CHAPTER 2

Accelerating Universe in Modified Theories of Gravity

2.1 Introduction

During the last couple of years, there is a modest progress in our understanding of the observed universe because of the advent of new cosmological precision tests, capable of providing physics at very large redshifts. The luminosity curve of type Ia supernovae [74], the large scale structure [75] and the anisotropy of the cosmic microwave background radiation [82] favor a spatially flat universe. The recent decade is witnessing a paradigm shift in cosmology from speculative to experimental science due to a large number of observational inputs and its analysis. It has been recently predicted that the present universe is passing through a phase of the cosmic acceleration. From COBE, it is also believed that the universe might have emerged from an inflationary phase in the past. A large number of cosmological models were proposed in Einstein's gravity including scalar field admissible from standard model of Particle Physics with an early inflationary scenario in the last three decades which work well. However, the recent prediction that the present universe is passing through an accelerated phase of expansion is interesting and a proper cause is yet to be understood. It is thought that the cause of the present acceleration of the universe might be due to dark energy in the universe. However, the concept of dark energy in the Einstein's gravity with normal matter or fields permitted from standard model of particle physics cannot be implemented. Consequently it is a challenging job in the theoretical physics to frame a satisfactory theory to determine cosmological evolution which could address the origin of dark energy fairly well. It is known from the cosmological observations that the dark energy content of the universe is about 70% to that of the total energy budget of the universe. As mentioned earlier the usual fields available in the standard model of the Particle Physics are not enough to account for the huge dark energy reservoir in the universe. A modification to the Einstein's field equation either in the gravitational or in the matter sector, perhaps is essential to accommodate the present cosmological observations. The issue of dark energy in the universe has been taken up in a gravitational theory and identified with a suitable cosmological constant [83, 84]. However, the vacuum energy density in such a theory remains constant in course of cosmic evolution and it is also true that there are known contributions into vacuum energy which are several orders of magnitude greater than the allowed cosmological values. These observations led us to look for an alternative model or a new physics [85]- [87]. It has been proposed recently that gravity itself, if properly modified, could account for the recent cosmic acceleration [88, 89]. The standard Einstein's gravity may be modified at low curvature by including the terms those are important precisely at low curvature. The simplest possibility is to consider a $\frac{1}{R}$-term in the Einstein's-Hilbert action (it may originate from M-theory) [90]. Carroll *et al.* [87] also considered such a theory to derive cosmological models for accommodating a late accelerating phase of the universe. Although a theory with $\frac{1}{R}$-term added to the Einstein's gravity accounts satisfactorily the present acceleration of the universe, it is realized that such an inclusion in the Einstein's theory leads to instabilities [91]. Subsequently, it has been shown that further addition of an R^2- term [92] or $ln(R)$ term [93] to the Einstein's gravitational action lead to an acceptable modified theory of gravity which may pass satisfactorily solar system tests, and free from instability problem. It is known that a modified gravity with a positive power of the curvature scalar (namely, R^2-term) [5], [93]-[95] in the Einstein-Hilbert action admits early inflation. The modified gravity with negative powers of the curvature in the Einstein-Hilbert action is recently becoming popular as it might effectively behave as a dark energy candidate. Consequently the theory might satisfactorily describe the recent cosmic acceleration [88]-[90]. So it is reasonable to explore a theory which could accommodate an inflationary scenario of the early universe including an accelerating phase of expansion at late time followed by a matter dominated phase. As a result, modified theory of gravity which contains both positive and negative powers of the curvature scalar (R) namely, $f(R) = R + \alpha R^m + \frac{\beta}{R^n}$ where α and β represent coupling constants with arbitrary constants,

m and n are considered for exploring a satisfactory cosmological model. It is known that the term R^m dominates and it permits power law inflation if $1 < m \leq 2$, in the large curvature limit. It may be mentioned here that an inflationary scenario driven by vacuum trace anomaly which corresponds to $m = 2$ and $\beta = 0$ was first obtained by Starobinsky [5] for describing early inflation. Recently, in the low curvature limit, a number of $f(R)$ models have been proposed in order to accommodate universe with late acceleration using a modified gravity namely, $f(R) = R - \frac{\lambda}{R^n}$ with $n > 0$ [87, 90]. In the metric approach, it has been shown that the model is not suitable because it does not permit a matter dominated era [96]. Recently, it is known that modified gravity namely, $f(R) = R + \alpha R^m$ is also not cosmologically viable because it does not permit a consistent scenario accommodating a matter dominated era at late time. In the above theory it is found that instead of matter dominated era, one ends up with a radiation era ($a(t) \sim \sqrt{t}$). On the other hand, R^m -model permits matter dominated universe ($a(t) \sim t^{2/3}$) but it fails to connect to a late accelerating phase. In *Ref.* [96], it was shown that the models of the types where lagrangian density, $f(R) = R - \frac{\lambda}{R^n}$ with $n > 0$ and $f(R) = \alpha R^m$ with $m \neq 1$ are not viable for a realistic cosmological scenario as they do not permit matter epoch even if it accommodate a late accelerating phase [97]. Recently, modified gravity with power law in R, i.e., $f(R)$ -gravity is examined and found that a large class of models including R^m-model does not permit matter dominated universe also. Capozziello *et al.* [98] criticized the claim made in *Ref.* [96]. Tsujikawa [99] derived observational signature of $f(R)$ dark energy models that satisfy cosmological and local gravity constraints fairly well. The modified $f(R)$-gravity is found to be consistent with realistic cosmology in some cases [100]. However, no definite physical criteria is known so far which could select a particular kind of theory capable of matching the data at all scales. However, modified gravity namely, $f(R) \sim e^R$; *or* $\log R$ may be useful to build a viable cosmological model as they permit a matter dominated phase before an accelerating phase of expansion. In this chapter cosmological solutions are obtained in a modified theory of gravity described by the non-linear terms in R in the Einstein-Hibert action. Different phases of expansion of the universe from early era to the present accelerating phase are explored, including prediction of the future evolution in the framework of higher derivative gravity. The corresponding field equation obtained from the above gravitational action is a fourth order differential equation of the scale factor $(a(t))$ of the universe obtained in the modified theory of gravity. As the field equation is highly non-linear and not simple enough to obtain an analytic solution we adopt here a numerical technique to solve it. The approach here is similar to that adopted earlier in *Ref.* [101] which was recently employed in *Ref.* [102]. In this approach the field equations are first expressed in terms of two functions namely, deceleration parameter (q) and Hubble parameter (H) and its derivatives, respectively, which are then solved numerically.

The chapter is organized as follows: in sec. 2.2, the relevant field equations in the modified theory of gravity are obtained, in sec. 2.3, the field equation is converted in terms of q and H and cosmological evolutions are predicted in different models depending upon the coupling parameters of the action adopting numerical technique. Finally in sec. 2.4, the results are summarized.

2.2 The Field Equations in Modified Gravity

Let us consider a gravitational action with non-linear terms in the scalar curvature (R), which is given by

$$I = -\int \left[\frac{1}{2}f(R) + L_m\right]\sqrt{-g}\, d^4x \tag{2.1}$$

where $8\pi G = c = 1$, g is the determinant of the four dimensional metric and R is the scalar curvature. Here $f(R)$ is a function of R and its higher power and L_m represents the matter lagrangian. Variation of the action (2.1) with respect to the metric yields

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T^c_{\mu\nu} + T^M_{\mu\nu}, \tag{2.2}$$

where $T^M_{\mu\nu}$ represents the contribution from matter fields scaled by a factor of $\frac{1}{f'(R)}$ and $T^c_{\mu\nu}$ denotes the contribution that originates from the curvature to the effective stress energy tensor. The energy momentum tensor $T^c_{\mu\nu}$ is given by

$$T^c_{\mu\nu} = \frac{1}{f'(R)}\left[\frac{1}{2}g_{\mu\nu}(f(R) - Rf'(R)) + f'(R)^{;\alpha\beta}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\nu}g_{\alpha\beta})\right], \tag{2.3}$$

where $(')$ represents the derivative with respect to the Ricci scalar (R). The role of the geometry alone in driving cosmological evolution is considered here, so we set $L_m = 0$ which leads to $T^M_{\mu\nu} = 0$. This will be used in the subsequent sections.

We consider an isotropic universe given by Robertson-Walker space-time

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{2.4}$$

where $a(t)$ is the scale-factor of the universe, the curvature constant $k = +1,\ 0,\ -1$ corresponds to closed, flat and open universe respectively. The scalar curvature is given by

$$R = -6\left[\dot{H} + 2H^2 + \frac{k}{a^2}\right] \tag{2.5}$$

where $H = \left(\frac{\dot{a}}{a}\right)$ is the Hubble parameter. For simplicity we consider here a flat Robertson-Walker spacetime and set $k = 0$. Using the metric (2.4) in the field eqs. (2.2) (see also Ref. [101]) we obtain the following equation :

$$3\frac{\dot{a}^2}{a^2} = \frac{1}{f'(R)}\left[\frac{1}{2}(f(R) - Rf'(R)) - 3\frac{\dot{a}}{a}\dot{R}f''(R)\right], \tag{2.6}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{f'(R)}\left[2\frac{\dot{a}}{a}\dot{R}f''(R) + \ddot{R}f''(R) + \dot{R}^2 f'''(R) - \frac{1}{2}(f(R) - Rf'(R))\right], \tag{2.7}$$

where an overdot indicates derivative with respect to the cosmic time t. The Ricci scalar (R) involves second order time derivative of the scale factor $(a(t))$. As the eq. (2.7) contains $\ddot{R}$ terms, consequently it is a system of fourth order differential equations of scale factor $(a(t))$. In the next sections we consider theories of gravity described by $f(R)$ of the forms: (I) $f(R) = R + \alpha R^2 - \frac{\mu^4}{R}$, (II) $f(R) = R + b\ ln(R)$ and (III) $f(R) = R + m\ e^{[-nR]}$ to explore cosmic evolution. In the above, μ , α , b , m and n are constants and μ has a dimension of $R^{\frac{1}{2}}$ [65] i.e. that of $(\text{time})^{-1}$, α has a dimension of R^{-1} i.e. $(\text{time})^2$.

2.3 Cosmological Solutions in Modified Gravities

Using eqs. (2.6) and (2.7) we obtain

$$\dot{H} = \frac{1}{2f'}\left[(H\dot{R} - \ddot{R})f'' - \dot{R}^2 f'''\right], \tag{2.8}$$

where $H = \left(\frac{\dot{a}}{a}\right)$ is the Hubble parameter. As both R and H are functions of $a(t)$ and its derivatives, eq. (2.8) is highly non-linear and a differential equation of fourth order in scale factor ($a(t)$) results. The field eq. (2.8) is highly non-linear and it is not simple to determine solution of the scale-factor of the universe in terms of known functions. A numerical technique is adopted here to explore viable cosmological solutions. A class of cosmological models based on different parameters of the modified gravitational action are permitted which will be discussed in the next section. Considering three different forms of $f(R)$ in the next sections to study cosmological evolution of the early, late and future evolution of the universe.

2.3.1 *Case I* $f(R) = R + \alpha R^2 - \frac{\mu^4}{R}$

In this case we consider modified gravitational action where $f(R)$ is given by

$$f(R) = R + \alpha R^2 - \frac{\mu^4}{R}. \tag{2.9}$$

The corresponding field eq. (2.8) now can be expressed as

$$\dot{H} = \frac{1}{1 + 2\alpha R + \frac{\mu^4}{R^2}} \left[\frac{\mu^4}{R^2} \left(\frac{\ddot{R}}{R} - \frac{H\dot{R}}{R} - 3\frac{\dot{R}^2}{R^2} \right) + \alpha(H\dot{R} - \ddot{R}) \right]. \tag{2.10}$$

The above equation is highly non-linear; however, it is possible to obtain asymptotic solutions corresponding to different epoch which accommodates

(i) an exponential expansion $a(t) \sim e^{H_o t}$ is permitted when $q = -1$ in the early era,

(ii) an accelerating universe with $a(t) \sim t^2$ at a later epoch admitting $q = -\frac{1}{2}$.

The variation of q with the exponent D related to the evolution of the scale factor ($a(t) \sim t^D$) is plotted in fig. (2.1). In fig. (2.2) a variation of Hubble parameter (H) with time for different D is plotted. The deceleration parameter (q) is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\dot{H}}{H^2} - 1. \tag{2.11}$$

Since, q is a function of H and its derivative, we can re-write the eq. (2.10) in terms of a second order differential equation in q and H to begin with. Since q contains terms with $\ddot{a}$, it is possible to replace terms with fourth order derivative of the scale factor in eq. (2.10) by $\ddot{q}[H]$. The functions q and H are, however, not independent. The time derivatives in the above equations may now be replaced by the derivatives with respect to H using eq. (2.11). The following non-linear differential equation is obtained

$$q'' + u(q,H)q'^2 + v(q,H)q' + w(q,H) = 0 \tag{2.12}$$

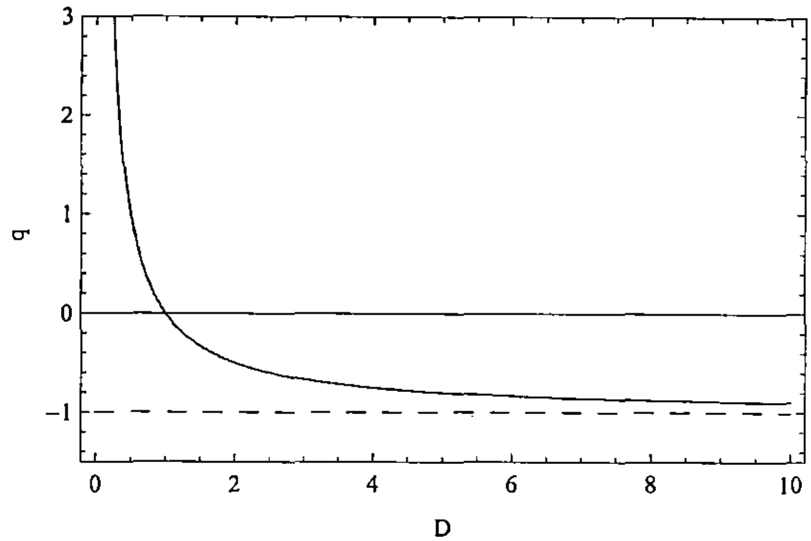

Figure 2.1: shows the plot of q vs D, where the dashing line is for exponential expansion.

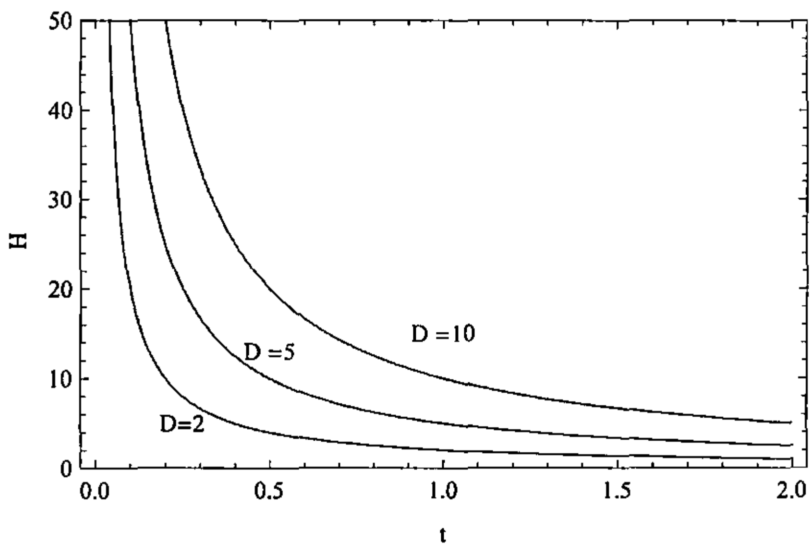

Figure 2.2: shows plot of H vs t for different value of D.

with

$$u(q,H) = -\frac{(2q+4)\mu^4 + 216\alpha(q-1)^4 H^6}{(q^2-1)[\mu^4 - 216\alpha(q-1)^3 H^6]},$$

$$v(q,H) = -\frac{(4q+7)\mu^4 + 216\alpha(8q+5)(q-1)^3 H^6}{(q+1)H[\mu^4 - 216\alpha(q-1)^3 H^6]},$$

$$w(q,H) = \frac{(q-1)[3\mu^4(2q+1) + 1296\alpha(q+1)(q-1)^3 H^6 - 36(q-1)^2 H^4]}{(q+1)H^2[\mu^4 - 216\alpha(q-1)^3 H^6]},$$

where the prime indicates a differentiation with respect to H and the functions u, v and w depend on the parameters α and μ^4. The above equation, although highly nonlinear, is a second order differential equation in q. Here both q and H are time dependent and cannot be solved exactly to obtain a known functional form. In this case the field equation are solved numerically following the approach adopted in Ref. [101, 102]. As $\frac{1}{H}$ is a measure of the age of the universe and H is a monotonically decreasing function of the cosmic time, eq. (2.12) may be used to study qualitatively the evolution of the universe in terms of q. Since eq. (2.12) is a second order differential equation, to solve it numerically two initial values (here, $q[H]$ and $q'[H]$), for a given value of H are assumed. Here, H_o represent the present value of H, is considered unity and pick up sets of values of q and q' for $H = 1$ (i.e. the present values) from the observationally consistent region [84, 103]. Variation of q with H is plotted for different configuration of the system. As the inverse of H gives an estimate for the cosmic age, future evolution is understood from the region $H < 1$ and the past from $H > 1$ in the (q-H) phase plane. Since the present universe is accelerating, we use a negative q at the present epoch, $H = H_0 = 1$ and the universe is consider to enter into this q negative phase (i,e, acceleration) of expansion, only in the recent past. The model may be useful to predict satisfactory the future course of the evolution of the universe. From the graphical plot of q vs H, the following points are noted:

(i) The variation of q with H is shown in fig. (2.3) for a given value of α, μ, $q[1]$ and $q'[1]$. In the fig. (2.3), H is plotted along the horizontal axis which implies that the time increases from right to left along the horizontal H axis. The upper half of the axis represents decelerating phase as $q > 0$ and the lower half represents the accelerating phase as $q < 0$ of the universe. It is evident that the universe entered into the present accelerating phase in the recent past, and the rate of acceleration will increase further

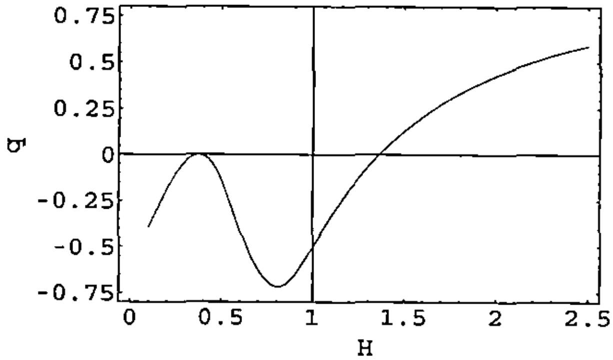

Figure 2.3: shows the plot q vs H for $\mu^4 = 12$ and $\alpha = 2$. Here we choose the initial conditions as $q[1] = -0.5$, $q'[1] = 1.655$.

to attains a maximum, thereafter it decreases. We note a diminishing tendency of the cosmic acceleration leading to an epoch when the universe expands without acceleration (which is transient and occurs at $H = 0.375$) followed by another phase of expansion. In this case, there exists another phase of expansion leading to accelerating universe once again. The universe transits from deceleration to acceleration phase at $H = 1.36$. We note that, in this case, the universe remains in the accelerating phase once it transits from decelerating phase.

(ii) the variation of q with H is shown in fig. (2.4) for different initial values of $q'[1]$. It is evident that the universe entered into the accelerating phase in the recent past followed by another phase of deceleration. The duration for which the universe transits from the present accelerating phase depends on the initial values of $q'[1]$ and the duration increases with increasing initial values of $q'[1]$. We note the following: (a) for $q'[1] = 1$ the universe transits from deceleration to acceleration at $H = 1.43$ and acceleration to deceleration at $H = 0.587$; (b) as initial values of q' is increased, the critical point shifts toward the left, i.e. it occurs at a later epoch.

(iii) In fig. (2.5) q Vs. H is plotted for different values of α. The figure shows dependence of the evolution of the universe on the coupling parameter (α). It is evident that there might be another sign change of q in the near future. It is also evident that the universe

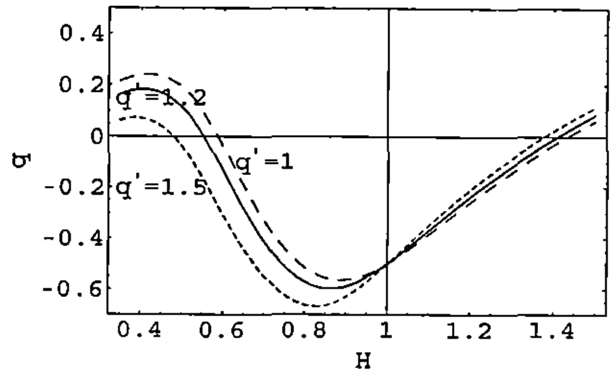

Figure 2.4: shows the plot of q vs H for different value of initial conditions $q'[1]$ with $q[1] = -0.5$, $\mu^4 = 12$ and $\alpha = 2$.

transits from decelerated phase to accelerated phase only in the recent past. The universe may transit once again to a decelerating phase followed by another accelerating phase if $\alpha > 0.815$ otherwise it will be always in the accelerating phase. However, the rate of acceleration depends on the coupling constant α. It is evident that as the values of α is increased the corresponding duration of the present accelerating phase diminishes, however, the late decelerating phase in future enhances. The universe in the remote future once again might enter into an accelerating phase as there will be one more sign flip in q. The plot for q vs H with $\alpha = 0.815$ is interesting as it decides whether the universe will transit to another phase of acceleration or not.

(iv) The variation of q with H for different values of coupling constant μ^4 is shown in fig. (2.6). It is evident that if the inverse Ricci scalar term in the action is absent, then it permits a universe which transits from a decelerating phase to an accelerating phase in the recent past ($H = 1.4$), allowing a further sign flip in q leading to a smooth transition from an accelerating to decelerating phase only. However, for $\mu^4 \neq 0$, an interesting evolutionary behaviour of the universe with three sign flips of q leading to a universe accommodating a transition from accelerating to decelerating phase followed by another phase transition from decelerating to accelerating phase and subsequently in future transition from deceleration to acceleration phase may result. However, the

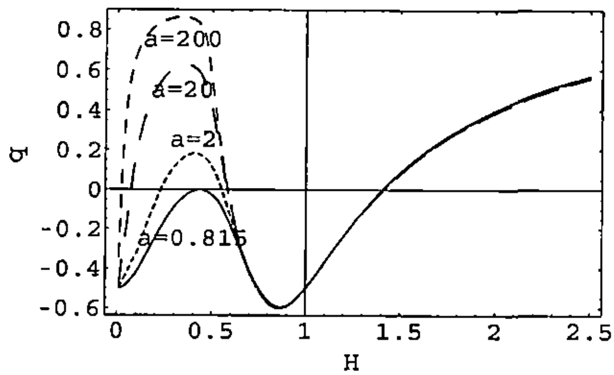

Figure 2.5: shows the plot of q vs H for different value of α (a represents α) with $\mu^4 = 12$, $q[1] = -0.5$, $q'[1] = 1.2$.

rate of acceleration changes with μ. As μ^4 increases, the period for the transition from deceleration to another acceleration phase in recent future is found to decrease and finally it vanishes at $\mu^4 = 26.46$.

Special Case : As a special case, we set $\alpha = 0$ and the Lagrangian polynomial becomes i.e., $f(R) = R - \frac{\mu^4}{R}$. Equation (2.12) now takes the form of a second order differential equation given by

$$q'' - \frac{2q+4}{q^2-1}\, q'^2 - \frac{(4q+7)}{(q+1)H}\, q' - \frac{3(q-1)(2q+1)}{(q+1)H^2} + \frac{36(q-1)^3H^2}{\mu^4(q+1)} = 0. \tag{2.13}$$

It may be mention here that similar case was considered by Das *et al.* [101]. Comparing the equation obtained by Das *et al.* at eq. (12) in Ref. [101] that with eq. (2.13) we note that some of the terms are missing as a result it leads to an incomplete conclusion. In this case q vs H curve is plotted in fig. (2.7) consequently a number of new phases of expansion is permitted differing significantly. The universe in the past may have started from a constant decelerating phase which then transits to an accelerating phase, thereafter once again because of sign flip in q, the universe may transit to a decelerating phase. For $\mu^4 = 0.01$, it is evident that the universe follows a constant decelerating phase and thereafter one sign flip in q is found. As μ^4 is increased the corresponding duration of the present accelerating phase is found to increase. For a small value of μ, the accelerating phase is found to be diminish. However, there exists another epoch where $q = 1$ is attained

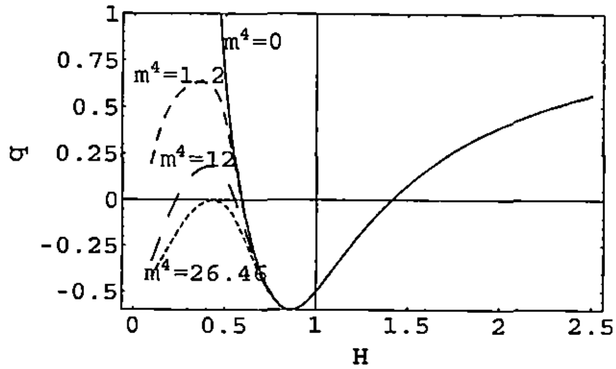

Figure 2.6: shows the plot of q vs H for different value of μ (m represents μ) with $\alpha = 2$, $q[1] = -0.5$ and $q'[1] = 1.2$.

before and after the present phase of acceleration. In all the cases the universe will end up with an accelerating phase having $q = -0.5$. As μ^4 increases, the time at which the universe enters for a transition from deceleration to acceleration phase advances and that from acceleration to deceleration found to occur at a later time. In this case the universe once again transits from deceleration to acceleration and this will occur at an earlier epoch as μ^4 is increased.

2.3.2 *Case II* $f(R) = R + b\, ln(R)$

In this case we consider higher derivative theory much discussed in recent times which has the form namely, $f(R) = R + b\, ln(R)$, to look for a physically relevant cosmological model. The field eq. (2.8) is then transformed into a second order differential equation in q, which is given by

$$q'' - \frac{q+3}{q^2-1}\, q'^2 - \frac{3}{(q+1)H}\, q' + \frac{2(q-1)^2}{(q+1)} \left[\frac{6}{b} - \frac{1}{H^2}\right] = 0. \tag{2.14}$$

The above equation is highly nonlinear, consequently we adopt a numerical technique to study the evolution of the universe as was done in *Case I*. Here we look for the evolution for different values of the coupling parameter b in the action. The curves in fig. (2.8) are plotted for different coupling constants b which are new and interesting from cosmological

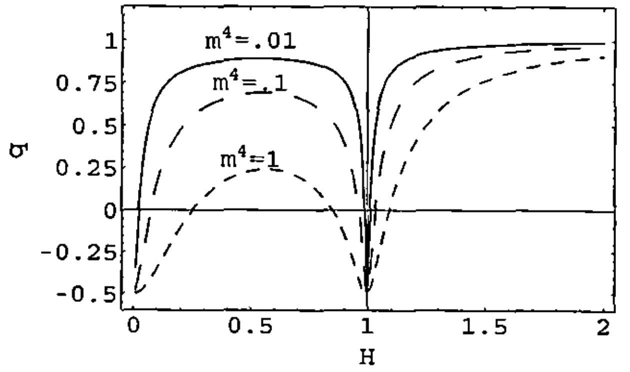

Figure 2.7: shows the plot of q vs H for different value of μ^4 (m represents μ). Here we choose the initial conditions as $q[1] = -0.5$, $q'[1] = 1.2$.

point of view. We note that the universe transits from decelerating phase to an accelerating phase in the recent past which thereafter might enter into decelerating phase once again in future. For $b < 0$ the duration for accelerating phase is found to shorten than that for $b > 0$. However, for b positive the duration of the accelerating phase is found to lengthen if b is smaller. In the case of a negative b, the universe is found to land up at a maximum possible acceleration at the present epoch, thereafter the rate of acceleration diminishes . Consequently the universe may transit to decelerating phase once again. For $b > 0$, it is evident from the plot that the rate of expansion of the universe will attain a maximum in near future.

2.3.3 *Case III* $f(R) = R + m\, e^{[-nR]}$

In this case we consider gravitational action given by $f(R) = R + m\ e^{[-nR]}$ [104]. Consequently, using eq. (2.8), we obtain the following second order differential equation in deceleration parameter and Hubble parameter:

$$q'' - \frac{1 - 6n(q+1)H^2}{q+1}\, q'^2 + \frac{8q + 5 - 24n(q^2-1)H^2}{(q+1)H}\, q' +$$

$$\frac{2(q-1)(3q+4)}{(q+1)H^2} - 24n(q-1)^2 - \frac{1}{3n^2 m(q+1)H^4}\left[e^{nR} - n\, m\right] = 0. \tag{2.15}$$

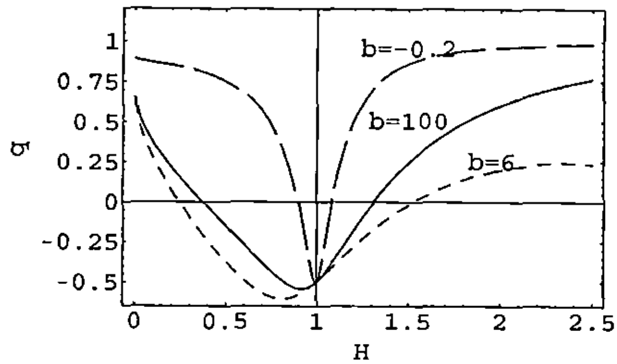

Figure 2.8: shows the plot of q vs H for different value of b with $q[1]$=-0.5, $q'[1]$=1.

The eq. (2.15) is highly non-linear, we adopt numerical technique to solve it as was done in earlier sections. We study qualitatively the evolution of the universe as follows:

(i) The variation of q with H for different values of n, a parameter in $f(R)$, is plotted in the fig. (2.9). It is evident that the epoch of transition of the universe from a decelerating to an accelerating phase depends on n. As n decreases, the time at which universe transits from decelerating to accelerating phase advances near to the present epoch. It is found that the duration of the present accelerating phase decreases with a decrease in n. When n is increased duration between two consecutive deceleration phases is found to increase.

(ii) The variation of q with H for different values of m, a parameter in $f(R)$, is plotted in fig. (2.10). It is evident from the plot that the universe at the present epoch entered from a decelerating to accelerating phase and subsequently the universe may switch over to decelerating phase once again in future. The smaller values of m leads to a shorter duration for the present accelerating phase.

From $(q - H)$ curve as drawn in fig. (2.4), we obtain sufficient data set from the numerical plot using MATHEMATICA [105]. Those numerical values may be used to find an analytic mathematical structure for q and H which can be determined using a polynomial function given by $q = \Sigma_0^n a_i H^i$. Using the polynomial relation we determine the mathematical function for $q(H)$ corresponding to fig. (2.4) with initial value $q'[1] = 1.2$. The corresponding approximate analytic function may be expressed as

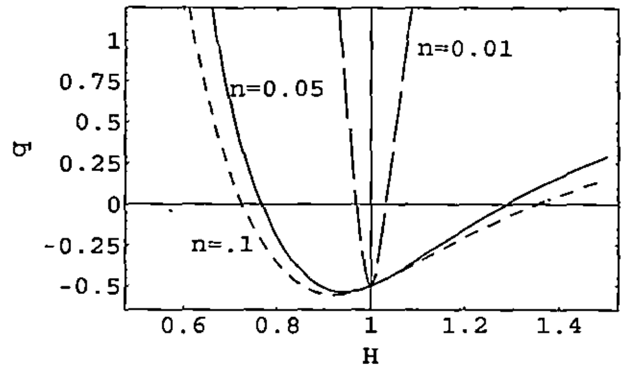

Figure 2.9: shows the plot of q vs H for different value of n with q[1]=-0.5, q'[1]=1 and $m = 4$.

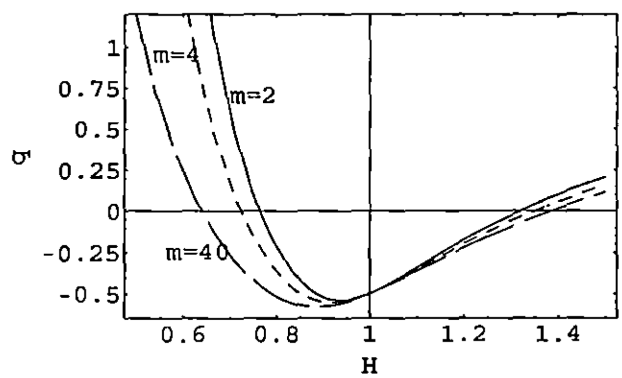

Figure 2.10: shows the plot of q vs H for different value of m with q[1]=-0.5, q'[1]=1 and $n = 0.1$.

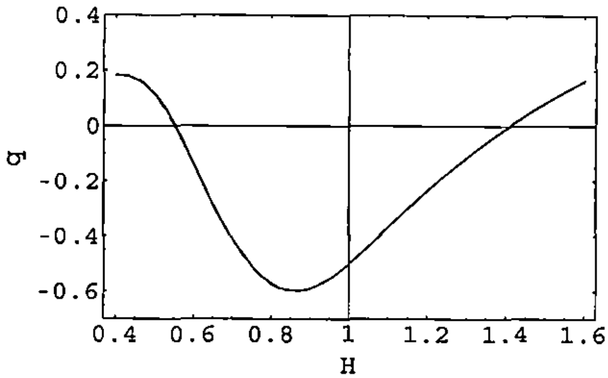

Figure 2.11: shows the dashed curve from the polynomial expression and the solid curve from fig. (2.4) with $q'[1] = 1.2$.

$$q[H] = 45.06381 - 515.24544H + 2478.58763H^2 - 6555.42602H^3 + 10556.26293H^4$$

$$-10835.40545H^5 + 7153.27419H^6 - 2949.71637H^7 + 693.08477H^8 - 70.97952H^9. \quad (2.16)$$

Thus an approximation solution for q is shown in eq. (2.16). We compare the curves obtained numerically with that corresponding to the curve fitted to an analytic function given in eq. (2.16). The two curves are shown in fig. (2.11) which is found exactly superimposed. This comparison holds good only when H is reasonably close to one. Thus in this section an outline is given to determine the approximate relation between q and H, which may be employed for determining solution for q for other curves also.

2.4 Discussion

In this chapter cosmological solutions are obtained in higher derivative theories of gravity without a cosmological constant. Three types of polynomial in scalar curvature (R) are taken up as toy models to study evolution of the universe. A number of phases of expansion in higher derivative theories of gravity without matter are obtained which are new and interesting for cosmological model building. As the field equations obtained from the gravitational action are highly non-linear it is not simple to obtain analytic solution in a closed form. Therefore a technique is adopted to solve numerically to understand the present and future evolution of the universe here. For this purpose the relevant field equations corresponding to each modified gravitational theory taken up here are rewritten in terms of two parameters namely, (i) Hubble parameter (H) and (ii) deceleration parameter (q) and its derivatives. The corresponding field equation in q is a second order differential equation which can be solved for initial conditions on q' and q. The variation of q and H are plotted for different values of the coupling constants of the higher derivative gravitational actions namely, (I) α, μ; (II) b; and (III) m and n, which are taken up in Sec. 2.3.1, 2.3.2 and 2.3.3 respectively to investigate present and future evolution of the universe. It is found that all the modified theories of gravity permit a cosmological scenario accommodating the present accelerating phase which is followed by a decelerating phase. The decelerating phase is important for matter creation in the universe as we know matter dominated / radiation dominated universe corresponds to a decelerating phase. The plot of q vs H shows that the universe might once again transit from the accelerating phase to another decelerating phase. The duration of the present accelerating phase is found to depend on coupling parameters in the gravitational action. In figs. (2.4)-(2.10), it is evident that the present accelerating phase of the universe might end and subsequently it will transit to a decelerating phase in future. In figs. (2.5)-(2.7) a number of evolutionary phases are noted with remarkable features. The plot shows that the universe transits from the present accelerating phase to a decelerating phase and then the decelerating phase end up leading to a change in evolution admitting another accelerating phase which finally ends up at $q = -1/2$. It is evident in $f(R) = R + \alpha R^2 - \frac{\mu^4}{R}$, theory for different α, μ values. In figs. (2.3), (2.5) and (2.6), it is evident that the present rate of acceleration of the universe decreases, which finally reach attains vanishing value, where the universe grows linearly with time as $a(t) \sim t$ for a sufficient period. Thereafter the universe once again may transit to an accelerating phase will an increasing rate of expansion.