CHAPTER 1

Introduction

1.1 Brief review

The modern cosmology began when the Einstein's general theory of Relativity (in short, GTR) was applied to understand the large scale structure of the universe. In 1917, Einstein wanted to obtain a static cosmological model of the universe in the framework of his new theory, as during that time information of the entities within the galaxy only was known to the astronomers, which showed static model of the universe. However, Einstein failed to obtain a static cosmological model with positive pressure and energy density in the frame work of GTR, with perfect fluid. Consequently, Einstein modified his field equations by introducing a repulsive term (cosmological constant) in order to obtain a static universe. In the absence of cosmological constant, an expanding universe solution was obtained by Friedmann in the year 1922, supposed to have only an academic interest at that time. Later, Hubble's discovery (1927) of redshift of the spectral lines of light from galaxies made it clear that the universe indeed is expanding. The cosmological constant introduced by Einstein, therefore, became redundant. Relativistic cosmological models based on perfect fluid assumptions then came up to accommodate an expanding universe solution. One of the acceptable model of the universe, known as Big Bang model is, therefore, obtained. It is, however, well known that the Big Bang model is also not free from problems fully. It is also understood that perfect fluid assumption fails to account for some of the observed facts of the universe. A semiclassical theory of gravity where space-time is described classically but matter by quantum fields may be important to address some of the issues in cosmology. The Particle Physics theories which are relevant at very high energy in the standard model may be important in cosmology to understand the dynamics of the universe as such huge energy perhaps was available shortly after the

Big Bang. It has been realized that the standard Big Bang model is fairly successful in explaining the 2.7K ***Cosmic Microwave Background Radiation (CMBR)***, the expansion of the universe, primodial nucleo synthesis, and the cosmic abundance of the light elements the weakly bond deuteron and ^{4}He (about 23% by mass). Nevertheless the model could not be considered as an acceptable model of the universe because it fails to explain some other observed features of the universe, namely; the horizon problem, the flatness problem, the small scale inhomogeneity, singularity problem etc.

Modern cosmology is based on the overlapping areas of Particle Physics and Cosmology. It permits a universe with rapid expansion in a very short time, known as inflation, which solves some of the outstanding problems in cosmology [1, 2]. Inflation opens up new avenues both in Particle Physics and Cosmology. Consequently it is now accepted that inflation is one of the essential ingredient to build a consistent cosmological model of the early universe. In the framework of inflationary universe scenario, the prediction of primordial density fluctuations which arise from Particle Physics scenario is in agreement with the result obtained from the analysis of ***Cosmic Background Explorer (COBE)*** data [3]. The observations from COBE predict that the present universe might have emerged from an early inflating phase in the past and then it settles down into the matter dominated phase through an intermediate radiation dominated phase. Several mechanisms have been proposed to generate the rapid expansion needed to solve some of the outstanding issues in Cosmology and in Particle Physics. Gliner [4] was the first to obtain inflation in GTR for an appropriate stress-energy tensor corresponding to matter with the properties of a vacuum. Later, inflationary solution using trace anomaly in GTR was obtained by Starobinsky [5]. However, the efficacy of the theory is known only after the seminal work done by Guth [1] in 1981. Guth in his original work employed temperature dependent phase transition mechanism in cosmology and had shown that necessary conditions favouring an inflationary universe might be realized easily. Guth's [1] model of inflation requires a universe trapped in a metastable state (supercooled) initially which thereafter decays through a process of bubble nucleation via quantum tunneling. In the model bubbles of true vacuum may be spontaneously formed in a sea of false vacua which thereafter rapidly expanded at the speed of light to accommodate a baby universe. However, it was realized later [6] that the above model of the universe fails to reheat the universe properly in its subsequent evolution to accommodate the present universe. In case inflationary epoch lasts long enough, it could solve the initial condition problem satisfactorily, but the collision rate between bubbles in this scenario is found to be exceedingly low which does not lead to a viable cosmological model. It has been realized that inflationary model based on a first order phase transition, could not provide a satisfactory explanation of how to exit from the inflationary phase without disturbing the good properties of the standard cosmological models [6, 7]. Consequently the problem of *graceful exit* of an inflationary universe was studied in a new model which is known as *'new inflation'* [2]. In the new inflation model a rapid expansion in a very short duration is obtained using a homogeneous scalar field, when it rolls down the potential energy hill instead of tunneling out of a false vacuum state. When the field rolls down along the potential slope, it vary very slowly compared to the expansion rate of the universe, leading to inflation. However, if the hill becomes steeper inflation ends and subsequently at the minimum of the potential because of the oscillations of the scalar field the universe reheats. It was shown that although new inflation does not produce a perfectly symmetric universe, a tiny quantum fluctuation of the scalar field that originates during this phase of expansion leads to the observed structure of the universe. Later Mukhanov and Chibisov [8] show that the quantum fluctuations of the scalar field originated during the evolution of the early universe might act as the primordial seed for the structure formation of the universe. ***Cosmic Microwave Background Radiation (CMBR)*** data supports the theoretical prediction that was obtained for structure formation, in the inflationary model.

Due to the shortcoming in the *new inflation*, cosmological models with second order phase transition mechanism have been employed to obtain a viable cosmology. However, these models are found to be plagued with several fine tuning problems [9]. The inflationary models of the early universe including the new inflationary model [2] makes use of a phase transition mechanism which require some fine tuning either in the potential $V(\phi)$ of the inflaton field ϕ or in the initial conditions of the universe. In 1983, Linde [10, 11] proposed a new model (known as the ***chaotic model***) to obtain an inflationary universe scenario which apparently need no specific fine tuning or phase transition mechanism. In the chaotic model, a scalar field is still needed but the initial state of the field is required to be non-thermal. The advantage of the model is that it is not essential to restrict to a particular initial configuration for the field,($\phi_i = 0$) as was required in the *new inflation* model. The homogeneous scalar field instead can take any value permitted by a random initial distribution of the field satisfying the constraint $V(\phi) \leq M_p^4$m imposed by Planck scale. The upper bound on potential is essential so that the quantum behaviour of gravity does not dominate and a classical description of the space-time still remains valid. Since the initial data for the model are randomly distributed, the scenario is known as the chaotic inflationary scenario. Thus, Linde's ***chaotic model*** is based on a random distribution of the values of the vacuum scalar field (also called inflaton field) ϕ at the time $t \sim t_p = M_p^{-1}$, where t_p represents the Planck time and M_p represents the Planck mass.

An inflationary universe scenario may be realized in a semiclassical theory where gravity is described classically while matter interns of quantum fields. It may be noted here that the necessary equation of state for inflation with a scalar field is permitted when its potential energy dominates over the kinetic energy. During this epoch, the vacuum energy behaves like an effective positive cosmological constant, which yields a huge expansion of the universe in a very short time. It is also shown [12] that the scenario may be realized even when ϕ is initially highly fluctuating ($\frac{1}{2}\dot{\phi}^2 >> V(\phi)$). Papantonopouls *et al.* [13] have shown that a favourable condition for inflation ($\frac{1}{2}\dot{\phi}^2 << V(\phi)$), may emerge quite naturally if an axion field ϵ is considered which has only derivative coupling to the field ϕ. Linde further assumed that subject to the condition $V(\phi) \leq M_p^4$ all the values of ϕ are equally probable to begin with, however, the problems of big-bang model can be solved satisfactorily in this framework if the observed part of the universe emerged out from a region having scalar field value initially $\phi_i(t = M_p^{-1}) \geq 3M_p$. The lower bound on ϕ ensures that there is sufficient inflation to solve the problems of Big Bang model. Subsequently it was also shown by Paul *et al.* [14] that Linde's chaotic model is fairly general and it can be extended even for a universe with initial anisotropy.

The quantum fluctuations of the field generated during inflation are capable of explain large scale structure formation of the universe. The fluctuation may be increasing or decreasing, consequently in the model in addition to chaotic inflation another type of inflationary scenario may be realized. In the case when the quantum fluctuation helps the field to grow, that in turn it corresponds to a field which is moving up the potential leading to a self reproducing universe. This behaviour of the scalar field admits a new scenario of the universe known as ***Eternal chaotic inflation*** [15].

In addition to exponential growth of the scale factor of the universe as discussed above leading to inflation another kind of inflation is characterized by a period in which scale factor of the universe $a(t)$ grows as t^D with $D \gg 1$, which is known as power law inflation. It may be mentioned here that inflationary universe scenario with exponential expansion is more attractive as it leads to a scale invariant Zeldovich density spectrum [16]. However, the later type i.e., power law inflation is more simple, which helps to explore easily different aspects of the universe. Power law inflation may be realized with Salam-Sezgin [17] exponential potential which usually originates in the context of string theory. Cosmological inflation without scalar field may be realized, if one considers viscous fluid. There are various reasons [18] to consider viscosity in the early universe. The important of the viscous universe is that it permit both the exponential and power law inflation [19]. The effective energy momentum tensor for an imperfect fluid changes in such a way that it leads to a negative pressure accommodating inflation. The origin of such an imperfect fluid may be understood in terms of various dissipative processes which might have played an important role in the evolution of the early universe. The various processes that may be responsible for viscosity in the early universe are that the decoupling of neutrinos during the radiation era, the decoupling of matter from radiation during the recombination era, creation of superstrings during the quantum era, particle collisions involving gravitons, cosmological quantum particle creation processes and during the formation of galaxies [18]. Thus it is also important to take up non-equilibrium thermodynamical processes in cosmology which might have played a crucial role in physics of the early universe.

A large number of inflationary models [1, 2, 4, 5, 6], [10]-[15], [20]-[40] in the context of ever changing fundamental theories have came up during the last three decades. The attractive idea of inflation was taken up to build up viable cosmological models and tested in a number of theories formulated in the last three decades leading to different cosmological models namely,

• (1980-1989): R^2 inflation [5], Old inflation [1], New inflation [2], Chaotic inflation [10, 11], Double inflation [20], Power law inflation [21], SUGRA inflation [22], Extended inflation [23].

• (1990-1999): Hybrid inflation [24], SUSY D-term inflation [25], Assisted inflation [26], Brane inflation [27]-[29].

• (2000-2011): Supernatural inflation [30], K inflation [31], D3-D7 inflation [32], DBI inflation [33], Racetrack inflation [34], Tachyon inflation [35], Hill top inflation [36], Landscape model [37], String model [38], Loop quantum cosmology [39], Emergent universe [40].

At present there seems to be no alternative to inflationary scenario for the early universe description. But in spite of all the attractive features of cosmological inflation, its mechanism of realization still remains *ad hoc*. It is not known when and how the universe has entered into the inflationary phase. However, an acceptable model of the universe which can explain the evolution of the universe right from the quantum gravity regime is yet to come up. As inflation might operate at Planck regime, it is interesting to explore cosmological models in the context of string theory which permits such a scale. It is, therefore, not surprising that M/String theory inspired models are under active consideration in cosmology at present mostly because of some interesting discoveries [27, 41]. The striking discovery that in ten dimensions, a supergravity theory coupled to Yang-Mills fields with a gauge group SO(32) or $E_8 \times E_8'$ is anomaly free has inspired considerable activities in this area. Since the quantum consistency of the theory is obtained in the critical dimension D=10, one has to look for a realistic compactification scheme. Candelas *et al* [42] set out to achieve this by requiring that the ten dimensions should compactify to $M_4 \times K$, where M_4 is maximally symmetric and K is a compact six dimensional manifold. It was also demanded that the four dimensional theory should lead to a realistic chiral fermion spectrum and should also have an unbroken $N = 1$ supersymmetry, so that hierarchy problem can be tackled. Candelas *et al* [42] obtained a solution with Minkowskian

($\lambda = 0$) space M_4 and K as a Ricci flat CalabiYau manifold, with SU(3) holonomy. While the discovery of Candelas *et al* [42] is striking, the success of the compactification scheme partly rests on its ability to be accommodated in a realistic dynamical theory of evolution. However, attempts [43]-[46] to build cosmological models based on the $N = 1$ Yang Mills supergravity action have not been successful. Weiss [47] has even questioned the consistency of a Ricci flat compact manifold with observed matter dominated universe. It is not easy to get inflation in this model. The difficulties stem from the Ricci flatness of the internal space and also from the absence of any dimensionless free parameter. In the absence of SUSY breaking, the naive potential has only one minimum at $g^2_{GUT} = 0$ and the realization of the cosmological evolution of the compactified theory to the correct vacuum, $g^2_{GUT} \sim 0(1)$, is a nontrivial problem. The available mechanism for SUSY breaking is to invoke gaugino condensation in the hidden E'_8 sector of the $E_8 \times E'_8$ theory. Ellis *et al* [43] have considered the tree level potential with some fine tuning of the condensation temperature and obtained an inflationary phase in the D=4 effective theory. However, the one loop effective potential does not have a minimum and the situation is not very clear. Maeda *et al* [44] have considered another SUSY breaking potential, given by Binetruy and Gaillard [48], which has only asymptotic validity. It is clear that further investigation is called for to understand the detailed dynamics both the processes, compactification from ten to four dimensions and a subsequent inflation in four dimensions.

Recently, there is a paradigm shift in implementing a cosmological model in the higher dimensions. The concept of extra dimension entered into Physics in an attempt that tried to unify electromagnetism and gravity as a single theory [49] before the advent of GTR was known. It is also realized that both GTR and quantum mechanics are not compatible with each other at high enough energies and at very small distances. There have been a number of attempts to consider quantization of gravity but a consistent quantum theory of gravity is yet to emerge. Consequently understanding the dynamics of the universe in higher dimensions is of considerable interest at the present time. A number of cosmological models in the framework of higher dimensions have also been proposed. A universe with product space with different dynamics for the evolution of the usual four dimensional scale factor and that of extra dimensions have been proposed [46, 48, 50, 51]. The following behaviour for the extra dimensional scale factor are considered: the extra dimensional scale factor (i) remains constant, (ii) decreases with time (iii) initially increases thereafter decreases and stop by some unknown quantum gravity effect not known, were taken up to build a consistent higher dimensional model of the universe. The earlier attempts [51, 52] fails to provide a consistent scenario of the universe because of unattractive evolution of the extra dimensions.

The recent idea is that the universe is a (3+1) dimensional brane embedded in a higher dimensional spacetime [53]. The brane world scenario is now taken up widely to construct a viable cosmological scenario where the usual matter field and force except for gravity are confined on the brane [27, 28]. In this new picture all the matter fields confined to the brane whereas gravity can propagate in the bulk. The scenario admits interesting cosmological implications, in particular, the prospects of inflation are enhanced on the brane due to a major modification to the Friedmann equation. While discussing the applications on the brane world, one often assumes Einstein gravity in the bulk and then projects the dynamics on to the brane. This leads to a high energy correction to the Friedmann equation leading to a modification in the expansion dynamics of the early universe. In the theory, gravity is regarded as higher dimensional, which reduces to an effective four dimensional theory. The brane world scenario not only resolves some of the longstanding problems of the big bang model but also opens up possibilities of solving the hierarchy problem in particle physics by considering large number of compactified extra dimensions making use of the string scale, that might become accessible to future laboratory experiments [28]. In the string theory, it is necessary to include the higher order curvature invariant terms to the Einstein-Hilbert (in short, EH) action, as the higher order terms occur naturally in the small slope expansion limit.

In the above, an inflationary phase is generally driven by a potential or vacuum energy of a scalar field and the inflaton, whose dynamics is governed by Klein-Gordon equation [54]. Recently, motivated by string theory, other non-standard scalar field actions have been used in cosmology. In the string theory higher order curvature terms are also permitted in the gravitational action which leads to a modified theory of gravity. One of the major mechanism to obtain inflation is to consider a modified theory of gravity e.g., R^2

gravity adds terms quadratic in the curvature scalar to the standard EH action. For a certain range of initial parameters, the quadratic terms will generate sufficient expansion without the need for the special matter term used in most inflationary mechanisms.

The possibility of having inflation without any inflaton field was first predicted by Starobinsky [5]. It was shown that the addition of some terms (e.g., R^2) to the EH action permits a de Sitter solution. In fact those terms may be obtained in the effective action of gravity as a result of integrating out conformal free matter fields. The possibilities for the R^2 theory to produce successful inflation was studied by Starobinsky [5], Vilenkin and others [55] in different contexts. The justification for adding the quadratic terms in the Einstein-Hilbert action is usually two folds. Firstly renormalization of quadratic field theories in curved space time usually admits such terms [56] although with coefficient much smaller than that required for a fully satisfactory inflationary picture. Secondly it is not clear why EH action should be linear in R only. Perhaps, nature does contain second order, cubic order or higher order in R -terms which are observationally not detected at our present epoch [57]. It was shown by Starobinsky [5] that an inflationary solution may be obtained using trace anomaly in the Einstein's field equation, which is equivalent to a $R + \alpha R^2$ theory. Zwiebach [58] has shown that string corrections due to Einstein action up to first order in slope parameter and fourth power of momenta should be proportional to Gauss-Bonnet (GB) terms (where GB $= R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$). However, it was realized subsequently that the field redefinition theorem of G.'t Hooft and Veltman [59] may be applicable in this case. On the Einstein's shell ($R_{ab} = 0$), an action with curvature square term of the form $R + \alpha R^2 + \beta R_{ab}^2$ can be transformed into R itself (neglecting higher order terms) by the field redefinition $g_{ab} = g_{ab} + \beta R_{ab} + g_{ab}\frac{\beta+2\alpha}{2-D}$, where D represent the number of dimension. Subsequently, Deser and cowokers [60] have shown that on the linearized Einstein's shell, the actions $R + \alpha'(GB)$ and $R + \alpha' R_{abcd}^2$ (where, α' is the inverse string tension [61]) are not different and this result generalizes to all higher-order ghost terms. GB terms arises naturally as the leading order of the α' expansion of heterotic supersting theory. The GB terms in the higher dimensions lead to ghost free propagator. This particular combination (GB terms) cancels out the square of the second derivative in four dimensions and it acts as simply a Euler number. However, it turns out that this term does not make any contribution to the dynamical equation in GR in four dimensions. However, in higher dimension (D>4), the GB term is important for the physical realization of second iteration of the self interaction of the gravity. The GB term in the theory modifies the gravitational action which is important to obtain viable cosmological scenario. Some interesting results appear in the literature with GB interaction e.g., avoidance of the naked singularities in the dilatonic braneworld scenarios and the address of the problems of fine tuning with scalar field and GB interaction have also been discussed [62]. Further in string induced gravity GB coupling terms with scalar dilaton field has been found [63] to play an important role for obtaining a nonsingular cosmological model. GB term is a topological invariant which in the four dimensional space time becomes a Euler number, hence to derive its effect it is coupled with a dynamic dilatonic scalar field, which in term helps in understanding the dynamics of the universe in four dimension. It is important to explore cosmological models in the presence of GB terms taking into account conceptual issues in cosmology as it has rich structure.

Recent cosmological observations predict that the present universe is passing through an accelerating phase of expansion which is remarkable. It has also been estimated from the cosmological observations that 4% matter in the universe is observable, 70% of the universe consists of dark energy and 26% matter is in the from of dark matter [64]. It is a challenge in theoretical physics to address such an accelerating phase of the universe as the known matter fields in the standard model fails to account for the observations. Recently two approaches have been mainly followed in accommodating present accelerating phase in the cosmological models. Models that are proposed (i) incorporating a modification of matter sector of the Einstein gravity including exotic type of matter and (ii) a modification to gravitational sector by including higher power of scalar curvature/ polynomial in R (similar to that incorporated for obtaining early inflation) or with a new physics. Experimental results of Large Hadron Collider (LHC) may add further knowledge in understanding the later issue. In the first approach a modification of the matter sector of the Einstein field equation is done considering field other than that of a scalar field, namely Chaplygin gas, phantom field, tachyon field etc. called exotic matter field. In the second approach a modification to the Einstein-Hilbert action is considered with higher order terms in scalar curvature. It is known that a modified theory which is also known as higher derivative theory of gravity works well for realizing early inflation [55, 57]. In analogy it may be interesting to explore a modification to the Einstein gravitational action with terms that might be important at extremely low curvature region in order to accommodate the present cosmic acceleration. Recently a modification to the Einstein-Hilbert action by considering modification to the gravitational sector of the action including quadratic (αR^2) and /or cubic (βR^3) terms, along with a new ($\frac{\delta}{R}$) term are employed for accommodating the present cosmological observation, where R is the Ricci Scalar and α, β, δ are dimensional parameters [65].

Generally speaking, modified gravity looks very attractive as there appears possibilities to unify and to address both the early inflation and the late time acceleration. Meanwhile, at the intermediate epoch the gravity may be approximated by General Relativity. The modified gravity may be suitable for describing dark matter and dark energy also. The present thesis contains a study on cosmological models of the universe both in the framework of four and higher dimensions taking into account some of the recent theories mentioned above. The observational constraints will also be taken into account to estimate the different parameters of the theory for a consistent cosmological model. The problem of reproducing an accelerating universe with usual matter field in a modified Einstein gravity will also be explored. Higher derivative theory of gravity will be taken up here to obtain viable cosmological model which accommodates the recent cosmological and astronomical observations.

It is also known that an inflationary solution can be realized taking into account imperfect fluid described by non-equilibrium thermodynamical processes. The recent accelerating phase may be understood in the frame work of a viscous universe to be studied here. Eckart was the first to propose such a theory for describing imperfect fluid [66]. However, it is realized that Eckart theory cannot give rise to a satisfactory theory for viscous fluid as it has numbers of shortcomings. The problems in the Eckart theory arises due to its limitations to consider terms up to first order deviations from equilibrium. Therefore the problem in Eckart theory may be overcome by taking terms up to second order deviation [67, 68] from equilibrium. In the realm of cosmology, especially bulk viscous phenomena have attracted considerable interest, since bulk viscosity is the only possible dissipative mechanism in a homogeneous isotropic universe.

Recently, cosmological models with bulk viscosity admitting an inflationary epoch of the early universe were taken up in the literature for investigating early universe [69]. In fact the effect of bulk viscosity in an expanding universe leads to a reduction in equilibrium pressure, which subsequently drives the effective pressure negative. This is important as it is the key condition needed for inflation. Thermodynamic states with negative pressure are metastable which are not excluded by laws of nature. In general, these thermodynamical states are connected with phase transitions and for certain physical systems the occurrence of negative pressure seems to be inevitable [70]. It is found that the system is hydrodynamically unstable for bubbles and cavity formations. Subsequently one may end up with spontaneous collapse [71]. It was shown by Whittaker [72] that in a stressed self-gravitating fluid described by GTR, the pressure may contribute to the effective gravitational mass. In the case of negative contribution its effect is repulsive resulting in an accelerating cosmic expansion. Thus cosmic fluid out of thermodynamical equilibrium with negative effective pressure provides an alternative mechanism for realizing inflation. A major point of interest in the study of bulk viscous universe is to explore an inflationary universe in the presence of a sufficiently large bulk viscous pressure. A number of literatures in GTR have been reported in which bulk viscosity favours an early inflationary universe scenario [69], which arises because of equation of states and transport equation used in the model.

Bulk viscosity has been widely interpreted as a phenomenological description of the matter creation process in the cosmic fluid also. The particle creation at the Planck era may be studied using bulk viscous stress. This is an interesting connection since irreversible processes are believed to have played a fundamental role in the context of time asymmetry [71]. The usual thermodynamic *arrow of time* translated in this context in terms of entropy generation due to matter creation could provide a natural explanation of the *arrow of time* in the cosmological domain. The modification of the traditional FRW equilibrium equation to include these effects is also important as it helps : (1) to explain the observed large entropy of the cosmic background radiation and (ii) to avoid the initial singular state existing in the standard model [72]. In recent years irreversible processes have become the subject of study once again in connection with a late accelerating universe [73], which will be taken up in the thesis.

It is also important to look for cosmological models where Einstein's cosmological constant Λ, Newton's gravitational constant G, etc may vary with time. The recent type Ia supernovae (SNe) observations [74] and anisotropy measurement of the cosmic microwave background radiation (CMBR) including analysis from the WMAP experiment [75] that made it clear a cosmological constant Λ is essential in addition to the GR theory. However, a fundamental problem in modern cosmology is that the present observed value of the cosmological constant (Λ) is 10^{-120} orders of magnitude smaller than the value predicted by quantum field theories [76]. A possible explanation for this huge discrepancy is based on the idea that vacuum energy density is not constant, but decays as the universe expands [77, 78]. The spacetime is considered to be strongly curved at the Planck time. As a result one may expect an initial huge value of Λ (vacuum energy density) of the order of l_p^{-2}, where $l_p = \sqrt{G}$ is the Planck length. But, as the universe expands, the observed cosmological constant should decay, thereby leading to a small value of Λ observed at present [79]. If vacuum energy density decays with the expansion of the universe, one should expect cosmological time variations of masses too. The variation of masses is equivalent to variation of the gravitational constant G. The idea of a time varying Newton's gravitational coupling constant $G(t)$ was first introduced by Dirac on the basis of Large Numbers hypothesis: very large (or small) dimensionless universal constants cannot occur in the basic laws of physics. Dirac claims that a coincidence between the value of Large Number arising in dimensionless combinations of physical and cosmological constants may appear naturally if one of the constants involved is time varying that was significant over cosmological time scales [80]. The Brans-Dicke theory of gravitation [81], among others [77], supports a variation of G in the EH action. The effect of $\Lambda(t)$ and $G(t)$ in the a higher derivative theory to construct a viable cosmological model will also be taken up here.

1.2 Aim of the Work

The objective of the proposed research work is to study some specific issues relevant for the cosmological model building in higher derivative theories of gravity. Such theories may be obtained from a gravitational action with a polynomial function in Ricci scalar (R) given by the Lagrangian $L = \Sigma_i \lambda_i R^i$, where Einstein action corresponds to the case $i = 1$ only, and $i \geq 2$ corresponds to higher derivative theory of gravity. The Einstein gravitational action including a Gauss-Bonnet (GB) term is also important for exploring the universe as the GB term has rich structure. Cosmological models with imperfect fluid and time varying G and Λ are active fields of research in recent times because of the observational facts that the universe is accelerating and the cosmological parameters are important to understand the dynamics of the early and late universe. We intend to explore cosmological models in the framework of such theories, taking into account the recent predictions from cosmological observations.

1.3 Summary of the Work

• In Chapter 1, a review of the cosmological models in the Einstein gravity, Brane-World including higher derivative gravity and with imperfect fluids are presented. The aims and objective of the thesis are also presented in this chapter.

• In Chapter 2, cosmological models in modified theories of gravity considering a Lagrangian density $L = f(R)$ which is a polynomial function of scalar curvature (R) in the Einstein-Hilbert action in vacuum are presented. The field equation obtained from the modified action corresponding to a Robertson-Walker metric is highly non-linear and not simple enough to obtain analytic solution. Consequently we adopt a numerical technique to study the evolution of the universe. In this method the dynamical equation in cosmology obtained from the modified theory of gravity is converted into an equation of deceleration parameter(q) and Hubble parameter (H), which is then taken up to study the different phase of evolution of the universe. A number of evolutionary phases of the universe including the present accelerating phase are found to exist in the higher derivative theories of gravity. The cosmological solutions obtained here are new and interesting. We study modified theory of gravity as a toy model to explore the past, the present and predict the future evolution. It is found that all the models taken up here can reproduce the current accelerating phase of expansion of the universe. The duration of the present accelerating phase is found to depend on the coupling constants of the gravitational action. The physical importance of the coupling parameters those considered in the action are also discussed.

• In Chapter 3, cosmological models in higher derivative theories of gravity in the presence of imperfect fluid and a time varying cosmological constant (Λ) is presented. Both power-law and exponential expansion of the universe are considered separately in the frame work of imperfect fluid described by Eckart, subsequent theory known as truncated and full Causal theories proposed by Israel and Stewart. The physical and geometrical features of the cosmologies in the presence of a time varying Λ is presented. The interesting feature of the models is that these cosmologies admit a late accelerating universe in some specific cases. The evolution of the temperature of the universe in the framework of the full causal theory is also determined.

• In Chapter 4, an isotropic universe described by Robertson Walker metric is considered which is filled with a bulk viscous fluid in a higher derivative theory of gravity including a variable gravitational and cosmological constant. Cosmological models with power-law and exponential expansions are presented here which are obtained in the presence of imperfect fluid described by full Israel Stewart theory. A class of new and interesting cosmological solutions relevant for model building including present accelerating phase are noted. In the case of power law, it is found that gravitational constant increases with time for a positive cosmological constant whereas it decreases for a negative cosmological constant. The evolution of temperature of a viscous universe is also determined, which supports the present CMBR observations from WMAP. We present some new cosmological solutions where the universe begins from singularity free state in flat model.

• In Chapter 5, cosmological solutions in a flat Roberstson-Walker metric are presented in the framework of a higher derivative theory of gravity, including $\alpha\ R^2$ terms to the Einstein-Hilbert action with variable gravitational and cosmological constants. Both the radiation dominated and matter dominated phase of evolution of the universe are studied. Some new and interesting cosmological solutions are obtained, which may be important in describing late universe. In the presence of R^2 term in Einstein Hilbert action some new and interesting cosmological solutions are noted.

• In Chapter 6, an exact cosmological solutions in the Randall- Sundrum type II (RS) brane-world model with or without Gauss-Bonnet (GB) terms, in the presence of a bulk viscous cosmological fluid is taken up. Cosmological models with power law and exponential evolution of the early universe is explored here in the presence of imperfect fluid described by truncated and full Causal theories proposed by Israel and Stewart. The effect of viscosity increases the rate of expansion of the universe during GB regime. The stability of the equilibrium points of the dynamical system associated with the evolution of the viscous fluid in the RS Brane in the presence of GB term is also studied.

• In Chapter 7, concluding remarks and future work is presented.