neq \frac{4}{3\gamma}$, the temperature of the universe evolves as $T = T_0 t^{(3D-4-\frac{6\rho_0\rho_2 D}{\Pi_2}+\frac{2\rho_0^{\frac{1}{4}}}{\beta\rho_2})}\left[\Pi_0 t^2 + \Pi_2\right]^{(2-\frac{3\rho_0 D}{\Pi_0}+\frac{3\rho_0\rho_2 D}{\Pi_2})}\left[t^2 + \rho_2\right]^{-6-\frac{\rho_0^{\frac{1}{4}}}{\beta\rho_2}}$. At a later stage of evolution of the universe ($t >> \Pi_2$ and $t >> \rho_2$), a power law variation of temperature is also permitted. Using Gibbs integrability condition the temperature of the universe is obtained which is $T = T_0 t^{\frac{-12(\gamma-1)}{\gamma}}$. Here the condition $D > \frac{4}{\gamma}$, ensure the negative value of bulk viscous stress.

(iii) For $s = \frac{m+1}{2}$, the temperature of the universe in GR ($\alpha = 0$) may be evaluated using eqs. (4.13) and eq. (4.7). The temperature evolves as

$$T = T_0 t^{\frac{-2(\gamma-1)}{\gamma(1-m)}}, \tag{4.23}$$

which is decaying if $\gamma > 1$, $m < 1$ or $\gamma < 1$, $m > 1$. Here the negative value of bulk viscous stress (Π) is obtain for $\Pi_0 > 0$ (i. e., $D > \frac{4}{3\gamma(1-m)}$). The condition $D > \frac{4}{3\gamma(1-m)}$ also implies that in an expanding universe the temperature decrease less rapidly in the presence of viscosity compared to that with out viscosity. The plot of T vs t in fig. (4.2) shows that at a given time, a universe with higher temperature corresponds to a lesser value of the cosmological constant. The scenario a universe with late acceleration. To estimate the temperature at the present epoch, consider $T_0 = 1 \times 10^{10}$K at $t = 1$ second [123] i.e., during the cosmological epoch of decoupling of neutrinos from the cosmic plasma.

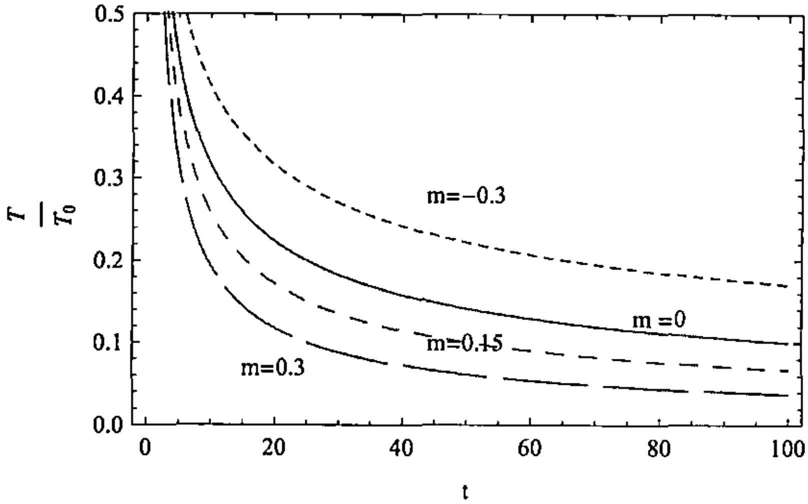

Figure 4.2: shows the plot of $\frac{T}{T_0}$ vs t for a different value of m with $D = \frac{3}{2}$ and $\gamma = \frac{4}{3}$.

Assuming the present age of the universe $\sim$ 13 billion year $\sim 4.1 \times 10^{17}$ second, $\gamma = \frac{4}{3}$ (radiation) and cosmological constant $\Lambda = 3 \times 0.079 \times H^2 (i.e., m = 0.079)$, the present temperature is estimated to be $T \sim 2.74$K, which is in fair agreement with observed value $T \sim 2.72$K from CMBR.

4.3.2 *Exponential models*

The set of eqs. (4.5), (4.7), (4.16)-(4.17) admit cosmological solution without singularity. Two such cases are discussed in the next section.

Case I : Consider Hubble parameter satisfying a differential equation

$$\dot{H} = \eta H - \frac{3}{2} H^2, \tag{4.24}$$

which is permitted by the field equations with $\eta = \sqrt{\frac{1-m}{6\alpha}}$. The solution is interesting for $\alpha > 0$ and $m < 1$, or $\alpha < 0$ and $m > 1$. In this case an ever expanding universe with no singularity result such a solution admits emergent universe scenario [124]. In this case the corresponding variation of gravitational constant and energy density are determined using eqs. (4.5) and (4.17), which are given by

$$G = G_0 \, exp[bH^{-2}], \tag{4.25}$$

$$\rho = \rho_0 H^4 exp[-bH^{-2}], \tag{4.26}$$

where $\rho_0 = \frac{81\alpha}{2}$ (with $8\pi G_0 = 1$) and $b = \frac{2m}{27\alpha}$ and $G_0 = const.$ It is evident that the model admits a constant G when $\Lambda = 0$. The bulk viscous stress is obtained from eq. (4.16), which yields

$$\Pi = (2-\gamma)\rho - \frac{1}{3}\rho H^{-3}\left[4\eta H^2 + 2b\eta - 3bH\right]. \tag{4.27}$$

We note that without a cosmological constant ($\Lambda = 0$), one obtains a realistic solution when Hubble parameter satisfies an upper bound $H < \frac{4\eta}{3(2-\gamma)}$. The temperature evolution of the universe in full causal theory is obtained from eq. (4.21), which yields

$$T = T_0\, e^{f(H)}\, e^{g(H)}\ \frac{\Pi^2}{\rho(H - \frac{2\eta}{3})^{-2}} \tag{4.28}$$

where we denote $f(H) = \int \frac{6\rho H}{\Pi} dt$ and $g(H) = \frac{2}{\beta}\int \rho^{1-s} dt$. The expression for temperature given in eq. (4.28) is found to satisfy the Gibbs integrability condition. We note the following:

(i) For $s = \frac{1}{2}$, $\beta = \frac{2\sqrt{3}}{3}$, in the absence of cosmological constant and for stiff fluid the temperature of the universe evolves as $T = T_0 H^2 (H - \frac{2\eta}{3})^{-2}$. However, $H \ll \frac{2\eta}{3}$, the temperature evolves as $T \sim H^2$.

On integrating eq. (4.24) the scale factor of the universe is obtained, which is given by

$$a(t)_{\pm} = \left[a_1 \pm a_2\, e^{\eta t}\right]^{\frac{2}{3}}. \tag{4.29}$$

The solution (a_+) is important for building emergent universe scenario [124]. It has no singularity and the universe originated from a static state in the infinite asymptotic past ($t \to -\infty$). In this case, we note that the solution ($a(t) = a(t)_+$) represents a universe which begins with a finite size in the past and grows exponentially. However, initially at $t \sim 0$, the universe is matter dominated which subsequently emerges to an accelerated phase of expansion. We note that the temperature of the universe decreases in this case. The decreasing mode of temperature obtained in the FIS theory may be relevant for the subsequent evolution.

Case II : The dynamical equation admits de Sitter solution with $H = \frac{2\eta}{3}$. The scale factor of the universe evolves as $a(t) = a_0 e^{\frac{2\eta}{3}t}$, for a sufficient inflation to solve cosmological problems, de Sitter phase should exit after an epoch $\Delta t > \frac{195}{2\eta}$. We note the de Sitter phase with demands

$$G = const.,\ \rho = const.,\ \Lambda = const.,\ \zeta = const.,\ \tau = const. \tag{4.30}$$

for $m \neq 1$. In this case that bulk viscosity also remains constant throughout the inflationary phase. The temperature of the universe in this case is found to be a constant similar to that obtained by Beesham in *Ref* [116] with $\beta = \frac{\gamma \rho^{1-s}}{(2-\gamma)\eta}$.

We also note that for $H = \frac{2}{3}\eta$ and $m = 1$, one ends up with $\Lambda = const.$, $G = const.$, $\rho = 0$, $\zeta = 0$, $\tau = 0$ and $T = 0$, which does not permit a physically relevant solution.

4.4 Distance Modulus Curves

We now probe late universe with exponential and power law expansion taking into account observational results. The distance modulus relation is $\mu = 5 \log d_L + 25$, where the luminosity distance is $d_L = r_1(1+z)\, a(t_0)$, z represents the redshift parameter, where $1 + z = \frac{a(t_0)}{a(t_1)}$. We determine r_1 from

$$\int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t_0} \frac{dt}{a(t)}. \tag{4.31}$$

At the late time, the scale factor becomes $a(t) \sim a_2 e^{\frac{2\eta t}{3}}$. For exponential expansion of a flat universe, the distance modulus relation is given by

$$\mu(z) = 5 \log(\frac{z(1+z)}{H_0}) + 25, \tag{4.32}$$

where $H_0 = \frac{2\eta}{3}$. For power law expansion $(a(t) \sim t^D)$ with $k = 0$, the distance modulus relation is given by

$$\mu(z) = 5 \log \left(\left(\frac{D}{H_0}\right)^{\frac{1}{D}} \frac{(1+z)}{D-1}((1+z)^{\frac{D-1}{D}} - 1) \right) + 25. \tag{4.33}$$

The observed values of $\mu(z)$ at different z parameters [125] given in table (4.1) are employed to draw the curves corresponding to the exponential and power law expansion of the universe discussed above. The plots are shown in figs. (4.3) and (4.4) which matches with observations perfectly well.

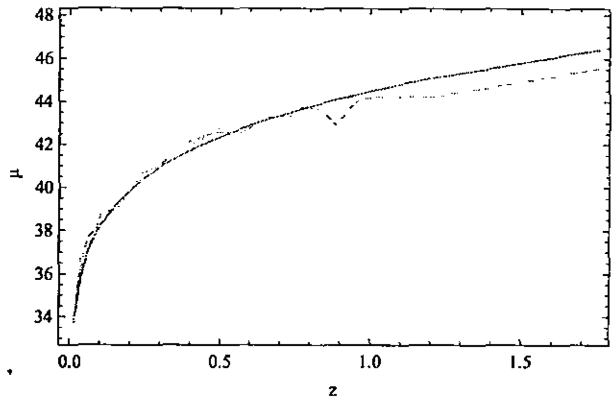

Figure 4.3: shows the plots of μ vs z for supernova data (dashing line) and for the exponential expansion (solid line) with $\eta = 0.00038$.

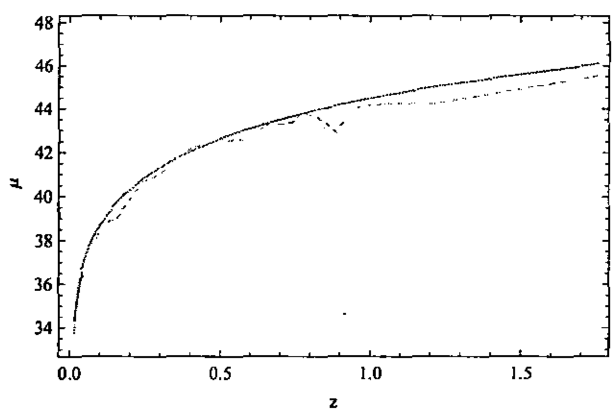

Figure 4.4: shows the plot of μ vs z for supernova data (dashing line) and for the power law expansion (solid line) with $H_0 = 10^{-5}$ and $D = \frac{4}{3}$.

z	supernova μ	Exponential μ	Power law μ
0.038	36.67	35.961	36.543
0.014	33.73	33.742	34.343
0.026	35.62	35.112	35.703
0.036	36.39	35.839	36.423
0.040	36.38	36.077	36.657
0.050	37.08	36.582	37.154
0.063	37.67	37.110	37.673
0.079	37.94	37.634	38.184
0.088	38.07	37.887	38.43
0.101	38.73	38.212	38.745
0.160	39.08	39.324	39.814
0.240	40.68	40.349	40.783
0.300	41.01	40.936	41.329
0.380	42.02	41.579	41.921
0.430	42.33	41.925	42.235
0.490	42.58	42.298	42.572
0.526	42.56	42.504	42.757
0.581	42.63	42.797	43.018
0.657	43.27	43.165	43.345
0.740	43.35	43.530	43.665
0.778	43.81	43.686	43.801
0.828	43.61	43.881	43.971
0.886	42.91	44.096	44.158
0.949	43.99	44.316	44.348
0.970	44.13	44.388	44.490
1.056	44.25	44.665	44.646
1.190	44.19	45.061	44.983
1.755	45.53	46.403	46.104

Table 4.1: shows the variation of distance modulus (μ) for Supernova explosion [125], power law model and exponential model for different value of redshift parameter (z).

4.5 Discussion

In this chapter both power law and exponential behaviour of the universe are studied in the presence of viscosity separately in a higher derivative theory of gravity considering a varying cosmological and a gravitational constant. We note that for a power law evolution of the universe, one obtains an increasing mode of gravitational constant with a positive Λ but a decreasing mode of gravitational constant results for a negative Λ, which is a new result. We note a physically realistic ($\rho_0 > 0$) solution for $\Lambda < 3H^2$. For $\Lambda = \frac{3}{2}H^2$ the variation of energy density ($\rho \sim t^{-4}$) is determined by R^2 term only. However, the energy density does not depend on the coupling parameter α in the gravitational action. We determine the characteristics temperature of a viscous universe from eq. (4.7), which also follows from Gibbs integrability condition eq. (4.13). Fig. (4.1) shows the variation of temperature for different values of γ in the presence and in the absence of viscosity permitting an accelerating universe ($q > 0$). It is evident that the higher value of γ leads to a universe with lesser temperature. The evolution of temperature of a viscous universe is found to be more than that in a universe without viscosity. Fig. (4.2) shows the variation of temperature for different values of Λ (determined by m). It is evident from fig. (4.2) that the rate of decrease of temperature is higher for larger values of m in the presence of viscosity. Here we obtain an interesting solution leads to the present temperature of the universe $T \sim 2.74$K, which is in fair agreement with observed value $T \sim 2.72$K, for a radiation dominated accelerating universe, in the presence of both time varying cosmological ($\Lambda > 0$) and gravitational constant (G) in GR. Cosmological solutions are obtained which originates from singularity free state. One of the solution corresponds to emergent universe [124] which is interesting. A de Sitter solution with $\Lambda \neq 3H^2$ is also obtained here. The figs. (4.3)-(4.4) show the plot of distance modulus (μ) Vs. red shift parameter (z). The figures indicate that power-law and exponential model support the present observational data perfectly well.