CHAPTER 7

Concluding Remarks and Future Work

In this thesis cosmological models are studied in higher derivative theories of gravity and presented different aspects of the models. Recent predictions from the observed supernovae light curve data, Wilkinson Microwave Anisotropy Probe (WMAP) data [75], etc indicate that the present universe is accelerating. It is also known that the universe might have emerged to the present state from an inflationary phase in the past. General relativity, in spite of being the most successful gravitational theory in the last hundred years, has left some of these problems without satisfactory explanation.

Recent cosmological and astronomical observations led us to believe that the Einstein field equation is not enough. Some of the cosmological issues are addressed incorporating either modification of the matter sector including exotic field, namely chaplygin gas, tachyon, etc or modification of the gravitational sector of the Einstein field equation, considering a Lagrangian which is a polynomial function of R. One of the major mechanism to obtain inflation is to consider a higher derivative theory of gravity e.g., R^2 - term including quadratic terms in the curvature tensors to the standard Einstein-Hilbert (in short, EH) action. The possibility of having inflation without any inflaton field was first predicted by Starobinsky [5], who found that the addition of R^2 term to the EH action admits a de Sitter solution.

In addition we consider gravitational theories with variable Λ and G, and viscosity in order to obtain suitable model of the universe. We consider the following models: (i) higher derivative theory without matter (ii) higher derivative theory with a time varying cosmological constant Λ in the presence of viscosity (iii) higher derivative theory of gravity with a cosmological constant Λ and gravitational constant both varying with time in the presence of viscosity (iv) higher derivative theory of gravity with time varying $\Lambda(t)$ and $G(t)$ (without viscosity) (v) bulk viscosity in the brane-world with GB-terms.

In this thesis early inflation, late acceleration and intermediate standard cosmological evolution of the universe in cosmological time scale has been explored. Inflation became established as a standard model of the early universe as the outcome of the research in cosmology last 30 years and it is now considered to be one of the most important developments in our cosmological understanding. Soon after the seminal paper by Guth, taken to mark the birth of the inflation paradigm, several versions of inflation were came up, considering its profound importance in cosmological model building. It is interesting to note that a large number of cosmological observations support the idea of inflation and the suitability of the inflation is not ruled out. On the scale > 100 Mpc our universe is homogeneous and isotropic and it is described by Robertson-Walker metric. The field equation is then written in terms of scale factors of the universe. The dynamical field equation obtained corresponding to the gravitational action considered here is highly non-linear, so it is not simple to determine analytic solution. We adopt numerical technique to determine the evolution of the universe. A number of phases of expansion of the universe are obtained where the evolution is studied in $(q - H)$ plane. We also determine approximate q as a function of H.

One of the attempt to incorporate an accelerating universe without higher derivative terms is to consider dark energy. However, the nature and its origin are not yet known in the standard model of particle physics. This demands a new physics with theoretical and experimental research work or a modification to the existing theories.

In this thesis specific cosmological issues related to obtain a viable cosmological models are presented. In chapter 2, cosmological solution are obtained in higher derivative theories of gravity without matter as a toy model. The role of the geometry alone in driving cosmological evolution is considered. As the field equation is highly non-linear a numerical technique is adopted to understand the evolution of the universe. A number phases of expansion of the universe are obtained here when the evolution is studied in $(q - H)$ plane. The present accelerating phase of the universe, early inflationary phase and prediction of the future possible evolution of the universe are analyzed.

In chapter 3, cosmological evolution of the universe in higher derivative theory of gravity with a time varying cosmological constant $\Lambda(t)$ in the presence of viscous fluid is studied. A viscous universe described by Eckart, truncated and full Israel-Stewart theories are taken up considering both power-law and exponential expansion of the universe. We note that present accelerating phase of expansion of the universe is followed by a decelerating phase in the past. It is found that there exists phases with a number of accelerating and decelerating phases which are important to build a cosmological scenario of the universe.

In chapter 4, evolution of the universe in higher derivative theory of gravity with a time varying cosmological $\Lambda(t)$ and gravitational constant G have studied in the presence of imperfect fluid describe by FIS theory. We determine temperature evolution in this case which also follows from Gibbs integrability condition. It is found that the temperature of a viscous universe is more than the universe without viscosity at a given instant. The present temperature of the universe in the model is $T \sim 2.74$K which is in fair agreement with observed value $T \sim 2.72$K from CMBR.

In chapter 5, R^2 theory is considered with varying gravitational constant (G), which is found to decrease initially and subsequently attains a constant value at a later stage. In the matter dominated universe (MDU) case the scale factor of the universe evolves as $a \sim t^{\frac{2}{3}}$, which is similar to that of an Einstein gravity during MDU. In this case a realistic variation of the parameter G is obtained where G is large initially in the early universe, which, subsequently attains a constant value. The above cosmological solutions are permitted either with (i) $\Lambda > 0$ if $\alpha < 0$ or (ii) with $\Lambda < 0$ if $\alpha > 0$. In the MDU, a new cosmological solution is found in R^2 theory which admits an accelerating universe (supported by supernova observation), with a decaying cosmological constant and a constant G. Since G couples geometry to matter, in an expanding universe, we hopefully expect $G = G(t)$. A variation of G with time has a considerable effect on the evolution of the solar system and on the other galaxy. $G(t)$ variation originates from the decaying of vacuum energy density. One expects a constant value of G at the scales of solar system and galaxies, since at these scales the matter density and the curvature (and most probably the vacuum density) are nearly constant. A physical realistic case with varying G is noted.

Finally in chapter 6, brane-world solution in the presence of viscosity is studied. The effect of extra-dimensions with viscous fluid increases the rate of expansion of the universe. It is well known that the inflationary scenario in the Randall-Sundrum type II (RS) brane-world cosmology model is influenced due to Gauss Bonnet terms. The effect of bulk viscosity both in the GB regime and in the RS regime are explored here and a satisfactory explanation of the late universe is obtained. The outcome from this analysis may help in the future to study the properties of matter for a cosmological model building of the early as well as late universe.

To conclude, the results included in this thesis have shown that higher derivative theory of gravity remains compelling candidate to describe cosmological evolution of the universe. Both the validity and viability of theses theories have still to be subjected to many theoretical and experimental tests. Perhaps in future experiments may reveal some of its predictions. The outcome from the result obtained in chapter 2 may be useful to study cosmological evolution of the universe in the modified theory of gravity in the presence of matter. In almost all the higher derivative theories of gravity considered in the thesis accommodates the present accelerating phase of the universe.