SOME CLASSIFIED PROBLEMS OF STRUCTURES UNDER MECHANICAL AND THERMAL LOADING

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*Resubmitted as per instruction from the Examiner.

CERTIFICATE

This is to certify that the research carried out by Mr.Bapi Karmakar for the preparation of the thesis entitled

"SOME CLASSIFIED PROBLEMS OF STRUCTURES UNDER MECHANICAL AND THERMAL LOADING".

has been supervised jointly by Professor Sitangsu Bimal Karanjai, Department of Mathematics, University of North Bengal, Raja Rammohanpur(Dist-Darjeeling),West Bengal, India and Professor Paritosh Biswas, Senior Academician in Higher Mathematics and Principal(Retired), Ananda Chandra College of Commerce, Jalpaiguri,West Bengal [Residence-Old Police Line, Jalpaiguri,West Bengal, India.]

The Honorable Examiner of the Thesis suggested resubmission with some valuable comments and observations.

Accordingly Sri Bapi Karmakar has made revision in the light of his comments and observations. In our opinion the Thesis is substantially improved and the Revised Thesis is now being submitted to the University of North Bengal for the Ph.D.(Science) degree as per advice of the Examiner and it has not been ~~not~~ submitted elsewhere for the same or any other degree .

12/5/11

(Professor Sitangsu Bimal Karanjai)

(Professor Paritosh Biswas)

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1. PREFACE

The present form of the revised thesis is based on the valuable comments of the honorable Examiner of the Thesis.

The earlier version of the thesis had some discrepancies as well as the deviation of the format from standard presentation.

The following changes have been incorporated in the present form of the Thesis.

(i) The title of the thesis could not be changed due to the question of re-registration of the thesis.

(ii) The standard format, as defined by the Examiner, has been applied to the present form of the thesis as far as possible.

(iii) The literature survey has been made to include the recent works available to the author.

(iv) The author is thankful to the honorable Examiner in pointing out the deficiency in the graphical representation of the numerical results in Chapter-I and Chapter-II with regard to the previous version of the Thesis. The comments helped much to identify the loopholes of the numerical results. The significant nonlinearity has been exhibited. In other problems also, recalculation and modification of the numerical results have been made. Also stress has been given on the nonlinearity aspect of the relevant problems.

(v) The author is also indebted to the Examiner in pointing out the importance of a temperature distribution in practical fields. Sufficient care has been taken to overcome the deficiency with suitable assumption of the temperature field.

(vi) The Author has employed his best efforts to modify the Thesis in the light of the suggestions within his ability.

(vii) Undoubtedly the Author has not only improved his knowledge in course of revision of the Thesis work and at the same time he can now differentiate between "work" and "Research Work". It is due to the valuable review-comments and suggestions of the learned Examiner.

The present work though offers a little contribution to knowledge in the scientific world as a whole, yet it is hoped that it may open up a few small roads, if not avenues, for some young future research workers.

2. NOTATIONS AND DEFINITIONS

The following are the Notations which have not otherwise been stated in the text of the thesis and arranged alphabetically:

$C_i\,(i = 1,2,3,....)$ = constants

$C = \dfrac{Eh}{(1-\nu^2)}$

$C_Q = G_c h_c$

$D = \dfrac{Eh^3}{12(1-\nu^2)}$ = flexural rigidity

$D = \dfrac{E_f h_f (h_f + h_c)^2}{2}$ = flexural rigidity

$e_1 = u,_r + \dfrac{u}{r} + \dfrac{1}{2}(\omega,_r)^2$ = First invariant of middle surface strains

$e_2 = \dfrac{u}{r} + \left\{ u,_r + \dfrac{1}{2}(\omega,_r)^2 \right\}$ = Second invariant of middle surface strains

E = Young's modulus

E_f = Young's modulus of the face material

$F(x,y)$ = stress function

G_c = shearing modulus of the core material

h = thickness of the shell

h_f = face thickness

h_c = core thickness

$K_x(=\dfrac{1}{R_1}), K_y(=\dfrac{1}{R_2})$ = principal curvature of the shell panel

m, n = integers

M_x, M_y, M_{xy} = moments

$M_T = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} zT(x,y,z)dz$ = Thermal Moment [thermal stress resultant]

$N_T = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} T(x,y,z)dz$ = Thermal Stress Couple

q = uniform external normal load per unit area

$R =$ Radius of the cylindrical shell
$R_1, R_2 =$ Principal radii of curvature of the shell panel
$T(x, y, z) = \tau_0(x, y) + z\tau(x, y) =$ temperature distribution within the shell panel
$x, y =$ independent in-plane variables
$u, v, W =$ displacement components along X, Y and Z direction respectively.
$W_0 =$ central deflection
$\nu =$ Possion's ratio of homogeneous material
$\nu_f =$ Possion's ratio of the face material
$\alpha_t =$ co-efficient of thermal expansion
$\sigma_x, \sigma_y =$ Normal stresses in the X and Y directions respectively
$\tau_{xy} =$ shear stress
$\gamma_{xy} =$ shear strain
$\varepsilon_x, \varepsilon_y =$ Normal strains in the X and Y directions respectively
$\nabla^2 =$ Laplacian operator $= \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$
(,)=order of partial differentiation

3. EXTENDED SYNOPSIS

In this chapter the Author wishes to report in details the work carried out by him during the period of investigation with an introduction to the present work, objectives and scope of the subject after having a literature survey on the topics of the thesis. The theory and formulation of the subject under consideration will be presented as per need. A summary report on the findings of the results of the present investigation has been presented. There are every possibility of exploring some avenues for further investigations in future on the present topics and as such some recommendations has been made for the said purpose.

3.1 INTRODUCTION TO THE PRESENT WORK.

In engineering and building designs, especially in several of the present day high-technology industries, high speed spacecrafts, nuclear power plants, offshore and ship building mechanics, storage and high-rise structures including applications in many branches of engineering mechanics and aeronautics, the widespread use of plates and shells are made for which there arises the need for reliance upon different methods of analysis.

Analytical techniques have serious limitations because of difficulties in closed-form solutions of nonlinear differential equations. Therefore despite the simplified nature of plates and shell theory and the effort that has been extended in this area, relatively few solutions are known, particularly when the structures behave geometrical nonlinearity.

Thin plates and shells of regular polygonal and irregular shapes made of isotropic, orthotropic and sandwich materials are often subjected to different kinds of mechanical and thermal loading and as a result such structural components are prone to deformations, bucking and vibrations for which proper analysis are required to be made and of great interest to designers, engineers, scientists and researchers.

Analysis of deformations, buckling and vibrations of different kinds of plates and shells could be made by the applications of linear and nonlinear theory. In the linear theory the components of strains $\varepsilon_{11}, \varepsilon_{22}$ and ε_{12} of the deflected middle surface have negligible magnitude but for nonlinear theory these

components cannot be neglected and are to be taken into consideration for deriving the basic governing nonlinear partial differential equations.
Attempts have been made during the course of the present investigation so as to fill in the gaps where some more emphasis has to be made or to investigate further the elastic behavior of structures often used in modern technology.

3.2. OBJECTIVES AND SCOPE

As already been stated that a closed form solution of governing differential equation, to study the nonlinear and sometimes even the linear static or dynamic analysis of structures, is at most difficult if not impossible.

The main objective of the present work would be to find ways and means so as to overcome the difficulties with the help of existing methodologies as far as possible.

Also the other aim would be to identify problems, which have either been overlooked, or to have a new search for the development of the existing ones. With the advancement of modern technologies and subsequent applications of them in practice creates problems anew. The present investigator hopes to add something new, which may be of little magnitude, yet for which an honest attempt will be made.

However, the main sphere of investigation will be restricted to justify the Title of the present thesis.

3.3 SUMMERY OF LITERATURE SURVEY

Use of analytical techniques in solving such problems has considerable limitations because of difficulties in having closed-form solutions of nonlinear differential equations involved therein. Therefore despite the simplified nature of plates and shells theory and the efforts that have been extended to this area, relatively few solutions are known, particularly when the structures have large deformations.

Moreover, thin isotropic, orthotropic and sandwich plate or shell structures of regular polygonal and irregular shapes are often subjected to different kinds of mechanical and thermal loading and as a result such structural components are prone to small or large deformations, bucking and vibrations, for which

proper analyses are of great interest to designers, engineers, scientists and researchers.
In deformation, buckling and vibration analysis of different kinds of plates and shells may be made by using both the linear and nonlinear theory. In the linear theory the components of strains $\varepsilon_{11}, \varepsilon_{22}$ and ε_{12} of the deflected middle surface have little significance but for nonlinear theory these components cannot be neglected and rather they are to be taken into consideration for deriving the basic governing equations leading to nonlinear partial differential equations of higher orders.

Worth mentioning research works on static and dynamic analysis of thin plates and many researchers using different boundary conditions have carried out researches on shell structures under mechanical, thermal and other types of loadings. Unfortunately, most of them are restricted to linear analysis only. Extensive references are cited in the works of S.Timoshenko and S.Woinowsky Krieger [1], N.J.Hoff [2], B.E.Gatewood [3],Witold Nowacki [4], D.J.Jones [5],B.A.Boley and J.H.Weiner [6] J.L.Nowinski [7], H.Parkus [8], E. A.Thornton [9], L.H.Donnel [182] and P.Biswas [10].

Elaborate discussions on the temperature and membrane stress distribution in an elastic plate with an insulated central elliptical hole have been made by K.S.Rao, M.N.Bapu Rao and T.Ariman[11] using linear theory. Also mention may be made of some other major research works in analysing the thermal stress distribution and vibrations of different structures [12- 18].G.Fanonneau and R.D.Marangoni [19] considered the effect of a thermal gradient on the natural frequencies of rectangular plates. In such cases under elevated temperature, the elastic coefficients of homogeneous materials are no longer constants but become functions of space variables [2] and so application of non-homogeneous theory becomes a necessity.

Based on this non-homogeneous theory several other papers may be cited of which mention may be made of the work of N.Ganesan [20] who considered linear vibration analysis of a rectangular plate subject to a thermal gradient and J.S.Tomar and A.K.Gupta [21,22] who considered such an analysis for orthotropic rectangular and elliptic plates of linearly varying thickness and of non-uniform thickness and temperature, respectively.

Some other papers [23 - 25] deal with finite deformations, post buckling behavior of heated rectangular plates with temperature-dependent material properties and thermo-elastic analysis in orthotropic elastic semi-space and

finite orthotropic slab [24-25]. T.R.Tauchert [26] presented an analysis on thermal shock of orthotropic rectangular plates.
The literature has also been enriched by the works on thermal post-buckling behavior of skew plates [27-28], thick elastic rectangular plate on an elastic foundation subject to a steady temperature distribution [29] and its extension to, a thick right-angled isosceles triangular plate [30]. Some other relevant works related with different structures or different technical methods may be found in the literature cited in references [31-43].

The above bulk of classical approach in Applied Mechanics rests on the assumption that the linear mathematical model has described the phenomenon involved. However, with the advent of modern technology and systems exposed to oppressive operational conditions induce large deflections, i.e., deflections that are of the same order as the plate and shell thickness and small compared to in-plane dimensions of the structures. Thus, when the deflections are no longer small in comparison with the thickness but small compared to the in-plane dimensions, the middle surface strains must be considered in deriving the differential equations of thin plates and shell structures. In this way one gets the nonlinear differential equations in the classical nonlinear theory.

For the analysis of large deflections of plates von Kármán's [44] coupled nonlinear partial differential equations have extensively been employed by many a earlier researcher. These equations which are of the fourth order with respect to the unknown deflection W and stress function F enables one to determine W and F. These equations can also be expressed in terms of displacement components u,v and W and conveniently been used for the nonlinear analysis of different kind of plates and shells. von Kármán's equations are generally difficult to deal with because of its coupled nonlinearity and as yet no general solutions of these equations are known. However, approximate and different numerical and computational methods have been adopted for the solution of such large deflections analysis of plates and shells.

There is a galaxy of outstanding research workers who employed von Kármán equations to the analysis nonlinear behavior of thin plates, both isotropic and orthotropic under mechanical and other kinds of loading and subsequently extended to shallow shells with the inclusion of curvature.

Several other authors extended von Kármán's equations plates and shells with the inclusion of thermal loading, both stationary (steady) and non-stationary (time-dependent) and further extension were made to sandwich plates and shells under mechanical and thermal loading [46-78].

Meanwhile C.Y.Chia [79] has published an excellent book entitled "Non-Linear Analysis of Plates" in which problems on orthotropic and laminated plates have been analyzed in addition to other problems with an Kármán extensive bibliography of other related works. Subsequently von Karman's equations have been extended with the inclusion of thermal loading in the static case for both plates and shells as have been nicely presented by W.Nowacki in his famous monograph [4]. Further studies by different authors using type field Kármán equations for different structures can be fond in Refs.[80-82].

As von Kármán's equations are in the coupled form and very difficult to deal with and yet having no closed-form solutions, H.M.Berger [83] proposed a pair of quasi-linear partial differential equations for the analysis of large deflection of isotropic plates. These equations being in the decoupled form have obvious advantages for getting solution of large deflection problems of elastic plates with much ease and computational effort. In Berger's method, the second strain invariant in the middle surface in the expression for total potential energy has been neglected. An application of the variational techniques of the Calculus of Variations to this simplified energy expression ultimately gives rise to the Berger's equations. Although no physical explanation of this method has been provided, yet results obtained by him and other authors agree well with those obtained from more precise analysis. However it has been shown by J.L.Nowinski and H.Ohnabe [84] and G.Prathap [85] that this method miserably fails i.e. this method gives absurd results for plates with movable edge boundary conditions. Considering the obvious advantages of Berger's method due to its quasi-linearity and de-coupled forms this method has been employed by many authors and further extended to the dynamic cases with and without thermal loading by William A. Nash and J.Modeer [86] followed by others [87-107]

Berger's method has further been extended with the inclusion of thermal loading in a good number of papers in the static and dynamic cases by many authors in the literature . Noteworthy mention may be made of the works of S.Basuli [108] who first extended Berger's method with the inclusion of thermal loading and investigated large deflection of some elastic plates under uniform load and heating. Intensive use of this method has been continued for a long period [108-143]

Over the years investigations on finite deformations and vibrations of Sandwich plates and shells under mechanical and thermal loading have been gaining importance due to wide applications in aerospace industry, high-speed aircrafts, missiles and in different components of structural mechanics.

However,Berger's equations have certain limitations and inaccuracies as discussed by several authors [84-85, 104] . These are most accurate for the immovable clamped edge conditions and fairly accurate for immovable simply supported edge conditions. Berger's assumptions yield absurd results for movable edge conditions. This is due to the fact that neglect of e_2 [the second strain invariant] for movable edge fails to imply freedom of rotation in the meridian planes where membrane stress exists. For movable edges the in-plane displacement u is never zero and thus Berger's equations lead to absurd results. On the other hand, for clamped edge conditions $u = 0$ and $dw/dr = 0$ at the boundary and Berger's equations are most accurate here. But for simply-supported edge conditions u is zero but $dw/dr \neq 0$ at the boundary and thus Berger's equations yield fairly accurate results. It is also interesting to note that under many loading conditions, especially uniform and under relatively smooth and regular boundary conditions–the distortional energy and its variation should be substantially smaller than the dilatation. Hence the Berger's assumption which too simplistically has been translated into assuming a Poisson's ratio of unity and has patently absurd . For this reason this assumption has always yielded reasonably good practical results for the uniform or smoothly varying loading. The circular is the best geometry, but as any in-plane large distortional changes even in rectangular plates is usually confined to the corners, reasonable results should also be expected there. On the other hand disparities such as movable boundary suggest large energy changes and the basic hypothesis becomes questionable. Consequently due to severe criticism of the application of Berger's equations B.Banerjee *et.al* [144,145] offered a new set of decoupled differential equations to investigate the nonlinear behaviors of elastic plates under different types of loading. The new set of differential equations are formed under a modified energy expression containing the expression for the in-plane stress σ_{rr}.

The differential equations as proposed in [144, 145] are decoupled and hence can be solved without any difficulty for any type of loading and are valid for both movable and immovable edge conditions with ease and with relative accuracy and further applied by them for spherical and cylindrical shells in the static and dynamic cases [146-147]. Following the same approach B.Banerjee alone [148] presented large deflection analysis of circular plates of variable thickness. Exponential variation of thickness useful in design and discussed fully by S.Timoshenko and S.Woinowsky-Krieger [1] was considered in this paper. Further S .Datta and B .Banerjee [149] and P.Bhattacharya and B.Banerjee [150] extended this modified Berger's method to large deflection analysis of sandwich plates. Subsequently, using this modified Berger's method D.N.Paliwal et.al. [151-157] considered problems on nonlinear deformations and vibrations of elastic plates and shells under mechanical and thermal loading and resting on Pasternak and Kerr type of elastic. However, using this method M.M.Banerjee *et al.*[158] investigated large amplitude free vibration of shallow spherical shell subjected to thermal gradient including effects of temperature dependent modulus of elasticity of material and expressed some reservations on the use of this modified Berger's method. This modified Berger's has also been further extended to heated sandwich plates [159-160]

In addition to the voluminous works of plate and shell problems using von Kármán's method, Berger's method and modified Berger's method, a great deal of remarkable progress has been made for the analysis of nonlinear static and dynamic behavior of plate and shell structures subjected to different types of loading-mechanical and thermal, by using other analytical methods. Such analytical methods are not always suitable to deal with problems on plates and shells having complex geometrical and boundary conditions. For such cases, numerical methods like finite element method, boundary element method, complex variables method and conformal mapping technique have been used by many researchers and scientists some of which have been cited in this thesis.

During the course of revision of this Thesis, the Author has made further literature survey and discovered some recent published works on topics related with those of the Thesis. These current works have helped the Author in many ways for the revision of this Thesis. These References [201-208] have been included in the Reference Section of the Thesis.

3.4 SUMMARY OF FORMULATIONS.

With usual notations, the total strain energy is given by

$$U = \frac{1}{2}\iiint(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_{xy}\varepsilon_{xy})\,dzdxdy$$

whereas the kinetic energy is

$$T_e = (\rho h/2)\iint\left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right)dxdy$$

and the work done is

$$W_k = \iint pw\,dxdy$$

Formulating the Lagrangian with the help of the above expressions and applying Hamilton's principle, a straightforward application of the variational calculus will yield the following equations of motion [192]

$$D\nabla^4 w = hS(F,w) - h\left(\frac{F,_{yy}}{R_x} + \frac{F,_{xx}}{R_y} - 2\frac{F,_{xy}}{R_{xy}}\right) + q - \rho\, hw,_{tt}$$

and

$$D\nabla^4 F = -\frac{E}{2}hS(w,w) + E\left(\frac{w,_{yy}}{R_x} + \frac{w,_{xx}}{R_y} - 2\frac{w,_{xy}}{R_{xy}}\right)$$

where the operator $S(w,F)$ stands for

$$S(w,F) \equiv \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - 2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2}$$

Here 'F' denotes the Airy-Stress function as found in the literature

$$\int_{-h/2}^{h/2}\sigma_{xx}dz = N_x = h\frac{\partial^2 F}{\partial y^2},\quad \int_{-h/2}^{h/2}\sigma_{yy}dz = N_y = h\frac{\partial^2 F}{\partial x^2},\quad \int_{-h/2}^{h/2}\sigma_{xy}dz = N_{xy} = -h\frac{\partial^2 F}{\partial x\partial y},$$

whereas

$$M_x = \int_{-h/2}^{h/2}\sigma_x z dz = -D\left[w,_{xx} + \nu\, w,_{yy}\right] \quad , \quad M_y = \int_{-h/2}^{h/2}\sigma\; z dz = -D\left[w,_{yy} + \nu\, w,_{xx}\right]$$

$$M_{xy} = \int_{-h/2}^{h/2}\sigma_{xy} z dz = -D(1-\nu)\, w,_{xy}$$

and the (,) notation signifies partial derivative with respect to the suffix

If we are concerned with the doubly-curved shells we may put $1/R_{xy} = 0$, and R_x and R_y are suitably chosen with proper signs for Gaussian curvature and one may get

$$D\nabla^4 w = hS(F,w) - h\left(\frac{F,_{yy}}{R_x} + \frac{F,_{xx}}{R_y} - 2\frac{F,_{xy}}{R_{xy}}\right) + q - \rho h w,_{tt}$$

and

$$D\nabla^4 F = -\frac{E}{2} hS(w,w) + E\left(\frac{w,_{yy}}{R_x} + \frac{w,_{xx}}{R_y} - 2\frac{w,_{xy}}{R_{xy}}\right)$$

with $1/R_{xy} = 0$.

But when plate problems will be considered we also put all $1/R_x$ and $1/R_y$ to be zero, i.e with $1/R_x$ and $1/R_y$ equal to zero i.e.,

$$D\nabla^4 w = hS(\phi, w) + q - \rho h w_{tt}$$
$$\nabla^4 \phi = -(E/2)S(w,w),$$

When thermal contribution is added to the problem the strain components will take the form

Median surface stress-strain relations are given by

$$\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y) - \frac{\alpha_t ET}{(1-\nu)}$$
$$\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x) - \frac{\alpha_t ET}{(1-\nu)}$$
$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

For a cylindrical shell Panel the von-Karman strain displacement equations are given by Donnell[182]

$$\varepsilon_x = u,_x + \frac{1}{2}(W,_x)^2 - zW,_{xx}$$

$$\varepsilon_y = u,_y + \frac{1}{2}(W,_y)^2 - zW,_{yy} - \frac{W}{R}$$

$$\gamma_{xy} = u,_x + v,_y + W,_x W,_y - 2zW,_{xy}$$

The forces N_{xx}, N_{yy}, N_{xy} and the moments M_{xx}, M_{yy}, M_{xy} can be expressed by the matrix equation

$$\begin{bmatrix} N_{xx}, N_{yy}, N_{xy} \\ M_{xx}, M_{yy}, M_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x, \sigma_y, \tau_{xy} \\ z\sigma_x, z\sigma_y, z\tau_{xy} \end{bmatrix} dz$$

The in-plane equations of equilibrium in the X and Y directions are

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0$$

These equations are identically satisfied by introducing the Airy stress function defined by the relations

$$N_{xx} = F,_{yy}$$
$$N_{yy} = F,_{xx}$$
$$N_{xy} = -F,_{xy}$$

Considering preceding equations one gets

$$\varepsilon_x = \frac{1}{Eh}(F,_{yy} - \nu F,_{xx}) - zW,_{xx} + \frac{\alpha_t N_T}{h}$$

$$\varepsilon_y = \frac{1}{Eh}(F,_{xx} - \nu F,_{yy}) - zW,_{yy} + \frac{\alpha_t N_T}{h}$$

$$\gamma_{xy} = -\frac{2(1+\nu)}{Eh} F_{,xy} - 2zW,_{xy}$$

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 - \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} - \frac{1}{R}\frac{\partial^2 W}{\partial x^2}$$

By virtue of the above equations one gets the following differential equation for the stress function in terms of deflection function

$$\nabla^4 F = Eh\left[\left(W_{,xy}^2\right) - W_{,xx} W_{,yy}\right] - \alpha_t E\left(\nabla^2 N_T\right) - \frac{Eh}{R} W_{,xx}$$

where

$$N_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} T(x,y,z)dz = \text{Thermal Stress Couple}$$

Considering the expressions for the moments from the equations as stated above and considering the following equation of equilibrium [1,page-379]

$$\frac{\partial^2}{\partial x^2}(M_{xx}) - 2\frac{\partial^2}{\partial x \partial y}(M_{xy}) + \quad \frac{\partial^2}{\partial y^2}(M_{yy}) = -\left[N_{xx}W_{,xx} + 2N_{xy}W_{,xy} + N_{yy}W_{,yy}\right]$$

one gets

$$D\nabla^4 W + \frac{\alpha_t E}{(1-\nu)}\left(\nabla^2 M_T\right) = \left[F_{,xx} W_{,yy} - 2F_{,xy}W_{,xy} + F_{,yy} W_{,xx}\right]$$

where

$$M_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} zT(x,y,z)dz = \text{Thermal Moment}$$

The above two equations constitute coupled nonlinear partial differential equations in the von Karman sense for determining the large thermal deflections of a shallow cylindrical shell panel.

If the plate is assumed to be comparatively thin and normal to the plane

$$\bar{\varepsilon}_x = \frac{1}{E}(\sigma_{xm} - \nu\sigma_{ym}) = \frac{1}{E}(\phi_{yy} - \nu\phi_{xx}) = u_x + \tfrac{1}{2}w_x^2$$

$$\bar{\varepsilon}_y = \frac{1}{E}(\sigma_{ym} - \nu\sigma_{xm}) = \frac{1}{E}(\phi_{xx} - \nu\phi_{yy}) = v_y + \tfrac{1}{2}w_y^2$$

$$\bar{\varepsilon}_{xy} = \frac{2(1+\nu)}{E}\sigma_{xym} = -\frac{2(1+\nu)}{E}\phi_{xy} = u_y + v_x + w_x w_y$$

Performing necessary integrations, the total strain energy can be expressed in terms of the first and second invariants of the middle surface of the plate

$$\xi = \frac{D}{2}\iint\left\{(\nabla^2 w)^2 + \frac{12}{h^2}e_1^2 - 2(1-\nu)\left[\frac{12}{h^2}e_2 + w_{xx}\,w_{yy} - w_{xy}^2\right]\right\}dx\,dy - \iint p\,dx\,dy$$

where $\quad e_1 = \bar{\varepsilon}_x = +\bar{\varepsilon}_y \quad , \quad e_2 = \bar{\varepsilon}_x\,\bar{\varepsilon}_y - \tfrac{1}{4}\bar{\varepsilon}_{xy}^2$

The kinetic energy of the plate is

$$T = \frac{\rho h}{2}\iint(u_t^2 + v_t^2 + w_t^2)^2\,dx\,dy$$

Using the Lagrangian $L = (T - \xi^{*})$ and applying Hamilton's principle we get the dynamic analogue of the von Karman equations (in the absence of time derivative), as:

$$u_{xx} + w_x\,w_{xx} + \nu(v_{xy} + w_y\,w_{xy}) + \tfrac{1}{2}(1-\nu)\,(u_{yy} + v_{xy} + w_x\,w_{yy} + w_y\,w_{xy}) = \frac{\rho(1-\nu^2)}{E}u_{tt}$$

$$v_{yy} + w_y\,w_{yy} + \nu(u_{xy} + w_x\,w_{xy}) + \tfrac{1}{2}(1-\nu)\,(u_{xy} + v_{xx} + w_x\,w_{xy} + w_y\,w_{xx}) = \frac{\rho(1-\nu^2)}{E}v_{tt}$$

$$\frac{h^2}{12}\nabla^4 w = u_x\,w_{xx} + \tfrac{1}{2}w_x^2\,w_{xx} + v_y\,w_{yy} + \tfrac{1}{2}w_y^2\,w_{yy} + \nu(v_y\,w_{xx} + \tfrac{1}{2}w_y^2\,w_{xx} + v_y\,w_{yy} + \tfrac{1}{2}w_x^2\,w_{yy}) +$$
$$(1-\nu)(u_y\,w_{xy} + v_x\,w_{xy} + w_x\,w_y\,w_{xy}) + \frac{\rho(1-\nu^2)}{E}(w_x\,u_{tt} + w_y\,v_{tt} + w_{tt}) + q$$

The above equations may be re-written in terms of stress resultants and moments for a plate with moderately large amplitude as

$$\frac{\partial}{\partial x}(\sigma_{xm}) + \frac{\partial}{\partial y}(\sigma_{xym}) = \rho h u_{tt} \quad , \quad \frac{\partial}{\partial y}(\sigma_{ym}) + \frac{\partial}{\partial x}(\sigma_{xym}) = \rho h v_{tt}$$

and

$$\xi^* = \iint\left\{-\frac{1}{2E}\left[(\phi_{xx}+\phi_{yy})^2 + 2(1+\nu)(\phi_{xy}^2 - \phi_{xx}\phi_{yy})\right] + \frac{D}{2}\left[(w_{xx}+w_{yy})^2 + 2(1-\nu)(w_{xy}^2 - w_{xx}w_{yy})\right]\right.$$
$$\left. + \frac{h}{2}\left[\phi_{yy}w_{xx}^2 + \phi_{xx}w_{yy}^2 - 2\phi_{xy}w_xw_y\right] - pw \right\} dx\,dy$$

$$\frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^2 M_{xy}}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial}{\partial x}(h\phi_{yy}w_x) + \frac{\partial}{\partial y}(h\phi_{xx}w_y) + \frac{\partial}{\partial x}(-h\phi_{xy}w_y) + \frac{\partial}{\partial y}(-h\phi_{xy}w_x) = \rho h w_{tt}$$

In general the preceding equations may represent the motion of the plate. Yet for practical purpose let us simplify the basic equations with the assumption that the effect of both the longitudinal and rotatory inertia forces can be neglected. The basic equations governing the nonlinear vibration of plates subjected to a normal load 'p' may be reduced to

$$\frac{\partial}{\partial x}(\sigma_{xm}) + \frac{\partial}{\partial y}(\sigma_{xym}) = 0 \quad , \quad \frac{\partial}{\partial y}(\sigma_{ym}) + \frac{\partial}{\partial x}(\sigma_{xym}) = 0$$

$$\nabla^4\phi = E(w_{xy}^2 - w_{xx}\,w_{yy})$$

$$L(w,\phi) \equiv D\nabla^4 w - h(\phi_{yy}\,w_{xx} + \phi_{xx}\,w_{yy} - 2\,\phi_{xy}\,w_{xy}) + \rho h w_{tt} - p = 0$$

Besides the expressions for total energy, the required basic governing equations can be derived.

METHODOLOGIES AVAILABLE IN THE LITERATURE.

A. METHODS OF APPROXIMATE SOLUTION.

Once the basic equations have been established it is now the time for investigation of plate problems. The linear approach may sometime look easier than that of nonlinear ones, but sometimes linear problems also involve other geometric nonlinearities or other factors that make the investigation a little harder when it becomes necessary to explore possibilities for an approximate solution. Thus our next purpose will be to identify such methods which are broadly in use.

B. RAYLEIGH,RITZ OR RAYLEIGH-RITZ METHOD.

The Ritz Method is simple and convenient for determining solutions to plate problems. It involves choosing the deflection function in advance in the form of a series as,

$$w = \sum_{i=1}^{n} C_i \; \varphi_i$$

where C_i's are undetermined parameters obtained by minimizing the total potential (V) satisfying the condition

$$\frac{\partial V}{\partial C_k} = 0, \qquad k = 1,2,3,\ldots..n$$

We are interested here only the important observations made on the use of this method rather than illustrating any particular problem here.

The problem of rectangular plates with all possible mixed boundary conditions has been included in an excellent paper by Warburton [193] which gave formulas for finding the natural frequencies of all twenty-one possible distinct combinations of simple boundary conditions, using the Rayleigh method with single-term deflection modes composed of products of "beam functions"

The concept of using beam functions with the Rayleigh-Ritz method to obtain highly accurate frequencies and mode shapes was set forth in the classic work of Young [194] Young used superposition of beam functions and determined the eigenvectors of the amplitude coefficients by the minimizing scheme of Ritz [195].

An important component in the application of R-R method is the selection of appropriate admissible functions for use in the series representing the deflection of the plate in concern. Different sets of functions have been proposed by different authors. Gram-Schmidt process [196] is the important tool to generate sets of Orthogonal Polynomial functions, the first member of each set satisfying the geometric and natural boundary conditions of an equivalent beam, the remainder of the set satisfying, automatically, only the geometric boundary conditions of the beam.
For example, if the deflection function is set as

$X_m(x)$ *aswellas* $Y_n(y)$ may be generated by using Gram-Schimdt process.

$$(w(x,y)=\sum_{m=1}^{p}\sum_{n=1}^{q}A_{mn}W_{mn}(x,y),\ W_{mn}(x,y)=X_m(x)Y_n(y)$$

Gram-Schimdt considered the starting function as $X_0(x)$ and generated the subsequent functions as

$$X_1(x)=(x-B_1)X_0(x),\quad X_k(x)=(x-B_k)X_{k-1}(x)-C_k\,X_{k-2}(x),\quad k>1$$

with

$$B_k=\int_a^b x\,w(x)X_{k-1}^2(x)\,dx\Big/\int_a^b X_{k-1}^2(x)\,dx$$

$w(x)$being the weighted unction and the polynomials $X_k(x)$ satisfy the orthogonality condition

$$\int_a^b w(x)X_k(x)X_l(x)dx=\begin{cases}0 & if\quad k\neq l\\ a_{kl} & if\quad k=l\end{cases}$$

Bhat [197] opted to choose the weighted function as unity, the interval as 0 to 1 and the coefficients of the polynomials are so chosen as to make the polynomials orthonormal,

$$\int_0^1 X_k^2(x)\,dx=1$$

Since the orthogonal polynomials satisfy only the geometric boundary conditions, except for the first member, they do not over restrain the structure, unlike the beam functions. Hence Bhut observes that the functions are able to closely approximate the true boundary conditions of the plate with the application of R-R[Rayleigh-Ritz] procedure.

Thus with a first polynomial $X_1(x)$ satisfying the geometric and natural boundary conditions of the equivalent beam function the subsequent terms may be obtained from $X_2(x)=(x-B_2)X_1(x)$ and $X_k(x)=(x-B_k)X_{k-1}(x)-C_k\,X_{k-2}(x),\ k>2$, where

$$B_k=\int_0^1 x\,X_{k-1}^2(x)\,dx\Big/\int_0^1 X_{k-1}^2(x)\,dx,$$

and

$$C_k=\int_0^1 x\,X_{k-1}(x)\,X_{k-2}(x)dx\Big/\int_0^1 X_{k-2}^2(x)dx.$$

In conclusion he adds that the characteristic orthogonal polynomials as proposed by him yield superior results for lower modes, particularly when the plate has free edges and are simple to construct and possess the orthogonal property which simplifies the analysis as in the case of beam functions[198] and simply-supported plate functions . Another important observation has been made by Liew and Lam[199] by using characteristic orthogonal polynomials in Rayleigh-Ritz method for flexural vibration, first introduced by Bhat.

Numerical results for skew plates depict that the convergence pattern for different modes of vibration for skew angles 15° and 45° stable convergence is reached with twenty-five terms used in the series for the expression of the transverse deflection in terms of two-dimensional orthogonal plate function. For fundamental mode, the convergence may be reached with lesser terms but for higher modes use of more than fifteen terms seems to be essential. The convergence study made by the authors [199] shows that for m=n=6 in the expression for $W(x,y)$ the convergence is very rapid for all classes where free edges exist and that the use of a single term starting function (suggested by the authors) yields more satisfactory results.

The results presented by Dickinson and Blasio [200] confirm that the Gram-Schmidt generated polynomial functions proposed by Bhat are very satisfactory for use in Rayleigh-Ritz method for the study of variety of plate problems.

C.GALERKIN METHOD

In finding the solution for the equation $L[w(x,y)] = p(x,y)/D$, if somehow we can find an exact solution

$$w(x,y) = w_o(x,y) = \sum_{j=1}^{n} a_j \, w_j(x,y) ,$$

then

$$L[w_o(x,y)] - p(x,y)/D = 0$$

But if it is not so then will yield some error given by

$$E_r(x,y) = \sum_{j=1}^{n} a_j L[w_j(x,y)] - p(x,y)/D$$

known as the error function. The Galerkin procedure requires that the error function be orthogonal to with all the approximate functions ϕ_j., i.e.,

$$\iint_R E_r(x,y)\,\phi_j(x,y)\,dx\,dy = 0$$

which in turn yields *'n'* simultaneous equations for determination of the unknown coefficients in equation

The Galerkin Method has some advantages over the Ritz Method and thus its application extends over a broader range.. However, there are many other methods in the literature which are omitted here for the sake of brevity

4. PROBLEMS CONSIDERED IN THE THESIS.

This thesis is divided into five chapters containing some classified problems of nonlinear mechanics.

The first chapter consists of two papers.

The first paper deals with nonlinear deformation analysis of elastic plates in the form of a square panel under mechanical loading using von Karman's equations in terms of displacement components under immovable edge boundary conditions and with the inclusion of curvature .It appears to the author that very little attention has been paid to use von Karman's equations with the inclusion of curvature for the analysis of a square panel except the work of Bhattacherjee and Banerjee [183].The drawback of this paper is its presentation in a very concise form.

To solve this problem, suitable expressions for u and v have been chosen. The solutions for u and v with immovable edges have first been determined. Expressing the load in terms of Fourier Series the final solution is obtained by Galerkin's error minimizing technique in the form of a cubic equation in terms of non-dimensional deformation $\left(\frac{W_0}{h}\right)$ and non-dimensional load parameter $\left(\frac{q_0 a^4}{Eh^4}\right)$. Results have been presented graphically at the end of the paper.

The second paper of this chapter is concerned with the non-linear deformation analysis of a square sandwich shell panel under mechanical loading. Here von Karman's non linear partial differential equations, also in terms of u, v and W with the inclusion of curvature and extended to sandwich shell panels, have been employed. The method of solution is same as used in the foregoing paper .Here also sufficient numerical results have been presented using the values of different parameter for a sandwich shell panel in tabular form.

The second chapter consists of one paper only and is concerned with the analysis of a sandwich rectangular panel under mechanical loading using von Karman's coupled non-linear partial differential equation in terms of deflection W and stress function F and extended to sandwich shell with the inclusion of curvature under both movable and immovable edge boundary conditions. Assuming appropriate form of the deflection function in conformity with simply supported edge boundary conditions, the stress function is first completely determined for both movable and immovable edges. The second of the von Karman's equation is now utilized using W and F and applying Galerkin's error minimizing technique a cubic equation is again determined showing variation of non-dimensional central deflection against variation of mechanical load parameter. Here also extensive numerical computations have been made using different values of parameters for a sandwich panel and presented in tabular form.

The third chapter consists of one paper and concerned with non linear deformation analysis of a shallow shell panel under thermal loading. Here the basic governing equations have been employed with the inclusion of thermal loading and curvature and expressed in terms of W and F. These equations have been solved using temperature distribution varying in the direction of z-axis and taken as linear in the form given in [4].Here also the stress function F is first completely solved and applying Galerkin's procedure a cubic equation has been obtained involving the central deflection and non dimensional load parameter Meridian surface membrane stresses have conveniently been determined from the analysis .Numerical results for non-dimensional thermal deformation and membrane stresses have been calculated and presented graphically.

The fourth chapter consists of two papers **the first one of which** is concerned with the non linear thermal vibrations of a circular plate under elevated temperature. Using modified Berger's equation as proposed by Banerjee and Dutta [144] and employed by Sinharay and Banerjee [145-147] in some of their problems, the basic governing equations for a circular plate have been derived and solved for a clamped circular plate. Application of Galerkin's procedure ultimately leads to the well known time differential equation from which ratio of non-linear and linear frequencies can be determined. Sufficient numerical results have been calculated and presented graphically.

The second paper of this chapter is concerned with the analysis of non-linear vibrations of a non homogeneous elastic shell under severe thermal loading. Here also basic governing equations have been derived. Considering a thin shell subjected to a steady thermal gradient, temperature distribution is assumed to be a linear function of radial distance. Following the same method as in the previous paper, the ratio of non-linear and linear frequencies have been calculated and presented graphically.

The last and fifth chapter of this thesis consists of one paper only.

Use of complex variable theory and conformal mapping technique has been adopted in this chapter. By this method linear and non-linear deformation and vibrations of regular and irregular shaped plates under both mechanical and thermal loading have conveniently been investigated by different authors in some of their problems [49,101,161-165].The essence of this method is to transform the governing differential equation in terms of complex co-ordinate ($z = x + iy, \bar{z} = x - iy$) and the domain can be conformally mapped onto a unit circle. It is to be noted that a one term approximation of the mapping function yields fairly accurate results with less computational
efforts. In this paper investigation has been made on the free non-linear vibrations of regular polygonal plates resting on non-linear elastic foundations with clamped edge boundary conditions and subjected to uniaxial compressive loads normal to all the edges. Some numerical results have been presented graphically for different kinds of plate shapes considering different values of mapping function co-efficient δ correspondence to each plate shape and followed by observation and discursions.

Last of all, it should be noted that in the compilation of the text of this thesis some equations have to be repeated in some of the chapters and whenever occur with the same equation number.

CHAPTER-I

NONLINEAR DEFORMATION ANALYSIS OF A SQUARE PANEL UNDER MECHANICAL LOADING*

1.1.1. INTRODUCTION

Nonlinear behavior of plates has been the subject of investigation of several investigators as fully discussed in [1].Among plates of various shapes, rectangular plates have received considerable attention in the literature compared to plates of other shapes [4-6,79]. However very little attention has been paid for investigation with inclusion of curvature.

In this paper, nonlinear deformation analysis of a square panel has been investigated for immovable edge boundary condition. Some graphical representation have been made and followed by observation and discussion.

1.1.2. BASIC GOVERNING DIFFERENTIAL RQUATIONS

Considered here a square panel of length $2a$. The displacement formulation of the differential equations governing the nonlinear(large) deformation of a square panel under mechanical load can be expressed as:

$$u_{,xx} + \frac{(1-\nu)}{2}u_{,yy} + \frac{(1+\nu)}{2}v_{,xy} = (K_x + \nu K_y)W_{,x} - \left\{W_{,xx} + \frac{(1-\nu)}{2}W_{,yy}\right\}W_{,x} - \frac{1}{2}(1+\nu)W_{,y}W_{,xy} \tag{1}$$

$$v_{,yy} + \frac{(1-\nu)}{2}v_{,xx} + \frac{(1+\nu)}{2}u_{,xy} = (K_y + \nu K_x)W_{,y} - \left\{W_{,yy} + \frac{(1-\nu)}{2}W_{,xx}\right\}W_{,y} - \frac{1}{2}(1+\nu)W_{,x}W_{,xy} \tag{2}$$

$$D\nabla^4 W = \left[q + \left(F_{,yy}W_{,xx} - 2F_{,xy}W_{,xy} + F_{,xx}W_{,yy}\right) + K_x F_{,yy} + K_y F_{,xx}\right] \tag{3}$$

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For a shell panel the membrane forces and displacement components are stated by

$$N_{xx} = C\left[u_{,x} + \frac{1}{2}W_{,x}^{2} + \nu\left\{v_{,y} + \frac{1}{2}W_{,y}^{2}\right\} - (K_x + \nu K_Y)W\right] \quad (4)$$

$$N_{yy} = C\left[v_{,y} + \frac{1}{2}W_{,y}^{2} + \nu\left\{u_{,x} + \frac{1}{2}W_{,x}^{2}\right\} - (K_y + \nu K_x)W\right] \quad (5)$$

$$N_{xy} = C\frac{(1-\nu)}{2}\left(u_{,y} + v_{,x} + W_{,x}W_{,y}\right) \quad (6)$$

The membrane forces and the stress function are related by

$$N_{xx} = F_{,yy} \quad (7)$$

$$N_{yy} = F_{,xx} \quad (8)$$

$$N_{xy} = -F_{,xy} \quad (9)$$

1.1.3. BOUNDARY CONDITIONS :

Clamped tangential edges, movable or immovable are considered, so that boundary conditions for a square panel of equal length and peripheral breadth $2a$,the deformation satisfying the simply supported boundary conditions are given by

$$W = 0 = W_{,xx} \text{ at } x = \pm a$$

$$W = 0 = W_{,yy} \text{ at } y = \pm a \quad (10)$$

1.1.4. METHOD OF SOLUTION

Here W is assumed in the form

$$W = W_0 Cos\left(\frac{\pi x}{2a}\right)Cos\left(\frac{\pi y}{2a}\right) \tag{11}$$

The inplane displacements u and v can be chosen in the following form [183]

$$u = Ax + A_1 Sin\left(\frac{\pi x}{2a}\right)Cos\left(\frac{\pi y}{2a}\right) + A_2 Sin\left(\frac{\pi x}{a}\right)Cos^2\left(\frac{\pi y}{2a}\right) + A_3 Sin\left(\frac{\pi x}{a}\right)Sin^2\left(\frac{\pi y}{2a}\right) \tag{12}$$

$$v = By + B_1 Sin\left(\frac{\pi y}{2a}\right)Cos\left(\frac{\pi x}{2a}\right) + B_2 Sin\left(\frac{\pi y}{a}\right)Cos^2\left(\frac{\pi x}{2a}\right) + B_3 Sin\left(\frac{\pi y}{a}\right)Sin^2\left(\frac{\pi x}{2a}\right) \tag{13}$$

where A and B are constants to be determined from movable and immovable edge boundary conditions and A_1, A_2, A_3 and B_1, B_2, B_3 are constant co-efficients to be determined.

To determine the constant co-efficients A_1, A_2, A_3 and B_1, B_2, B_3, we compare the co-efficients of trigonometric functions after inserting equations (11),(12) and (13) in equations (1) and (2).

The following relations connecting the co-efficients can be obtained :

$$(3-\nu)A_1 + (1+\nu)B_1 = 4\frac{W_0 a}{\pi}\left(K_x + \nu K_y\right) \tag{14}$$

$$A_2 + A_3 = \frac{\pi W_0^{\,2}}{16a}(1-\nu) \tag{15}$$

$$(3-\nu)(A_2 - A_3) + (1+\nu)(B_2 - B_3) = \frac{\pi W_0^{\,2}}{4a} \tag{16}$$

$$(1+\nu)A_1 + (3-\nu)B_1 = 4\frac{W_0 a}{\pi}\left(K_y + \nu K_x\right) \tag{17}$$

$$B_2 + B_3 = \frac{\pi W_0^{\,2}}{16a}(1-\nu) \tag{18}$$

$$(3-\nu)(B_2 - B_3) + (1+\nu)(A_2 - A_3) = \frac{\pi W_0^{\,2}}{4a} \tag{19}$$

Solving the above six relations, one gets the values of the six co-efficients in the forms:

$$A_1 = \frac{W_0 a}{2\pi}\left[K_x(3+\nu) - K_y(1-\nu)\right] \tag{20}$$

$$B_1 = \frac{W_0 a}{2\pi}\left[K_x(\nu-1) + K_y(\nu+3)\right] \tag{21}$$

$$A_2 = B_2 = \frac{\pi W_0^{\;2}}{32a}(2-\nu) \tag{22}$$

$$A_3 = B_3 = -\frac{\pi \nu W_0^2}{32a} \tag{23}$$

1.1.5. DETERMINATION OF CONSTANTS A AND B

The constants A and B are now determined by using conditions for immovable edges which are

$$\int_{-a}^{a} \left(F_{,yy}\right)_{x=\pm a} dy = 0 \tag{24}$$

$$\int_{-a}^{a} \left(F_{,xx}\right)_{y=\pm a} dx = 0 \tag{25}$$

Performing necessary integrations one gets,

$$2aA + 2a\nu B = \pi\left(A_2 + A_3\right) - \frac{1}{8}W_0^{\;2}\frac{\pi^2}{a} \tag{26}$$

$$2a\nu A + 2aB = \pi\left(B_2 + B_3\right) - \frac{1}{8}W_0^{\;2}\frac{\pi^2}{a} \tag{27}$$

Solving (26) and (27) one gets,

$$A = B = \frac{1}{2a(1+\nu)}\left[\frac{\pi^2 W_0^{\;2}}{16a}(1-\nu) - \frac{1}{8}W_0^{\;2}\frac{\pi^2}{a}\right] \tag{28}$$

The intensity of load q is expressed in the following trigonometric form

$$q = q_0 Cos\left(\frac{\pi x}{2a}\right) Cos\left(\frac{\pi y}{2a}\right) \tag{29}$$

where

$$q_0 = \frac{1}{ab}\int_{-a}^{a}\int_{-a}^{a} q Cos\left(\frac{\pi x}{2a}\right) Cos\left(\frac{\pi y}{2a}\right) dxdy \tag{30}$$

Applying now Galerkin Procedure in equation (3), one gets the following cubic equation connecting load and deformation parameters

$$C_1\left(\frac{W_0}{h}\right)^3 - C_2\left(\frac{W_0}{h}\right)^2 + C_3\left(\frac{W_0}{h}\right) - C_4\left(\frac{q_0 a^4}{Eh^4}\right) = 0 \tag{31}$$

where,

$$C_1 = \frac{\pi^4}{128} \tag{31.1}$$

$$C_2 = \frac{5}{6}\left(\frac{a}{h}\right)^2\left(\frac{h}{R_1} + \frac{h}{R_2}\right) \tag{31.2}$$

$$C_3 = \left[\frac{\pi^4}{48(1-\nu^2)} - \frac{1}{4(1-\nu^2)}\left(\frac{a}{h}\right)^2\left[\left\{\left(\frac{a}{R_1}\right)^2 + \left(\frac{a}{R_2}\right)^2\right\}(3+\nu^2) + 2\left(\frac{a}{R_1}\right)\left(\frac{a}{R_2}\right)(\nu^2 + 4\nu - 1)\right]\right.$$
$$\left. + \frac{1}{(1-\nu^2)}\left(\frac{a}{h}\right)^2\left\{\left(\frac{a}{R_1}\right)^2 + \left(\frac{a}{R_2}\right)^2 + 2\nu\left(\frac{a}{R_1}\right)\left(\frac{a}{R_2}\right)\right\}\right]$$

(31.3)

$$C_4 = 1 \tag{31.4}$$

1.1.6. NUMERICAL RESULTS

Numerical results have been computed for variations of non-dimensional deformations $\left(\frac{W_0}{h}\right)$ [shown horizontally] against variations of non-dimensional load parameter $\left(\frac{q_0 a^4}{Eh^4}\right)$ [shown vertically] and presented graphically [Figures 1.1(a)-1.1(f)] considering the following set of values

$\nu = 0.3$

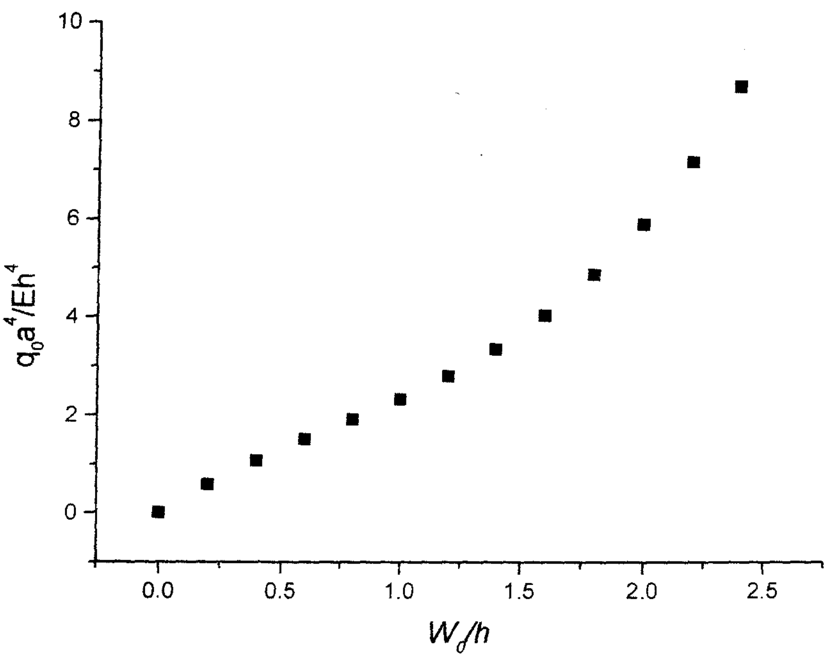

Fig.1.1(a) $\frac{W_0}{h}$ vs. $\frac{q_0 a^4}{Eh^4}$ for values of $a = b = 10, R_1 = R_2 = 100$

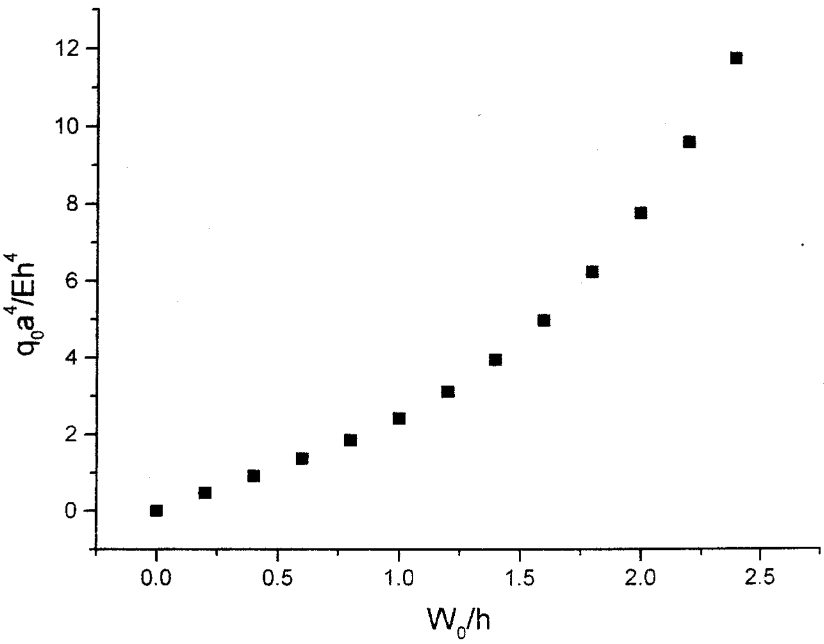

Fig.1.1(b) $\frac{W_0}{h}$ vs. $\frac{q_0 a^4}{Eh^4}$ for values of $a = b = 10$, $R_1 = R_2 = 200$

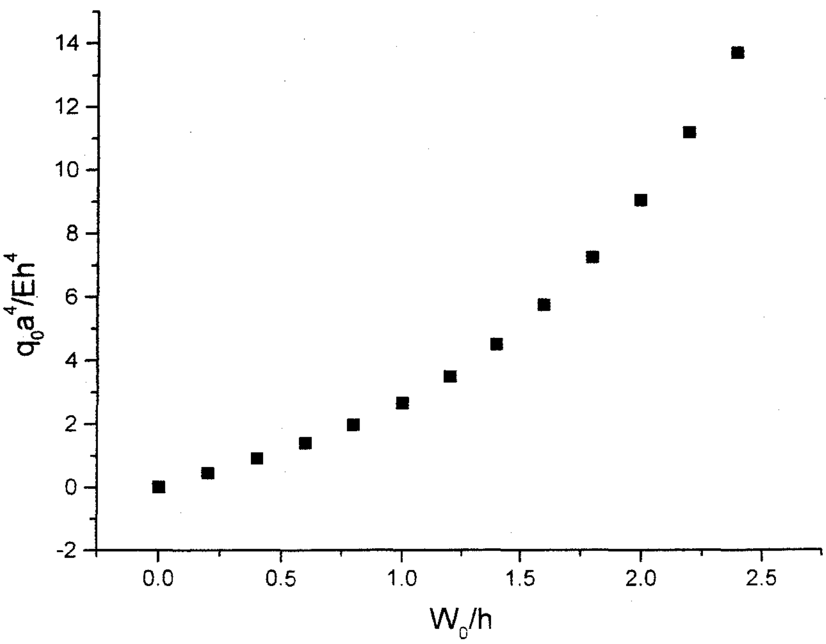

Fig.1.1(c) $\frac{W_0}{h}$ vs. $\frac{q_0 a^4}{Eh^4}$ for values of $a = b = 10, R_1 = R_2 = 400$

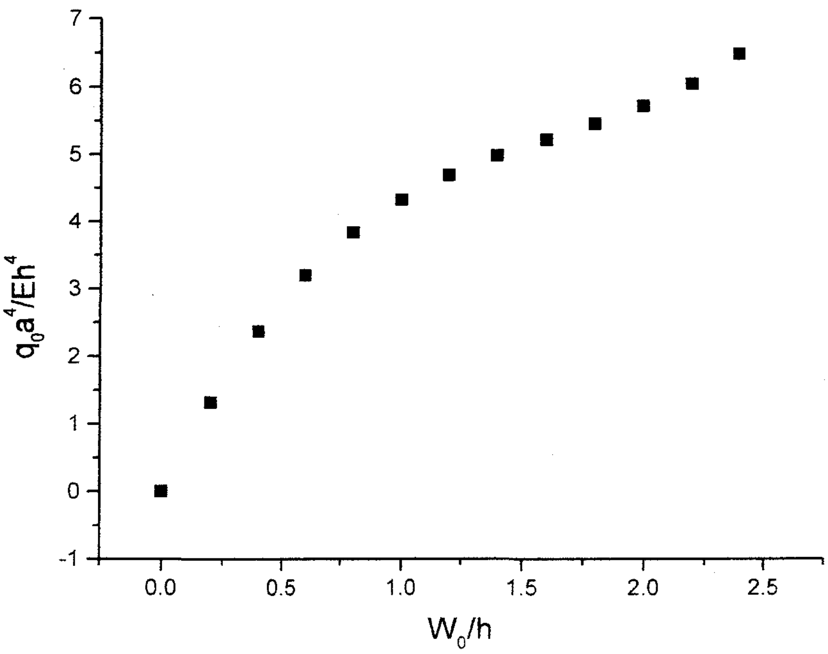

Fig.1.1(d) $\frac{W_0}{h}$ vs. $\frac{q_0 a^4}{Eh^4}$ for values of $a = b = 15, R_1 = R_2 = 100$

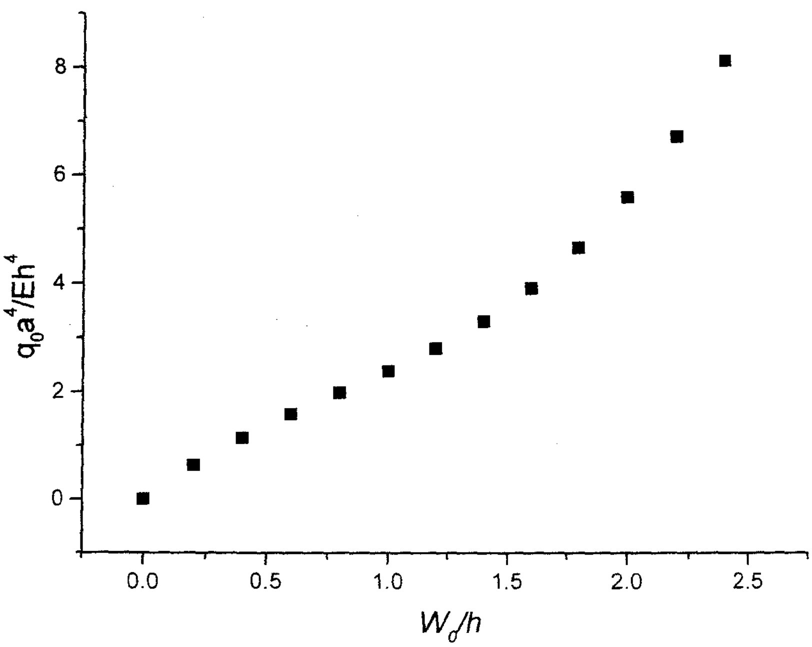

Fig.1.1(e) $\frac{W_0}{h}$ vs. $\frac{q_0 a^4}{Eh^4}$ for values of $a = b = 15, R_1 = R_2 = 200$

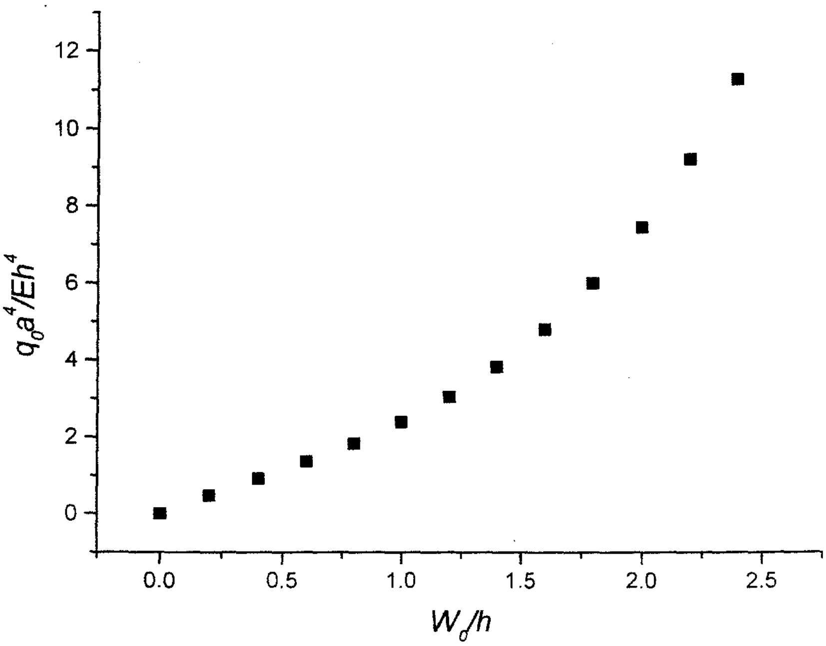

Fig.1.1(f) $\frac{W_0}{h}$ vs. $\frac{q_0 a^4}{Eh^4}$ for values of $a = b = 15, R_1 = R_2 = 400$

1.1.7. OBSERVATION AND DISCUSSIONS:

i) The cubic equation (31) should be solved by Cardon's method as available in any book on Classical Algebra. But in the present case, it would be a Complicated exercise if that method of solution is adopted. Instead, for a given value of non-dimensional deflection $\left(\frac{W_0}{h}\right)$, the load distributions $\left(\frac{q_o a^4}{Eh^4}\right)$ parameter can conveniently be determined from the cubic equation(31) for different variations of parameters as considered in the above six figures.

ii) From the above six graphical figures one can observe that as load parameter $\left(\frac{q_0 a^4}{Eh^4}\right)$ increases the non-dimensional deflection $\left(\frac{W_0}{h}\right)$ increases but slowly.

iii) From the above Fig.1.1(a)-Fig.1.1(c) one can observe that for a square panel the load bearing capacities are increased when radii of curvature $(R_1, R_2)$are increased to attain the same level of deflection $\left(\frac{W_0}{h}\right)$.

iv) Same nature of variations are also noted for a square panel from Fig.1.1(d)-Fig.1.1(f).

v) Keeping the radii of curvature (R_1, R_2) fixed, the load bearing capacities are different when dimensions of the square panel are changed to attain the same level of deflection $\left(\frac{W_0}{h}\right)$.The load bearing capacities are enormously high when the length of the square panel is decreased.

vi) In general slow nonlinearity is observed when graphical representation is made. A better observations can be made if the results are graphically represented simultaneously in a single graphical representation, as shown in Figure-1.1(g)

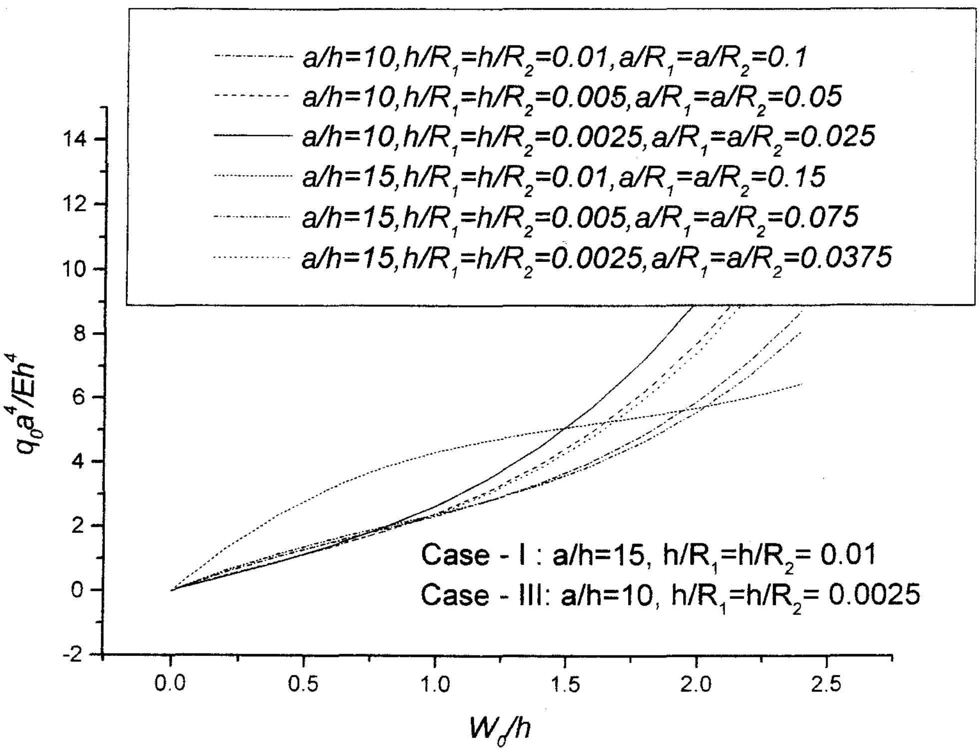

Fig.1.1(g) $\frac{W_0}{h}$ vs. $\frac{q_0 a^4}{Eh^4}$ for different values of a, b and R_1, R_2.

For a square panel as load increases the deflection also increases slowly for all the cases with R increasing up to a load value $\frac{q_0 a^4}{Eh^4} < 2$. But after the load parameter becomes greater than 2, the deflection increases slowly for deflection from 0.0 – 0.8. For the same load function, for Case-I the deflection is minimum and for Case-III the deflection is maximum. This trend continues till the load function attains the approximate value 2 and thereafter this trend is reversed and the deflection increases comparatively slowly with the increase of Load Function $\frac{q_0 a^4}{Eh^4}$ That is it requires a much more load to deflect the surface by certain amount than it requires for the range of Deflection from 0.0- 0.8.

Therefore we can conclude that in the neighbourhood of the value of the Load Function $\frac{q_0 a^4}{Eh^4} \approx 2.0$ all the results for all the cases (except for Case – I) nearly converge. In the light of the present geometrical configuration $\frac{q_0 a^4}{Eh^4} \approx 2.0$ may be treated as a critical load.

CHAPTER-I [Second Paper]

NONLINEAR DEFORMATION ANALYSIS OF A SQUARE SANDWICH PANEL UNDER MECHANICAL LOAD*

1.2.1. INTRODUCTION

The second paper of this chapter presents an analysis for the nonlinear deformations of a square sandwich panel under mechanical load . Outstanding research works in this field, using different approach and methods of solution, can be found in many references [16, 131-142, 149, 183-189]. Here the basic governing equations expressed in terms of displacement components in the von Karman sense for a sandwich panel have been employed for the analysis of the title problem. Some numerical results have been computed and followed by observation and discussion.

1.2.2. BASIC GOVERNING EQUATIONS

In this paper, we consider a square panel of length $2a$ along the X - axis and peripheral width along the Y - axis, Z - axis is taken normally upwards, origin being located at the centre of the panel at the middle surface. The displacement formulation of the nonlinear partial differential equations in terms displacement components can be expressed as [74,184]

$$u_{,xx}+\frac{(1-\nu_f)}{2}u_{,yy}+\frac{(1+\nu_f)}{2}v_{,xy}=(K_x+\nu_f K_y)W_{,x}-\left\{W_{,xx}+\frac{(1-\nu_f)}{2}W_{,yy}\right\}W_{,x}-\frac{(1+\nu_f)}{2}W_{,y}W_{,xy} \tag{32}$$

$$v_{,yy}+\frac{(1-\nu_f)}{2}v_{,xx}+\frac{(1+\nu_f)}{2}u_{,xy}=(K_y+\nu_f K_x)W_{,y}-\left\{W_{,yy}+\frac{(1-\nu_f)}{2}W_{,xx}\right\}W_{,y}-\frac{(1+\nu_f)}{2}W_{,x}W_{,xy} \tag{33}$$

$$D\nabla^4 W=\left(1-\frac{D}{C_Q}\nabla^2\right)\left[q+F_{,yy}W_{,xx}-2F_{,xy}W_{,xy}+F_{,xx}W_{,yy}+K_xF_{,yy}+K_yF_{,xx}\right] \tag{34}$$

Presented at the Fourth National Conference on Applicable Mathematics in Wave Mechanics and Vibrations(WMVC-2008),November.14-16,2008 held at Jaipur Engineering College and Research Centre,Jaipur(Rajsthan),India.
Appeared in the E-Proceedsings of the Conference, Pages 105-118.

The stress function and displacement components are related by

$$F,_{yy} = 2\frac{E_f h_f}{(1-\nu_f^2)}\left[u,_x + \frac{1}{2}W,_x{}^2 + \nu_f\left\{v,_y + \frac{1}{2}W,_y{}^2\right\} - (K_x + \nu_f K_y)W\right] \tag{35}$$

$$F_{,xx} = 2\frac{E_f h_f}{(1-\nu_f^2)}\left[v,_y + \frac{1}{2}W,_y{}^2 + \nu_f\left\{u,_x + \frac{1}{2}W,_x{}^2\right\} - (K_y + \nu_f K_x)W\right] \tag{36}$$

$$F,_{xy} = -\frac{E_f h_f}{(1-\nu_f^2)}\left[u,_x + v,_y + W,_x W,_y\right] \tag{37}$$

1.2.3. BOUNDARY CONDITION AND METHOD OF SOLUTION:

For a simply supported panel the deformation satisfying the boundary conditions (10), is assumed in the form (11).The inplane displacements u and v are also chosen in the forms (12) and (13).

To determine the constant co-efficients A_1, A_2, A_3 and B_1, B_2, B_3,we compare the co-efficients of Trigonometric functions after inserting equations (11),(12) and (13) in equations (32) and (33).

The following relations connecting the co-efficients are given by

$$(3-\nu_f)A_1 + (1+\nu_f)B_1 = 4\frac{W_0 a}{\pi}(K_x + \nu_f K_y) \tag{38.1}$$

$$A_2 + A_3 = \frac{\pi W_0^2}{16a}(1-\nu_f) \tag{38.2}$$

$$(3-\nu_f)(A_2 - A_3) + (1+\nu_f)(B_2 - B_3) = \frac{\pi W_0^2}{4a} \qquad (38.3$$

$$(1+\nu_f)A_1 + (3-\nu_f)B_1 = 4\frac{W_0 a}{\pi}(K_y + \nu_f K_x) \tag{38.4}$$

$$B_2 + B_3 = \frac{\pi W_0^2}{16a}(1-\nu_f) \tag{38.5}$$

$$(3-\nu_f)(B_2 - B_3) + (1+\nu_f)(A_2 - A_3) = \frac{\pi W_0^2}{4a} \tag{38.6}$$

Solving the above six relations, one gets the values of the six co-efficients in the forms:

$$A_1 = \frac{aW_0}{2\pi}\left[K_x(3+\nu_f) - K_y(1-\nu_f)\right] \tag{39.1}$$

$$B_1 = \frac{aW_0}{2\pi}\left[K_x(\nu_f - 1) + K_y(3+\nu_f)\right] \tag{39.2}$$

$$A_2 = B_2 = \frac{\pi W_0^2}{32a}(2-\nu_f) \tag{39.3}$$

$$A_3 = B_3 = -\frac{\pi \nu_f W_0^2}{32a} \tag{39.4}$$

1.2.4. DETERMINATION OF CONSTANTS A AND B

The constants A and B are now determined by using conditions for immovable edges which are given by (24) and (25).

Performing necessary integrations one gets,

$$2aA + 2a\nu_f B = \pi(A_2 + A_3) - \frac{1}{8}\frac{\pi^2 W_0^2}{a} \tag{40.1}$$

$$2a\nu_f A + 2aB = \pi(B_2 + B_3) - \frac{1}{8}\frac{\pi^2 W_0^2}{a} \tag{40.2}$$

Solving equations (40.1) and (40.2) one gets

$$A = B = \frac{1}{2a(1+\nu_f)}\left[\frac{\pi^2 W_0^2}{16a}(1-\nu_f) - \frac{1}{8}\frac{\pi^2 W_0^2}{a}\right] \tag{41}$$

We now express the intensity of load q in the Trigonometric form as stated in (29) and (30)

Applying Galerkin's procedure in equation (34),one gets the following cubic equation connecting deformation and load parameters in the form

$$C_1'\left(\frac{W_0}{h_c}\right)^3 - C_2'\left(\frac{W_0}{h_c}\right)^2 + C_3'\left(\frac{W_0}{h_c}\right) - C_4'\left(\frac{q_0 a^4}{E_f h_c^4}\right) = 0 \tag{42}$$

where

$$C_1' = \frac{\pi^4}{64}\left(\frac{h_c}{a}\right)^2\left[1+\frac{\pi^2}{4}\left(\frac{h_f}{a}\right)^2\left(\frac{E_f h_f}{G_c h_c}\right)\left(1+\frac{h_c}{h_f}\right)^2\right] \tag{42.1}$$

$$C_2' = \frac{32}{9(1-\nu_f)}\left(\frac{h_c}{R_1}+\frac{h_c}{R_2}\right)+\frac{8\pi^2}{9(1-\nu_f)}\left(\frac{h_f}{a}\right)^2\left(\frac{E_f h_f}{G_c h_c}\right)\left(\frac{h_c}{R_1}+\frac{h_c}{R_2}\right)\left(1+\frac{h_c}{h_f}\right)^2$$

$$+\frac{7\pi^2}{18}\left(\frac{h_f}{a}\right)^2\left(\frac{E_f h_f}{G_c h_c}\right)\left(\frac{h_c}{R_1}+\frac{h_c}{R_2}\right)\left(1+\frac{h_c}{h_f}\right)^2-\frac{16}{9}\left(\frac{1+\nu_f}{1-\nu_f}\right)\left(\frac{h_c}{R_1}+\frac{h_c}{R_2}\right)$$

$$-\frac{1}{3}\left(\frac{h_c}{R_1}+\frac{h_c}{R_2}\right)-\frac{4\pi^2}{9}\left(\frac{1+\nu_f}{1-\nu_f}\right)\left(\frac{h_f}{a}\right)^2\left(\frac{E_f h_f}{G_c h_c}\right)\left(\frac{h_c}{R_1}+\frac{h_c}{R_2}\right)\left(1+\frac{h_c}{h_f}\right)^2 \tag{42.2}$$

$$C_3' = \frac{\pi^4}{8}\left(\frac{h_f}{a}\right)^2\left(1+\frac{h_c}{h_f}\right)^2$$

$$-\frac{1}{2(1-\nu_f^2)}\left[(\nu_f-1)\left\{\nu_f\left(\frac{a^2}{R_1^2}+\frac{a^2}{R_2^2}\right)+2\left(\frac{a}{R_1}\right)\left(\frac{a}{R_2}\right)\right\}+(\nu_f+3)\right.$$

$$\left.\times\left\{\left(\frac{a}{R_1}\right)^2+\left(\frac{a}{R_2}\right)^2+2\nu_f\left(\frac{a}{R_1}\right)\left(\frac{a}{R_2}\right)\right\}\right]+\frac{2}{(1-\nu_f^2)}\left\{\left(\frac{a}{R_1}\right)^2+\left(\frac{a}{R_2}\right)^2+2\nu_f\left(\frac{a}{R_1}\right)\left(\frac{a}{R_2}\right)\right\}$$

$$+\frac{\pi^2}{2(1-\nu_f^2)}\left(\frac{E_f h_f}{G_c h_c}\right)\left(\frac{h_f}{h_c}\right)^2\left(1+\frac{h_c}{h_f}\right)^2\left\{\left(\frac{h_c}{R_1}\right)^2+\left(\frac{h_c}{R_2}\right)^2+2\nu_f\left(\frac{h_c}{R_1}\right)\left(\frac{h_c}{R_2}\right)\right\}$$

$$-\frac{\pi^2}{8(1-\nu_f^2)}\left(\frac{E_f h_f}{G_c h_c}\right)\left(\frac{h_f}{h_c}\right)^2\left(1+\frac{h_c}{h_f}\right)^2\left[(\nu_f-1)\left\{\nu_f\left(\frac{h_c^2}{R_1^2}+\frac{h_c^2}{R_2^2}\right)+2\left(\frac{h_c}{R_1}\right)\left(\frac{h_c}{R_2}\right)\right\}\right.$$

$$\left.+(\nu_f+3)\left\{\left(\frac{h_c}{R_1}\right)^2+\left(\frac{h_c}{R_2}\right)^2+2\nu_f\left(\frac{h_c}{R_1}\right)\left(\frac{h_c}{R_2}\right)\right\}\right] \tag{42.3}$$

$$C_4' = \left(\frac{a}{h_f}\right)\left(\frac{h_c}{a}\right)^3\left[1+\frac{\pi^2}{4}\left(\frac{E_f h_f}{G_c h_c}\right)\left(\frac{h_f}{a}\right)^2\left(1+\frac{h_c}{h_f}\right)^2\right] \tag{42.4}$$

1.2.5. NUMERICAL RESULTS

Numerical results have been computed for variations of non-dimensional deflection $\left(\frac{W_0}{h_c}\right)$ [shown horizontally] against variations of non-dimensional loading parameter $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ [shown vertically] and presented graphically [Figures 1.2(a)-1.2(f)] considering the following set of values[184]:

$E_f = 10\times10^6$ psi , $G_c = 12000$ psi

$h_c = 0.2$ in , $h_f = 0.02$ in , $D = 4840$ ib-in

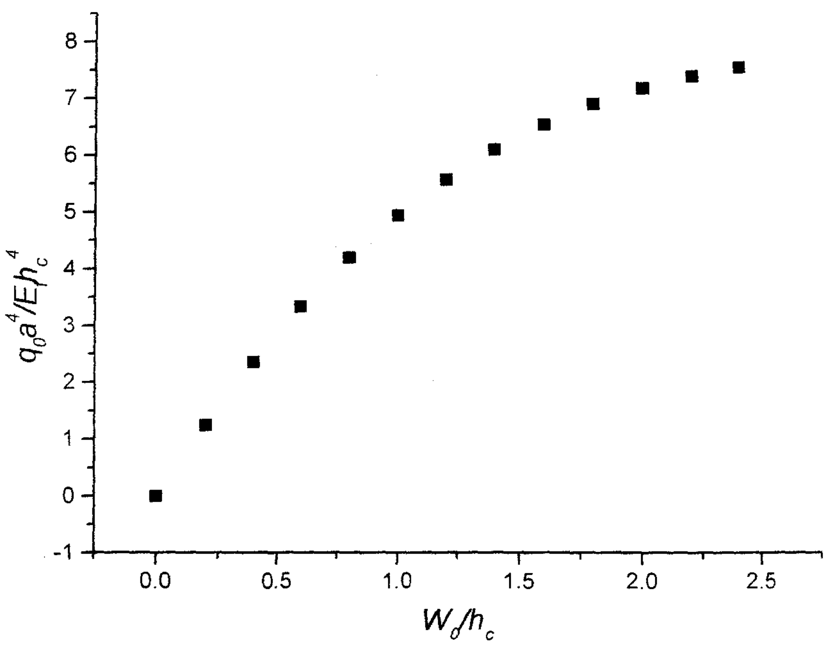

Fig.1.2(a) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=10, $R_1 = R_2 = 100$

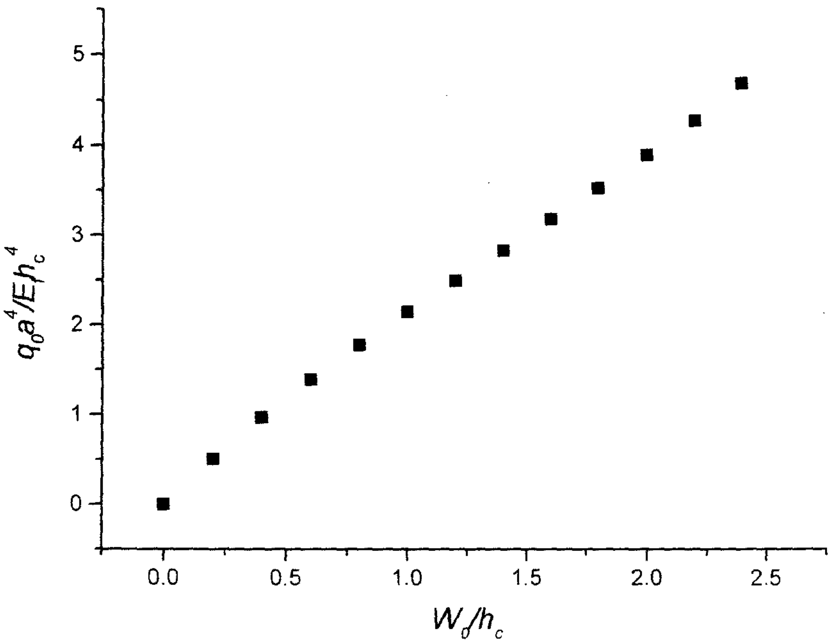

Fig.1.2(b) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=10, $R_1 = R_2 = 200$

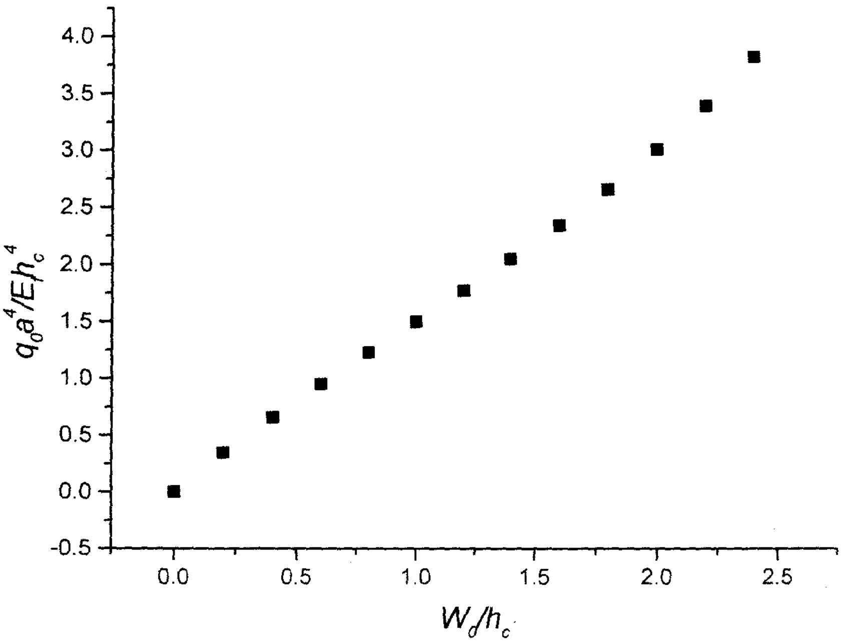

Fig.1.2(c) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0a^4}{E_fh_c^4}\right)$ for values of a=10, $R_1 = R_2 = 400$

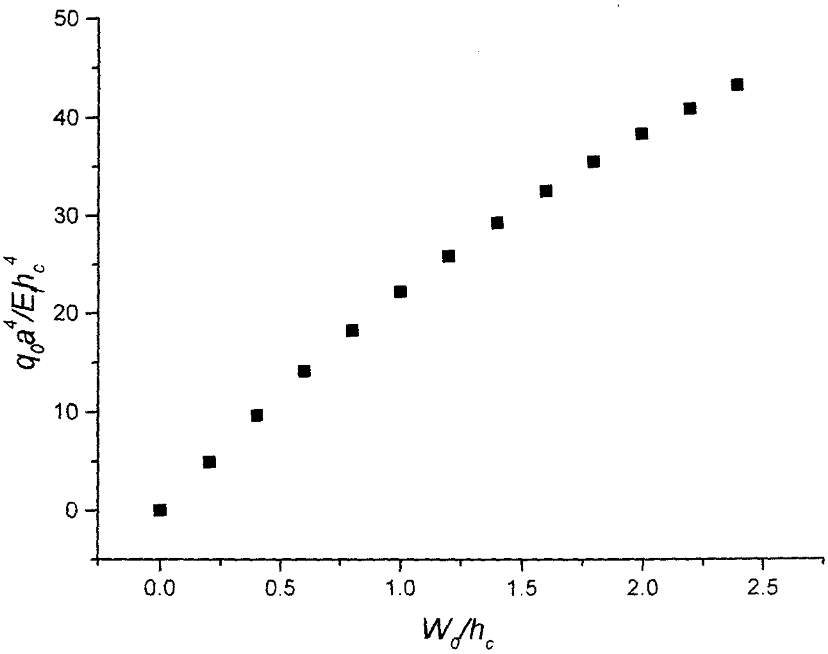

Fig.1.2(d) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=15, $R_1 = R_2 = 100$

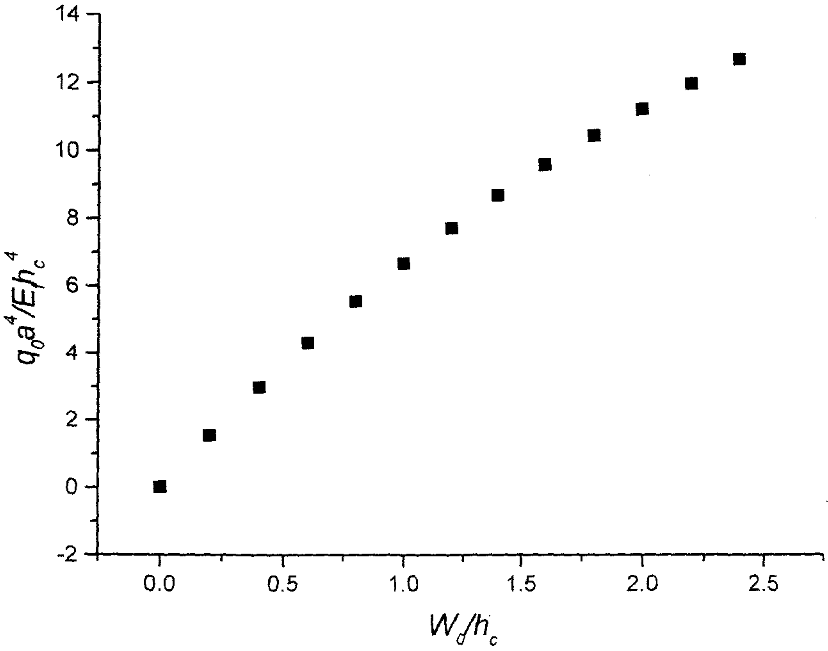

Fig.1.2(e) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0a^4}{E_fh_c^4}\right)$ for values of a=15, $R_1 = R_2 = 200$

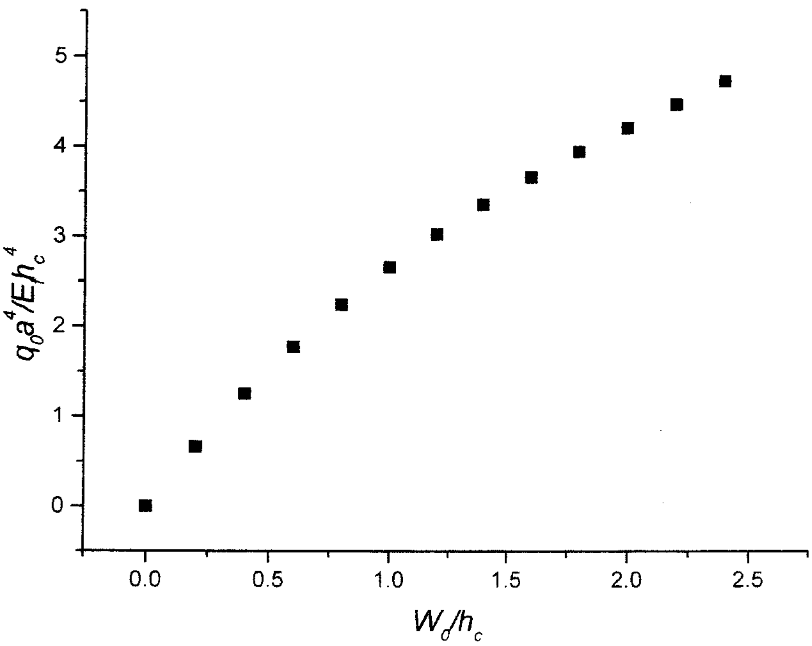

Fig.1.2(f) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0a^4}{E_fh_c^4}\right)$ for values of a=15, $R_1 = R_2 = 400$

1.2.6. OBSERVATION AND DISCUSSIONS

i) The cubic equation (42) should be solved by Cardon's method as available in any book on Classical Algebra. But in the present case, it would be a complicated exercise if that method of solution is adopted. Instead, for a given value of non-dimensional deflection $\left(\frac{W_0}{h_c}\right)$, the load distributions $\left(\frac{q_o a^4}{E_f h_c^4}\right)$ parameter can conveniently be determined from the cubic equation(42) for different variations of parameters as considered in the above six figures..

ii) From the above six figures, we observe that load parameter $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ increases with the increase of non-dimensional deflection $\left(\frac{W_0}{h_c}\right)$.

iii) From figures{I(a), 1(b), and 1(c)} and {1(d),1(e) and 1(f)} we observe that for a square sandwich panel the load $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ bearing capacities are increased when radii of curvature (R_1, R_2) are decreased to attain the same level of deflection $\left(\frac{W_0}{h_c}\right)$.

iii) Keeping the radii of curvature (R_1, R_2) fixed, the load $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ bearing capacities are seen to be different when dimensions of the shell panel are changed to attain the same level of deflection $\left(\frac{W_0}{h_c}\right)$.

From tables {1(a),1(d)},{1(b),1(e)} and {1(c),1(f)}we observed that load bearing capacities are increased when length of the square sandwich panel is increased.

vi) In general slow nonlinearity is observed when graphical representation is made.

CHAPTER II

NONLINEAR DEFORMATION ANALYSIS OF A SANDWICH SHELL PANEL UNDER MECHANICAL LOADING*

2.1.1 INTRODUCTION

Sandwich plates or shells consist of two thin load-bearing sheets called faces separated by a light weight core of low thickness. The face sheets are usually made of metal or fiber-reinforced plastics. Common core materials include foamed plastic, light-weight metallic honeycomb and light-weight corrugated sheets. The core serves to increase the load-bearing capacity and also to increase the bending resistance of the composite cross-section and as such sandwich plates and shells are widely used in modern design and aerospace industry, particularly in flight structures .Outstanding research works in this field ,using different approach and method of solution, can be found in many references [16,131-142,149,183-189].

2.1.2BASIC GOVERNING EQUATIONS

A sandwich shell panel of length $2a$ along the X -axis and peripheral width $2b$ along the Y -axis is considered, origin being located at the centre of the shell panel in the middle surface.

Basic governing equations derived in the von Karman sense and extended to sandwich shell panel can be expressed as [74,184]:

$$D\nabla^4 W = \left[1 - \frac{E_f h_f (h_f + h_c)^2 \nabla^2}{2G_c h_c}\right]\left(q + F_{,yy}W_{,xx} - 2F_{,xy}W_{,xy} + F_{,xx}W_{,yy} + K_x F_{,yy} + K_y F_{,xx}\right) \tag{43}$$

$$\nabla^4 F = 2E_f h_f \left(W^2_{,xy} - W_{,xx}W_{,yy} - K_x W_{,yy} - K_y W_{,xx}\right) \tag{44}$$

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Presented at the Fourth National Conference on Applicable Mathematics in Wave Mechanics and Vibrations(WMVC-2008),November.14-16,2008 held at Jaipur Engineering College and Research Centre,Jaipur(Rajsthan),India.
Appeared in the E-Proceedsings of the Conference, Pages 105-118.

2.1.3 BOUNDARY CONDITIONS

The deflection function W satisfy the simply supported edge movable boundary conditions

$$W = 0 = W_{,xx} \text{ at } X = -a, a \tag{45.1}$$

$$W = 0 = W_{,yy} \text{ at } Y = -b, b \tag{45.2}$$

2.1.4 METHOD OF SOLUTION

Here, W satisfying the above boundary conditions is assumed in the form:

$$W = W_0 Cos\left(\frac{\pi x}{2a}\right) Cos\left(\frac{\pi y}{2b}\right) \tag{46}$$

Inserting equation (46) into (44) yields,

$$\nabla^4 F = C_1 Cos\left(\frac{\pi x}{a}\right) + C_2 Cos\left(\frac{\pi y}{b}\right) + C_3 Cos\left(\frac{\pi x}{2a}\right) Cos\left(\frac{\pi y}{2b}\right) \tag{47}$$

where,

$$C_1 = -\frac{1}{16}\frac{W_0^2 \pi^4 E_f h_f}{a^2 b^2} \tag{47.1}$$

$$C_2 = -\frac{1}{16}\frac{W_0^2 \pi^4 E_f h_f}{a^2 b^2} \tag{47.2}$$

$$C_3 = \frac{W_0 \pi^2 E_f h_f}{2}\left(\frac{K_x}{a^2} + \frac{K_y}{b^2}\right) \tag{47.3}$$

The solution of equation (47) is

$$F = F_c + F_p \tag{48}$$

where F_C is the complementary function and F_p is the particular integral given by,

$$F_c = \frac{Lx^2}{2} + \frac{My^2}{2} + Nxy \tag{48.1}$$

$$F_p = b_1 Cos\left(\frac{\pi x}{a}\right) + b_2 Cos\left(\frac{\pi y}{b}\right) + b_3 Cos\left(\frac{\pi x}{2a}\right) Cos\left(\frac{\pi y}{2b}\right) \tag{48.2}$$

where L, M and N are arbitrary constants to be determined from movable edge boundary conditions and b_1, b_2 and b_3 are known constants given by

$$b_1 = \frac{1}{2} C_1 \left(\frac{a}{\pi}\right)^4$$

$$= -\frac{1}{32} W_0^{\,2} E_f h_f \left(\frac{a}{b}\right)^2 \tag{48.3}$$

$$b_2 = \frac{1}{2} C_2 \left(\frac{b}{\pi}\right)^4$$

$$= -\frac{1}{32} W_0^{\,2} E_f h_f \left(\frac{b}{\pi}\right)^2 \tag{48.4}$$

$$b_3 = \frac{8C_3}{\pi^4 \left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2}$$

$$= \frac{4W_0 E_f h_f}{\pi^2} \frac{\left(\frac{K_x}{a^2} + \frac{K_y}{b^2}\right)}{\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2} \tag{48.5}$$

2.1.5 DETERMINATION OF CONSTANTS L, M AND N [MOVABLE EDGES]

The constants L, M and N contribute directly to in-plane stresses N_{xx}, N_{yy} and N_{xy} respectively and will be determined by using the in-plane boundary conditions [183].

Since the above in-plane stresses can be expressed in terms of stress function $F(x, y)$, the following set of relaxed boundary conditions for movable edges are enforced [74]

$$\int_{-b}^{b}\left[\left(F_{,yy}\right)_{x=a,-a}\right]dy = 0 \tag{49.1}$$

$$\int_{-a}^{a}\left[\left(F_{,xx}\right)_{y=b,-b}\right]dx = 0 \tag{49.2}$$

$$\int_{-b}^{b}\left[\left(F_{,xy}\right)_{x=-a,a}\right]dy = 0 \tag{49.3}$$

$$\int_{-a}^{a}\left[\left(F_{,xy}\right)_{y=-b,b}\right]dx = 0 \tag{49.4}$$

Using the above set of boundary conditions in equation (48) one gets

$$L = M = N = 0 \tag{50}$$

Therefore, the stress function is completely determined for movable edge conditions.

Expressing the load q in the sinusoidal form

$$q = q_0 Cos\left(\frac{\pi x}{2a}\right)Cos\left(\frac{\pi y}{2b}\right) \tag{51}$$

where

$$q_0 = \frac{1}{ab}\int_{-a}^{a}\int_{-b}^{b} q Cos\left(\frac{\pi x}{2a}\right)Cos\left(\frac{\pi y}{2b}\right)dxdy \tag{51.1}$$

and operating ∇^2 in the right hand side of equation (43) one gets,

$$D\nabla^4 W = q_0 Cos\left(\frac{\pi x}{2a}\right)Cos\left(\frac{\pi y}{2b}\right) + F_{,yy}W_{,xx} - 2F_{,xy}W,xy + F_{,xx}W_{,yy} + K_x W_{,yy} + K_y W_{,xx}$$

$$-\frac{E_f h_f (h_f + h_c)^2}{2G_c h_c}\left\{\frac{q_0\pi^2}{4}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)Cos\left(\frac{\pi x}{2a}\right)Cos\left(\frac{\pi y}{2b}\right) + \left(F_{,yy}W_{,xxxx} + 2F_{,xyy}W_{,xxx} + F_{,xxyy}W_{,xx}\right.\right.$$
$$+ F_{,yy}W_{,xxyy} + 2F_{,yyy}W_{,xxy} + F_{,yyyy}W_{,xx}\Big) - 2\Big(F_{,xxxy}W_{,xy} + 2F_{,xxy}W_{,xxy} + F_{,xy}W_{,xxxy} + F_{,xy}W_{,xyyy} + 2F_{,xyy}W_{,xyy}$$
$$+ F_{,xyyy}W_{,xy}\Big) + \Big(F_{,xxxx}W_{,yy} + 2F_{,xxx}W_{,xyy} + F_{,xx}W_{,xxyy} + F_{,xxyy}W_{,yy} + 2F_{,xxy}W_{,yyy} + F_{,xx}W_{,xxyy}\Big) + K_x\Big(F_{,xxyy}$$
$$\left.+ F_{,yyyy}\Big) + K_y\left(F_{,xxyy} + F_{,xxxx}\right)\right\}$$

(52)

Applying now Galerkin's procedure in equation (52) one gets after a lengthy but simple calculations ,the following cubic equation in non-dimensional form:

$$P\left(\frac{W_0}{h_c}\right)^3 + Q\left(\frac{W_0}{h_c}\right)^2 + R\left(\frac{W_0}{h_c}\right) + S = 0 \qquad (53)$$

where,

$$P = (E_f h_f)\frac{\pi^6}{2048}\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(1+\frac{h_c}{h_f}\right)^2\left(\frac{a}{b}+\frac{b}{a}\right)\left(\frac{h_c}{h_f}\right)^2\left\{\left(\frac{h_f}{a}\right)^4+\left(\frac{h_f}{b}\right)^4\right\}$$

$$+(E_f h_f)\frac{\pi^4}{256}\left(\frac{h_f}{a}\right)\left(\frac{h_f}{b}\right)\left(\frac{h_c}{h_f}\right)^2\left\{\left(\frac{a}{b}\right)^2+\left(\frac{b}{a}\right)^2\right\} \qquad (53.1)$$

$$Q = -\frac{1}{18}(E_f h_f)\left\{\left(\frac{b}{a}\right)\left(\frac{h_c}{R_1}\right)+\left(\frac{a}{b}\right)\left(\frac{h_c}{R_2}\right)\right\}$$
$$-\frac{\pi^2}{12}(E_f h_f)\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(1+\frac{h_c}{h_f}\right)^2\left(\frac{h_f}{a}\right)\left(\frac{h_f}{b}\right)\left\{\left(\frac{h_c}{R_1}\right)+\left(\frac{h_c}{R_2}\right)\right\}$$

$$-\frac{2}{3}(E_f h_f)\left(\frac{h_c}{R_1}\right)\frac{\left\{1+\left(\frac{K_y}{K_x}\right)\left(\frac{a}{b}\right)^2\right\}}{\left\{1+\left(\frac{a}{b}\right)^2\right\}^2}\left[\left(\frac{a}{b}\right)+\pi^2\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(\frac{h_f}{a}\right)\left(\frac{h_f}{b}\right)\right. \qquad (53.2)$$
$$\left.\times\left\{1+\left(\frac{a}{b}\right)^2\right\}\left\{1+\left(\frac{h_c}{h_f}\right)\right\}^2\right]$$

$$R=\left(E_f h_f\right)\left(\frac{a}{R_1}\right)\frac{\left\{1+\left(\frac{K_y}{K_x}\right)\left(\frac{a}{b}\right)^2\right\}}{\left\{1+\left(\frac{a}{b}\right)^2\right\}^2}\left[\left(\frac{a}{b}\right)\left(\frac{a}{R_1}\right)+\left(\frac{b}{a}\right)\left(\frac{a}{R_2}\right)\right]$$

$$+\frac{\pi^2}{8}\left(E_f h_f\right)\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(\frac{h_c}{R_1}\right)\left(\frac{h_f}{h_c}\right)^2\left\{1+\left(\frac{h_c}{h_f}\right)\right\}^2\left\{1+\left(\frac{a}{b}\right)^2\right\}\frac{\left\{1+\left(\frac{K_y}{K_x}\right)\left(\frac{a}{b}\right)^2\right\}}{\left\{1+\left(\frac{a}{b}\right)^2\right\}^2}$$

$$\times\left[\left(\frac{a}{b}\right)\left(\frac{h_c}{R_1}\right)+\left(\frac{b}{a}\right)\left(\frac{h_c}{R_2}\right)\right]$$

$$+\frac{\pi^4}{32}\left(E_f h_f\right)\left(\frac{b}{a}\right)\left(\frac{h_f}{a}\right)^2\left\{1+\left(\frac{h_c}{h_f}\right)\right\}^2\left\{1+\left(\frac{a}{b}\right)^2\right\}^2 \qquad (53.3)$$

$$S=-\left(\frac{q_0 a^4}{E_f h_c^4}\right)\left(\frac{h_f}{a}\right)^4\left(\frac{h_c}{h_f}\right)^3\left[\left(\frac{a}{h_f}\right)\left(\frac{b}{h_f}\right)\right.$$

$$\left.+\frac{1}{8}\pi^2\left(\frac{E_f h_f}{G_c h_c}\right)\left(1+\frac{h_c}{h_f}\right)^2\left(1+\frac{a^2}{b^2}\right)\left(\frac{b}{a}\right)\right] \qquad (53.4)$$

2.1.6 NUMERICAL RESULTS

The tabular representations of numerical results have been presented graphically [Figures 2.1(a)-2.1(f)], showing variations of non-dimensional deflections $\left(\frac{W_0}{h_c}\right)$ against non-dimensional loading parameters $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ considering the following set of values [184]:

$E_f = 10 \times 10^6$ psi

$G_c = 12000$ psi

$h_c = 0.2$ in

$h_f = 0.02$ in

$D = 4840$ ib-in

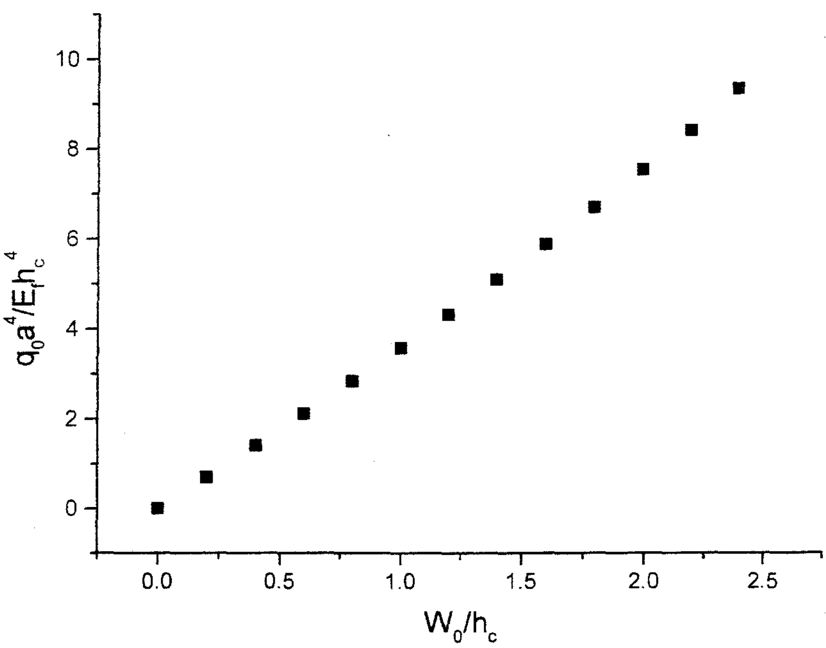

Fig.2.1(a) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=b=10, $R_1 = R_2 = 100$

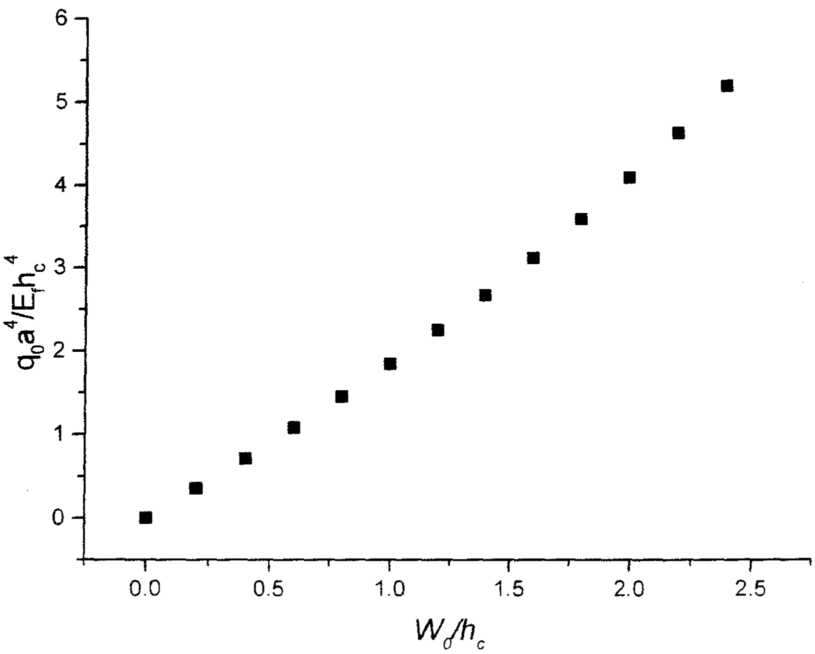

Fig.2.1(b) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=b=10, $R_1 = R_2 = 200$

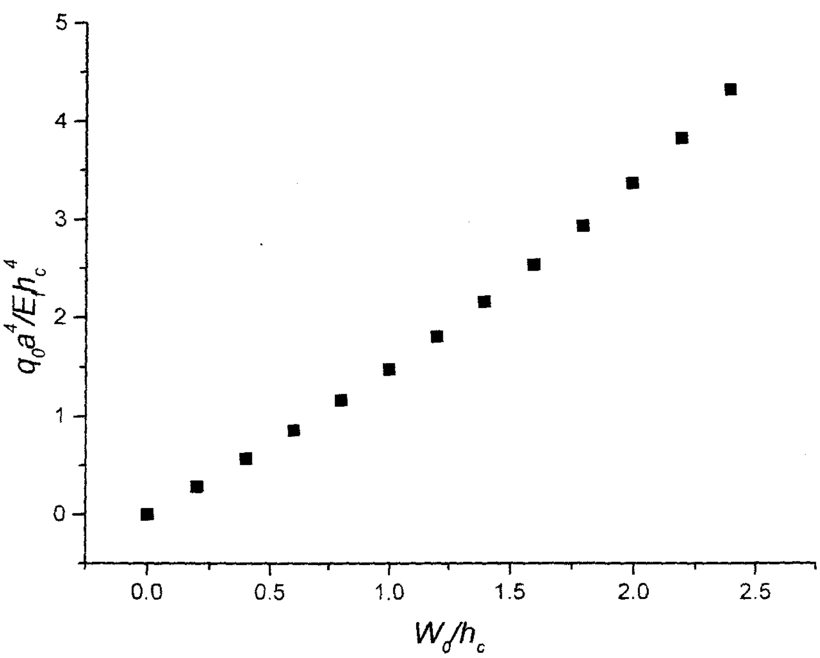

Fig.2.1.(c) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=b=10, $R_1 = R_2 = 400$

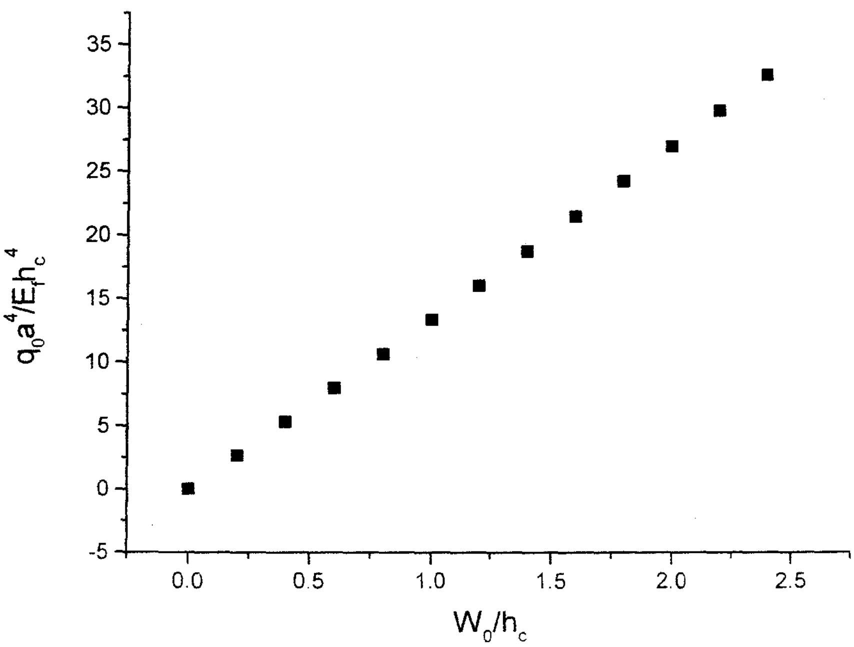

Fig.2.1.(d) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=b=15, $R_1 = R_2 = 100$

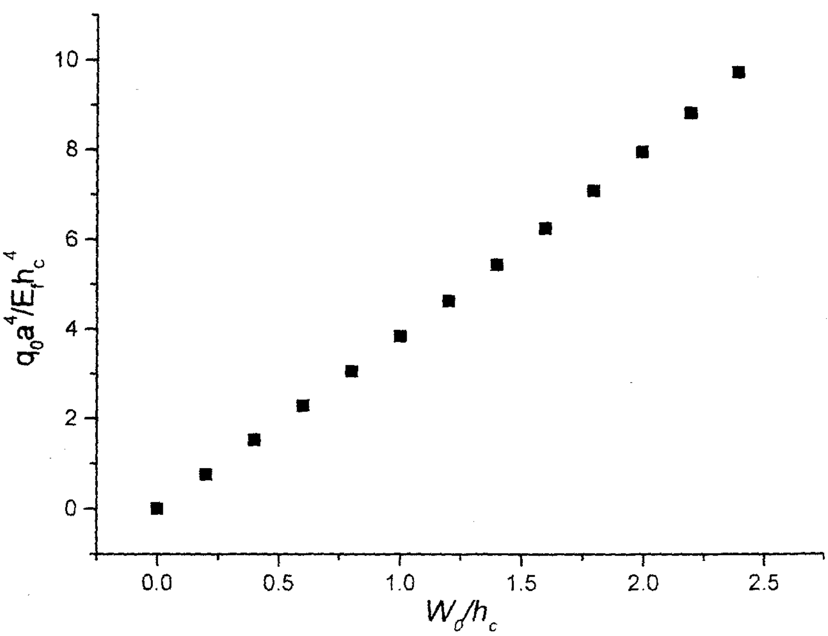

Fig2.1.(e) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=b=15, $R_1 = R_2 = 200$

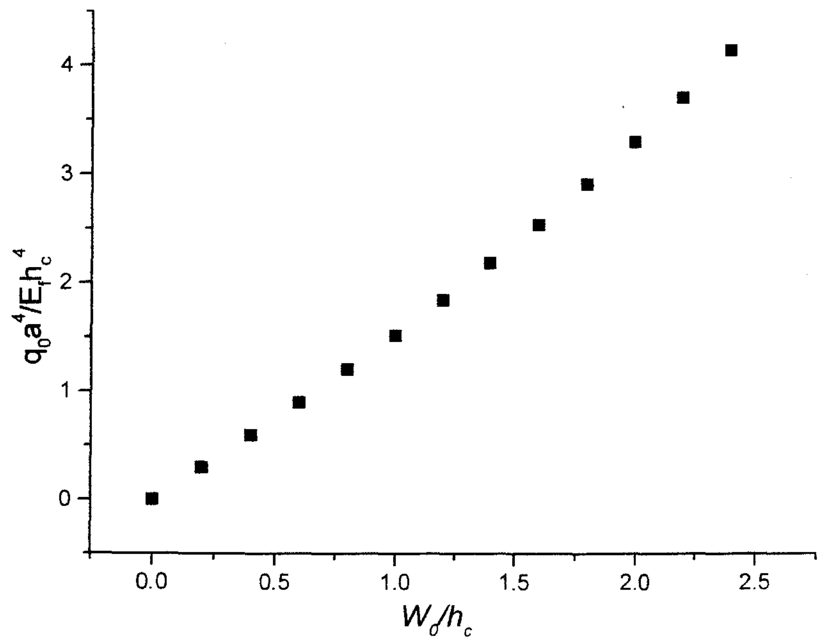

Fig.2.1.(f) $\left(\frac{W_0}{h_c}\right)$ vs. $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ for values of a=b=15, $R_1 = R_2 = 400$

2.1.7 OBSERVATIONS AND DISCUSSIONS

i) The cubic equation (53) should be solved by Cardon's method as available in any book on Classical Algebra. But to avoid complications in obtaining the solution a graphical representation of the result would be proper. For a given value of non-dimensional deflection $\left(\frac{W_0}{h_c}\right)$,the load distribution $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ can easily be determined from the cubic equation (53) for different variations of parameters as shown in Figures [2.1(a)- 2.1(f)]

ii) From the above six figures the general observations are made. The non-dimensional load $\left(\frac{q_0 a^4}{E_f h_c}\right)$ increases with the increase of non-dimensional deflections $\left(\frac{W_0}{h_c}\right)$. The nonlinearity effect is not so much significant for moderately large deflections made in the present study. However, it has been verified that the nonlinearity effect is significant for large or very large deflections with values of non-dimensional deflection parameter $\left(\frac{W_0}{h_c}\right)$ ranging up to 5.5.

iii) From figures [2.1(a)-2.1(c)] and [2.1(d)-2.1(f)] we see that for a square sandwich shell panel the load bearing capacity is increased when radii of curvature (R_1, R_2) are decreased to attain the same level of deflection $\left(\frac{W_0}{h_c}\right)$.

iv) Keeping the radius of curvature fixed the load bearing capacity is seen to be different when dimensions of the shell panel are changed to obtain the same level of deflections $\left(\frac{W_0}{h_c}\right)$. Load function is enormously high for the case of $a=b=15$ than those in the case of $a=b=10$, as may be seen from the figures (2.1.(a) and 2.1.(d)},{2.1.(b) and 2.1.(e)) and{2.1.(c) and 2.1.(f)}. In other words we can say that the load bearing capacities are enormously elevated when the length of the sandwich shell panel. is increased

v) In general, a slow nonlinearity is observed when graphical representation is made.

2.2.1 DETERMINATION OF THE CONSTANTS L, M AND N [IMMOVABLE EDGES]

The constants L, M and N are now determined from in-plane boundary conditions for immovable-edges which are [74]

At $X = +a, -a$ $\quad F_{,xy} = 0$

and $$u = \int_{-a}^{a}\int_{-b}^{b}\left[\frac{1}{Eh}\left(F_{,yy} - \nu_f F_{,xx}\right) - \frac{1}{2}\left(W_{,x}\right)^2 + K_x W\right]dxdy = 0 \tag{54.1}$$

At $Y = +b, -b$ $\quad F_{,xy} = 0$

and $$v = \int_{-a}^{a}\int_{-b}^{b}\left[\frac{1}{Eh}\left(F_{,xx} - \nu_f F_{,yy}\right) - \frac{1}{2}\left(W_{,y}\right)^2 + K_y W\right]dxdy = 0 \tag{54.2}$$

Using the condition $F_{,xy} = 0$ at $X = +a, -a\,; Y = +b, -b$

one gets $\quad N = 0$

Using the condition (54.1) one gets

$$M - \nu_f L = b_3\left(\frac{1}{b^2} - \nu_f\frac{1}{a^2}\right) + \frac{1}{32}Eh\pi^2\left(\frac{W_0}{a}\right)^2 - 4K_x W_0\frac{Eh}{\pi^2} \tag{55}$$

Using the condition (54.2) one gets

$$L - \nu_f M = b_3\left(\frac{1}{a^2} - \nu_f\frac{1}{b^2}\right) + \frac{1}{32}Eh\pi^2\left(\frac{W_0}{b}\right)^2 - 4K_y W_0\frac{Eh}{\pi^2} \tag{56}$$

Solving the equations (55) and (56) one gets

$$L = \frac{1}{32}\frac{E_f h_f W_0^2\pi^2}{\left(1-\nu_f^2\right)}\left(\frac{\nu_f}{a^2} + \frac{1}{b^2}\right) + \frac{b_3}{a^2} - 4\frac{E_f h_f W_0}{\pi^2\left(1-\nu_f^2\right)}\left(\nu_f K_x + K_y\right) \tag{57}$$

$$M = \frac{1}{32}\frac{E_f h_f W_0^2\pi^2}{\left(1-\nu_f^2\right)}\left(\frac{1}{a^2} + \frac{\nu_f}{b^2}\right) + \frac{b_3}{b^2} - 4\frac{E_f h_f W_0}{\pi^2\left(1-\nu_f^2\right)}\left(K_x + \nu_f K_y\right) \tag{58}$$

With the above values of L, M and N the stress function $F(x, y)$ is completely determined.

We now turn to equation (43).Using the expressions for W and F and applying Galerkin's procedure one gets the formidable-looking but simple cubic equation in terms of $\left(\frac{W_0}{h_c}\right)$ from which load-deflection results can easily be computed with different variations of parameters involved in the

analysis in the form given below:

$$P_1\left(\frac{W_0}{h_c}\right)^3+Q_1\left(\frac{W_0}{h_c}\right)^2+R_1\left(\frac{W_0}{h_c}\right)+S_1=0 \tag{59}$$

where,

$$\begin{aligned}
P_1 &= \frac{\pi^4}{128}\frac{(E_f h_f)}{(1-\nu_f^2)}\left(\frac{h_f}{a}\right)^2\left(\frac{h_c}{h_f}\right)^2\left[\left\{1+\nu_f\left(\frac{a}{b}\right)^2\right\}\left(\frac{b}{a}\right)+\left\{\nu_f+\left(\frac{a}{b}\right)^2\right\}\left(\frac{a}{b}\right)\right]\\
&+\frac{\pi^6}{1024}\frac{(E_f h_f)}{(1-\nu_f^2)}\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(\frac{h_f}{a}\right)^4\left(\frac{h_c}{h_f}\right)^2\left\{1+\left(\frac{a}{b}\right)^2\right\}\left\{1+\left(\frac{h_c}{h_f}\right)\right\}^2\\
&\times\left[\left\{1+\nu_f\left(\frac{a}{b}\right)^2\right\}\left(\frac{b}{a}\right)+\left\{\nu_f+\left(\frac{a}{b}\right)^2\right\}\left(\frac{a}{b}\right)\right]\\
&+\frac{\pi^6}{2048}(E_f h_f)\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(\frac{h_c}{h_f}\right)^2\left(\frac{a}{b}+\frac{b}{a}\right)\left(1+\frac{h_c}{h_f}\right)^2\left\{\left(\frac{h_f}{a}\right)^4+\left(\frac{h_f}{b}\right)^4\right\}\\
&+\frac{\pi^4}{256}(E_f h_f)\left(\frac{h_f}{a}\right)\left(\frac{h_f}{b}\right)\left(\frac{h_c}{h_f}\right)^2\left\{\left(\frac{a}{b}\right)^2+\left(\frac{b}{a}\right)^2\right\}
\end{aligned} \tag{59.1}$$

$$\begin{aligned}
Q_1 &= -\left[\frac{\pi^2}{12}(E_f h_f)\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(\frac{h_f}{a}\right)\left(\frac{h_f}{b}\right)\left(1+\frac{h_c}{h_f}\right)^2\left\{8\left(1+\frac{a^2}{b^2}\right)\frac{\left(1+\frac{K_y}{K_x}\frac{a^2}{b^2}\right)}{\left(1+\frac{a^2}{b^2}\right)^2}\left(\frac{h_c}{R_1}\right)\right.\right.\\
&\left.+\left(\frac{h_c}{R_1}+\frac{h_c}{R_2}\right)\right\}+\frac{\pi^2}{8}(E_f h_f)\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(\frac{h_c}{R_1}\right)\left(1+\frac{h_c}{h_f}\right)^2\left(1+\frac{a^2}{b^2}\right)\\
&\times\left\{\frac{\left(\frac{h_f}{a}\right)^2}{(1-\nu_f^2)}\left\{\left(1+\nu_f\frac{K_Y}{K_x}\right)\left(\frac{b}{a}\right)+\left(\nu_f+\frac{K_y}{K_x}\right)\left(\frac{a}{b}\right)\right\}-\frac{\left(1+\frac{K_y}{K_x}\frac{a^2}{b^2}\right)}{\left(1+\frac{a^2}{b^2}\right)}\left\{\left(\frac{b}{a}\right)\left(\frac{h_f}{b}\right)^2+\left(\frac{a}{b}\right)\left(\frac{h_f}{a}\right)^2\right\}\right\}
\end{aligned}$$

$$+\frac{1}{18}(E_f h_f)\left\{\left(\frac{h_c}{R_1}\right)\left(\frac{b}{a}\right)+\left(\frac{h_c}{R_2}\right)\left(\frac{a}{b}\right)\right\}+\frac{(E_f h_f)}{(1-\nu_f^{\,2})}\left\{\frac{1}{2}\left(\frac{b}{a}\right)\left\{\left(\frac{h_c}{R_1}\right)\left(1+\nu_f\frac{a^2}{b^2}\right)+\left(\frac{h_c}{R_2}\right)\left(\nu_f+\frac{a^2}{b^2}\right)\right\}\right.$$

$$\left.+\left(\frac{h_c}{R_1}\right)\left\{\left(\frac{b}{a}\right)\left(1+\nu_f\frac{K_y}{K_x}\right)+\left(\frac{a}{b}\right)\left(\nu_f+\frac{K_y}{K_x}\right)\right\}\right\}-\frac{4}{3}(E_f h_f)\left(\frac{a}{b}\right)\left(\frac{h_c}{R_1}\right)\frac{\left(1+\frac{K_y}{K_x}\frac{a^2}{b^2}\right)}{\left(1+\frac{a^2}{b^2}\right)^2}\Bigg]$$

(59.2)

$$R_1=\frac{\pi^4}{32}(E_f h_f)\left(\frac{b}{a}\right)\left(\frac{h_f}{a}\right)^2\left(1+\frac{a^2}{b^2}\right)^2\left(1+\frac{h_c}{h_f}\right)^2+(E_f h_f)\left(\frac{a}{R_1}\right)\frac{\left(1+\frac{K_y}{K_x}\frac{a^2}{b^2}\right)}{\left(1+\frac{a^2}{b_2}\right)^2}\left\{\left(\frac{a}{b}\right)\left(\frac{a}{R_1}\right)+\left(\frac{b}{a}\right)\left(\frac{a}{R_2}\right)\right\}$$

$$-\frac{64}{\pi^4}(E_f h_f)\left(\frac{a}{R_1}\right)\frac{\left(1+\frac{K_y}{K_x}\frac{a^2}{b^2}\right)}{\left(1+\frac{a^2}{b^2}\right)^2}\left\{\left(\frac{a}{b}\right)\left(\frac{a}{R_1}\right)+\left(\frac{b}{a}\right)\left(\frac{a}{R_2}\right)\right\}$$

$$+\frac{64}{\pi^4}\frac{(E_f h_f)}{(1-\nu_f^{\,2})}\left(\frac{a}{R_1}\right)\left\{\left(\frac{b}{R_1}\right)\left(1+\nu_f\frac{K_y}{K_x}\right)+\left(\frac{b}{R_2}\right)\left(\nu_f+\frac{K_y}{K_x}\right)\right\}$$

$$+\frac{\pi^2}{8}(E_f h_f)\left(\frac{E_f}{G_c}\right)\left(\frac{h_f}{h_c}\right)\left(\frac{h_c}{R_1}\right)\left(\frac{h_f}{h_c}\right)^2\left(1+\frac{h_c}{h_f}\right)^2\left(1+\frac{a^2}{b^2}\right)\frac{\left(1+\frac{K_y}{K_x}\frac{a^2}{b^2}\right)}{\left(1+\frac{a^2}{b^2}\right)^2}\left\{\left(\frac{a}{b}\right)\left(\frac{h_c}{R_1}\right)+\left(\frac{b}{a}\right)\left(\frac{h_c}{R_2}\right)\right\}$$

(59.3)

$$S_1=-\left(\frac{q_0 a^4}{E_f h_c^{\,4}}\right)\left(\frac{h_f}{a}\right)^4\left(\frac{h_c}{h_f}\right)^3\left[\left(\frac{a}{h_f}\right)\left(\frac{b}{h_f}\right)\right.$$

$$\left.+\frac{1}{8}\pi^2\left(\frac{E_f h_f}{G_c h_c}\right)\left(1+\frac{h_c}{h_f}\right)^2\left(1+\frac{a^2}{b^2}\right)\left(\frac{b}{a}\right)\right] \qquad (59.4)$$

2.2.2 NUMERICAL RESULTS

The graphical representations of numerical results have been presented graphically [Figures 2.2(a)-2.2(f)] , showing variations of non-dimensional deflections $\left(\frac{W_0}{h_c}\right)$ against non-dimensional loading parameters $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$, considering the set of values [184]: $E_f = 10\times10^6$ psi, $G_c = 12000$ psi, $h_c = 0.2$ in, $h_f = 0.02$ in , $D = 4840$ ib-in

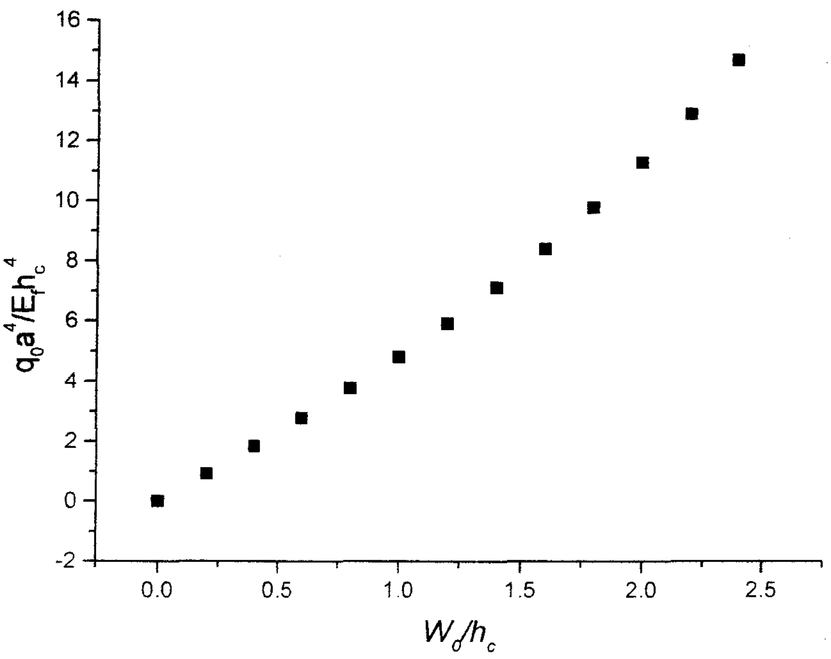

Fig.2.2.(a) [For values of a=b=10, $R_1 = R_2 = 100$]

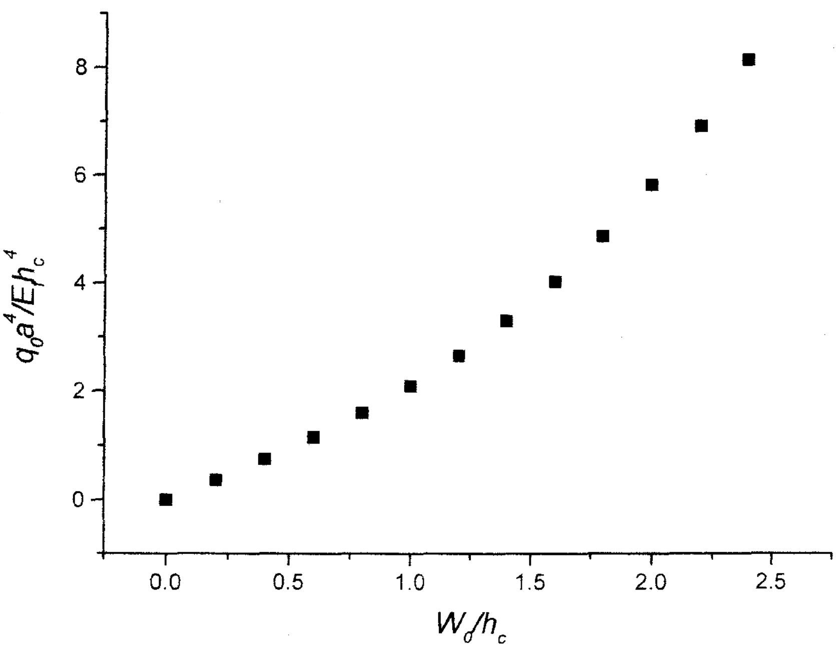

Fig.2.2(b) [For values of a=b=10, $R_1 = R_2 = 200$]

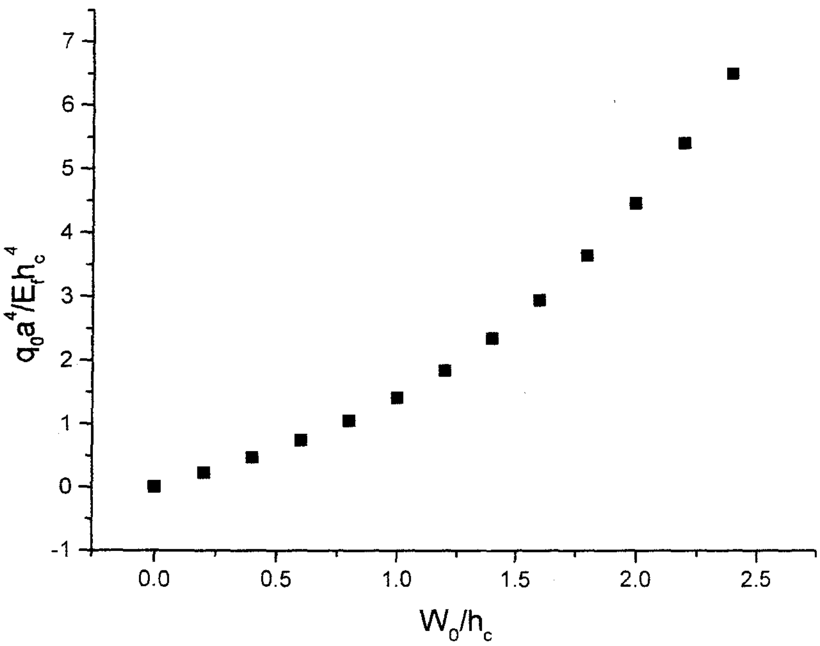

Fig.2.2.(c) [For values of a=b=10, $R_1 = R_2 = 400$]

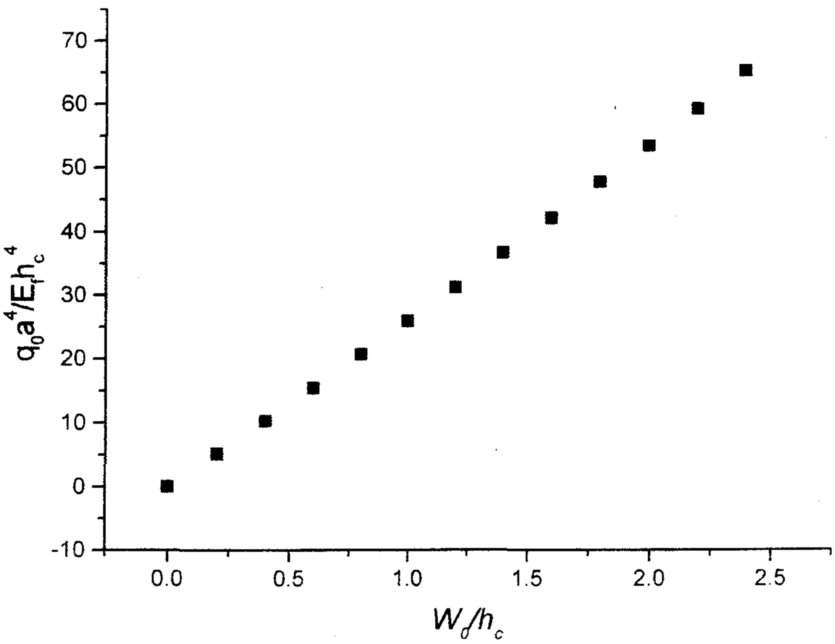

Fig.2.2(d) [For values of a=b=15, $R_1 = R_2 = 100$]

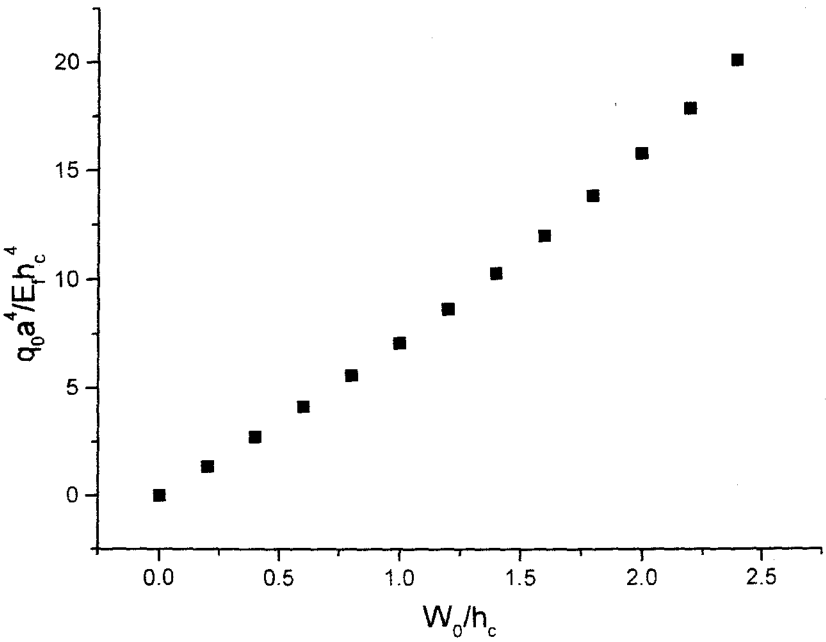

Fig.2.2(e) [For values of a=b=15, $R_1 = R_2$ =200]

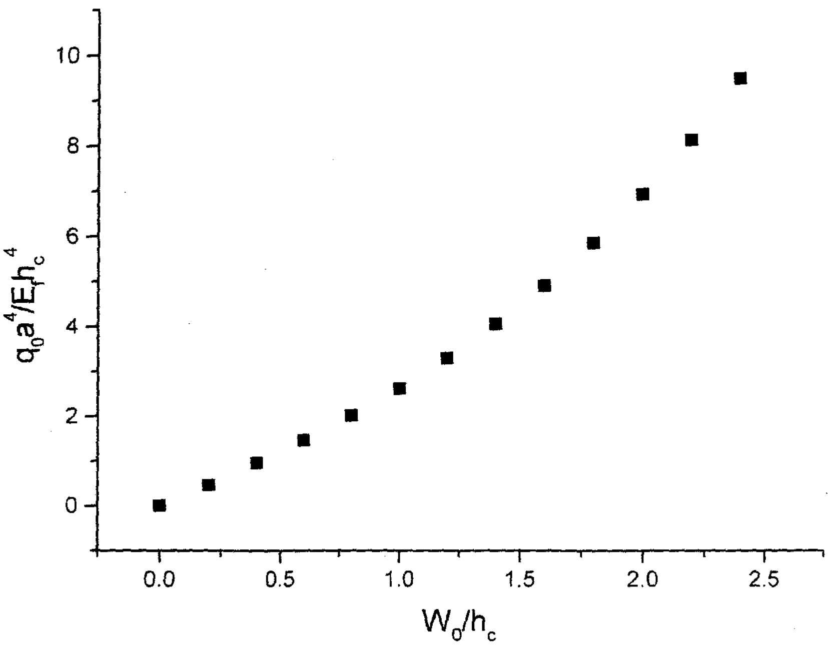

Fig.2.2(f) [For values of a=b=15, R_1 = R_2 =400]

2.2.3 **OBSERVATIONS AND DISCUSSIONS**

From the graphical figures [2.2(a)-2.2(f)] for the case of immovable edge boundary condition the following observations are made :

i) General behavior is the same as in the case of movable edge boundary condition.

ii) Comparing the Figures [2.1.(a)- 2.1.(f)] with the figures [2.2.(a) to 2.2.(f)] it is observed that load-function $\left(\frac{q_0 a^4}{E_f h_c^4}\right)$ is comparatively high for the case of immovable edge condition than for those in the case of movable edge condition to attain the same level of deflection $\left(\frac{W_0}{h_c}\right)$ keeping radii of curvatures (R_1, R_2) the same. The results comply with our expectations.

iii) To conclude the observations, it may be stated in a nut shell that the nonlinear effect on the deformation of simply-supported sandwich shell panels is not so much predominant for immovable edge conditions but for the movable edge condition the effect is comparatively more predominant.

CHAPTER III

THERMAL STRESSES AND NONLINEAR THERMAL DEFORMATION ANALYSIS OF A SHALLOW SHELL PANEL*

3.1 INTRODUCTION

Modern aerospace structures such as high speed aircrafts, missiles, space vehicles and mechanical and nuclear structures are often subject to severe thermal loads and reveal a clearly nonlinear response. In such cases the associated strains and stresses are usually determined from the von Karman field equations extended to thermal loading which are coupled nonlinear partial differential equations involving transverse deflection and membrane stress functions.

Several authors [4,174-176] have employed the method in some of the thermo elastic plate problems .The purpose of the present paper is to further generalize the equations for shell problems for the case of a rectangular panel under thermal loading. After determining the stress function, the normal displacement component has been obtained for simply-supported edges in terms of cubic equations by using Galerkin's procedure.

Numerical results for deflections and membrane stresses in non-dimensional forms have been computed for a simply-supported and immovable edge conditions and presented in graphical forms.

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3.2 BASIC RELATIONS AND DERIVATIONS OF GOVERNING EQUATIONS

Median surface stress-strain relations are given by

$$\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y) - \frac{\alpha_t ET}{(1-\nu)} \tag{60.1}$$

$$\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x) - \frac{\alpha_t ET}{(1-\nu)} \tag{60.2}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} \tag{60.3}$$

For a cylindrical shell Panel the von-Karman strain displacement equations are given by Donnell[182]

$$\varepsilon_x = u_{,x} + \frac{1}{2}(W_{,x})^2 - zW_{,xx} \tag{61.1}$$

$$\varepsilon_y = u_{,y} + \frac{1}{2}(W_{,y})^2 - zW_{,yy} - \frac{W}{R} \tag{61.2}$$

$$\gamma_{xy} = u_{,x} + v_{,y} + W_{,x}W_{,y} - 2zW_{,xy} \tag{61.3}$$

The forces N_{xx}, N_{yy}, N_{xy} and the moments M_{xx}, M_{yy}, M_{xy} can be expressed by the matrix equation

$$\begin{bmatrix} N_{xx}, N_{yy}, N_{xy} \\ M_{xx}, M_{yy}, M_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x, \sigma_y, \tau_{xy} \\ z\sigma_x, z\sigma_y, z\tau_{xy} \end{bmatrix} dz \tag{62}$$

The in-plane equations of equilibrium in the X and Y directions are

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{63.1}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0 \tag{63.2}$$

These equations are identically satisfied by introducing the Airy stress function defined by the relations

$$N_{xx} = F_{,yy} \tag{64.1}$$

$$N_{yy} = F_{,xx} \tag{64.2}$$

$$N_{xy} = -F_{,xy} \tag{64.3}$$

Considering equations (60.1-60.3, 61.1-61.3 and 62) one gets

$$\varepsilon_x = \frac{1}{Eh}\left(F_{,yy} - \nu F_{,xx}\right) - zW_{,xx} + \frac{\alpha_t N_T}{h} \tag{65.1}$$

$$\varepsilon_y = \frac{1}{Eh}\left(F_{,xx} - \nu F_{,yy}\right) - zW_{,yy} + \frac{\alpha_t N_T}{h} \tag{65.2}$$

$$\gamma_{xy} = -\frac{2(1+\nu)}{Eh}F_{,xy} - 2zW_{,xy} \tag{65.3}$$

Taking the second derivatives of (61.1-61.3),the following relation is obtained

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 - \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} - \frac{1}{R}\frac{\partial^2 W}{\partial x^2} \tag{66}$$

By virtue of equations (65.1-65.3) and (66) one gets the following differential equation for the stress function in terms of deflection function

$$\nabla^4 F = Eh\left[\left(W_{,xy}\right)^2 - W_{,xx}W_{,yy}\right] - \alpha_t E\left(\nabla^2 N_T\right) - \frac{Eh}{R}W_{,xx} \tag{67}$$

where

$$N_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} T(x,y,z)dz = \text{Thermal Stress Couple} \tag{67.1}$$

Driving the expression for the moments from equations (60.1-60.3),(61.1-61.3) and (63.1-63.2) and considering the following equation of equilibrium [1,page-379]

$$\frac{\partial^2}{\partial x^2}(M_{xx}) - 2\frac{\partial^2}{\partial x \partial y}(M_{xy}) + \frac{\partial^2}{\partial y^2}(M_{yy}) = -\left[N_{xx}W_{,xx} + 2N_{xy}W_{,xy} + N_{yy}W_{,yy}\right] \tag{68}$$

one gets

$$D\nabla^4 W + \frac{\alpha_t E}{(1-\nu)}\left(\nabla^2 M_T\right) = \left[F_{,xx}W_{,yy} - 2F_{,xy}W_{,xy} + F_{,yy}W_{,xx}\right] \tag{69}$$

where

$$M_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} zT(x,y,z)dz = \text{Thermal Moment} \tag{69.1}$$

Equations (67) and (69) constitute the two coupled nonlinear partial differential equations in the von Karman sense for determining the large thermal deflections of a shallow cylindrical shell panel.

3.3 RECTANGULAR PANEL SIMPLY-SUPPORTED WITH IMMOVABLE EDGE CONDITIONS

To approach the particular problem concerning a rectangular panel simply-supported at the edges origin being located at one corner of the shell in the middle surface. Let a,b be the length and peripheral width of the shell and are taken as the X and Y axis, Z - axis being normally downwards. The distribution of temperature in the direction of Z - axis is taken as linear in the form[4]

$$T(x,y,z) = \tau_0(x,y) + z\tau(x,y) \tag{70}$$

where

$$\tau_0(x,y) = \frac{T_1 + T_2}{2} \tag{70.1}$$

$$\tau(x,y) = \frac{T_1 - T_2}{h} \tag{70.2}$$

and

$$T_1 = T\left(x, y + \frac{h}{2}\right), \tag{70.3}$$

$$T_2 = T\left(x, y, -\frac{h}{2}\right) \tag{70.4}$$

Since M_T is constant one can express it in the form of the Fourier series.

$$M_T = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} a_{mn} Sin\left(\frac{\pi mx}{a}\right) Sin\left(\frac{\pi ny}{b}\right) \tag{71}$$

where

$$a_{mn} = \frac{16M_T}{mn\pi^2} \tag{71.1}$$

Compatible with simply-supported edges, the deflection function W is assumed in the form

$$W = W_0 Sin\left(\frac{\pi x}{a}\right) Sin\left(\frac{\pi y}{b}\right) \tag{72}$$

Subject to

$$W = 0 = W,_{xx} + \frac{M_T}{D(1-\nu)} \quad \text{at } x = 0, a \tag{72.1}$$

$$W = 0 = W,_{yy} + \frac{M_T}{D(1-\nu)} \quad \text{at } y = 0, b \tag{72.2}$$

Since N_T is a constant and appears in the boundary conditions for in-plane displacements, we take $\nabla^2(N_T) = 0$ in equation (67) from which the stress function is obtained in the form

$$\begin{aligned} F(x,y) = {} & A\frac{x^2}{2} + B\frac{y^2}{2} + \frac{EhW_0^2}{32}\left\{\frac{a^2}{b^2}Cos\left(\frac{2\pi x}{a}\right) + \frac{b^2}{a^2}Cos\left(\frac{2\pi y}{b}\right)\right\} \\ & + \frac{EhW_0}{Ra^2\pi^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2} Sin\left(\frac{\pi x}{a}\right) Sin\left(\frac{\pi y}{b}\right) \end{aligned} \tag{73}$$

where A and B are arbitrary constants to be determined from in-plane boundary conditions.

In accordance with the conditions occurring in airplane structures the shell is considered rigidly framed ,all edges remaining unaltered after deformation. The elongations of the shell in the directions of X and Y are independent Y and X respectively.[Ref.1,page-426]

From equations (61.1-61.3) and (65.1-65.3) one gets

$$u,_x = \frac{1}{Eh}\left(F,_{yy} - \nu F,_{xx}\right) - \frac{1}{2}\left(W,_x\right)^2 + \frac{\alpha_t N_T}{h} \tag{74}$$

$$v,_y = \frac{1}{Eh}\left(F,_{xx} - \nu F,_{yy}\right) - \frac{1}{2}\left(W,_y\right)^2 + \frac{\alpha_t N_T}{h} + \frac{W}{R} \tag{75}$$

Considering the preceding statement, one gets

$$u = \int_0^a (u,_x)_{y=0}\, dx = 0 \tag{76}$$

$$v = \int_0^b (v,_y)_{x=0}\, dy = 0 \tag{77}$$

Performing necessary integrations one gets,

$$u = \frac{1}{Eh}\left(B - \frac{EhW_0^{\,2}\pi^2}{8a^2} - \nu A \right) + \frac{\alpha_t N_T}{h} = 0 \tag{78}$$

$$v = \frac{1}{Eh}\left(A - \frac{EhW_0^{\,2}\pi^2}{8b^2} - \nu B \right) + \frac{\alpha_t N_T}{h} = 0 \tag{79}$$

Solving (78) and (79) one get

$$A = \frac{EhW_0^{\,2}\pi^2}{8(1-\nu^2)}\left(\frac{\nu}{a^2} + \frac{1}{b^2} \right) - \frac{E\alpha_t N_T}{(1-\nu)} \tag{80.1}$$

$$B = \frac{EhW_0^{\,2}\pi^2}{8(1-\nu^2)}\left(\frac{1}{a^2} + \frac{\nu}{b^2} \right) - \frac{E\alpha_t N_T}{(1-\nu)} \tag{80.2}$$

Applying Galerkin's procedure in equation (69) a cubic equation is obtained for central deflection in the following non-dimensional form:

$$C_1''\left(\frac{W_0}{h}\right)^3 - C_2''\left(\frac{W_0}{h}\right)^2 + C_3''\left(\frac{W_0}{h}\right) - C_4'' = 0 \tag{81}$$

where

$$C_1'' = \frac{\pi^4}{8}\left[\frac{1+\nu\left(\frac{b}{a}\right)^2}{\left(\frac{b}{h}\right)^2(1-\nu^2)} + \frac{\nu+\left(\frac{b}{a}\right)^2}{(1-\nu^2)\left(\frac{a}{h}\right)^2} + \frac{\left\{1+\left(\frac{b}{a}\right)^4\right\}}{2\left(\frac{b}{h}\right)^2} \right] \tag{81.1}$$

$$C_2'' = \frac{96}{9}\left[\frac{\left(\frac{b}{a}\right)^4}{\left(\frac{R}{h}\right)\left\{1+\left(\frac{b}{a}\right)^2\right\}^2} \right] \tag{81.2}$$

$$C_3'' = \frac{\pi^4}{12(1-\nu^2)} \frac{\left\{1+\left(\frac{b}{a}\right)^2\right\}^2}{\left(\frac{b}{h}\right)^2} - \frac{\alpha_t (T_1+T_2)\pi^2}{(1-\nu)} \left\{1+\left(\frac{b}{a}\right)^2\right\} \tag{81.3}$$

$$C_4'' = \frac{4}{3}\alpha_t (T_1 - T_2) \frac{\left\{1+\left(\frac{b}{a}\right)^2\right\}}{(1-\nu)} \tag{81.4}$$

Median surface membrane stresses N_{xx} and N_{yy} are given by

$$\frac{(N_{xx})_{\frac{a}{2},\frac{b}{2}}}{Eh} = C_5''\left(\frac{W_0}{h}\right)^2 - C_6''\left(\frac{W_0}{h}\right) - \frac{\alpha_t (T_1+T_2)}{2(1-\nu)} \tag{82.1}$$

$$\frac{(N_{yy})_{\frac{a}{2},\frac{b}{2}}}{Eh} = C_7''\left(\frac{W_0}{h}\right)^2 - C_8''\left(\frac{W_0}{h}\right) - \frac{\alpha_t (T_1+T_2)}{2(1-\nu)} \tag{82.2}$$

where

$$C_5'' = \frac{\pi^2}{8(1-\nu^2)\left(\frac{b}{h}\right)^2}\left\{\nu+\left(\frac{b}{a}\right)^2\right\} + \frac{\pi^2}{8\left(\frac{a}{h}\right)^2} \tag{83.1}$$

$$C_6'' = \frac{\left(\frac{b}{a}\right)^2}{\left(\frac{R}{h}\right)\left\{1+\left(\frac{b}{a}\right)^2\right\}^2} \tag{83.2}$$

$$C_7'' = \frac{\pi^2}{8(1-\nu^2)\left(\frac{b}{h}\right)^2}\left\{1+\nu\left(\frac{b}{a}\right)^2\right\} + \frac{\pi^2}{8\left(\frac{b}{h}\right)^2} \tag{83.3}$$

$$C_8'' = \frac{\left(\frac{b}{a}\right)^4}{\left(\frac{R}{h}\right)\left\{1+\left(\frac{b}{a}\right)^2\right\}^2} \tag{83.4}$$

3.4 NUMERICAL RESULTS

Numerical results and discussions for variations of Non-dimensional central deflection for variations of different parameters have been presented graphically at the end of the paper considering the following set of values:

$\nu = 0.3$

$\pi = 3.142857143$

$$\alpha_t (T_1 - T_2) = 12 \times 10^{-6} (T_1 - T_2) \tag{84}$$

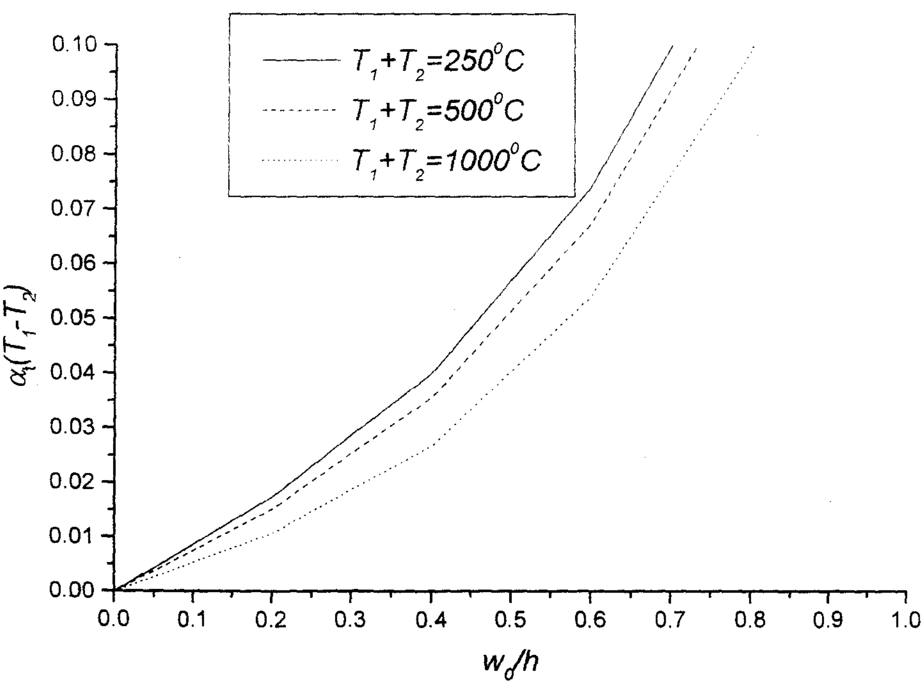

Fig.3.1(a) Shows comparative variations of $\alpha_t (T_1 - T_2)$ with $\left(\frac{W_0}{h}\right)$ for $\frac{b}{a} = 1, \frac{a}{h} = \frac{b}{h} = 10, \frac{R}{h} = 100$

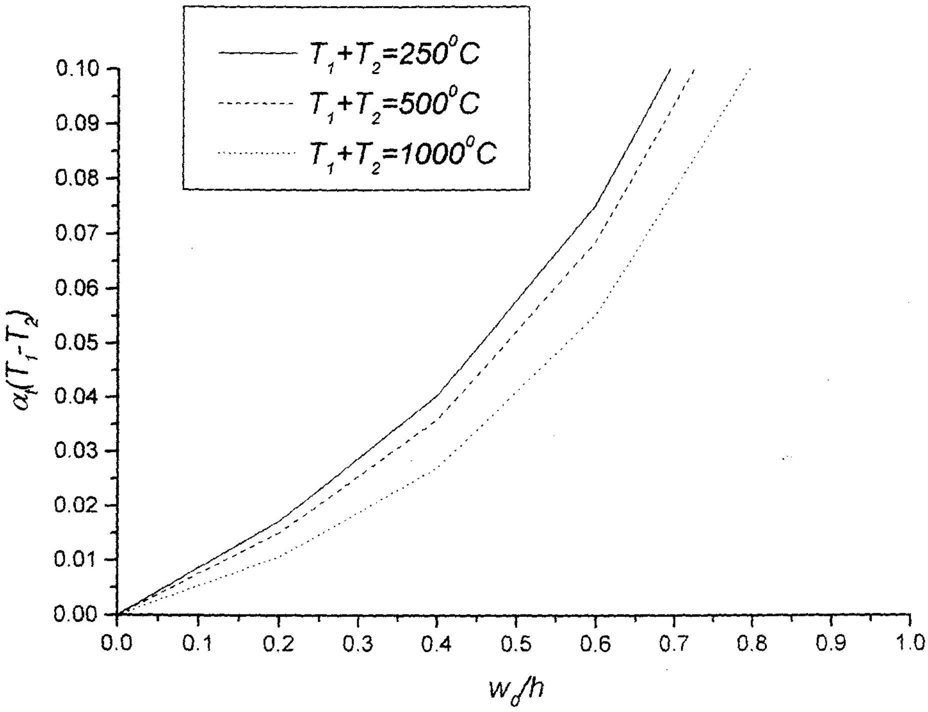

Fig.3.1(b) Shows comparative variations of $\alpha_1(T_1-T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $\frac{b}{a}=1, \frac{a}{h}=\frac{b}{h}=10, \frac{R}{h}=200$

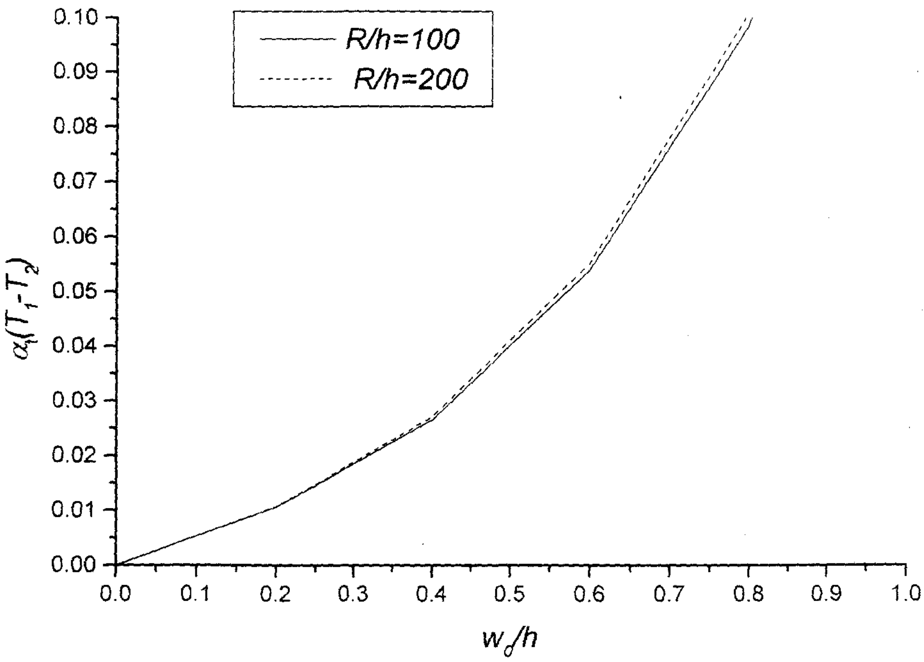

Fig.3.1(c) Shows comparative variations of $\alpha_t(T_1-T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $\frac{b}{a}=1, \frac{a}{h}=\frac{b}{h}=10, T_1+T_2=1000^0 C$

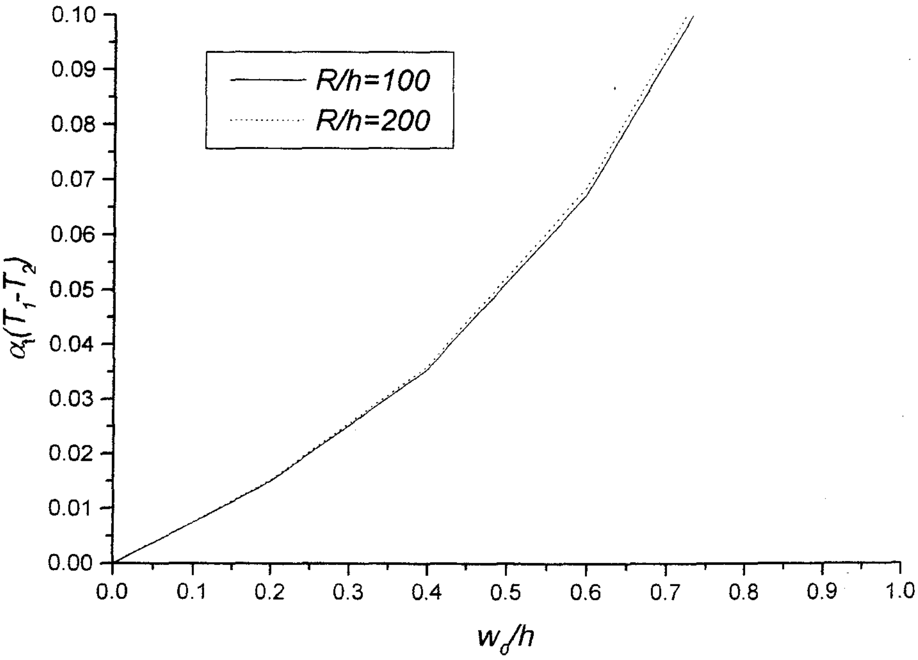

Fig.3.1(d) Shows comparative variations of $\alpha_t(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for

$\frac{b}{a} = 1, \frac{a}{h} = \frac{b}{h} = 10, T_1 + T_2 = 500^0 C$

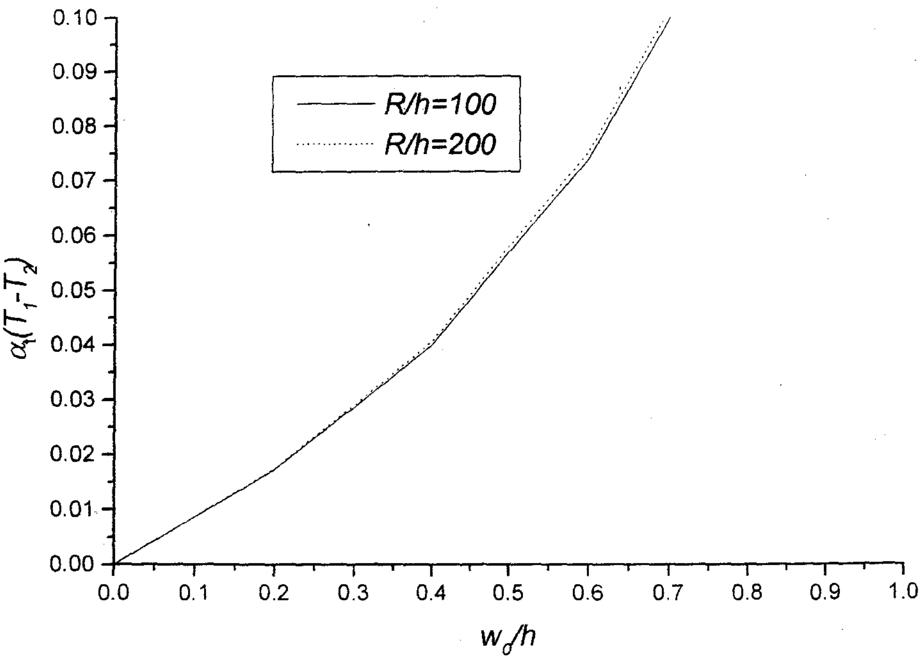

Fig.3.1(e) Shows comparative variations of $\alpha_t(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $\frac{b}{a} = 1, \frac{a}{h} = \frac{b}{h} = 10, T_1 + T_2 = 250^0 C$

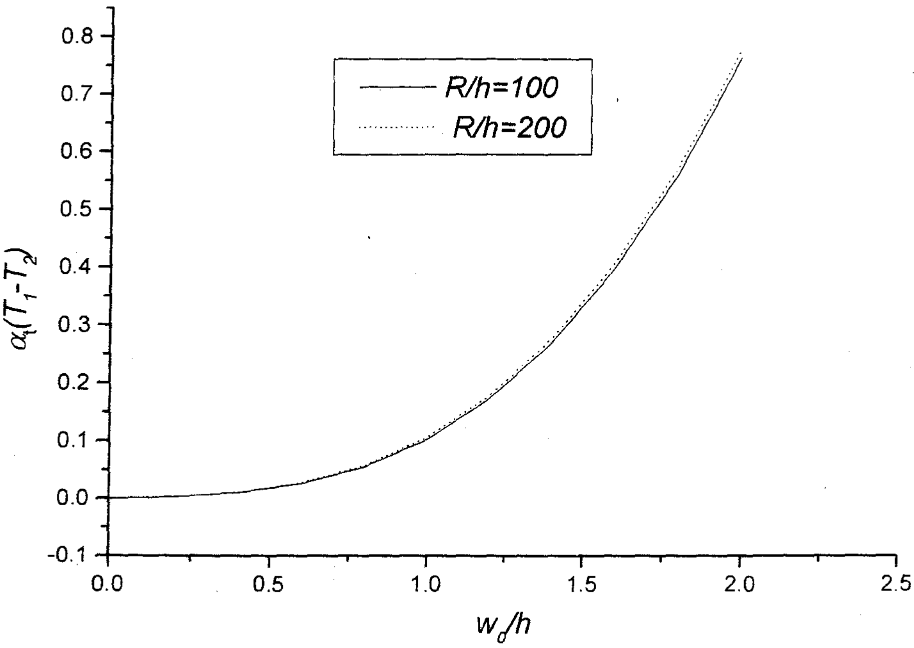

Fig.3.1(f) Shows comparative variations of $\alpha_t(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $\frac{a}{h} = 10, \frac{b}{h} = 20, T_1 + T_2 = 1000^0 C$

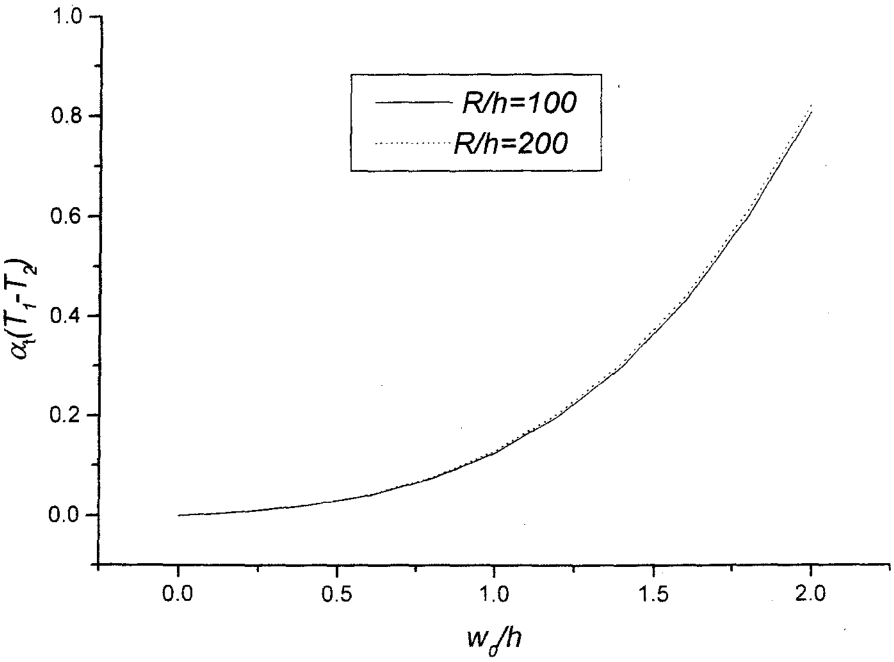

Fig.3.1(g) Shows comparative variations of $\alpha_t(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $\frac{a}{h} = 10, \frac{b}{h} = 20, T_1 + T_2 = 500^0 C$

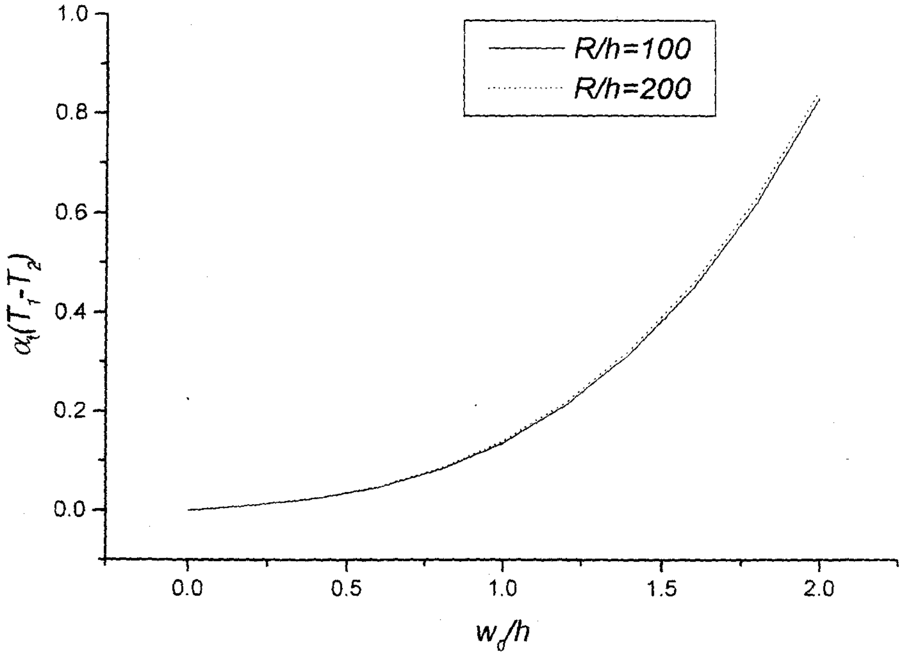

Fig.3.1(h) Shows comparative variations of $\alpha_t(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for

$\frac{a}{h} = 10, \frac{b}{h} = 20, T_1 + T_2 = 250^0 C$

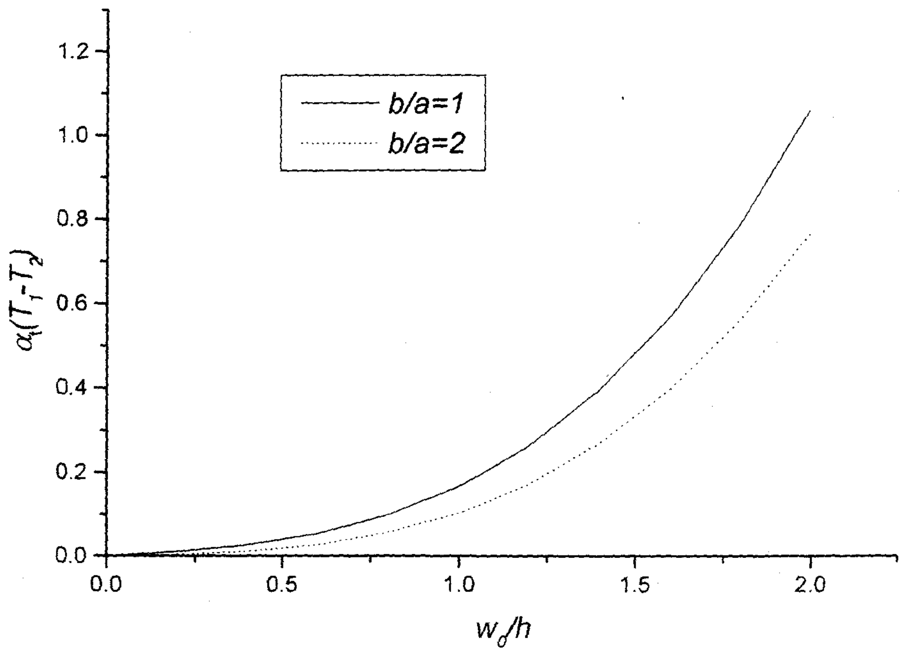

Fig.3.1(i) Shows comparative variations of $\alpha_t(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $T_1 + T_2 = 1000^0 C, \frac{R}{h} = 100$

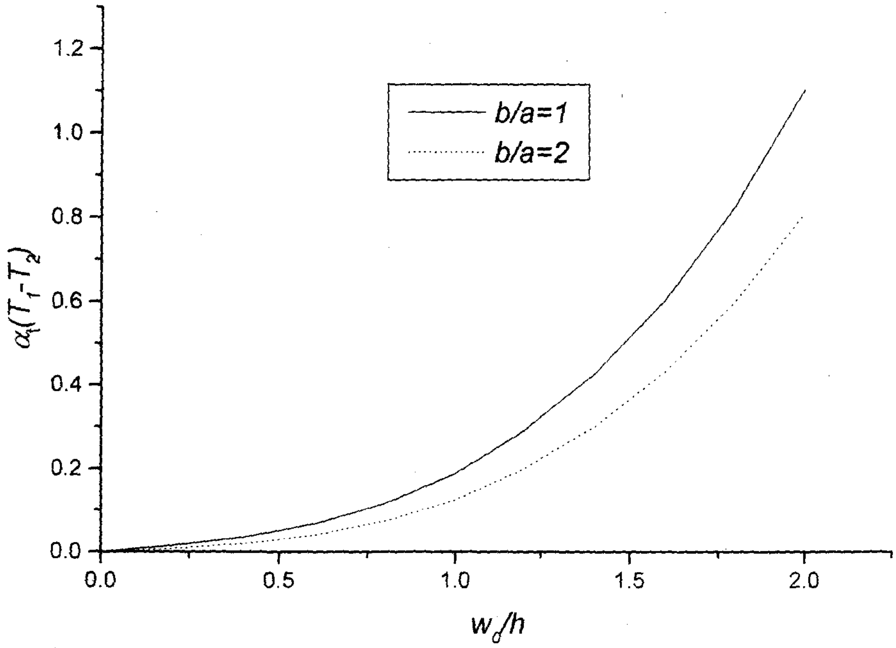

Fig.3.1(j) Shows comparative variations of $\alpha_t(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $T_1 + T_2 = 500^0 C, \frac{R}{h} = 100$

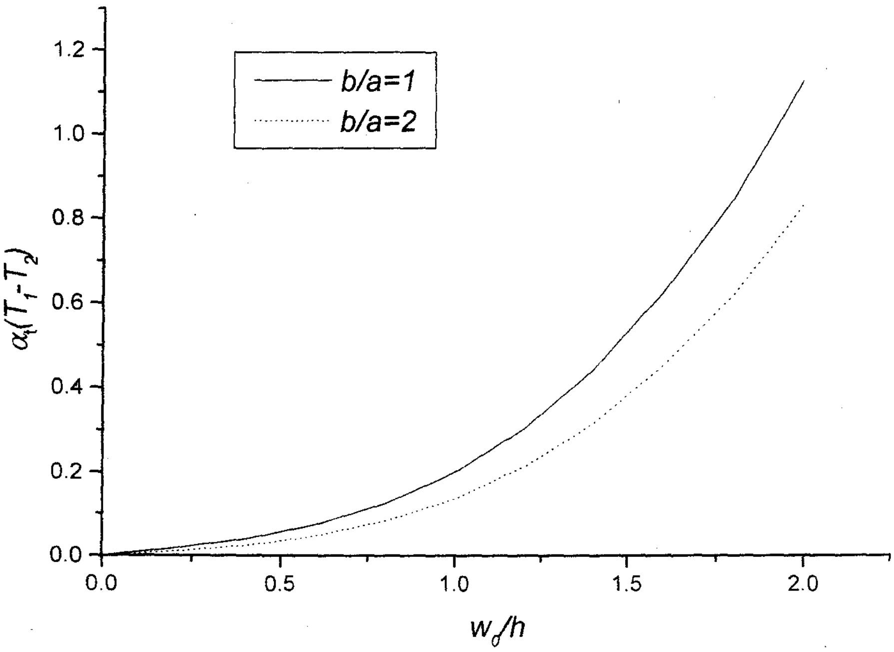

Fig.3.1(k) Shows comparative variations of $\alpha_t(T_1-T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $T_1+T_2=250^0 C, \frac{R}{h}=100$

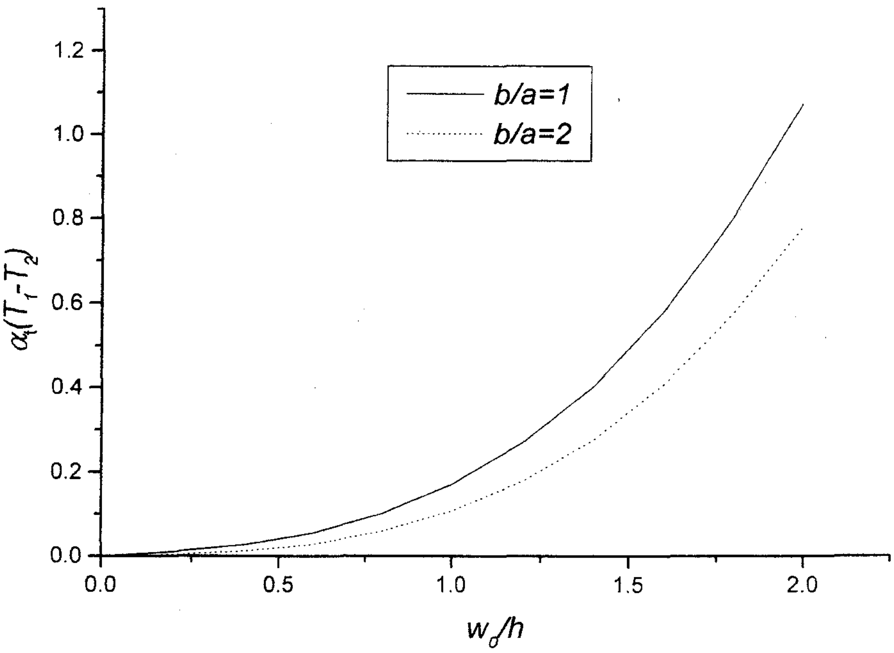

Fig.3.1(l) Shows comparative variations of $\alpha_t(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $T_1 + T_2 = 1000^0 C, \frac{R}{h} = 200$

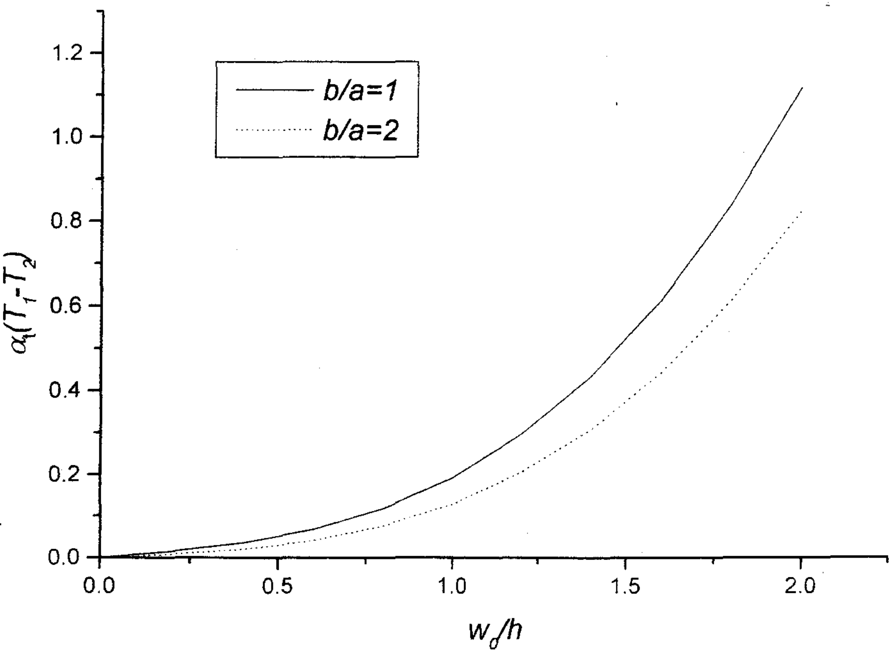

Fig.3.1(m) Shows comparative variations of $\alpha_l(T_1 - T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $T_1 + T_2 = 500^0 C, \frac{R}{h} = 200$

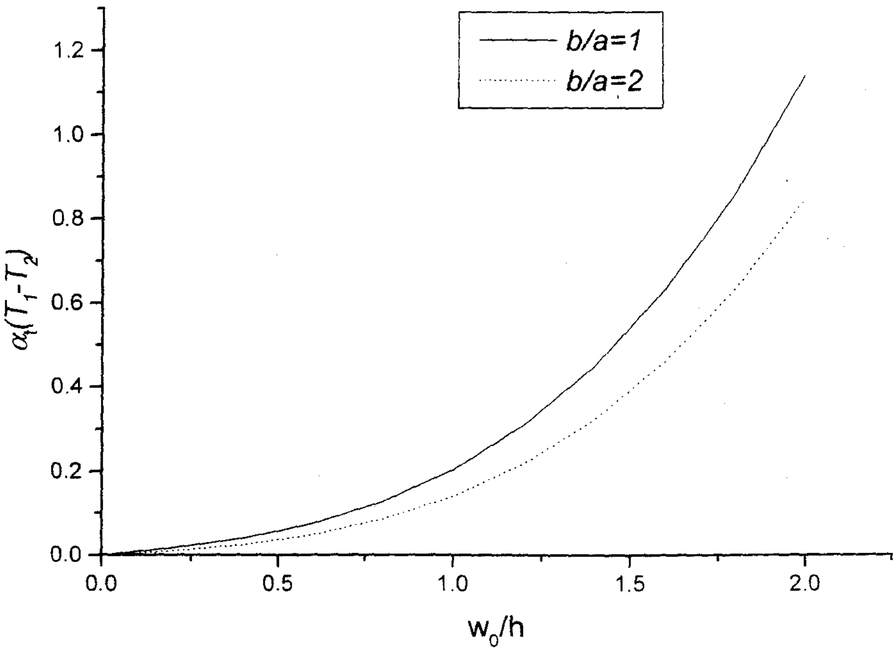

Fig.3.1(n) Shows comparative variations of $\alpha_t(T_1-T_2)$ vs. $\left(\frac{W_0}{h}\right)$ for $T_1+T_2=250^0C, \frac{R}{h}=200$

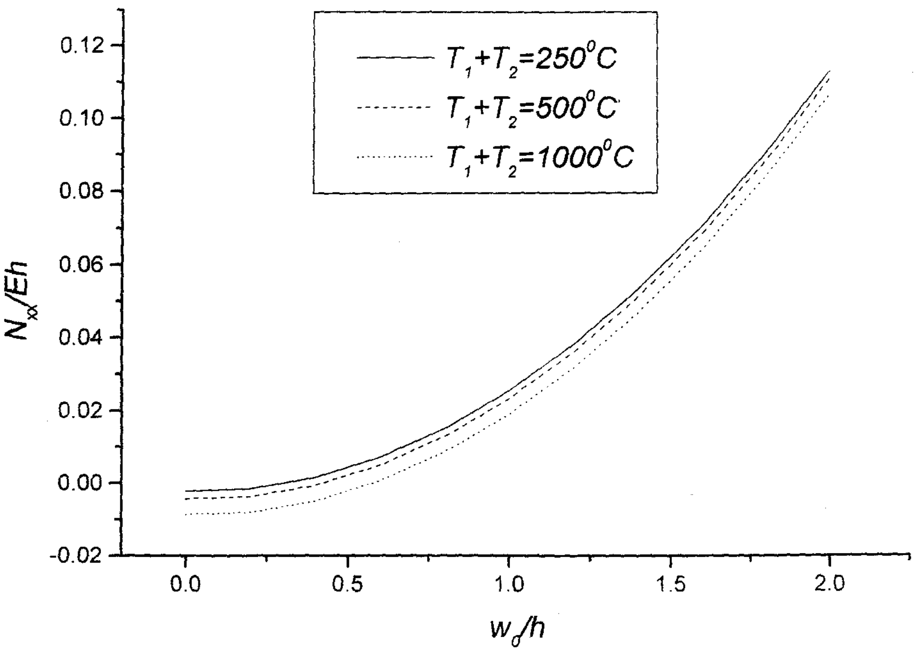

Fig.3.2 (a) Shows comparative variations of (N_{xx})/Eh vs. $\left(\frac{W_0}{h}\right)$ for

$\frac{b}{a}=1, \frac{a}{h}=\frac{b}{h}=10, \frac{R}{h}=100$

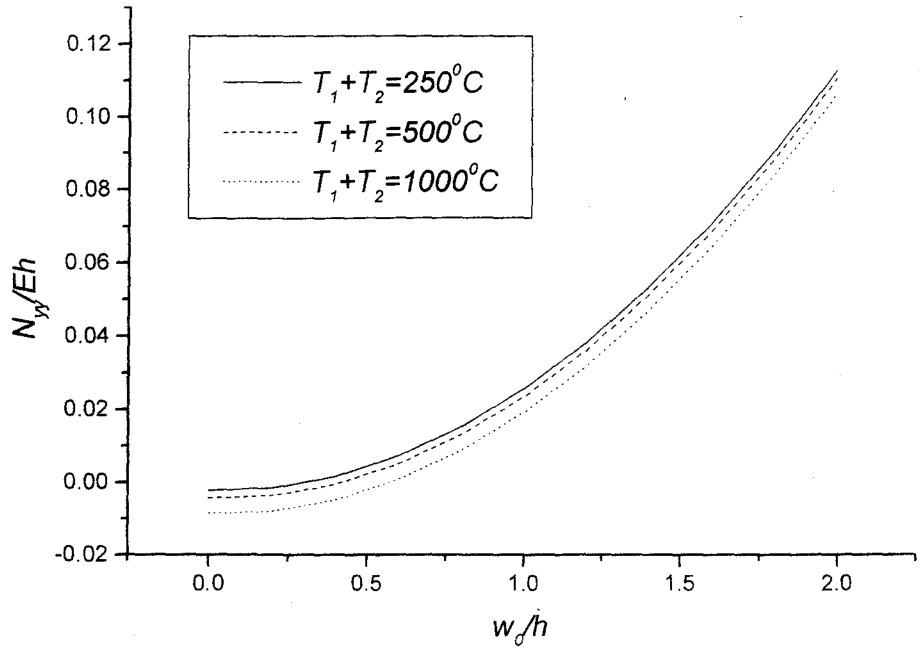

Fig.3.2 (b) Shows comparative variations of $(N_{yy})/Eh$ vs. $\left(\frac{W_0}{h}\right)$ for

$$\frac{b}{a}=1, \frac{a}{h}=\frac{b}{h}=10, \frac{R}{h}=100$$

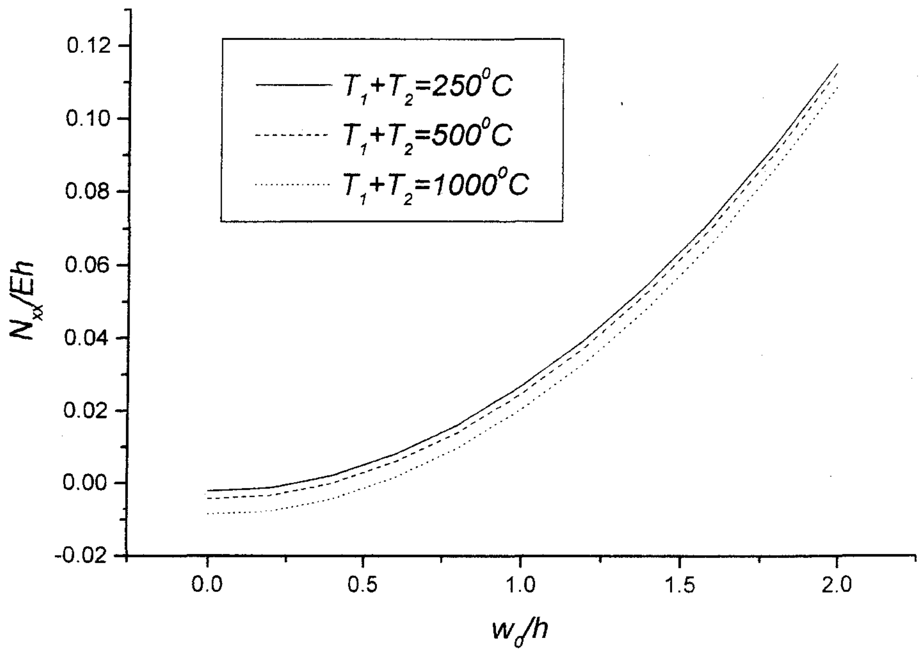

Fig.3.2 (c) Shows comparative variations of (N_{xx})/Eh vs. $\left(\frac{W_0}{h}\right)$ for

$\frac{b}{a}=1, \frac{a}{h}=\frac{b}{h}=10, \frac{R}{h}=200$

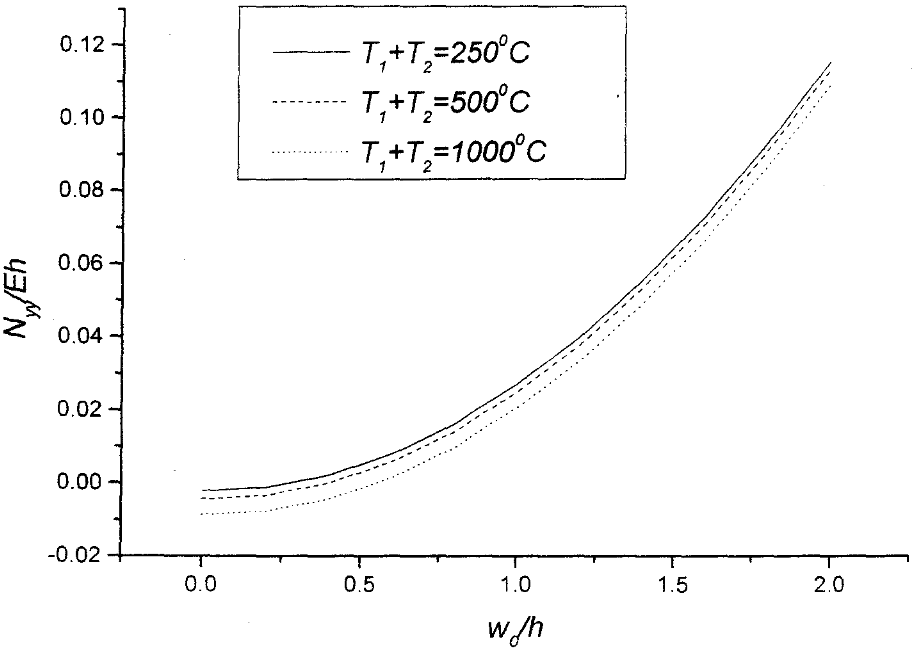

Fig.3.2 (d) Shows comparative variations of (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$ for $\frac{b}{a}=1, \frac{a}{h}=\frac{b}{h}=10, \frac{R}{h}=200$

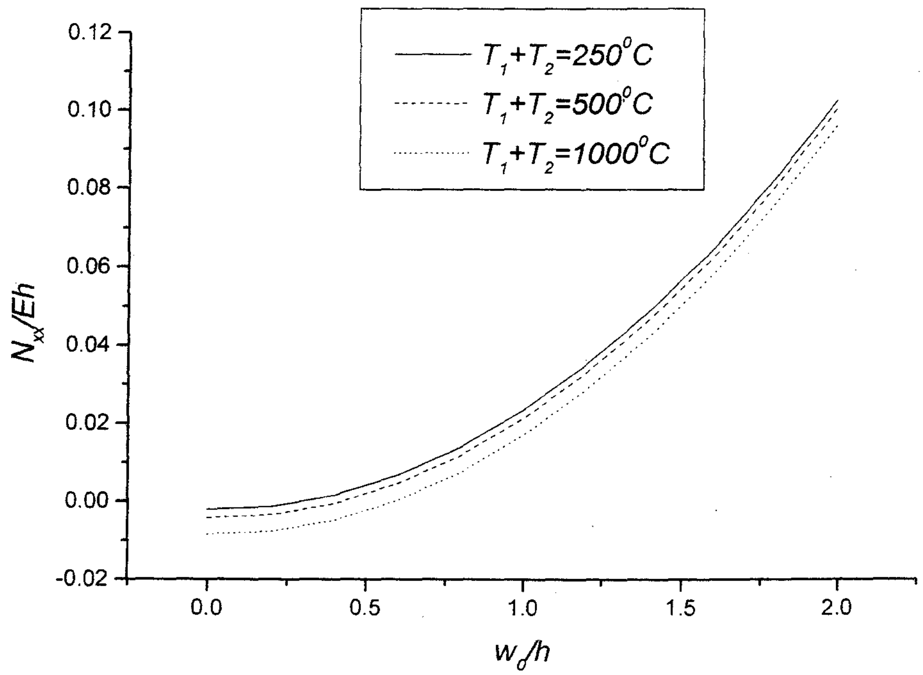

Fig.3.2 (e) Shows comparative variations of (N_{xx})/Eh vs. $\left(\frac{W_0}{h}\right)$ for $\frac{a}{h}=10, \frac{b}{h}=20, \frac{R}{h}=100$

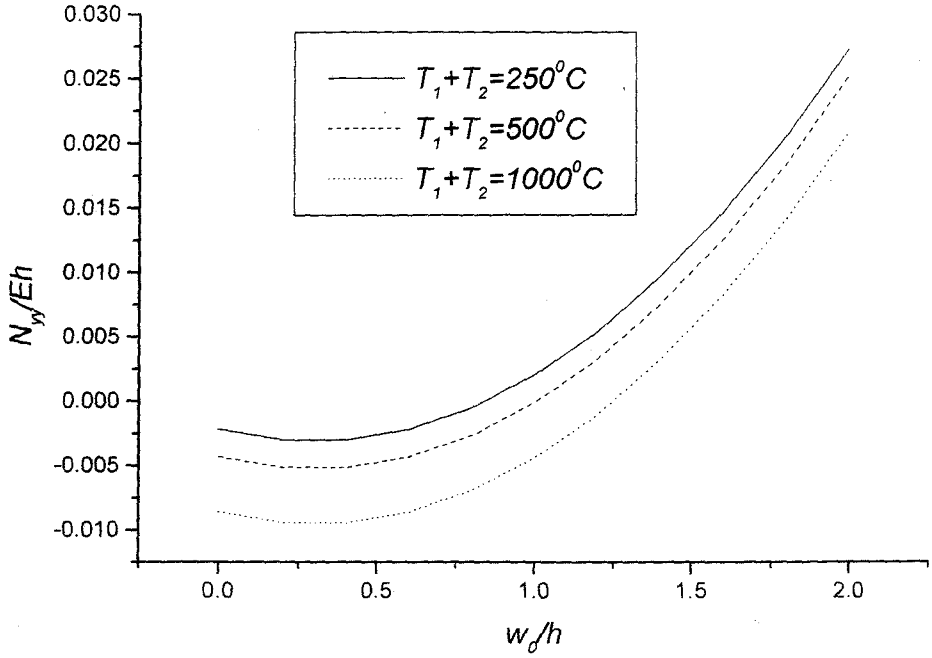

Fig.3.2 (f) Shows comparative variations of (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$ for

$\frac{a}{h}=10, \frac{b}{h}=20, \frac{R}{h}=100$

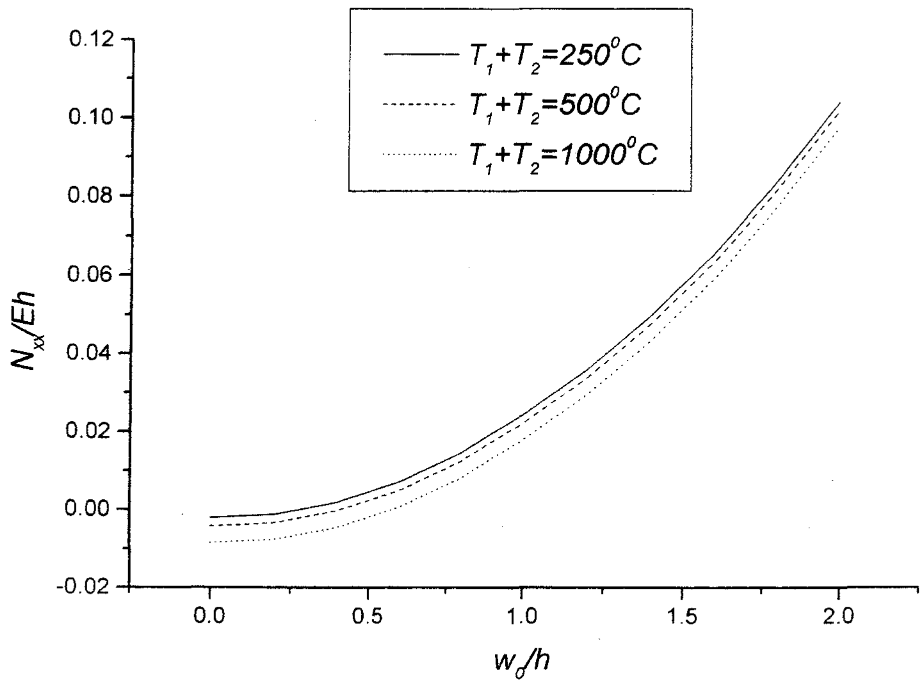

Fig.3.2 (g) Shows comparative variations of (N_{xx})/Eh vs. $\left(\frac{W_0}{h}\right)$ for $\frac{a}{h}=10, \frac{b}{h}=20, \frac{R}{h}=200$

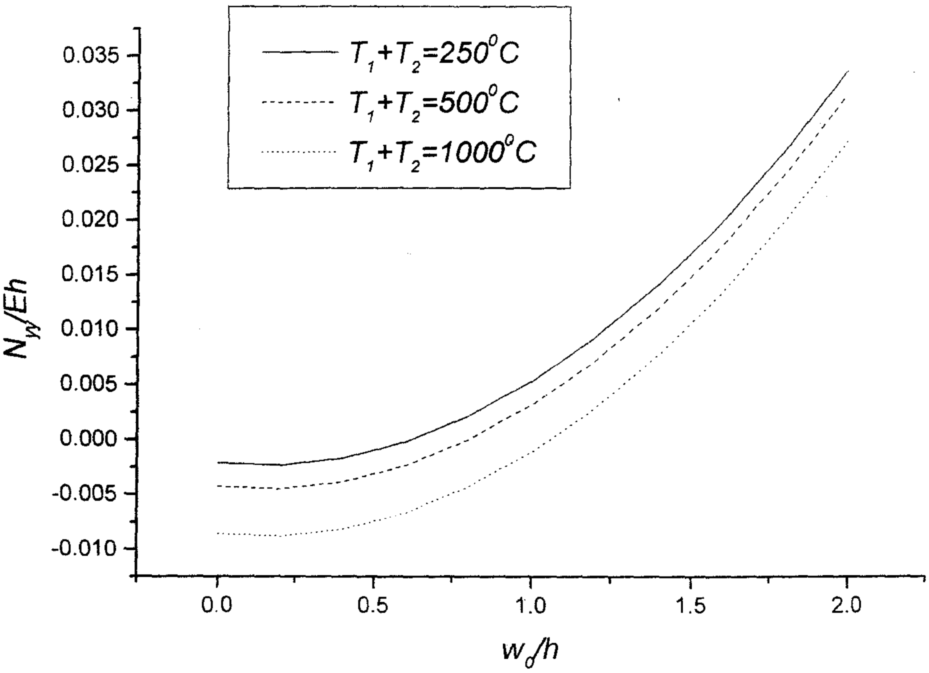

Fig.3.2 (h) Shows comparative variations of (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$ for $\frac{a}{h}=10, \frac{b}{h}=20, \frac{R}{h}=200$

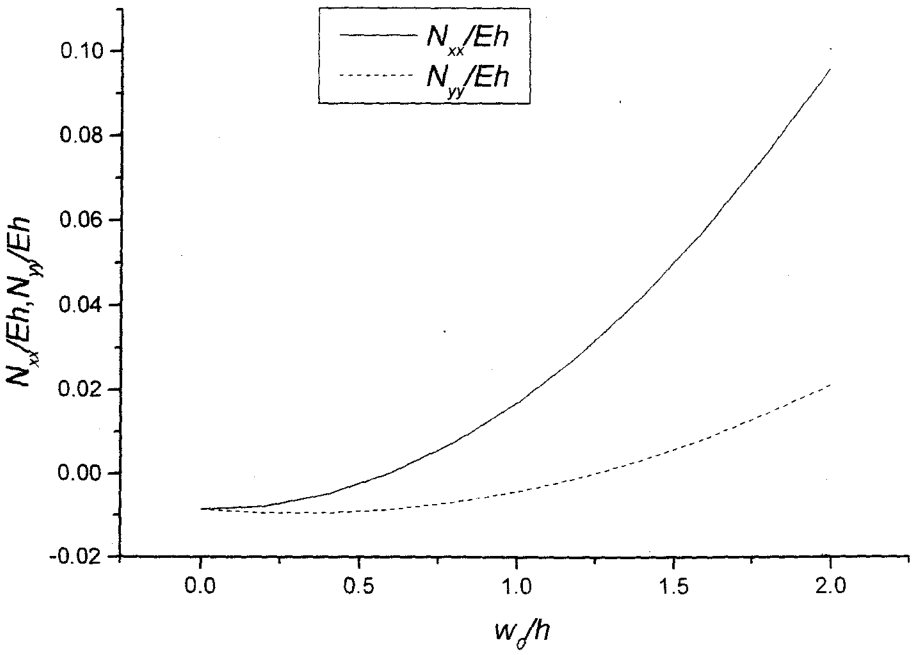

Fig.3.2 (i) Shows comparative variations of (N_{xx})/Eh, (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$ for

$\frac{a}{h}=10, \frac{b}{h}=20, \frac{R}{h}=10, T_1+T_2=1000^0 C$

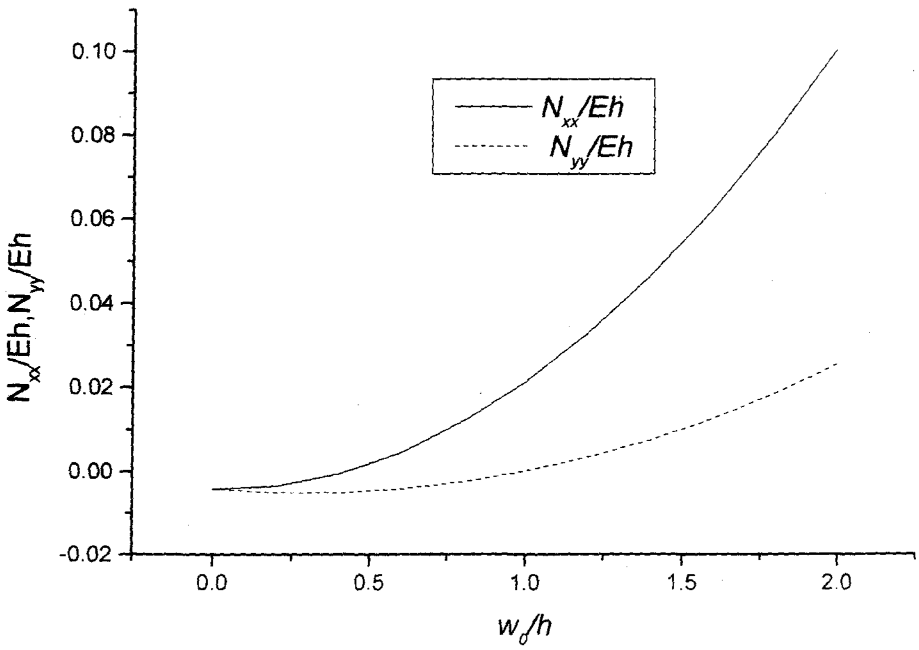

Fig.3.2 (j) Shows comparative variations of (N_{xx})/Eh, (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$ for

$\frac{a}{h} = 10, \frac{b}{h} = 20, \frac{R}{h} = 10, T_1 + T_2 = 500^0 C$

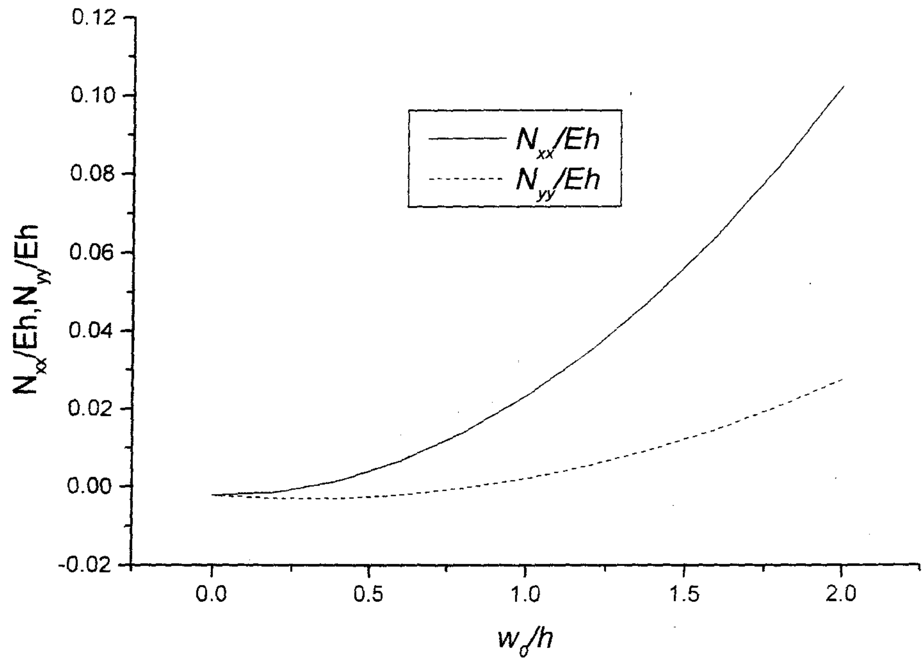

Fig.3.2 (k) Shows comparative variations of(N_{xx})/Eh, (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$for

$\frac{a}{h}=10, \frac{b}{h}=20, \frac{R}{h}=10, T_1+T_2=250^0 C$

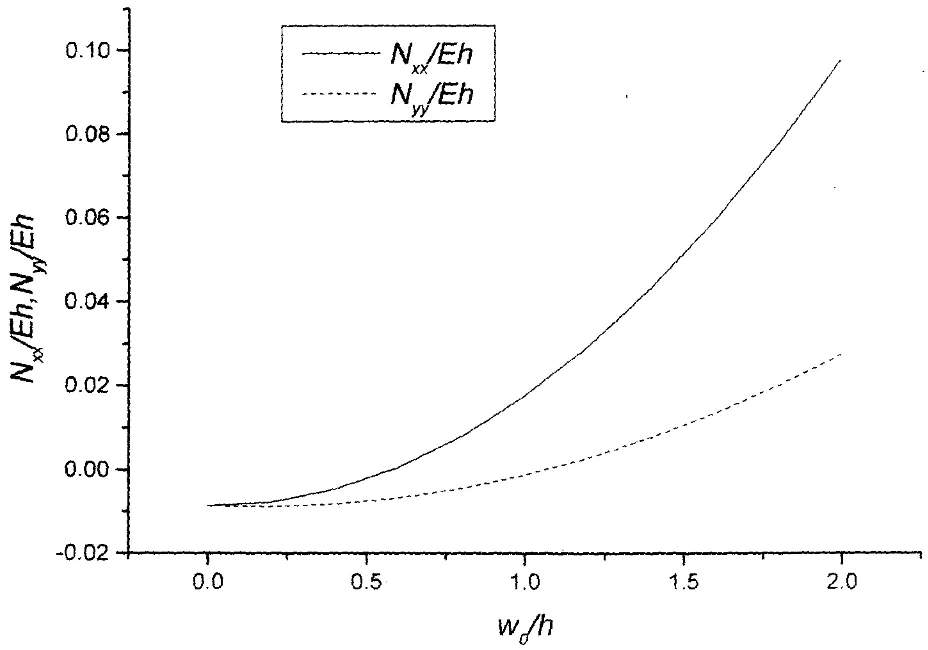

Fig.3.2 (l) Shows comparative variations of (N_{xx})/Eh, (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$ for $\frac{a}{h}=10, \frac{b}{h}=20, \frac{R}{h}=20, T_1+T_2=1000^0 C$

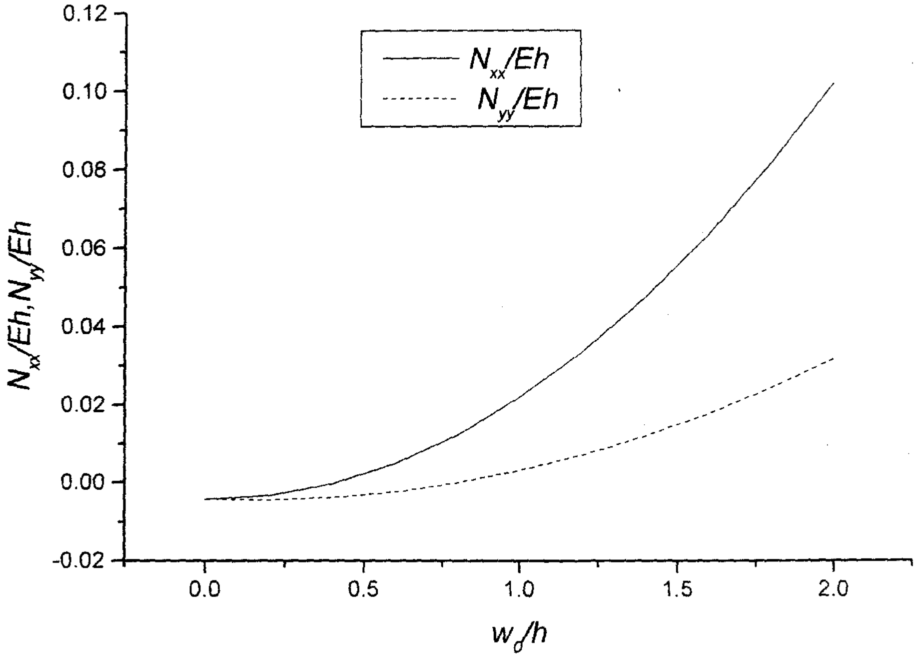

Fig.3.2 (m) Shows comparative variations of(N_{xx})/Eh, (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$ for

$\frac{a}{h} = 10, \frac{b}{h} = 20, \frac{R}{h} = 20, T_1 + T_2 = 500^0 C$

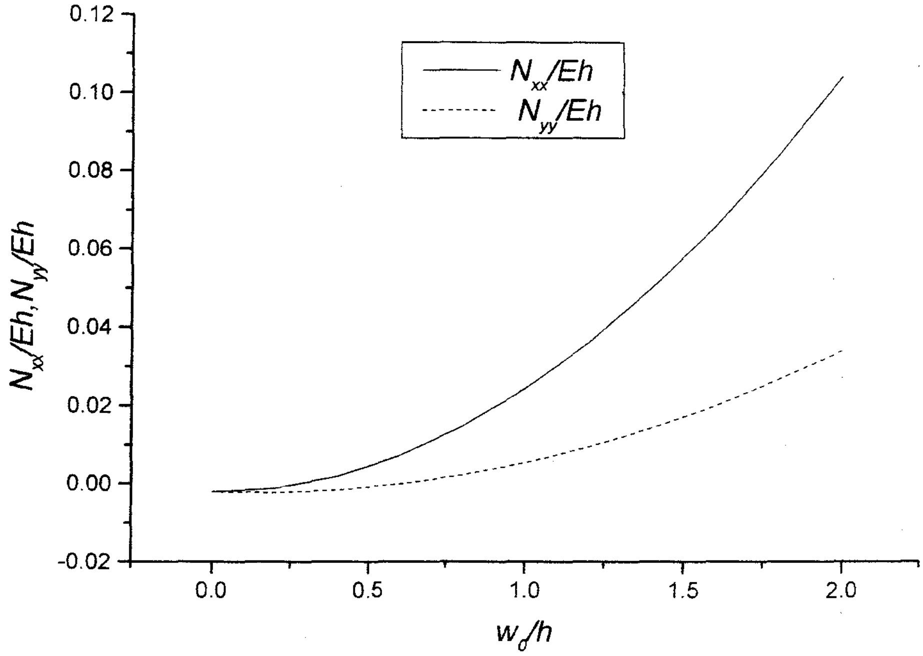

Fig.3.2 (n) Shows comparative variations of (N_{xx})/Eh, (N_{yy})/Eh vs. $\left(\frac{W_0}{h}\right)$ for $\frac{a}{h}=10, \frac{b}{h}=20, \frac{R}{h}=20, T_1+T_2=250^0 C$

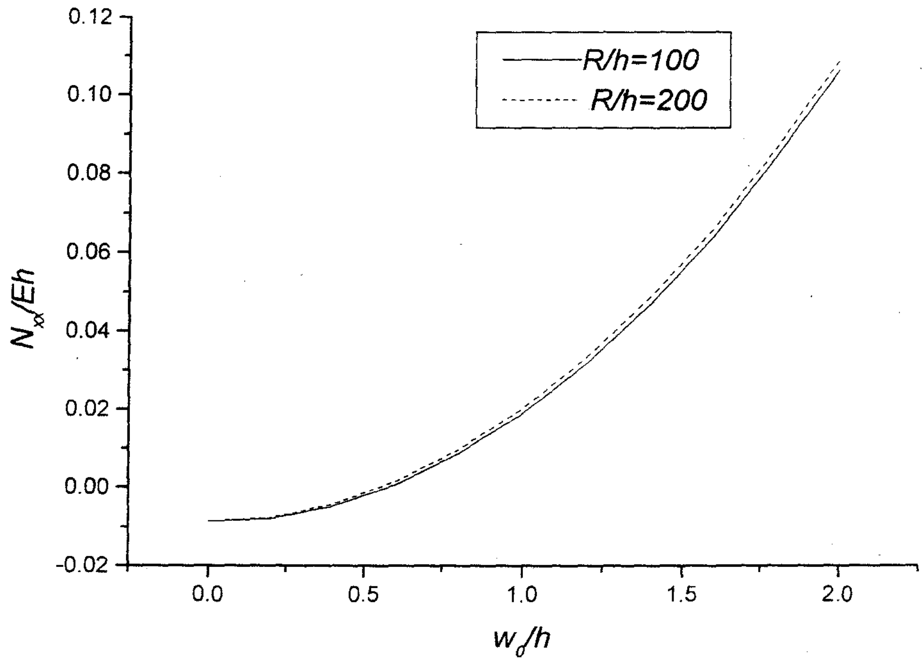

Fig.3.2 (o) Shows comparative variations of(N_{xx})/Eh, vs. $\left(\frac{W_0}{h}\right)$ for

$$\frac{b}{a}=1,\frac{a}{h}=\frac{b}{h}=10,T_1+T_2=1000^0C$$

3.5 OBSERVATIONS AND DISCUSSIONS:

1. From Fig.3.1(a) and Fig.3.1(b)

   i) one can observe that if the temperature $T_1 + T_2$ is kept fixed, non-dimensional thermal parameter $\alpha_t(T_1 - T_2)$ increases with the increase of the non-dimensional central deflection. $\left(\frac{W_0}{h}\right)$ for both the cases of $\left(\frac{R}{h}\right) = 100$ and 200.

   ii) Again one can observe further that with the increase of temperature ($T_1 + T_2$), non-dimensional thermal parameter $\alpha_t(T_1 - T_2)$ diminishes for the same deflection and the increment is accelerated with the increase of the non-dimensional central deflection $\left(\frac{W_0}{h}\right)$.

2. From Fig.3.1(c), Fig.3.1(d),Fig.3.1(e)

   i) One can observe that for a square shallow shell panel if $\left(\frac{R}{h}\right)$ remains fixed, non-dimensional thermal parameter $\alpha_t(T_1 - T_2)$ increases with the increase of the non-dimensional central deflection. $\left(\frac{W_0}{h}\right)$.

   ii) Again one can observe that with the increase of $\left(\frac{R}{h}\right)$, non-dimensional thermal parameter $\alpha_t(T_1 - T_2)$ increases for a particular deflection and the difference in increment increases insignificantly with the increase of the non-dimensional central deflection $\left(\frac{W_0}{h}\right)$. This also shows that keeping the difference in temperature constant the increase in R also insignificant for the deflection. It may be concluded that the variation in R plays a little role for the deflected surface.

3. Fig.3.1(f),Fig.3.1(g) and Fig.3.1(h),one can observe that these figures depict the nature of variation of the non-dimensional central deflection. $\left(\frac{W_0}{h}\right)$ with the non-dimensional thermal parameter $\alpha_t(T_1 - T_2)$ for rectangular shallow shell panel. The behavior of the rectangular shallow shell panel's dependence on R is alike the case for a square shallow shell panel. But the only difference

is that the amount of deflection is more in case of a rectangular shallow shell panel if other conditions remaining the same.

4. From Fig.3.1(i) - Fig.3.1(n)

i) One can observe that for a square and a rectangular shallow shell panel, if $\left(\frac{R}{h}\right)$ is kept fixed , non-dimensional thermal parameter $\alpha_t(T_1 - T_2)$ increases with the increase of the non-dimensional central deflection $\left(\frac{W_0}{h}\right)$. Also for a particular value of non-dimensional thermal parameter $\alpha_t(T_1 - T_2)$ the non-dimensional central deflection $\left(\frac{W_0}{h}\right)$ is much more in a rectangular shallow shell panel than that of a square shallow shell panel.

ii) Again one can observe that for a particular value of non-dimensional thermal parameter $\alpha_t(T_1 - T_2)$, non-dimensional central deflection $\left(\frac{W_0}{h}\right)$ decreases with the increase of $\left(\frac{R}{h}\right)$.

5. From Fig.3.2(a),Fig.3.2(b), Fig.3.2(c) and Fig 3.2.(d) one can observe that for a square shallow shell panel,if the temperature $T_1 + T_2$ is kept fixed, the nature of median surface membrane stresses $\left(\frac{N_{xx}}{Eh}\right)$ and $\left(\frac{N_{yy}}{Eh}\right)$ are same.

6. From Fig.3.2(e) and Fig.3.2(g)

i) One can observe that for having the same non-dimensional central deflection $\left(\frac{W_0}{h}\right)$ the median surface membrane stresses $\left(\frac{N_{xx}}{Eh}\right)$ and $\left(\frac{N_{yy}}{Eh}\right)$ decreases with the increase of temperature $T_1 + T_2$.

From Fig. 3.2 (f) and 3.2(h)

ii) It is interesting to note that for all temperature $T_1 + T_2$ the median surface membrane stresses $\left(\frac{N_{xx}}{Eh}\right)$ and $\left(\frac{N_{yy}}{Eh}\right)$ decreases with the increase of non-dimensional central deflection $\left(\frac{W_0}{h}\right)$ to a certain stage and then the median surface membrane stresses $\left(\frac{N_{xx}}{Eh}\right)$ and $\left(\frac{N_{yy}}{Eh}\right)$ increase with the increase of non-dimensional central deflection $\left(\frac{W_0}{h}\right)$.

7) From Fig.3.2(i),Fig.3.2(j), Fig.3.2(k) and Fig 3.2.(l),Fig.3.2(m),Fig.3.2(n) one can observe that for a rectangular shallow shell panel that if the temperature T_1+T_2 is kept fixed, the median surface membrane stresses $\left(\frac{N_{xx}}{Eh}\right)$ are higher than $\left(\frac{N_{yy}}{Eh}\right)$ with the increase of non-dimensional central deflection. $\left(\frac{W_0}{h}\right)$.

8) From Fig.3.2(o) one can observe that for a square shallow shell panel that if the temperature T_1+T_2 is kept fixed, the nature of median surface membrane stresses $\left(\frac{N_{xx}}{Eh}\right)$ increases with the increase of non-dimensional central deflection $\left(\frac{W_0}{h}\right)$ and $\left(\frac{R}{h}\right)$

CHAPTER IV

NONLINEAR THERMAL VIBRATIONS OF A CIRCULAR PLATE UNDER ELEVATED TEMPERATURE*

4.1.1 INTRODUCTION

Problems of mechanics of thermal vibrations have been analysed by many authors by using the classical von Karman field equations and Berger's approximation extended to the dynamic case with the inclusion of thermal loading [112,176-178]. Merits(advantages) and demerits (disadvantages) of the two methods have been discussed in the literature by many authors.

In this paper "a new approach" proposed by Banerjee and Dutta [144] and employed by Sinharay and Banerjee [146] and others [151-159] will be applied to derive the basic governing equations with the inclusion of thermal loading in the dynamic case and to make an empirical study of the nonlinear thermal vibrations of a circular plate with clamped immovable edges.

The basic governing equations so derived have been solved by Galerkin's procedure .Numerical computations have been presented graphically and followed by observations and discussions.

4.1.2 DERIVATION OF GOVERNING FIELD EQUATIONS

We consider a circular plate of radius 'a'and thickness 'h' and subjected to a temperature distribution [Nowacki,1962, Ref.4]

$$T(r,z)=\psi_0(r)+z\psi(r), \tag{85}$$

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The potential energy due to bending and stretching undergoing large deflections in the absence of any external mechanical load, may be expressed in the form

$$V=\frac{D}{2}\int_0^a\left[(W_{,rr})^2+\frac{2\nu}{r}W_{,r}W_{,rr}+\left(\frac{1}{r}W_{,r}\right)^2+\frac{12}{h^2}\left\{e_1{}^2+2e_2(\nu-1)\right\}\right]rdr \tag{86}$$

where

$$e_1=u_{,r}+\frac{u}{r}+\frac{1}{2}(W_{,r})^2=\text{First invariant of middle surface strains} \tag{86.1}$$

$$e_2=\frac{u}{r}+\left\{u_{,r}+\frac{1}{2}(W_{,r})^2\right\}=\text{Second invariant of middle surface strains} \tag{86.2}$$

Putting the expressions for e_1 and e_2 into equation (86) and rearranging one gets,

$$V=\frac{D}{2}\int_0^a\left[(W_{,rr})^2+\frac{2\nu}{r}W_{,r}W_{,rr}+\left(\frac{1}{r}W_{,r}\right)^2+\frac{12}{h^2}\left\{\bar{e}_1{}^2+\frac{u^2}{r^2}(1-\nu^2)\right\}\right]rdr \tag{87}$$

where,

$$\bar{e}_1=u_{,r}+\frac{1}{2}(W_{,r})^2+\nu\frac{u}{r} \tag{87.1}$$

The kinetic energy K.E. and W_T, the energy contribution due to heating effect in the plate are given by

$$\text{K.E.}=\frac{\rho h}{2}\iint_s\left\{(u_{,t})^2+(W_{,t})^2\right\}rdrdt \tag{88}$$

$$W_T=-\iint_s\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{\alpha_t ET}{(1-\nu)}\left(\bar{e}_1-z\nabla^2W\right)dxdydz \qquad \text{[Basuli 1968, Ref. 108]}$$

$$=-\iint_s\left[\bar{e}_1N_T-\nabla^2WM_T\right]dxdy \tag{89}$$

where

$$N_T=\frac{\alpha_t E}{(1-\nu)}\int_{-\frac{h}{2}}^{\frac{h}{2}}\tau_0(x,y)dz \tag{89.1}$$

$$M_T = \frac{\alpha_t E}{(1-\nu)} \int_{-\frac{h}{2}}^{\frac{h}{2}} z\tau(x,y)dz \tag{89.2}$$

Transforming the expression for W_T in polar co-ordinates,we get

$$W_T = -\iint_s \left[\overline{e_1} N_T - \nabla^2 W M_T\right] r dr d\theta \tag{90}$$

For a circular plate,

$$W_T = -\int_0^a \left[\overline{e_1} N_T - \nabla^2 W M_T\right] r dr \tag{91}$$

where

$$N_T = \frac{\alpha_t E}{(1-\nu)} \int_{-\frac{h}{2}}^{\frac{h}{2}} T(r,z)dz = \frac{\alpha_t E}{(1-\nu)} \psi_0(r) h, \tag{91.1}$$

$$M_T = \frac{\alpha_t E}{(1-\nu)} \int_{-\frac{h}{2}}^{\frac{h}{2}} zT(r,z)dz = \frac{\alpha_t E h^3}{12(1-\nu)} \psi(r), \quad \text{[Nowacki (1962), Ref.4]} \tag{91.2}$$

Following the 'new approach'(1981),the term $(1-\nu^2)\frac{u^2}{r^2}$ is replaced by $\frac{\lambda}{4}(W_{,r})^4$ in the expression (87) where λ is a factor depending on the Poission's ratio of the plate material .This is done for possible decoupling of the equation.

$$V = \frac{D}{2} \int_0^a \left[(W_{,r})^2 + \frac{2\nu}{r} W_{,r} W_{,rr} + \left(\frac{1}{r} W_{,r}\right)^2 + \frac{12}{h^2}\left\{\overline{e_1}^2 + \frac{\lambda}{4}(W_{,r})^4\right\}\right] r dr \tag{92}$$

The Lagrangian equation is

$$L = T - V' = T - V - W_T \ , \tag{93}$$

where $V' = V + W_T$ (93.1)

and

$$V' = \frac{D}{2} \int_0^a \left[(W_{,rr})^2 + \frac{2\nu}{r} W_{,r} W_{,rr} + \left(\frac{1}{r} W_{,r}\right)^2 + \frac{12}{h^2}\left\{\overline{e_1}^2 + \frac{\lambda}{4}(W_{,r})^4\right\}\right] r dr - \int_0^a \left[\overline{e_1} N_T - \nabla^2 W M_T\right] r dr$$

(94)

Since the inertia effect due to radial displacement u is negligible in equation (88),therefore it reduces to

K.E.= $\frac{\rho h}{2}\int_0^a\int_0^t (W_{,t})^2 r dr dt$ (95)

Therefore(93) becomes

$$L = \frac{\rho h}{2}\int_0^a\int_0^t (W_{,t})^2 r dr dt - \frac{D}{2}\int_0^a\left[(W_{,rr})^2 + \frac{2\nu}{r}W_{,r}W_{,rr} + \left(\frac{1}{r}W_{,r}\right)^2 + \frac{12}{h^2}\left\{\overline{e}_1^{\,2} + \frac{\lambda}{4}(W_{,r})^4\right\}\right] r dr$$
$$+ \int_0^a \left[\overline{e}_1 N_T - \nabla^2 W M_T\right] r dr \tag{96}$$

As the total energy is to be minimum, the integrand F must be satisfy the Euler's Variational equations which are as follows:

$$\frac{\partial F}{\partial u} - \frac{\partial}{\partial r}\left(\frac{\partial F}{\partial u_{,r}}\right) = 0 \tag{97}$$

$$\frac{\partial F}{\partial W} - \frac{\partial}{\partial r}\left(\frac{\partial F}{\partial W_{,r}}\right) - \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial W_{,t}}\right) + \frac{\partial^2}{\partial r^2}\left(\frac{\partial F}{\partial W_{,rr}}\right) = 0 \tag{98}$$

where

$$F = \frac{\rho h r}{2}(W_{,t})^2 - \frac{Dr}{2}\left[\left\{(W_{,rr})^2 + \frac{2\nu}{r}W_{,r}W_{,rr} + \left(\frac{1}{r}W_{,r}\right)^2\right\} + \frac{12}{h^2}\left\{\overline{e}_1^{\,2} + \frac{\lambda}{4}(W_{,r})^4\right\}\right]$$
$$+ \left\{r\overline{e}_1 N_T - r\nabla^2 W M_T\right\} \tag{99}$$

We shall now apply Euler's variational equations (97) and (98) on (99) and get

$$\frac{\partial F}{\partial u} = \nu\left[N_T - \frac{12D}{h^2}\overline{e}_1\right] \tag{100}$$

$$\frac{\partial F}{\partial u_{,r}} = r\left[N_T - \frac{12D}{h^2}\overline{e}_1\right] \tag{101}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial F}{\partial u_{,r}}\right) = \frac{\partial}{\partial r}\left[r\left\{N_T - \frac{12D}{h^2}\overline{e}_1\right\}\right] \tag{102}$$

Taking $k = \left[N_T - \frac{12D}{h^2}\bar{e}_1 \right]$ and using (100),(101) and (102) in (97),one gets

$$vk - \frac{\partial}{\partial r}(rk) = 0 \qquad (103)$$

Integrating (103),one get

$$\bar{e}_1 = \frac{N_T h^2}{12D} + \frac{Cf(t) r^{\nu-1} h^2}{12D} \qquad (104)$$

Again using Euler's Variational equations (98) on (99) one gets

$$\frac{\partial F}{\partial W} = 0 \qquad (105)$$

$$\frac{\partial}{\partial r}\left(\frac{\partial F}{\partial W_{,r}}\right) = -Dr\left[\frac{\nu}{r}W_{,rrr} + \frac{1}{r^2}W_{,rr} - \frac{1}{r^3}W_{,r} + \frac{6\lambda}{h^2}(W_{,r})^2\left\{\nabla^2 W + 2W_{,rr}\right\}\right]$$

$$-Cf(t)r^{\nu}\left[W_{,rr} + \frac{\nu}{r}W_{,r}\right] - (M_T)_{,r} \qquad (106)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial F}{\partial W_{,t}}\right) = \rho h r W_{,tt} \qquad (107)$$

$$\frac{\partial^2}{\partial r^2}\left(\frac{\partial F}{\partial W_{,rr}}\right) = -Dr\left[\frac{2}{r}W_{,rrr} + W_{,rrrr} + \frac{\nu}{r}W_{,rrr}\right] - r\left[\nabla^2 M_T + \frac{1}{r}(M_T)_{,r}\right] \qquad (108)$$

Using (105),(106),(107) and (108) in (98) one gets

$$\rho h W_{,tt} + D\nabla^4 W - \frac{6\lambda D}{h_2}(W_{,r})^2\left\{\nabla^2 W + 2W_{,rr}\right\} - Cf(t)r^{\nu-1}\left\{W_{,rr} + \frac{\nu}{r}W_{,r}\right\} + \nabla^2 M_T = 0$$

(109)

where C is a constant of integration and $f(t)$ is some unknown function of time.

4.1.3 METHOD OF SOLUTION

For free vibrations

$M_T = 0$ and $N_T \neq 0$, [Mazumdar,et al (1980), Rfe.177] (110)

For a clamped circular plate of radius $'a'$, the deflection function W satisfying the boundary conditions

$W = 0 = W_{,r}$ at $r = a$ (111)

and $u = 0$ at $r = 0, a$ (112)

is assumed in the form

$$W = AF(t)\left(1 + 2P\frac{r^2}{a^2} + Q\frac{r^4}{a^4}\right), P = 1 = Q \text{ (for clamped edges)} \qquad (113)$$

$'A'$ being the amplitude of vibrations.

Considering (87.1),(104) and (113) and integrating over the area of the plate one gets

$$r^2 u = \frac{N_T h^2}{12D}\frac{r^{\nu+1}}{\nu+1} + \frac{Cf(t)h^2}{12D}\frac{r^{2\nu}}{2\nu} - \frac{8A^2F^2(t)}{a^4}\left[r^{\nu+3}\left\{\frac{P^2}{(\nu+3)} - \frac{2PQ}{a^2}\frac{r^2}{(\nu+5)} + \frac{Q^2}{a^4}\frac{r^4}{(\nu+7)}\right\}\right]$$
$$+ A_1' \qquad (114)$$

where A_1' is a constant of integration. Considering the in-plane boundary conditions $u = 0$ at $r = 0, a$, the constant A_1' is eliminated and one finally gets

$$Cf(t) = \frac{192A^2F^2(t)a^{-(\nu+1)}}{h^2}D\nu\left[\frac{P^2}{\nu+3} - 2\frac{PQ}{\nu+5} + \frac{Q^2}{\nu+7}\right] - 2a^{1-\nu}N_T\frac{\nu}{\nu+1} \qquad (114.1)$$

For clamped edges, we put $P = Q = 1$ and get

$$Cf(t) = 192DA^2F^2(t)\frac{\nu a^{-(\nu+1)}}{h^2}\left[\frac{1}{\nu+3} - 2\frac{1}{\nu+5} + \frac{1}{\nu+7}\right] - 2N_T a^{1-\nu}\frac{\nu}{\nu+1} \qquad (115)$$

Applying Galerkin's procedure in equation (109) and eliminating $Cf(t)$ with the help of (115),one gets the well-known time-differential equation in the form:

$$\ddot{F}(t) + \alpha F(t) + \beta F^3(t) = 0 \qquad (116)$$

where

$$\alpha = \left(\frac{D}{\rho h a^4}\right)\left[\frac{320}{3} - 2560\left(\frac{N_T}{D}\right)\frac{\nu a^2}{(\nu+1)(\nu+3)(\nu+5)(\nu+7)}\right] \qquad (116.1)$$

$$\beta = 10\left(\frac{D}{\rho h a^4}\right)\left(\frac{A}{h}\right)^2\left[\frac{256\lambda}{35} + 196608\frac{\nu}{(\nu+3)^2(\nu+5)^2(\nu+7)^2}\right] \qquad (116.2)$$

In the limiting case in the absence of temperature, the result (116) is in exact agreement with that found in the literature [146,155]

The solution of equation (116) with the normalized conditions
$F(0)=1, \dot{F}(0)=0$
is given by Nash and Modeer [1959, Ref. 86] in the form

$$F(t) = Cn(\omega^* t, R), \text{ where } \omega^{*2} = \alpha + \beta, R^2 = \frac{\beta}{2(\alpha+\beta)} \qquad (117)$$

Cn being the Jocobian Elliptic Function.

The corresponding time-period is given by

$$T' = \frac{2\pi}{\Omega_0} \qquad (118)$$

where Ω_0 is obtained from (23) by dropping the nonlinear so that $\Omega_0 = \sqrt{\alpha}$
Relative time-period is given by

$$\frac{T^*}{T'} = \frac{2K}{\pi}\left(1+\frac{\beta}{\alpha}\right)^{-\frac{1}{2}} \qquad (119)$$

4.1.4 NUMERICAL RESULTS AND DISCUSSIONS

Numerical results have been computed and presented graphically with the non-dimensional amplitude $\left(\frac{A}{h}\right)$ along the horizontal axis and the relative time periods $\left(\frac{T^*}{T'}\right)$ along the vertical axis as shown in the figures considering the following set of values

$\alpha_t = 1.2\times10^{-5}, \nu = 0.3, \lambda = 2\nu^2$ (for clamped movable edge), (120)

and $\lambda = \nu^2$ (for simply supported edge). [Ref.144] (121)

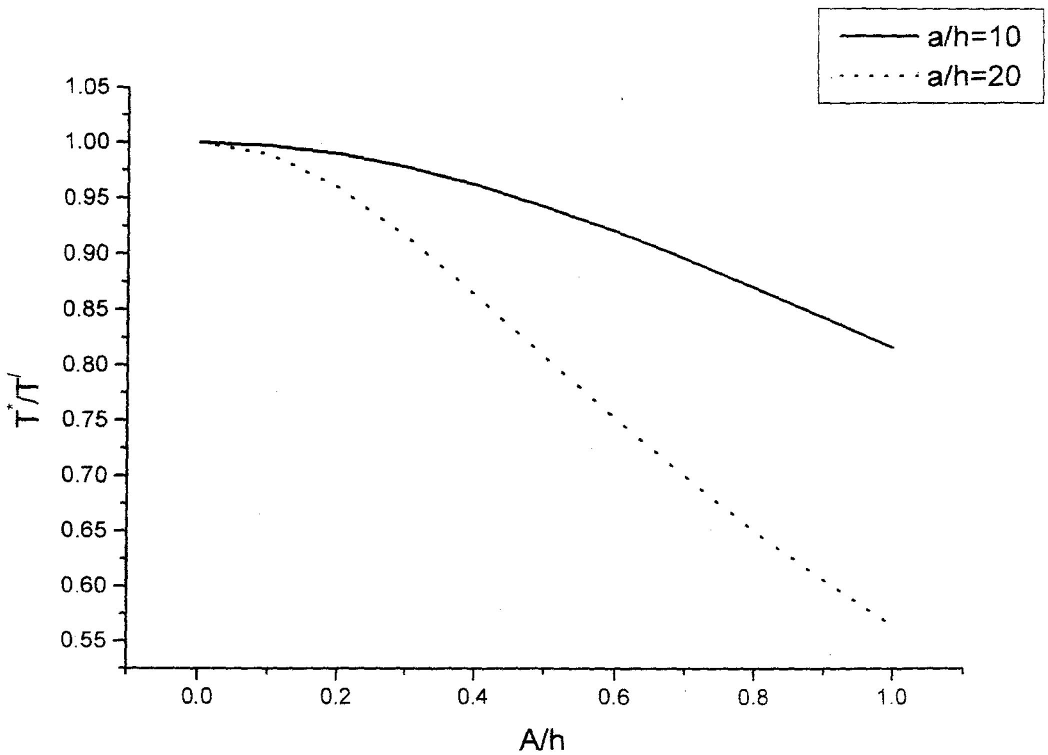

Fig.4.1(a) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for fixed $\tau_0 = 250K$ and $\left(\frac{a}{h}\right) = 10$ and 20

,

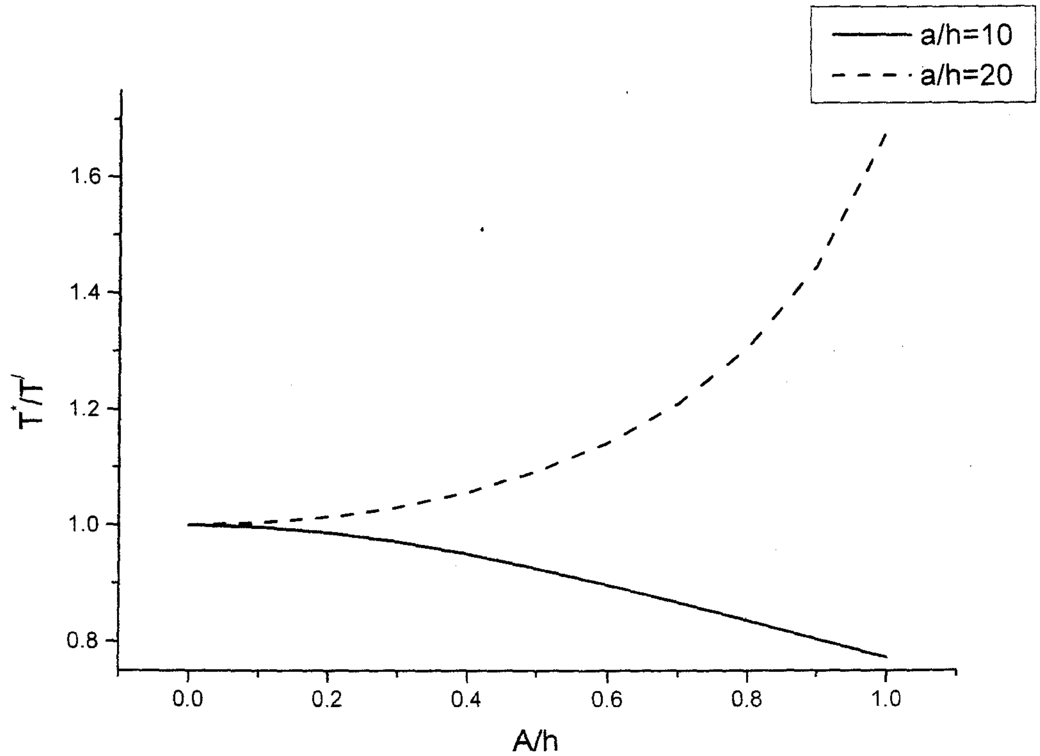

Fig.4.1(b) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for fixed τ_0 =500K and $\left(\frac{a}{h}\right) = 10$ and 20

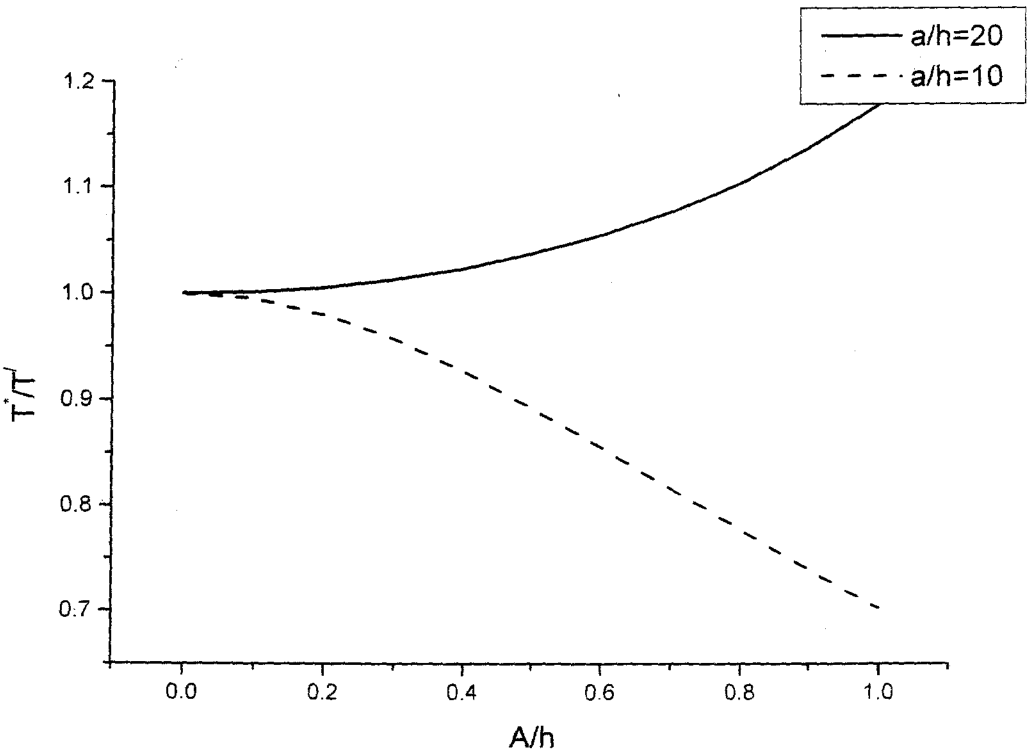

Fig.4.1(c) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for fixed $\tau_0 = 750\text{K}$ and $\left(\frac{a}{h}\right) = 10$ and 20

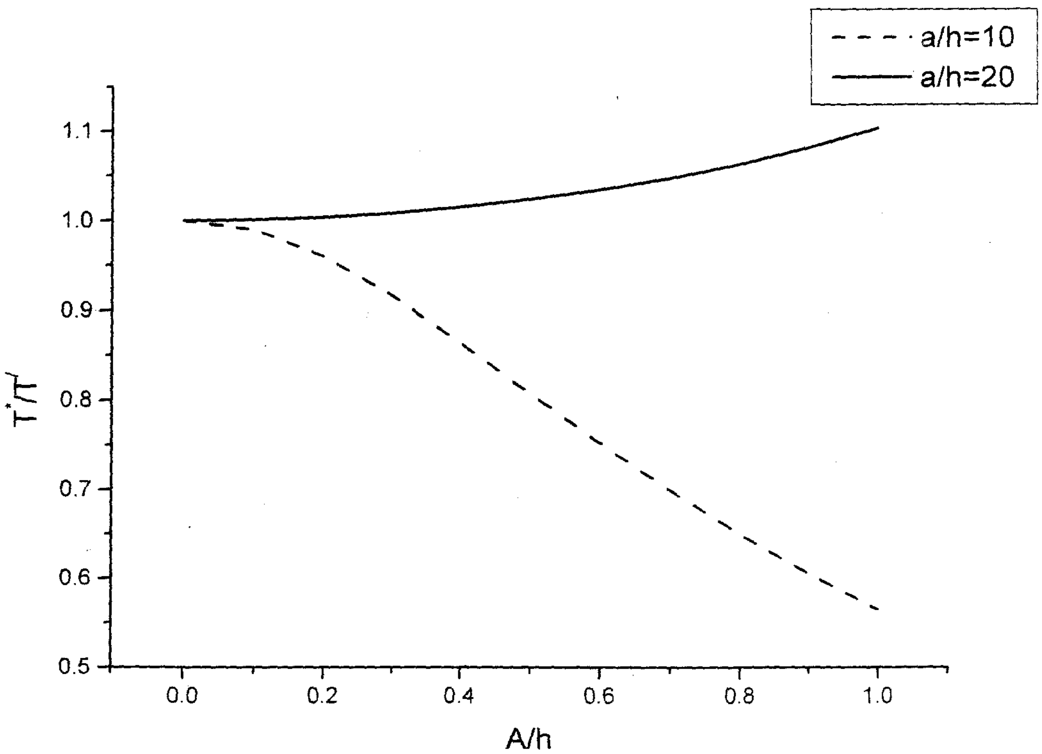

Fig.4.1(d) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for fixed $\tau_0 = 1000\text{K}$ and $\left(\frac{a}{h}\right) = 10$ and 20

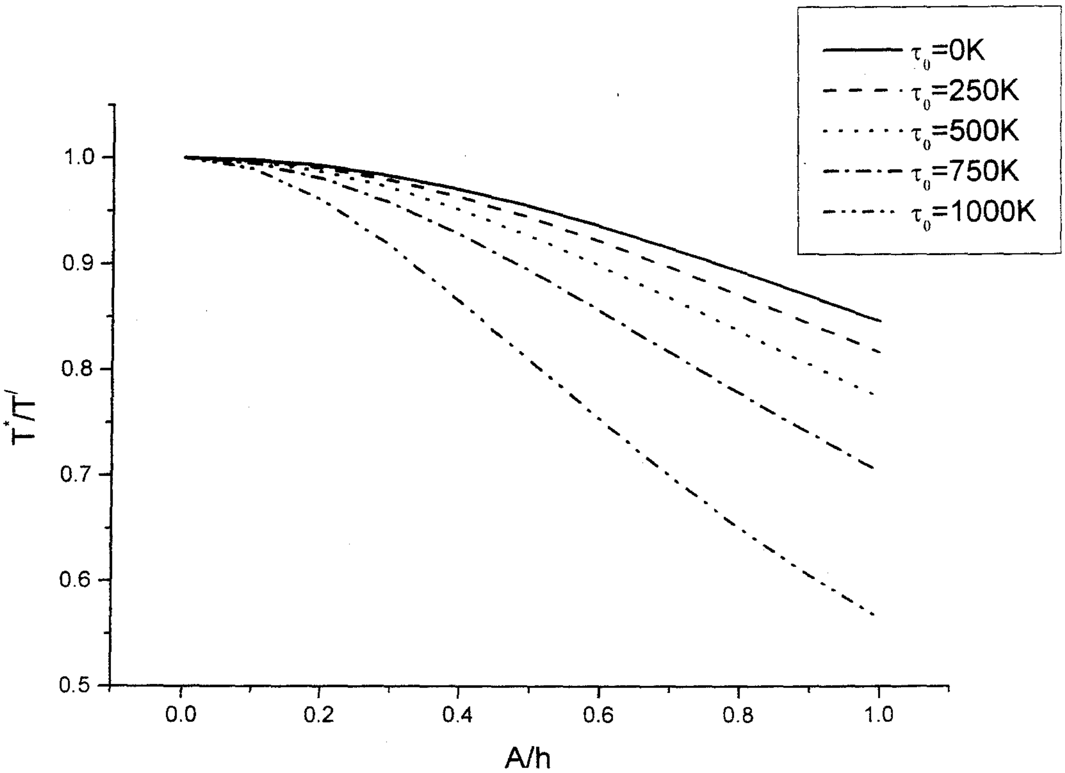

Fig.4.1(e) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for $\left(\frac{a}{h}\right) = 10,\ \tau_0 = 0\text{K},\ \tau_0 = 250\text{K},\ \tau_0 = 500\text{K},\ \tau_0 = 750\text{K},\ \tau_0 = 1000\text{K}$

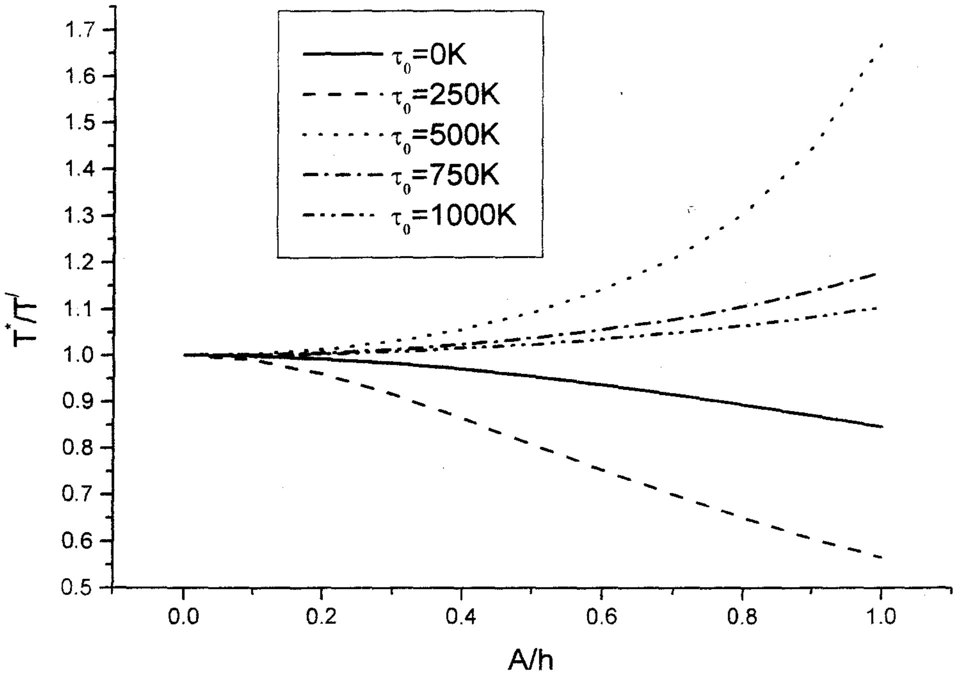

Fig.4.1(f) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for $\left(\frac{a}{h}\right) = 20$, $\tau_0 = 0\text{K}$, $\tau_0 = 250\text{K}$, $\tau_0 = 500\text{K}$, $\tau_0 = 750\text{K}$, $\tau_0 = 1000\text{K}$

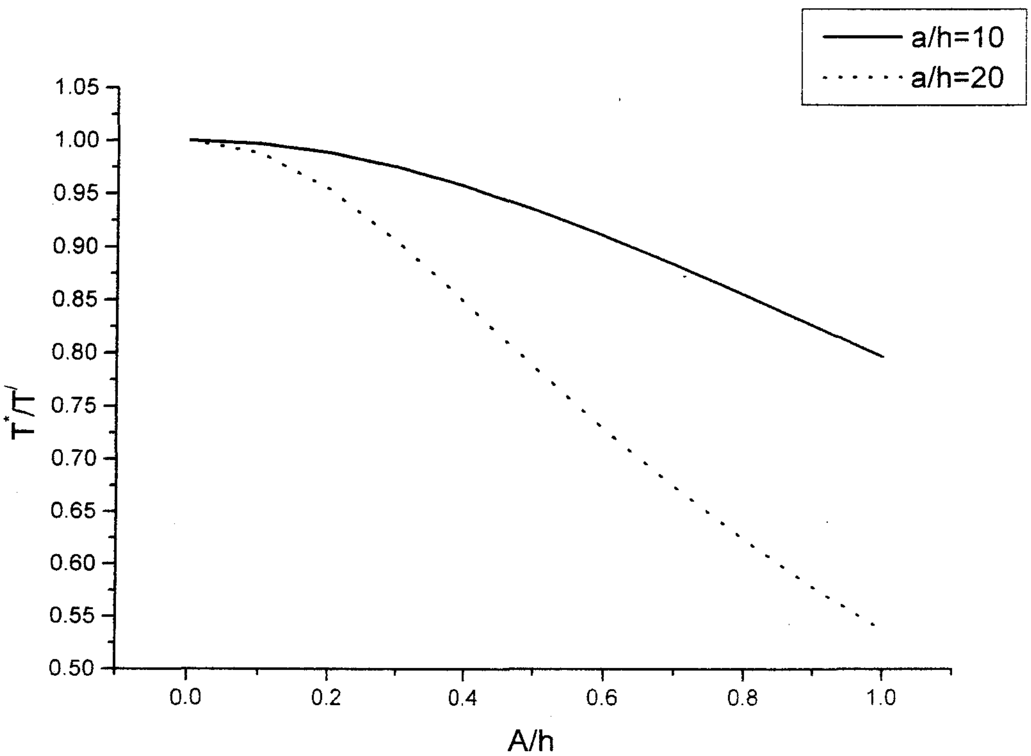

Fig.4.2(a) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for fixed τ_0 =250K and $\left(\frac{a}{h}\right) = 10$ and 20.

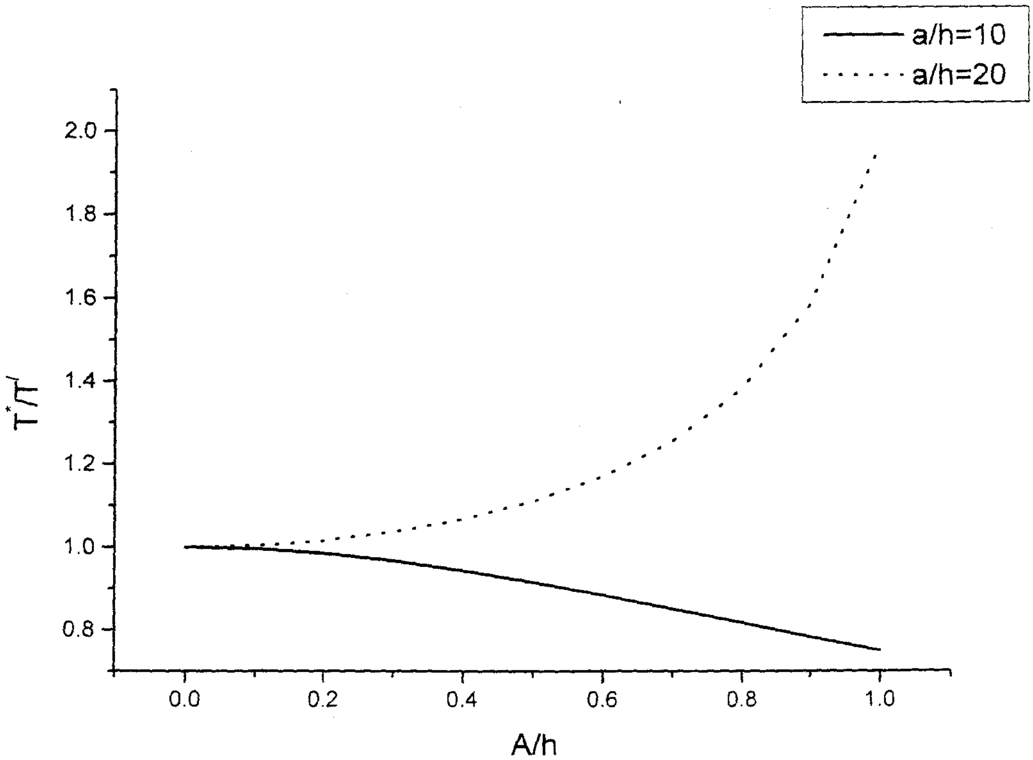

Fig.4.2(b) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for fixed $\tau_0 = 500\text{K}$ and $\left(\frac{a}{h}\right) = 10$ and 20.

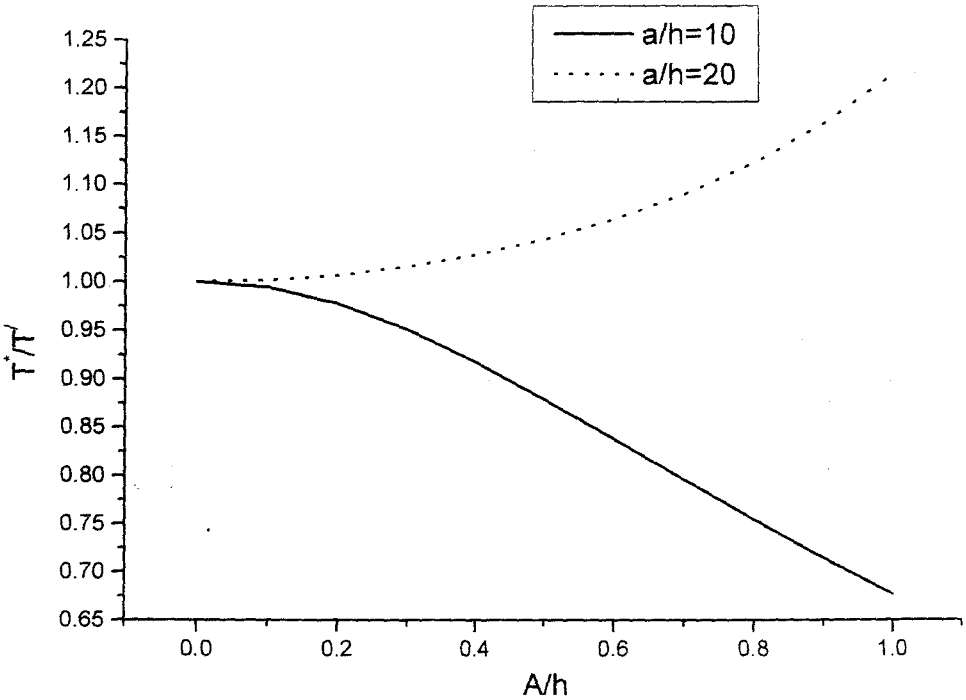

Fig.4.2(c) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for fixed $\tau_0 = 750\text{K}$ and $\left(\frac{a}{h}\right) = 10$ and 20.

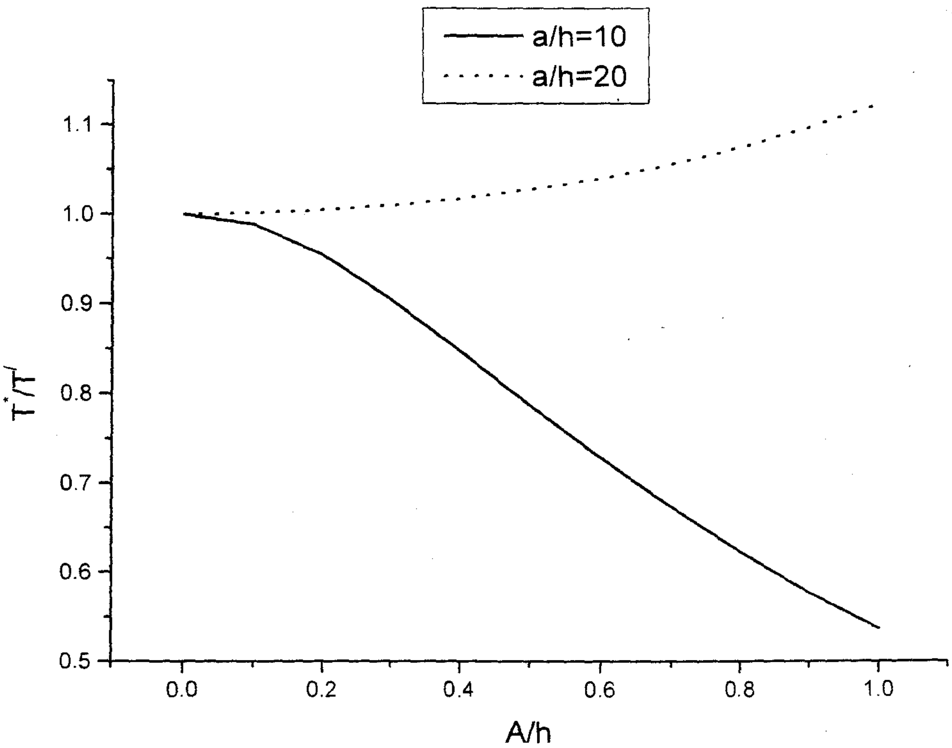

Fig.4.2(d) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for fixed $\tau_0 = 1000$K and $\left(\frac{a}{h}\right) = 10$ and 20.

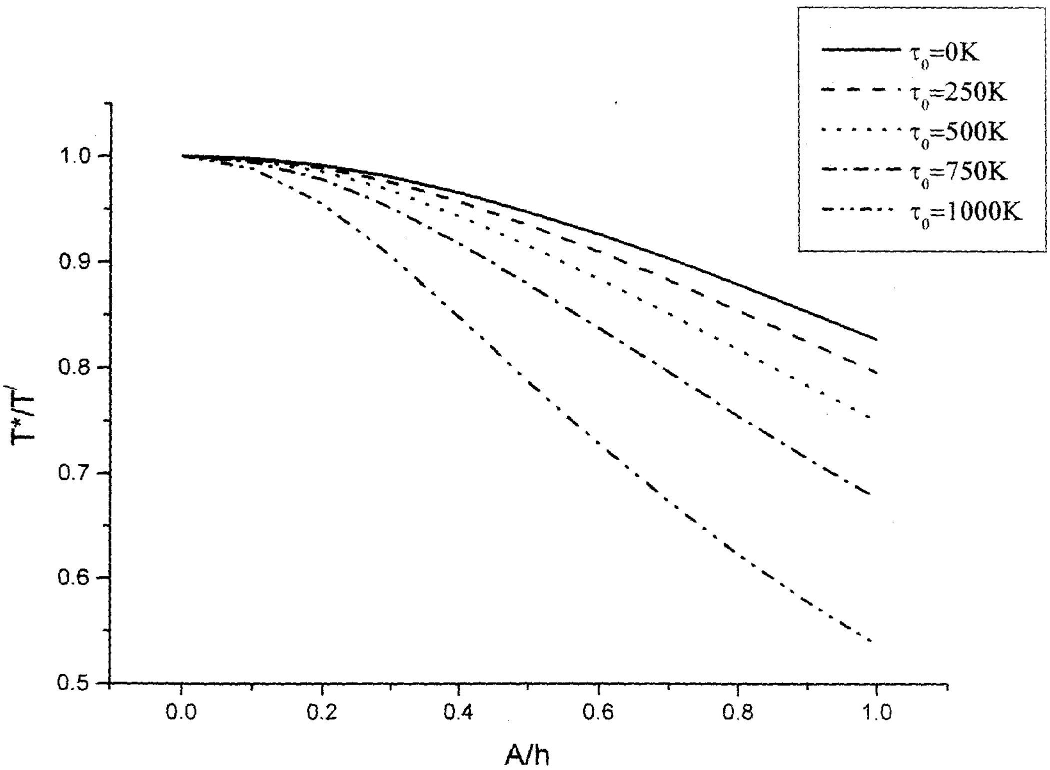

Fig.4.2(e) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for $\left(\frac{a}{h}\right) = 10,\ \tau_0 = 0\text{K},\ \tau_0 = 250\text{K},\ \tau_0 = 500\text{K},\ \tau_0 = 750\text{K},\ \tau_0 = 1000\text{K}$

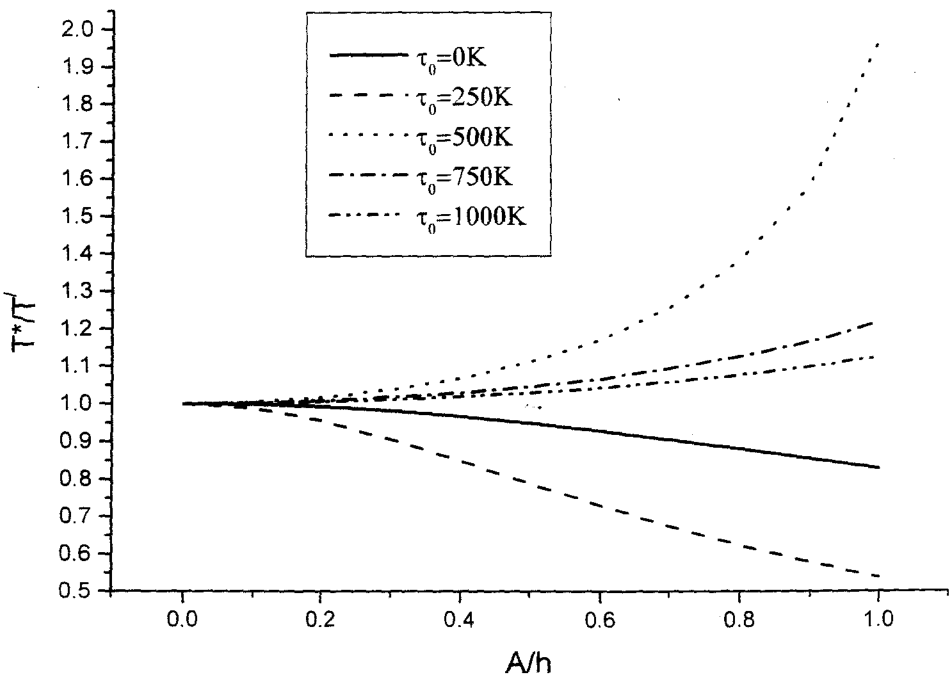

Fig.4.2(f) Shows comparative variations of $\left(\frac{A}{h}\right)$ vs. $\left(\frac{T^*}{T'}\right)$ for $\left(\frac{a}{h}\right)=20,\ \tau_0=0\text{K},\ \tau_0=250\text{K},\ \tau_0=500\text{K},\ \tau_0=750\text{K},\ \tau_0=1000\text{K}$

4.1.5 OBSERVATION AND DISCUSSIONS

(A) For Simply-supported cases:

i)From Fig.4.1(a) one can observe that if the temperature parameter τ_0 remains fixed at the level of 250K,the relative time period $\left(\frac{T^*}{T'}\right)$ diminish with the increase of non-dimensional amplitudes $\left(\frac{A}{h}\right)$ for both the cases of aspect ratios $\left(\frac{a}{h}\right)=10$ and 20.

ii) From Fig.4.1(b) one can observe that if the temperature parameter τ_0 remains fixed at the level of 500K,reverse situation develops i.e. the relative time period $\left(\frac{T^*}{T'}\right)$ diminish with the increase of non-dimensional amplitudes $\left(\frac{A}{h}\right)$ for the aspect ratios $\left(\frac{a}{h}\right) = 10$ but increases with the increase of non-dimensional amplitudes $\left(\frac{A}{h}\right)$ for the cases of aspect ratio $\left(\frac{a}{h}\right)$ is taken as 20. It is interesting to note that for aspect ratio ratios $\left(\frac{a}{h}\right) =$ 20 the diminishing tendency of the relative time period $\left(\frac{T^*}{T'}\right)$ is retained for the temperature level 250K.

iii) Similar situation also exists for higher temperature levels i.e. τ_0=750K and 1000K as shown in Fig.4.1(c) and 4.1(d).So there seems to be a transition phase at certain level in between 250K - 500K as aspect ratio $\left(\frac{a}{h}\right)$ changes from 10 to 20.

iv) From Fig.4.1(e) one can observe that if the temperature parameter τ_0 remains fixed, the relative time period $\left(\frac{T^*}{T'}\right)$ diminish with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$ for the cases of aspect ratio $\left(\frac{a}{h}\right) = 10$.

v) From Fig.4.1(f) one can observe that the relative time period $\left(\frac{T^*}{T'}\right)$ diminishes with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$ for a temperature range from τ_0=0K to 250K. But for the cases of temperature ranging from the transition level to τ_0=500K,750K or 1000K, one can observe that the relative time period $\left(\frac{T^*}{T'}\right)$ increases with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$ for the cases of aspect ratio $\left(\frac{a}{h}\right) = 20$.

vi) One more interesting point observed from this figure that while there is a jumping phenomenon in between 250K – 500K, the tendency of diminishing of the values of the relative time period $\left(\frac{T^*}{T'}\right)$ retains when the temperature increases.

vii) The same argument can be made by keeping the temperature constant but varying the aspect ratio $\left(\frac{a}{h}\right)$ from 10 to 20 or above.

(B) For Clamped Edges:

viii) The same observations may be made also for clamped edge conditions from Fig.4.2(a)-4.2(f) as in the case for simply supported edge conditions. The only difference is that the increase of the relative time period $\left(\frac{T^*}{T'}\right)$ for simply supported edges is a little more than for clamped edges with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$. However, the difference is not of much significance.

(C) General Oservations:

A clinical observations on the jumping phenomenon should have been made to explain the phenomenon but the present author will keep in mind for further investigation on this aspect in his future research activities. Because it will need much more equipments and time to examine the phenomenon

NONLINEAR VIBRATIONS OF NON-HOMOGENEOUS ELASTIC SHELLS UNDER SEVERE THERMAL LOADING*

4.2.1 INTRODUCTION

In many fields of engineering structures viz. high-speed spacecrafts ,nuclear power and chemical plants ,off shore and shipbuilding structures ,storage and building structures and the like, plates and shells of different shapes find enormous applications as integral structural components .Such structural components one likely to be subjected to various kinds of static loads or excitations such as mechanical, seismic ,blast ,hydrodynamic ,aerodynamic with or without thermal loading .Engineers and Scientists all over the world are exerting relentless efforts to design and construct economic ,efficient and smart structures with very low failure probability .In view of this ,for modeling ,analyzing and designing ,structural engineers should be well-acquainted with the static and dynamic behavior of such structural elements with different boundary and loading conditions.

For precise analysis of such problems one needs to consider middle surface strains leading to nonlinear equations .Compared to linear analysis ,the literature on large (nonlinear) thermal deflections ,buckling and vibrations of elastic plates and shells is some what scanty due to high nonlinearity of the classical von Karman field equations extended to thermal loading for static and dynamic cases.

Since classical von Karman equations [44] are highly non-linear in the coupled form without any possibility of having closed form solutions besides being difficult to deal with H.M.Berger [83] proposed a set of quassi-linear partial differential equations in the decoupled forms. These equations were to extended to the static and dynamic cases by many authors with the inclusion of thermal loading[112,125,130 ,177].

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Since Berger's equations face severe criticisms owing to the absurdity of results in several cases ,such as for movable-edge boundary conditions[84], a separate set of differential equations ,also in the decoupled forms ,were proposed by Banerjee and Sinharay[147] as a new approach demanding elimination of the inapplicability contained in the Berger's method . A number of problems were ,however ,solved giving satisfactory results [153, 155].

In this paper the new approach has been employed for the case of non-linear vibrations of non-homogeneous elastic shells under severe thermal loading. Such non homogeneity develops due to the fact that at elevated temperature modulus of elasticity of materials like Titanium alloys (at $1000^0 F$) becomes temperature dependent and becomes function of space variables [2,19,179]. The basic governing equations have been derived based on the new approach [147] and solved by applying Galerkin's procedure .Some numerical results have been presented.

4.2.2 DERIVATION OF DYNAMIC FIELD EQUATIONS FOR A HEATED SPHERICAL SHELL.

Consider a thin spherical shell of radius R subjected to a steady thermal gradient .The temperature distribution T is assumed to be a linear function of the radial distance r and the modulus of elasticity E is assumed to be a linear function of temperature T.As a result the stiffness parameter D is a linear function of r,so that

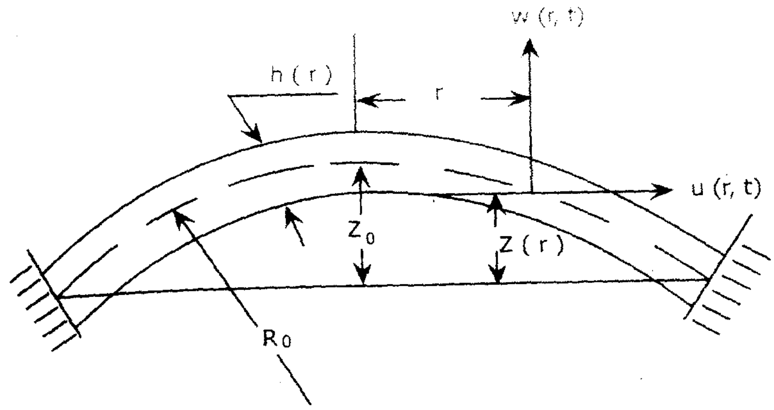
$$T = T_0\left(1-\frac{r}{R}\right) \quad \textbf{(122)}$$

$$E = E_0(1-mT)$$
$$= E_0\left\{1-\beta\left(1-\frac{r}{R}\right)\right\} \quad \textbf{(123)}$$

where $\beta = mT_0$,T_0 is the reference temperature, E_0 is the reference modulus of elasticity and m is the slope of variation of E with T.[19,179]
The flexural rigidity D is given by the expression

$$D(r) = D_0\left\{1-\beta\left(1-\frac{r}{R}\right)\right\} \quad \textbf{(124)}$$

where D_0 is the reference flexural rigidity.



D-1

For the spherical shell with clamped edges, the co-ordinate system is employed as shown in the adjoining diagram [D-1] .The vertical component of the displacement of the middle surface of the shell is denoted by W, considered to be positive in the direction shown .The radial displacement of a point in the middle surface is denoted by u,measured horizontally as shown in the figure. The elevation of the middle surface of the shell above the base plane is denoted by $z = z(r)$,such that

$$z = \frac{R^2}{2R_0}\left(1 - \frac{r^2}{R^2}\right) \qquad (125)$$

from which $z,_r$ is calculated.

$$z,_r = -\frac{r}{R_0} \qquad (125.1)$$

The total potential energy of bending and stretching may written as

$$V = \frac{1}{2}\iint D(r)\left[\left(\nabla^2 W\right)^2 - 2\frac{(1-\nu)}{r}W,_r W,_{rr} + \frac{12}{h^2}\left\{e^2 + 2e_2(\nu - 1)\right\}\right] r dr d\theta \qquad (126)$$

where

$$e = u,_r + \frac{u}{r} + \frac{1}{2}(\omega,_r)^2 + \omega,_r z,_r \qquad (126.1)$$

$$e_2 = \frac{u}{r}u,_r + \frac{u}{2r}(\omega,_r)^2 + \frac{u}{r}\omega,_r z,_r \qquad (126.2)$$

where e and e_2 are the first and second invariants and h is the thickness of the shell.

We now re-write the potential energy in the form :

$$V=\frac{1}{2}\iint D(r)\left[\left(\nabla^2 W\right)^2-2\frac{(1-\nu)}{r}W_{,r}W_{,rr}\right]rdrd\theta$$
$$+\frac{1}{2}\iint D(r)\left[\frac{12}{h^2}\bar{e}_1^{\,2}+\frac{12}{h^2}\lambda\left\{\frac{1}{2}(W_{,r})^2+W_{,r}z_{,r}\right\}^2\right]rdrd\theta \tag{127}$$

where $\bar{e}_1=u_{,r}+\nu\frac{u}{r}+\frac{1}{2}(W_{,r})^2+W_{,r}z_{,r}$ (127.1)

and the term $\lambda\left\{\frac{1}{2}(W_{,r})^2+W_{,r}z_{,r}\right\}^2$ has been replaced by the term

$(1-\nu^2)\frac{u^2}{r^2}$ contained in the equation (127).

Such replacement is physically possible as explained in [147].

The kinetic energy of the shell is given by,

$$T'=\frac{\rho h}{2}\iint\left\{(u_{,t})^2+(W_{,t})^2\right\}rdrd\theta \tag{128}$$

where ρ is the density of the material of the shell.

Since the inertial effect due to radial displacement u is negligible in equation (128) reduces to

$$T'=\frac{\rho h}{2}\iint(W_{,t})^2\,rdrd\theta \tag{129}$$

Forming the Lagrangian function $L=T'-V$, one gets

$$L=\frac{\rho h}{2}\iint(W_{,t})^2\,rdrd\theta-\frac{1}{2}\iint D(r)\left[\left(\nabla^2 W\right)^2-2\frac{(1-\nu)}{r}W_{,r}W_{,rr}\right]rdrd\theta$$
$$-\frac{1}{2}\iint D(r)\left[\frac{12}{h^2}\bar{e}_1^{\,2}+\frac{12}{h^2}\lambda\left\{\frac{1}{2}(W_{,r})^2+W_{,r}z_{,r}\right\}^2\right]rdrd\theta \tag{130}$$

As the total energy is to be minimum, the integrand F must be satisfy the Euler's variational equations which are as follows

$$\frac{\partial F}{\partial u}-\frac{\partial}{\partial r}\left(\frac{\partial F}{\partial u_r}\right)=0 \tag{131.1}$$

$$\frac{\partial F}{\partial W} - \frac{\partial}{\partial r}\left(\frac{\partial F}{\partial W_r}\right) - \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial W_t}\right) + \frac{\partial^2}{\partial r^2}\left(\frac{\partial F}{\partial W_{rr}}\right) = 0 \tag{131.2}$$

where

$$F = \frac{\rho h}{2}(W_{,t})^2 r - \frac{1}{2}D(r)\left[(\nabla^2 W)^2 - 2\frac{(1-\nu)}{r}W_{,r}W_{,rr}\right]$$

$$-\frac{1}{2}D(r)\left[\frac{12}{h^2}\bar{e}_1^{\,2} + \frac{12}{h^2}\lambda\left\{\frac{1}{2}(W_{,r})^2 + W_{,r}z_{,r}\right\}^2\right] \tag{131.3}$$

We shall now apply Euler's variational equations (131.1) and (131.2) on (131.3)

$$\frac{\partial F}{\partial u} = -12D\frac{\bar{e}_1\nu}{h^2} \tag{134.1}$$

$$\frac{\partial F}{\partial u_r} = -12D\frac{\bar{e}_1 r}{h^2} \tag{134.2}$$

From equation (131.1) using (134.1) and (134.2) one gets

$$-12D\frac{\bar{e}_1\nu}{h^2} + \frac{\partial}{\partial r}\left(12D\frac{\bar{e}_1 r}{h^2}\right) = 0 \tag{135}$$

putting $k = 12D\frac{\bar{e}_1}{h^2}$,in equation (135) one gets,

$$k\nu = \frac{\partial}{\partial r}(kr) \tag{136}$$

From (136) one gets

$$\bar{e}_1 = Cf(t)\frac{r^{\nu-1}h^2}{12D} \tag{137}$$

$$\frac{\partial F}{\partial W} = 0 \tag{138}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial F}{\partial W_r}\right) = -\frac{\partial D}{\partial r}\left(\frac{1}{r}W_{,r} + \nu W_{,rr}\right) - D\left(-\frac{1}{r^2}W_{,r} + \frac{1}{r}W_{,rr} + \nu W_{,rrr}\right)$$

$$-Cf(t)\left[\nu r^{\nu-1}W_{,r} + r^{\nu}W_{,rr} - \frac{1}{R_0}(\nu+1)r^{\nu}\right]$$

$$-6\frac{\lambda}{h^2}\frac{\partial D}{\partial r}\left[r(W_{,r})^3 + 2\frac{r^3}{R_0^{\,2}}W_{,r} - 3\frac{r^2}{R_0}(W_{,r})^2\right]$$

$$-6D\frac{\lambda}{h^2}\left[(W_{,r})^3 + 3r(W_{,r})^2 W_{,rr} + \frac{2}{R_0^{\,2}}\left(3r^2 W_{,r} + r^3 W_{,rr}\right) - \frac{3}{R_0}\left\{2r(W_{,r})^2 + 2r^2 W_{,r}W_{,rr}\right\}\right] \tag{139}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial F}{\partial W_{,t}}\right)=\rho hrW_{,tt} \tag{140}$$

$$\frac{\partial^2}{\partial r^2}\left(\frac{\partial F}{\partial W_{,rr}}\right)=-\frac{\partial^2 D}{\partial r^2}\left(rW_{,rr}+\nu W_{,r}\right)-\frac{\partial D}{\partial r}\{2(1+\nu)W_{,rr}+2rW_{,rrr}\}-D\{(2+\nu)W_{,rrr}+rW_{,rrrr}\} \tag{141}$$

Using (138-141) in equation (131.2) one gets

$$D(r)\left[\nabla^4 W-\frac{12\lambda}{h^2}\left\{W_{,r}^{\,2}W_{,rr}-\frac{3r}{R_0}W_{,r}W_{,rr}-\frac{3}{R_0}W_{,r}^{\,2}+\frac{r^2}{R_0^{\,2}}W_{,rr}+\frac{3r}{R_0^{\,2}}W_{,r}+\frac{1}{2r}W_{,r}^{\,3}+\frac{1}{2}W_{,r}^{\,2}W_{,rr}\right\}\right]$$

$$+\frac{dD(r)}{dr}\left[2\left(W_{,rrr}+\frac{W_{,rr}}{r}\right)-\frac{1}{r^2}W_{,r}+\frac{\nu}{r}W_{,rr}-\frac{12\lambda}{h^2}\left\{\frac{W_{,r}^{\,3}}{2}-\frac{3r}{2R_0}W_{,r}^{\,2}+\frac{r^2}{R_o^{\,2}}W_{,r}\right\}\right]$$

$$\frac{d^2D(r)}{dr^2}\left[W_{,rr}+\frac{\nu}{r}W_{,r}\right]+\rho hW_{,tt}$$

$$-Cf(t)\left[r^{\nu-1}W_{,rr}+\nu r^{\nu-2}W_{,r}-\frac{r^{\nu-1}}{R_0}(1+\nu)\right]=0 \tag{142}$$

where C is a constant and $f(t)$ is a function of time.

4.2.3 METHOD OF SOLUTION

For a spherical shell with clamped immovable edges one may assume

$$W=AW_0(t)\left(1-\frac{r^2}{R^2}\right)^2 \tag{143}$$

where A stands for the maximum deflection in the positive direction. Inserting equation (143) into equation (137) and integrating over the surface area of the shell one gets the constant in the form

$$\frac{Cf(t)h^2}{12D_0\beta}\left[1+\frac{(1-\beta)}{\beta}\log(1-\beta)\right]=\frac{64A^2W_0^{\,2}(t)R^{-1-\nu}}{(5-\nu)(7-\nu)(9-\nu)}+\frac{8AW_0(t)R^{1-\nu}}{R_0(5-\nu)(7-\nu)} \tag{144}$$

To solve (142) one may use Galerkin's error minimizing technique. Substituting equation (143) into equation (142) and using equation (144) one gets ,after lengthy but simple calculations ,the following time differential equation of the form

$$W_0(t)_{,tt}+P_1^{'}W_0(t)+Q_1^{'}W_0^{\,2}(t)+R_1^{'}W_0^{\,3}(t)=0 \tag{145}$$

where

$$P_1' = \frac{10D_0}{\rho h R^4}\left[10.66666667 + 3.2\frac{\lambda R^4}{h^2 R_0^{\ 2}} - \beta\left(2.742857143 + 1.219047619\nu + 0.983549783\frac{\lambda R^4}{h^2 R_0^{\ 2}}\right)\right.$$
$$\left. + \frac{768\beta^2 R^4}{R_0^{\ 2} h^2\{\beta + (1-\beta)\log(1-\beta)\}}\frac{1}{(\nu+3)(\nu+5)(5-\nu)(7-\nu)}\right]$$

(145.1)

$$Q_1' = \frac{10D_0}{\rho h R^4}\left(\frac{A}{h}\right)\left[9.6\frac{\lambda R^2}{hR_0} - 3.462137862\beta\frac{\lambda R^2}{hR_0} + \beta^2\left\{\frac{R^2}{hR_0\{\beta + (1-\beta)\log(1-\beta)\}}\right.\right.$$
$$\left.\left.\times\left(\frac{6144}{(\nu+3)(\nu+5)(5-\nu)(7-\nu)(9-\nu)} + \frac{12288}{(\nu+3)(\nu+5)(\nu+7)(5-\nu)(7-\nu)}\right)\right\}\right]$$

(145.2)

$$R_1' = \frac{10D_0}{\rho h R^4}\left(\frac{A}{h}\right)^2\left[7.314285714\lambda - 2.94958375\beta\lambda\right.$$
$$\left. + \frac{\beta^2}{\{\beta + (1-\beta)\log(1-\beta)\}}\frac{98304}{(\nu+3)(\nu+5)(5-\nu)(\nu+7)(7-\nu)(9-\nu)}\right]$$

(145.3)

4.2.4 EVALUTION OF λ

The factor λ which occurs in the foregoing equations can be determined from minimum potential energy and given by [147]

$\lambda = 2\nu^2$ (for clamped edges) (146)

$\lambda = \nu^2$ (for simply supported edges) (147)

4.2.5 SOLUTION OF THE TIME DIFFERENTIAL EQUATION

The solution of the time differential equation (145) with initial condition $W_0 = 1$ and $\frac{dW_0}{dt} = 0$ at $t = 0$ has been given by A.P. Bhattacharjee[180] and obtained the ratio of the non-linear and linear vibrational frequencies as

$$\frac{\omega^*}{\omega} = \left[1 + \left(\frac{A}{h}\right)^2\left\{\frac{3}{4}\frac{C_3}{C_1} - \frac{5}{6}\left(\frac{C_2}{C_1}\right)^2\right\}\right]^{\frac{1}{2}} \tag{148}$$

where ω^* and ω are the non-linear and linear frequencies.

4.2.6 NUMERICAL RESULTS AND DISCUSSIONS

Variations of the non-dimensional frequency ratios for the nonlinear and linear vibrations have been presented for different variations of non-dimensional amplitude $\left(\frac{A}{h}\right)$ and temperature co-efficient β.Results for both geometries for different values of ξ where $\xi=\left(\frac{R^2}{2R_0h}\right)$ have been presented in graphical forms. Clamped movable edge conditions have been considered in the analysis and the results for ξ=1 are not in agreement with those of ref.[147] when temperature co-efficient β=0 due to incorrect form of equation (7) of ref.[147].Correct form should be the equation (10) of the present paper.

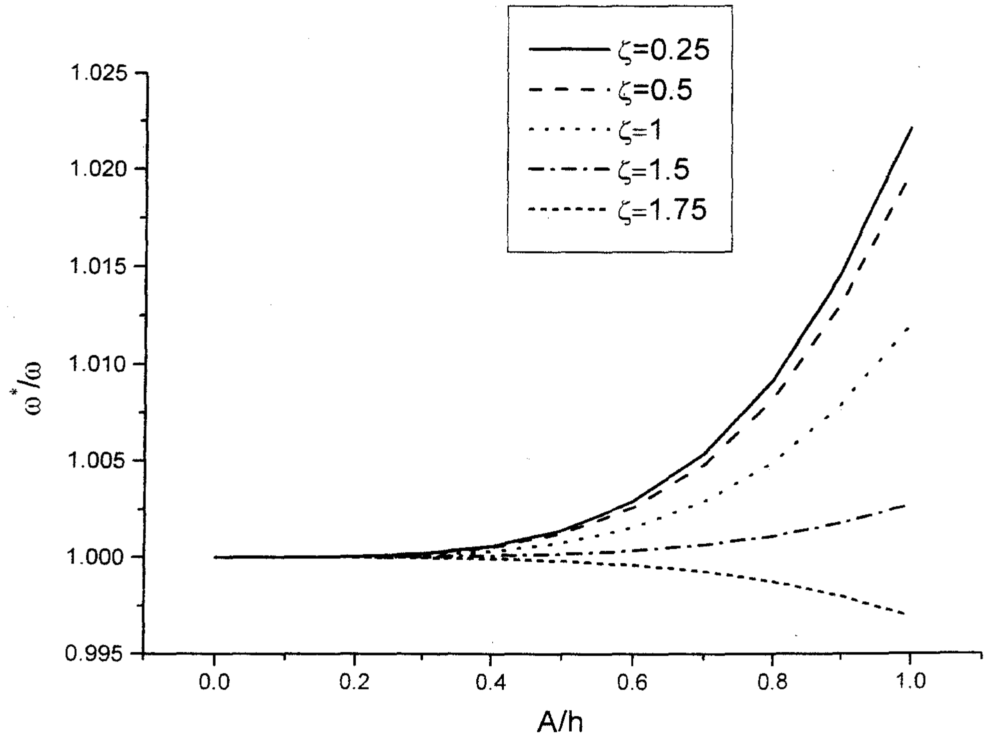


Fig.4.3(a) Shows comparative variations of $\left(\dfrac{\omega^{*}}{\omega}\right)$ vs. $\left(\dfrac{A}{h}\right)$ for temperature co-efficient β=0 and non-dimensional curvature ξ=0.25,0.5,1,1.5,1.75.

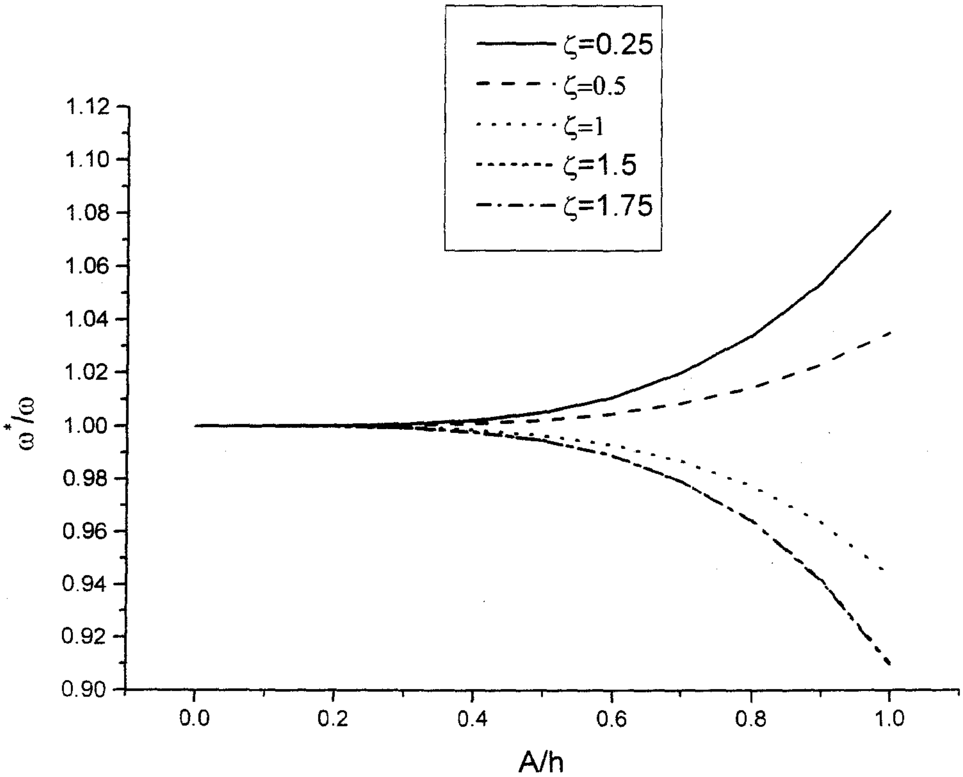


Fig.4.3(b) Shows comparative variations of $\left(\frac{\omega^*}{\varpi}\right)$ vs. $\left(\frac{A}{h}\right)$ for temperature co-efficient β=0.5 and non-dimensional curvature ξ=0.25,0.5,1,1.5,1.75.

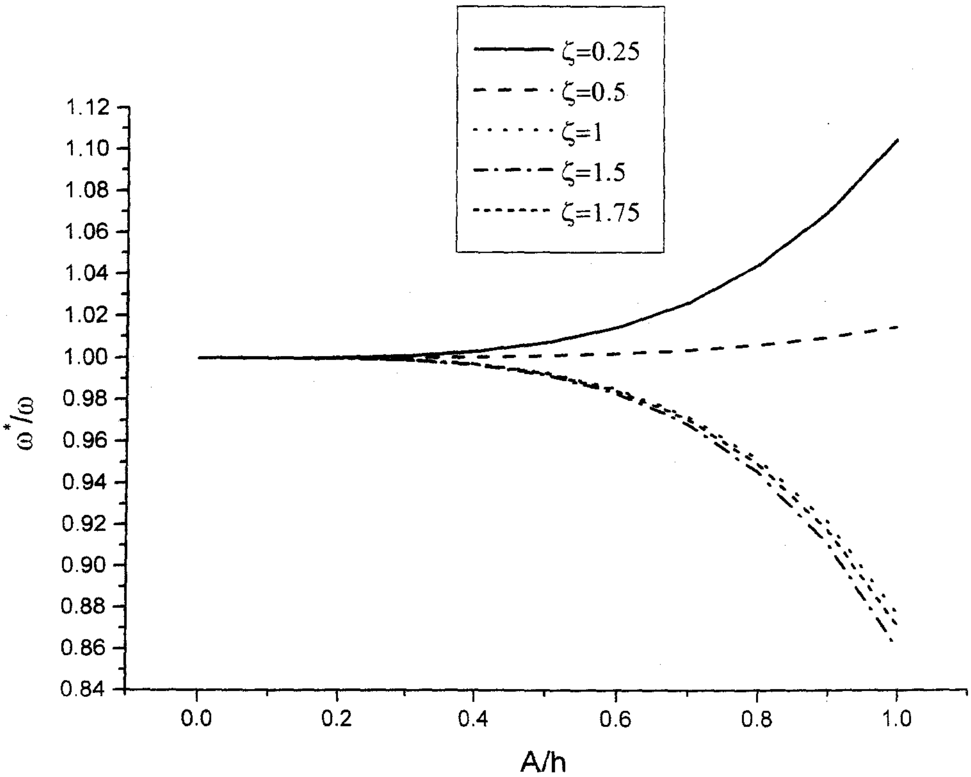


Fig.4.3(c) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for temperature co-efficient β=0.9 and non-dimensional curvature ξ=0.25,0.5,1,1.5,1.75.

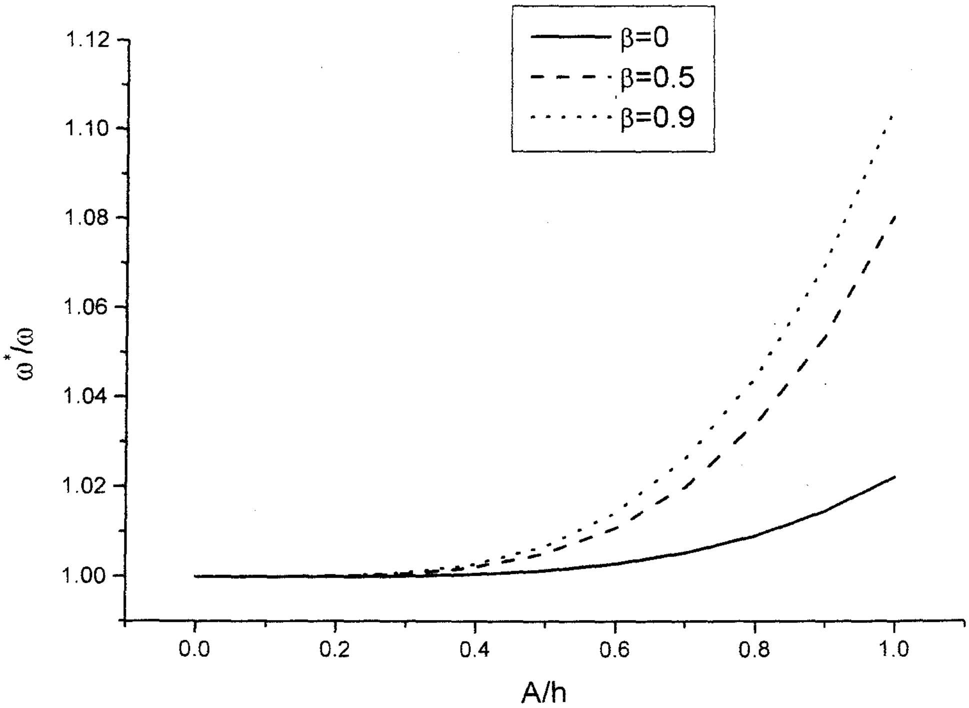


Fig.4.3(d) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for non-dimensional curvature ξ=0.25 and for temperature co-efficient β=0,0.5,0.9.

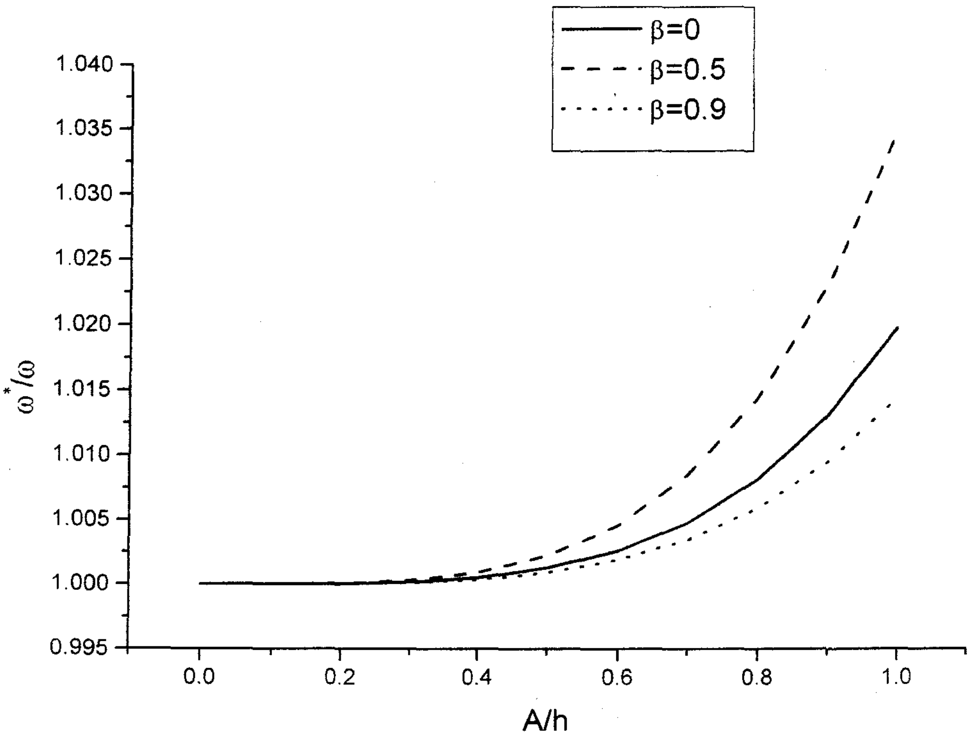


Fig.4.3(e) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for non-dimensional curvature ξ=0.5 and for temperature co-efficient β=0,0.5,0.9.

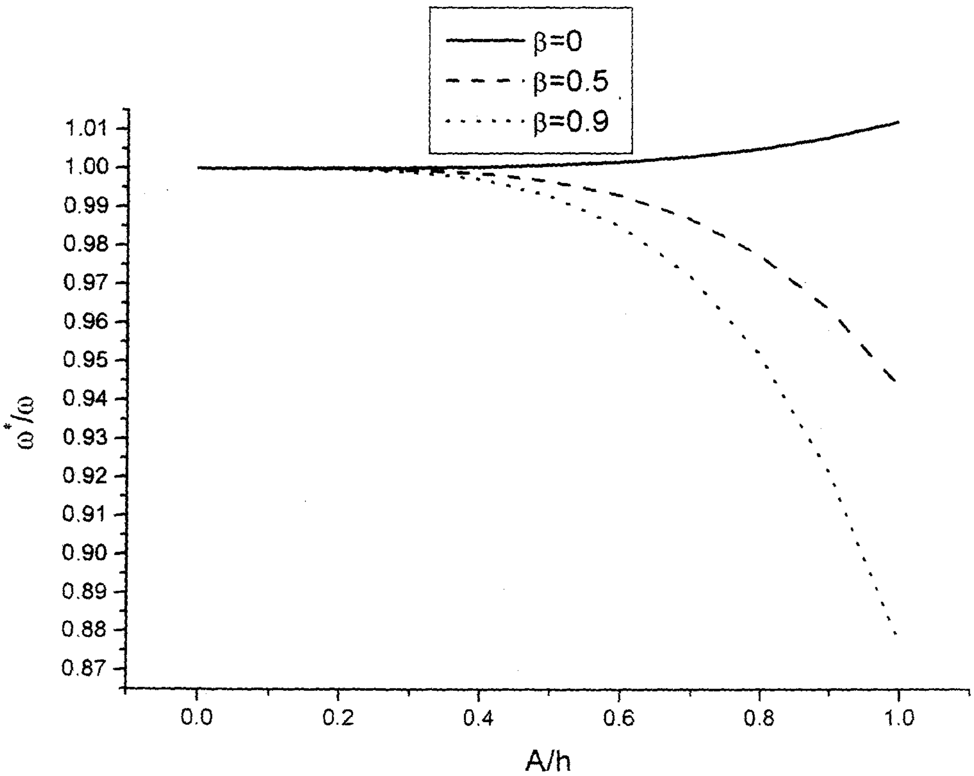


Fig.4.3(f) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for non-dimensional curvature ξ=1 and for temperature co-efficient β=0,0.5,0.9.

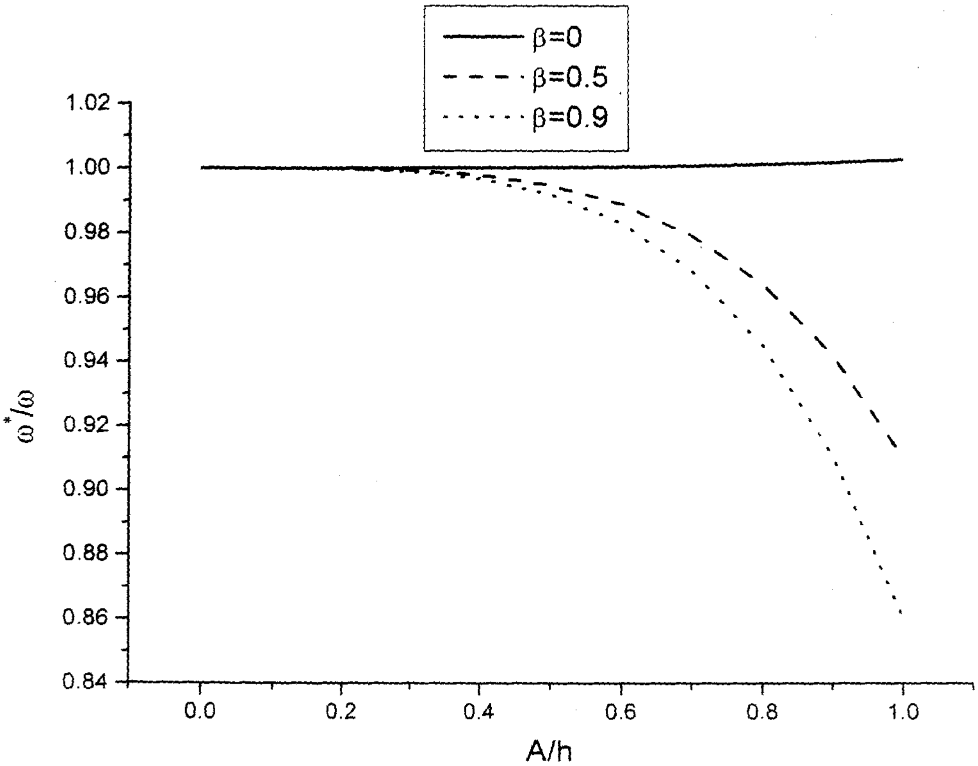


Fig.4.3(g) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for non-dimensional curvature ξ=1.5 and for temperature co-efficient β=0,0.5,0.9.

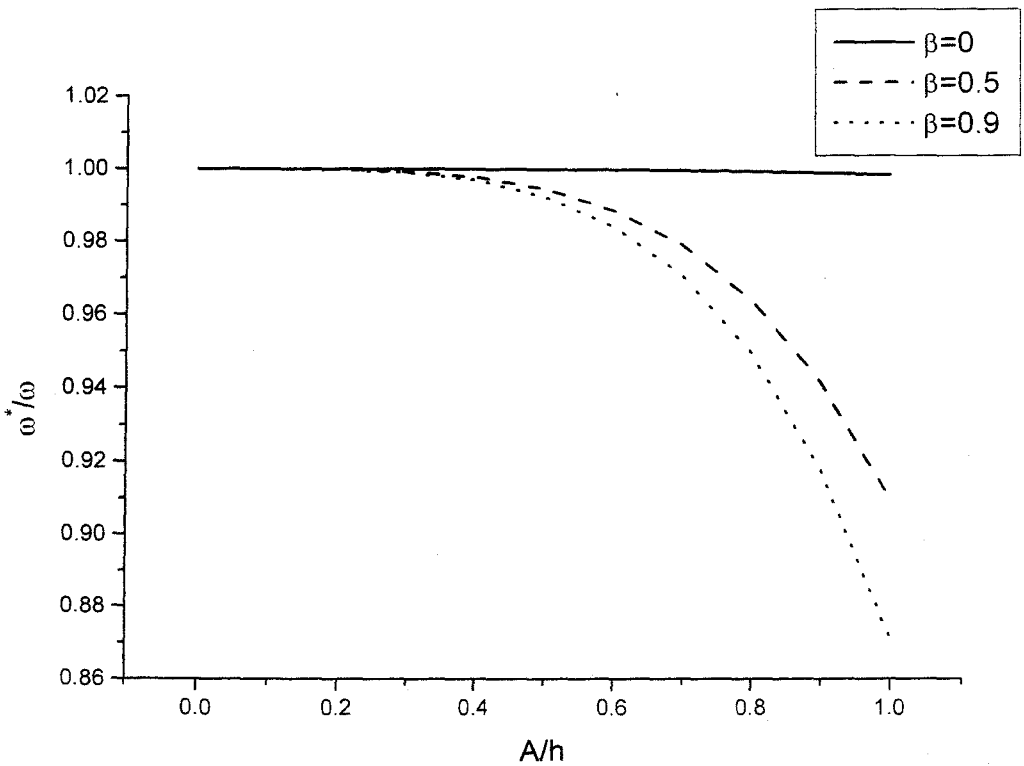


Fig.4.3(h) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for non-dimensional curvature ξ=1.75 and for temperature co-efficient β=0,0.5,0.9.

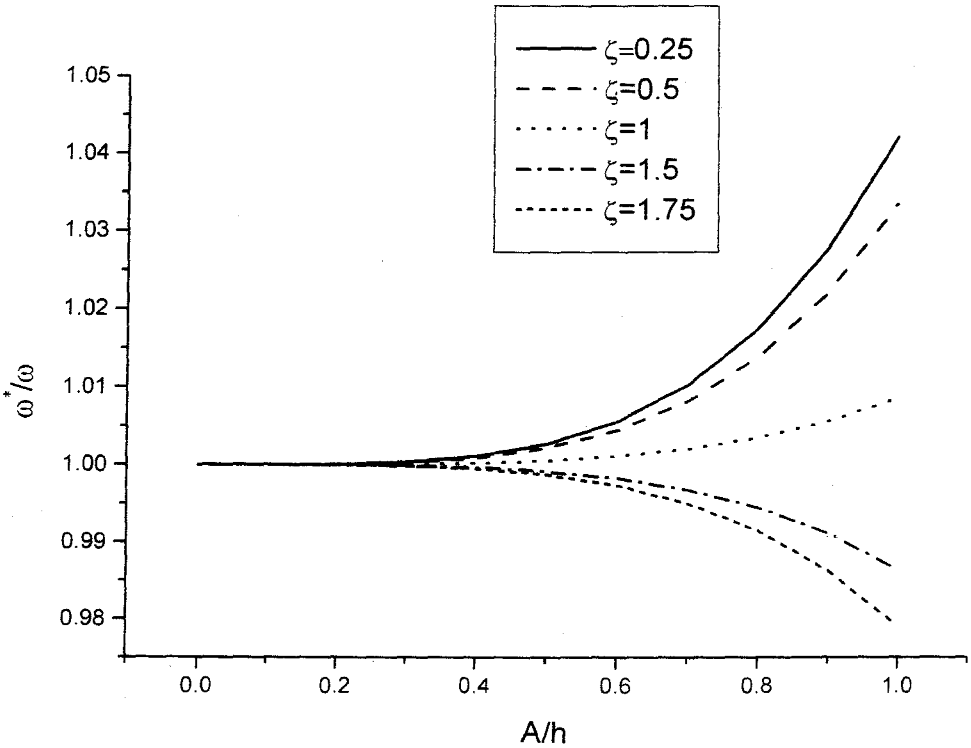


Fig.4.4(a) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for temperature co-efficient $\beta=0$ and for non-dimensional curvature $\xi=0.25,0.5,1,1.5,1.75$.

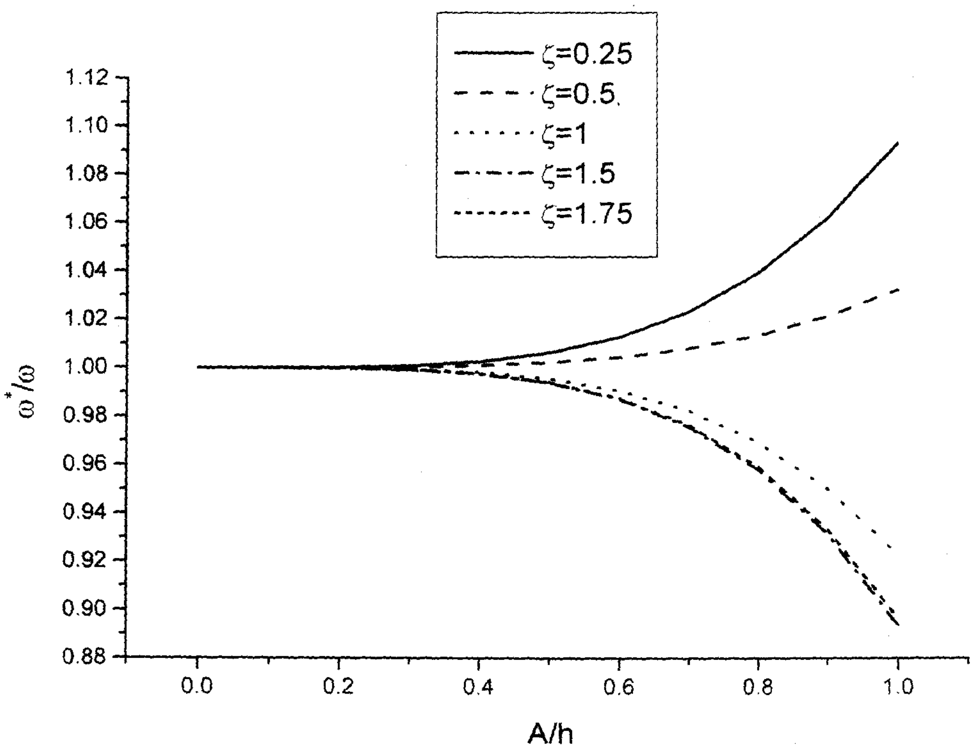


Fig.4.4(b) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for temperature co-efficient β=0.5 and for non-dimensional curvature ξ=0.25,0.5,1,1.5,1.75.

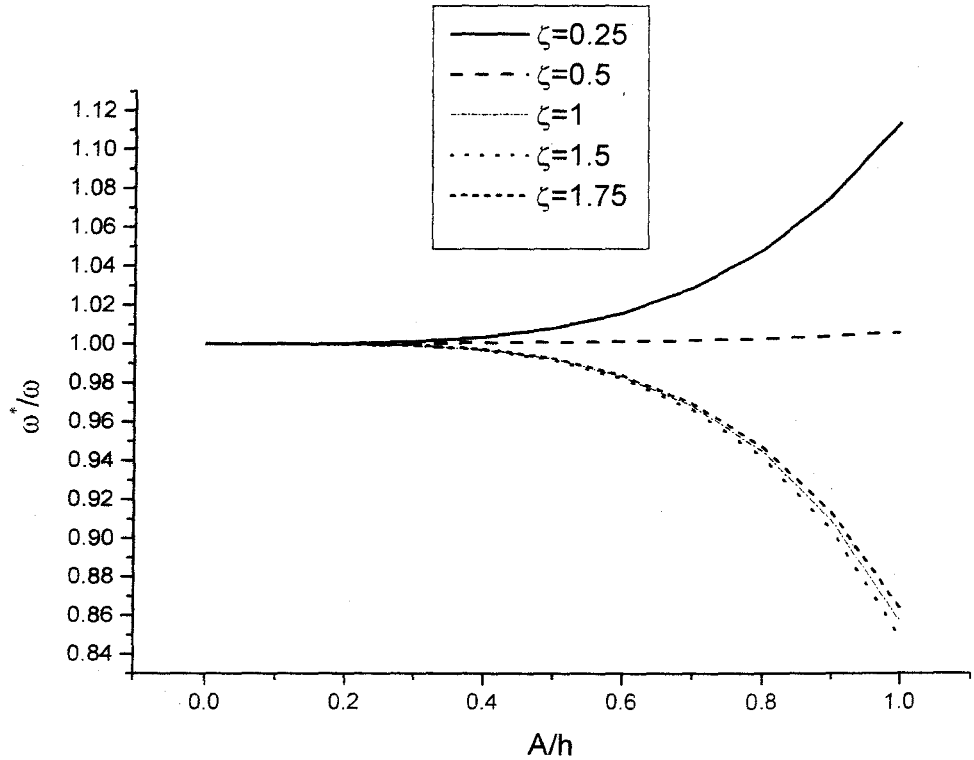


Fig.4.4(c) Shows comparative variations of $\left(\dfrac{\omega^*}{\omega}\right)$ vs. $\left(\dfrac{A}{h}\right)$ for temperature co-efficient β=0.9 and for non-dimensional curvature ξ=0.25,0.5,1,1.5,1.75.

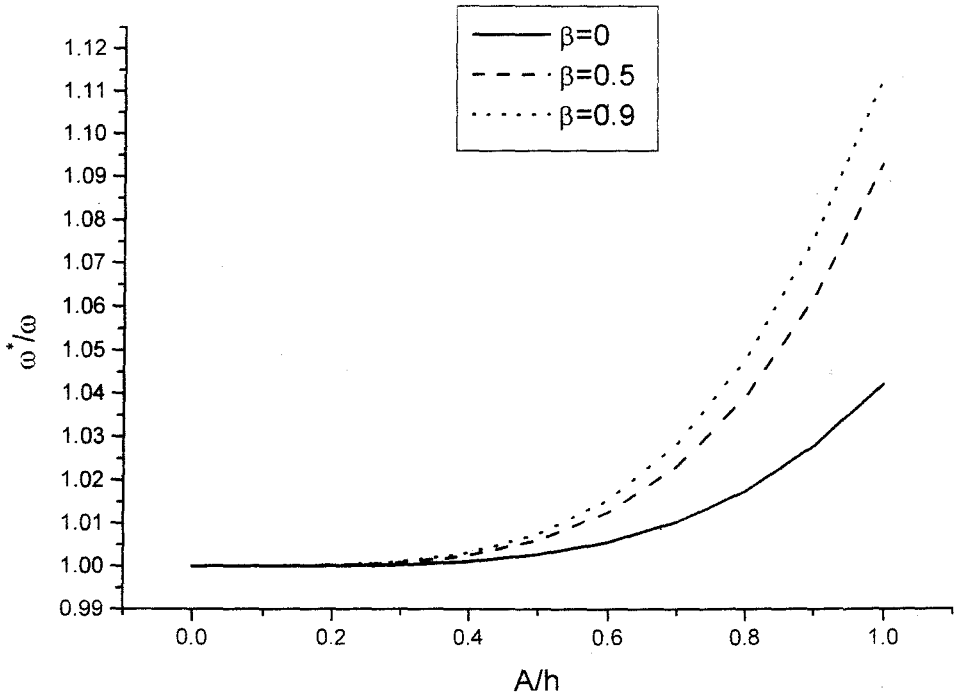


Fig.4.4(d) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for non-dimensional curvature ξ=0.25 and for temperature co-efficient β=0,0.5,0.9.

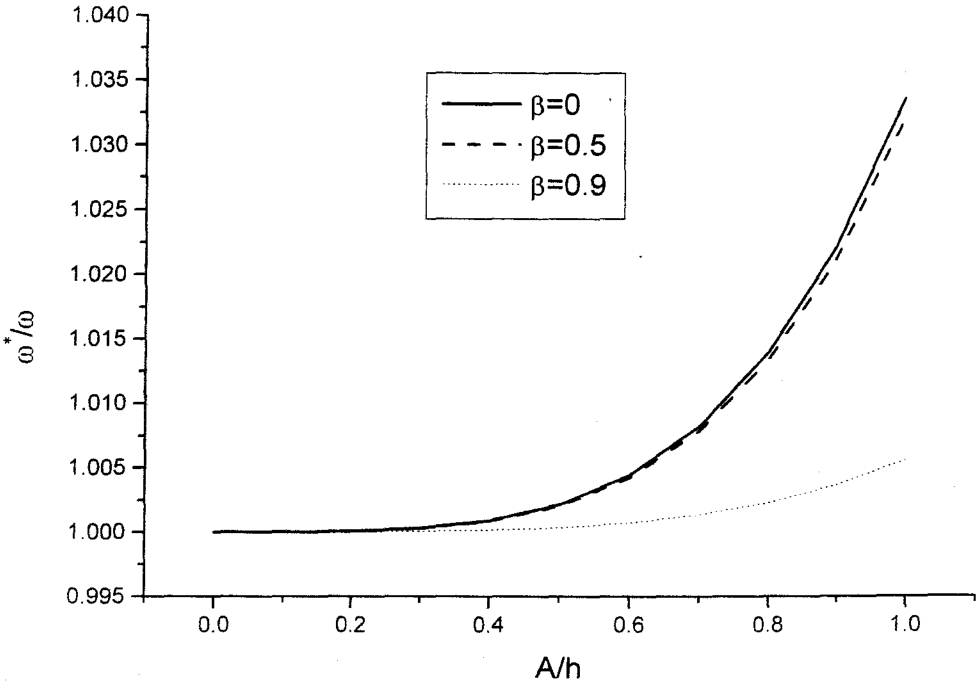


Fig.4.4(e) Shows comparative variations of $\left(\dfrac{\omega^*}{\omega}\right)$ vs. $\left(\dfrac{A}{h}\right)$ for non-dimensional curvature ξ=0.5 and for temperature co-efficient β=0,0.5,0.9.

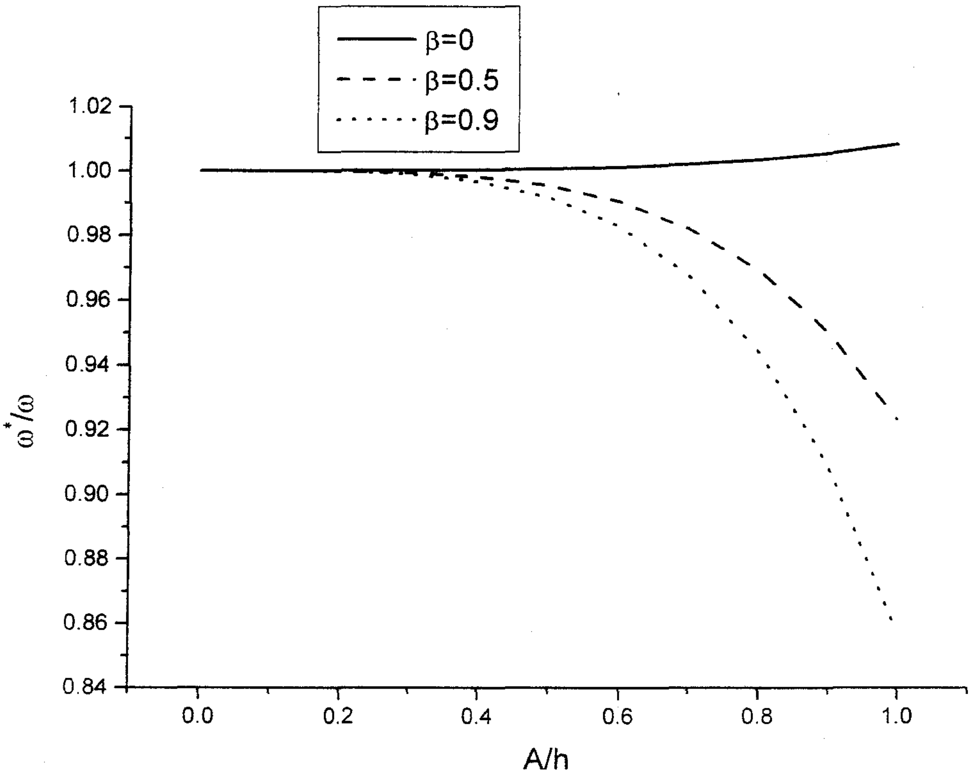


Fig.4.4(f) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for non-dimensional curvature $\xi=1$ and for temperature co-efficient $\beta=0,0.5,0.9$.

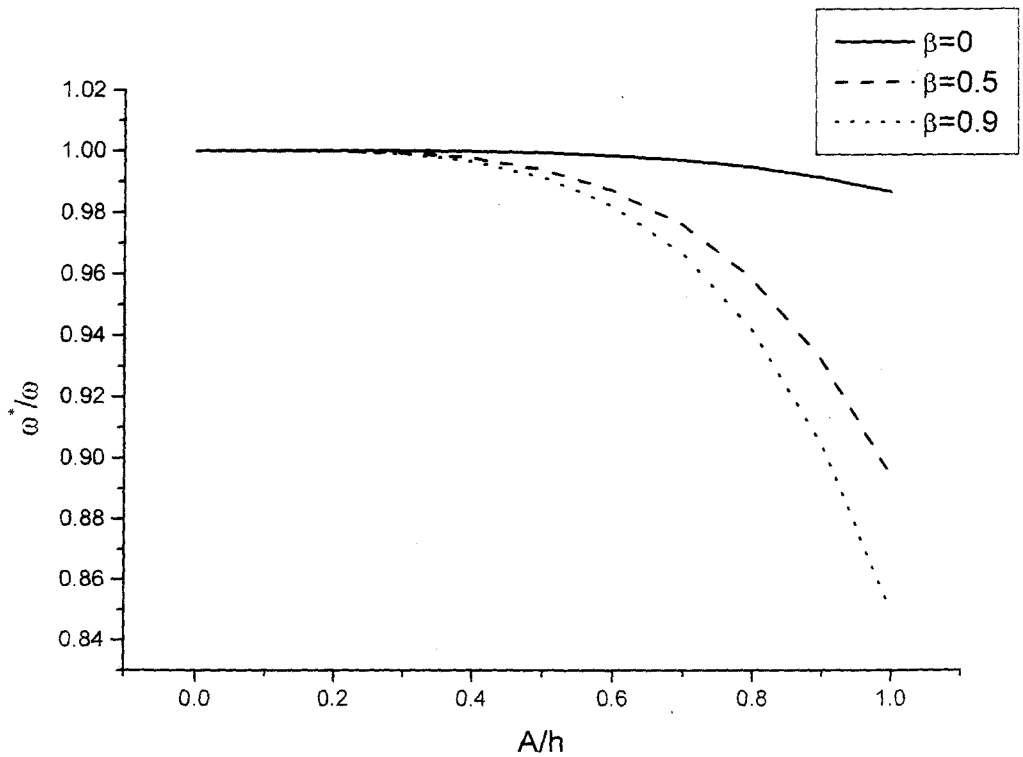


Fig.4.4(g) Shows comparative variations of $\left(\dfrac{\omega^*}{\omega}\right)$ vs. $\left(\dfrac{A}{h}\right)$ for non-dimensional curvature ξ=1.5 and for temperature co-efficient β=0,0.5,0.9.

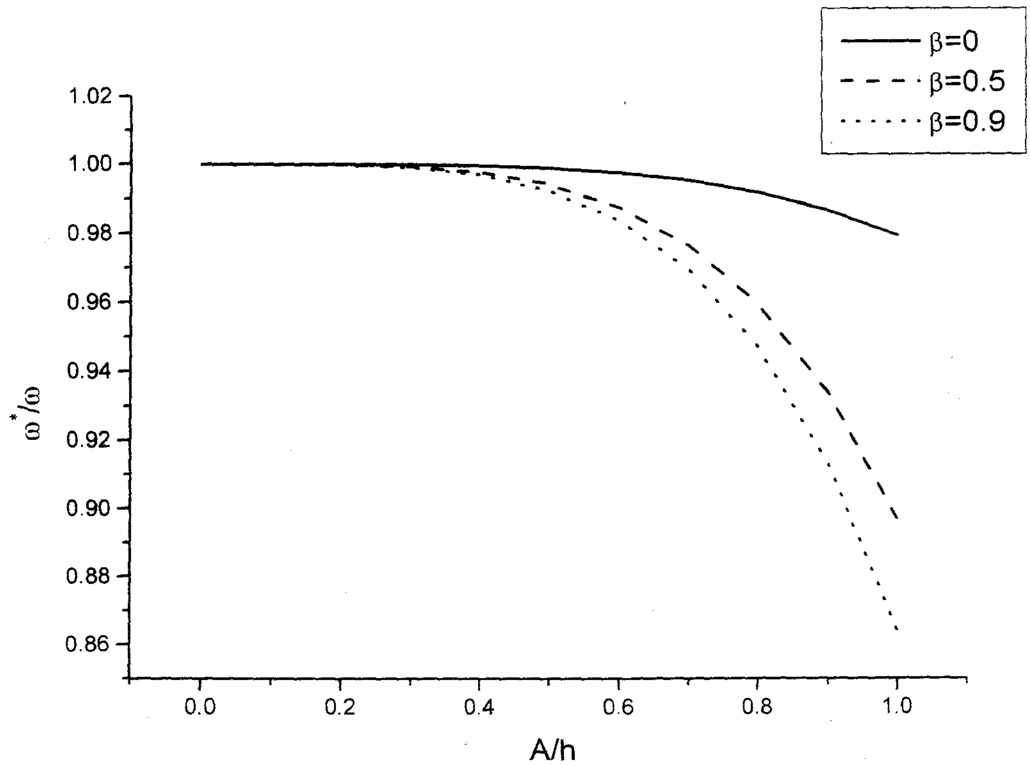


Fig.4.4(h) Shows comparative variations of $\left(\frac{\omega^*}{\omega}\right)$ vs. $\left(\frac{A}{h}\right)$ for non-dimensional curvature ξ=1.75 and for temperature co-efficient β=0,0.5,0.9.

4.2.7 OBSERVATIONS AND DISCUSSIONS:

Case-I : Figures from 4.3(a) to 4.3(h) relate results for Simply-Supported Cases.

i)From Fig.4.3(a) one can observe when non-dimensional curvature ξ increases ,the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ also increase at a decreasing rate upto a certain value of non-dimensional curvature ξ after which the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ begin to decrease with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.So there exists a transition phase between increase and decrease in the values of the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ at certain value of non-dimensional curvature ξ between 1.5 and 1.75 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

ii) From Fig.4.3(b) one can observe that when non-dimensional curvature ξ increases ,the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ also increase at a decreasing rate upto a certain value of non-dimensional curvature ξ after which the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ begin to decrease with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.So there exists a transition phase between increase and decrease in the values of the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ at certain value of non-dimensional curvature ξ between 0.5 and 1.75 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

iii) From Fig.4.3(c) one can observe the same nature of Fig.4.3(b).The only difference is that non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ increase and decreases more for β=0.9 than β=0.5 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

iv) From Fig.4.3(d) one can observe with the increase of temperature co-efficient β the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ increase with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

v) From Fig.4.3(e) one can observe with the increase of temperature co-efficient β the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ increase with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$ upto certain level of temperature co-efficient β ,after which the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ begin to decrease with increasing rate. So there exists a transition phase between increase and decrease in the values of temperature co-efficient β between 0.5 and 0.9 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

vi) From Fig.4.3(f) and 4.3(g) one can observe that with the increase of temperature co-efficient β the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ increase with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$ upto certain level of temperature co-efficient β,after which the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ begin to decrease. So there exists a transition phase between increase and decrease in the values of temperature co-efficient β between 0 and 0.9 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.The only difference is that for non-dimensional curvature ξ=1,the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ increase more than ξ=1.5 for temperature co-efficient β=0 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$and for non-dimensional curvature ξ=1.5,the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ decrease more than ξ=1 for temperature co-efficient β=0.5 and 0.9 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

vii) From Fig.4.3(h) one can observe with the increase of temperature co-efficient β the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$decrease with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$ for non-dimensional curvature ξ =1.75.

Case-II. Figures 4.4(a) to 4.4(h) relate results for Clamped-edges

viii) From Fig.4.4(a)one can observe when non-dimensional curvature ξ increases ,the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$also increase at a decreasing rate upto a certain value non-dimensional curvature ξ after which the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ begin to decrease with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.So there exists a transition phase between increase and decrease in the values of the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ at certain value of non-dimensional curvature ξ between 1 and 1.75 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

ix) From Fig.4.4b) one can observe that when non-dimensional curvature ξ increases ,the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ also increase at a decreasing rate up to a certain value non-dimensional curvature ξ after which the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ begin to decrease with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.So there exists a transition phase between increase and decrease in the values of the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ at certain value of non-dimensional curvature ξ between 0.5 and 1.75 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

x) From Fig.4.4(c) one can observe that the same nature of Fig.4.4b).The only difference is that non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ increase and decreases more for β=0.9 than for β=0.5 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

xii) From Fig.4.4(d) one can observe that with the increase of temperature co-efficient β the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ increase with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

xiii) From Fig.4.4(e) one can observe that with the increase of temperature co-efficient β the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ decrease with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

xiv)) From Fig.4.4(f) one can observe that with the increase of temperature co-efficient β the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ increase with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$ upto a certain level of temperature co-efficient β,after which the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ begin to decrease. So there exists a transition phase between increase and decrease in the values of temperature co-efficient β between 0 and 0.9 with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

xv) From Fig.4.4(g) and 4.4(h) one can observe that with the increase of temperature co-efficient β the non-dimensional frequency ratios $\left(\frac{\omega^*}{\omega}\right)$ decrease with the increase of non-dimensional amplitude $\left(\frac{A}{h}\right)$.

CHAPTER V

NONLINEAR VIBRATIONS OF IRREGULAR-SHAPED PLATES RESTING ON NONLINEAR ELASTIC FOUNDATION*

5.1 INTRODUCTION:

Nonlinear vibrations of elastic plates have great important to the engineering and designers, when these easily deformable structures vibrate at large amplitude, the classical bending theory becomes inadequate and it is necessary to allow moderately large deflections.

In this paper the free vibrations of irregular-shaped plates under clamped-edge boundary condition resting on non-linear elastic foundation and subject to uni-axial compressive loads normal to all the edges have been investigated by the method of complex variables [17, 42, 49,101,161- 170, 173]. The essence of the method is to transform the basic governing differential equation [171] in terms of complex co-ordinates $(z, \bar{z}, z = x + iy)$ and the domain can be conformally mapped onto a unit circle. It is to be noted that a one-term approximation of the mapping function yields fairly accurate results with less computational effort.

Presented at the 9th.International Conference on Vibration Problems(ICOVP-2009) held at Indian Institute of Technology(IIT),Kharagpur, West Bengal(India), January 19- 22, 2009.. The Paper is published in the E-Proceedings of the Conference, Session-X-B.

5.2 BASIC GOVERING EQUATION

One can describe the large amplitude vibrations of a plate resting on non linear elastic foundation by the non-linear differential equation [171]

$$D\nabla^4 W + \rho h\frac{\partial^2 W}{\partial t^2} + k_1 W + k_3 W^3 = Z \tag{149}$$

where,

k_1, k_3 = foundation parameters and

Z =resultant transverse load given by

$$Z = N_{xx}W_{,xx} - [2N_{xy}W_{,xy}] + N_{yy}W_{,yy}$$

$$N_{xx} = k\left(\frac{1}{2}W_{,x}^2 + \frac{1}{2}\nu W_{,y}^2\right) - P_x$$

$$N_{yy} = k\left(\frac{1}{2}W_{,y}^2 + \frac{1}{2}\nu W_{,x}^2\right) - P_y$$

$$N_{xy} = k'(W_{,x}W_{,y})$$

$$k = \frac{Eh}{(1-\nu^2)}, k' = \frac{Eh}{(1+\nu)} \tag{150}$$

5.3 TRANSFORMATION OF EQUATION (149) INTO COMPLEX CO-ORDINATES:

Let $z = x + iy$ $(i = \sqrt{-1})$, then $\bar{z} = x - iy$, so that

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}, \frac{\partial}{\partial y} = i(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}) \tag{151}$$

Considering $p_x = p_y = P$, equation (149) transform into

$$16D\frac{\partial^4 W}{\partial^2 z\partial \bar{z}^2}+\rho h\frac{\partial^2 W}{\partial t^2}+k_1W+k_3W^3$$

$$=\frac{Eh}{(1+\nu)}\left[\left\{\left(\frac{\partial W}{\partial z}\right)^2+\left(\frac{\partial W}{\partial \bar{z}}\right)^2\right\}\left(\frac{\partial^2 W}{\partial z^2}+\frac{\partial^2 W}{\partial \bar{z}^2}\right)+4\frac{\partial W}{\partial z}\frac{\partial W}{\partial \bar{z}}\frac{\partial^2 W}{\partial z\partial \bar{z}}\left(\frac{1+\nu}{1-\nu}\right)\right]$$

$$+\frac{2Eh}{(1+\nu)}\left[\left(\frac{\partial W}{\partial z}\right)^2-\left(\frac{\partial W}{\partial \bar{z}}\right)^2\right]\left(\frac{\partial^2 W}{\partial z^2}-\frac{\partial^2 W}{\partial \bar{z}^2}\right)-P\nabla^4 W \qquad (152)$$

Let $z=f(\xi)$ be the analytic function, called the mapping function, which maps the shape in the z-plane onto a unit circle into ξ-plane, where

$\xi=re^{i\theta},\bar{\xi}=re^{-i\theta},r$ – being the radius of the circle.

Now, equation (152) takes the form:

$$16D\times\left[\frac{\partial^4 W}{\partial\xi^2\partial\bar{\xi}^2}\left(\frac{dz}{d\xi}\right)^4\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^4-\frac{\partial^3 W}{\partial\xi^2\partial\bar{\xi}}\left(\frac{dz}{d\xi}\right)^4\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^3\frac{d^2\bar{z}}{d\bar{\xi}^2}-\frac{\partial^3 W}{\partial\xi\partial\bar{\xi}^2}\left(\frac{dz}{d\xi}\right)^3\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^4\frac{d^2 z}{d\xi^2}\right.$$

$$\left.+\frac{\partial^2 W}{\partial\xi\partial\bar{\xi}}\left(\frac{dz}{d\xi}\right)^3\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^3\frac{d^2 z}{d\xi^2}\frac{d^2\bar{z}}{d\bar{\xi}^2}\right]+\left(\rho h\frac{\partial^2 W}{\partial t^2}+k_1W+k_3W^3\right)\left(\frac{dz}{d\xi}\right)^6\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^6.$$

$$=\frac{Eh}{(1+\nu)}\left[\left\{\frac{\partial^2 W}{\partial\xi^2}\left(\frac{dz}{d\xi}\right)^2-\left(\frac{\partial W}{\partial\xi}\right)\left(\frac{dz}{d\xi}\right)\left(\frac{d^2 z}{d\xi^2}\right)\right\}\left\{3\left(\frac{\partial W}{\partial\xi}\right)^2\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^6-\left(\frac{\partial W}{\partial\bar{\xi}}\right)^2\left(\frac{dz}{d\xi}\right)^2\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^4\right\}\right]$$

$$+\frac{Eh}{(1+\nu)}\left[\left\{\frac{\partial^2 W}{\partial\bar{\xi}^2}\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^2-\left(\frac{\partial W}{\partial\bar{\xi}}\right)\left(\frac{d\bar{z}}{d\bar{\xi}}\right)\left(\frac{d^2\bar{z}}{d\bar{\xi}^2}\right)\right\}\left\{3\left(\frac{\partial W}{\partial\bar{\xi}}\right)^2\left(\frac{dz}{d\xi}\right)^6-\left(\frac{\partial W}{\partial\xi}\right)^2\left(\frac{dz}{d\xi}\right)^4\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^2\right\}\right]+$$

$$\frac{Eh}{(1+\nu)}\left[4\left(\frac{(1+\nu)}{(1+\nu)}\right)\frac{\partial^2 W}{\partial\xi\partial\bar{\xi}}\frac{\partial W}{\partial\xi}\frac{\partial W}{\partial\bar{\xi}}\left(\frac{dz}{d\xi}\right)^4\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^4\right]-4P\frac{\partial^2 W}{\partial\xi\partial\bar{\xi}}\left(\frac{dz}{d\xi}\right)^5\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^5. \qquad (153)$$

5.4 BOUNDARY CONDITIONS AND SOLUTION:

The boundary conditions for clamped immovable edges are

$$W=0=\frac{dW}{dr}\quad\text{at } r=1 \qquad (154)$$

In conformity of the above boundary condition the deflection function may be chosen as [49]

$$W=(1-\xi\bar{\xi})^2F(t) \qquad (155)$$

where $F(t)$ is some unknown function of time t.

Let $z = a\delta\xi$ (156)

be the first term of the mapping function where δ is the mapping function co-efficient having different values for different plate shapes given in **Table-1** and " a " is some characteristic dimension of the plate shape. Inserting (155) and (156) into equation (153),and applying Galerkine procedure one gets after lot of integrations the following Duffing's cubic equation in the form

$$F''(t) + A_1' F(t) + B_1' F^3(t) = 0 \tag{157}$$

where,

$$A_1' = \left[\frac{16h\chi_0}{3a^4\delta^4(\chi_0 - 2\chi_2 + \chi_4)} + \frac{(1-\nu^2)}{Eh^2}k_1 - \frac{8(1-\nu^2)(\chi_0 - 2\chi_2)}{Eh^2a^4\delta^4(\chi_0 - 2\chi_2 + \chi_4)}P \right] \frac{Eh}{\rho(1-\nu^2)} \tag{158}$$

$$B_1' = \left[\frac{(\chi_0 - 6\chi_2 + 15\chi_4 - 20\chi_6 + 15\chi_8 - 6\chi_{10} + \chi_{12})(1-\nu^2)}{(\chi_0 - 2\chi_2 + \chi_4)Eh^2} k_3 \right] \frac{Eh}{\rho(1-\nu^2)}$$

$$+ \frac{16}{ha^4\delta^4} \left[\left(\frac{\chi_4 - 2\chi_0 + \chi_8}{\chi_0 - 2\chi_2 + \chi_4} \right)(1-\nu) + 2\left(\frac{\chi_2 - 4\chi_4 + 5\chi_6 - 2\chi_8}{\chi_0 - 2\chi_2 + \chi_4} \right)(1+\nu) \right] \frac{Eh}{\rho(1-\nu^2)} \tag{159}$$

$$\chi_n = \int_0^1 r^n (1-r^2)^2 r dr = \frac{8}{(n+2)(n+4)(n+6)} \tag{160}$$

The solution of equation (157)) has been derived in [172] following the method of Nash and Modeer [86] and is expressed in the form

$$\left(\frac{\omega_{NL}}{\omega_L} \right) = \left(1 + \frac{B_1'}{A_1'} \right)^{\frac{1}{2}} \tag{161}$$

5.5 MAPPING FUNCTION CO-EFFICIENTS [Ref. 161]

Regular Polygon	Values of Coefficients δ
Equilateral Triangle	1.353
Square	1.080
Pentagon	1.0526
Hexagon	1.0376
Heptagon	1.0279
Octagon	1.0219
Circle	1

5.6NUMERICAL RESULTS AND DISCUSSION:

Numerical Results have been presented graphically (Fig.5.1-Fig.5.11) showing variations of non-dimensional frequencies $\left(\frac{\omega_{NL}}{\omega_L}\right)$ against the compressive load parameter P for the cases of (i) Circle, (ii) Square and (iii) Equilateral Triangular Plates considering corresponding values of the mapping function Coefficients δ and some dimension "a" of the plate shape and k_1 and k_3

$E = 21\times10^5$, $h = 0.5$, $\nu = .3$, $\rho = 1$

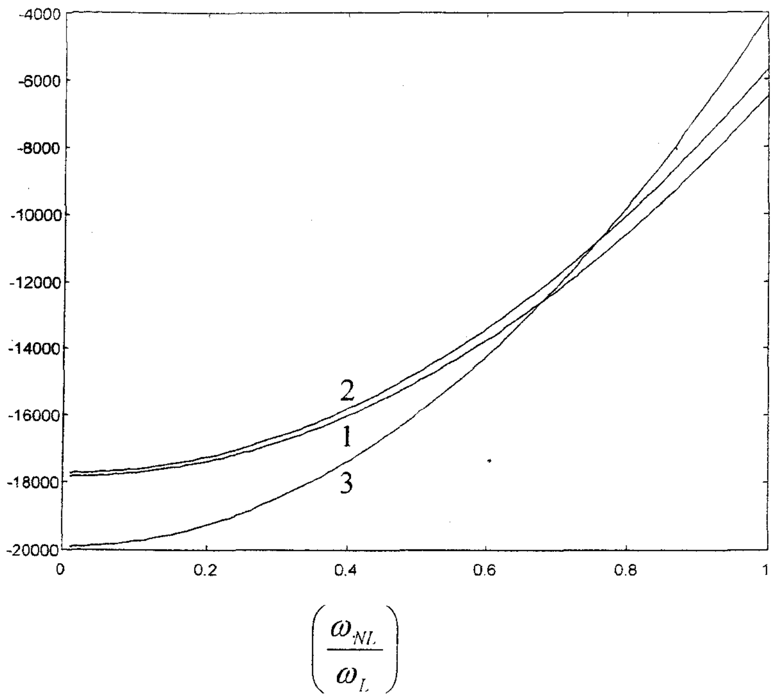


Fig.5.1 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for a=10, k_1=500, k_3=50.Here 1,2 and 3 represent circle, square and equilateral triangle respectively.

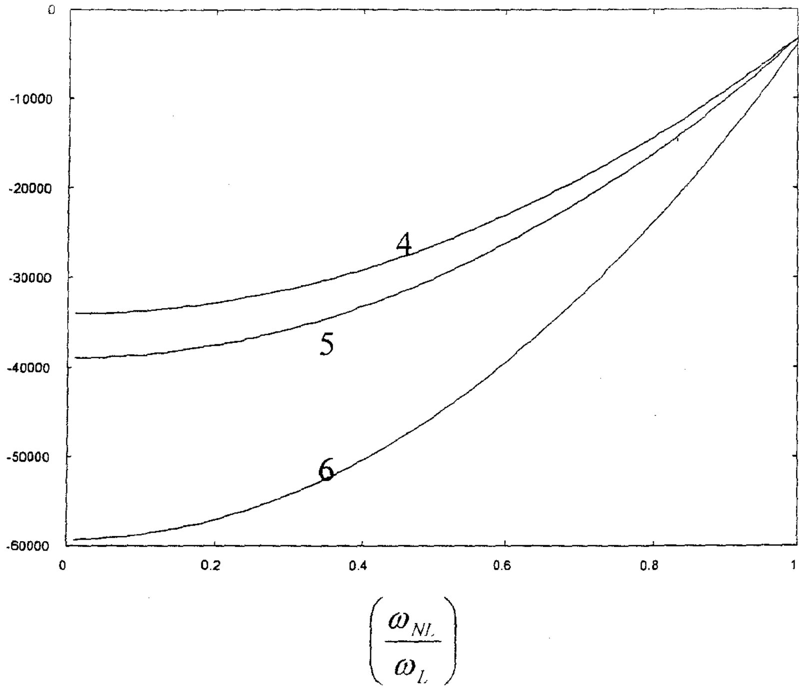


Fig.5.2 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for a=20, k_1=500, k_3=50.

Here 4,5 and 6 represent circle, square and equilateral triangle respectively.

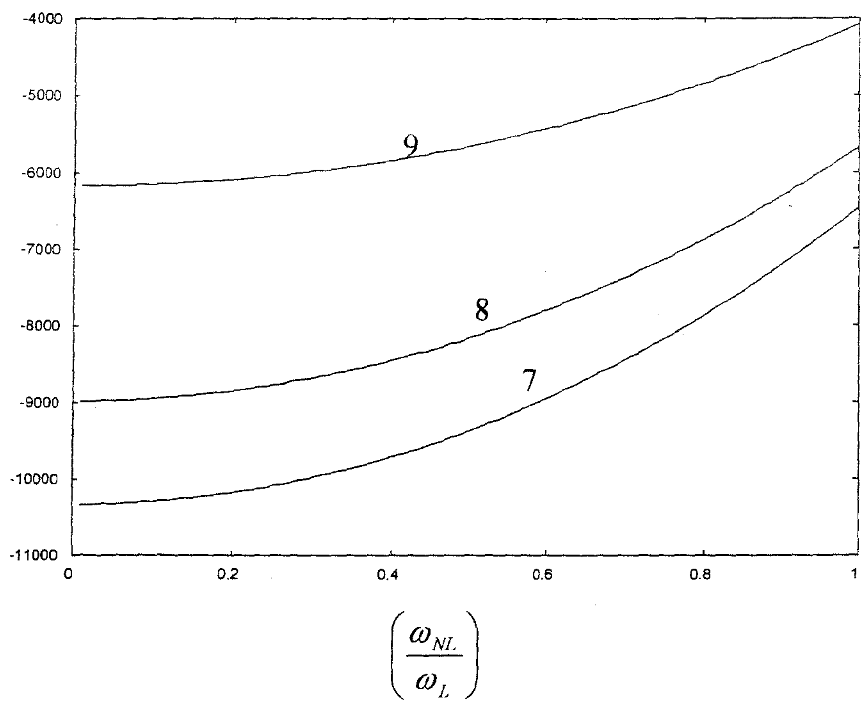
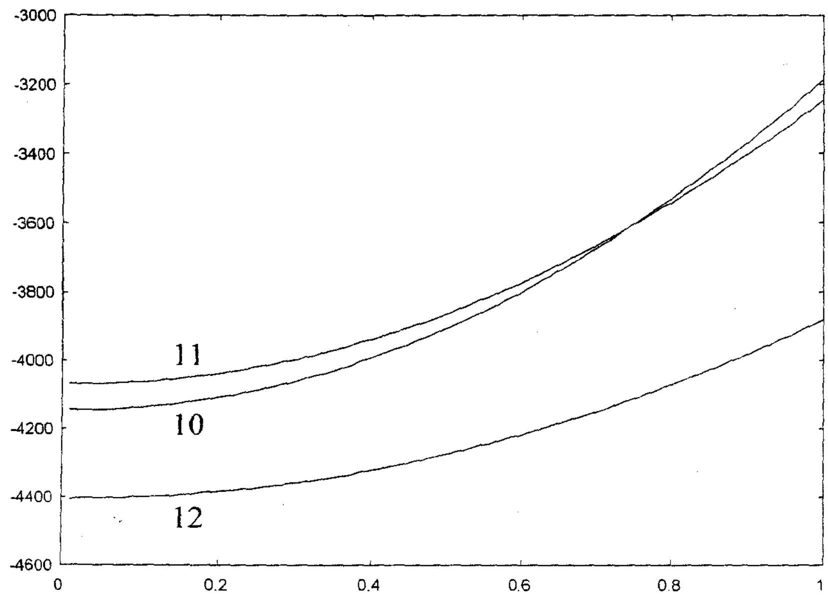


Fig.5.3 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for a=10, k_1=0, k_3=50

Here 7,8 and 9 represent circle, square and equilateral triangle respectively.



$$\left(\frac{\omega_{NL}}{\omega_L}\right)$$

Fig.5.4 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for a=20, k_1=0, k_3=50

Here 10,11 and 12 represent circle, square and equilateral triangle respectively.

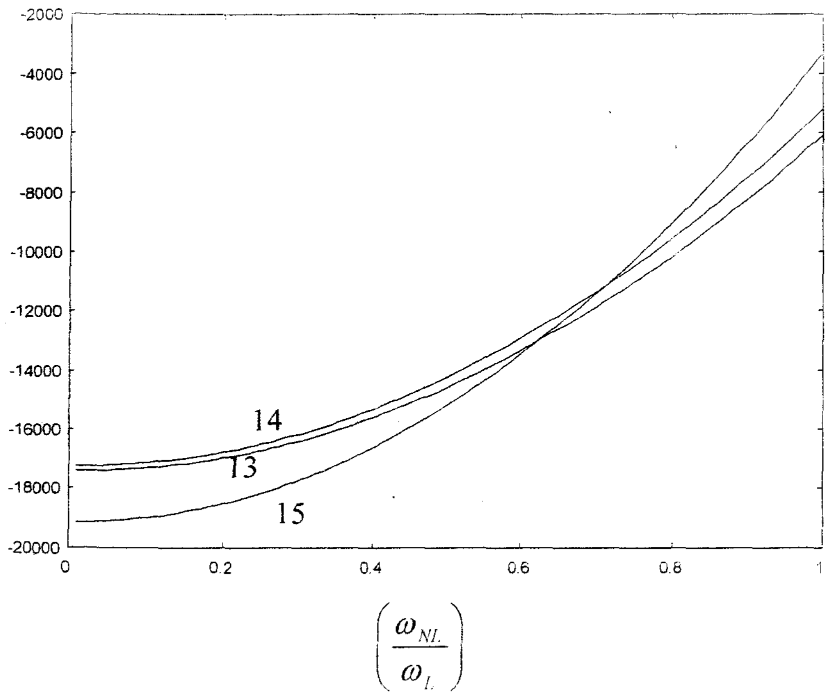


Fig.5.5 Shows comparative variations of $\left(\dfrac{\omega_{NL}}{\omega_L}\right)$ vs. P for a=10, k_1=500, k_3=0

Here 13,14 and 15 represent circle, square and equilateral triangle respectively.

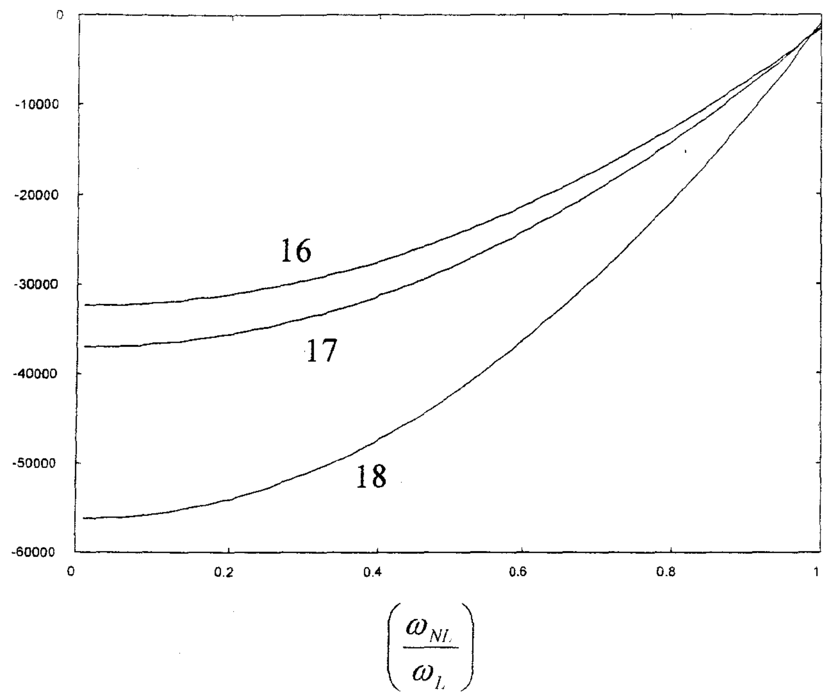


Fig.5.6 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for a=20, k_1=500, k_3=0

Here 16,17 and 18 represent circle, square and equilateral triangle respectively.

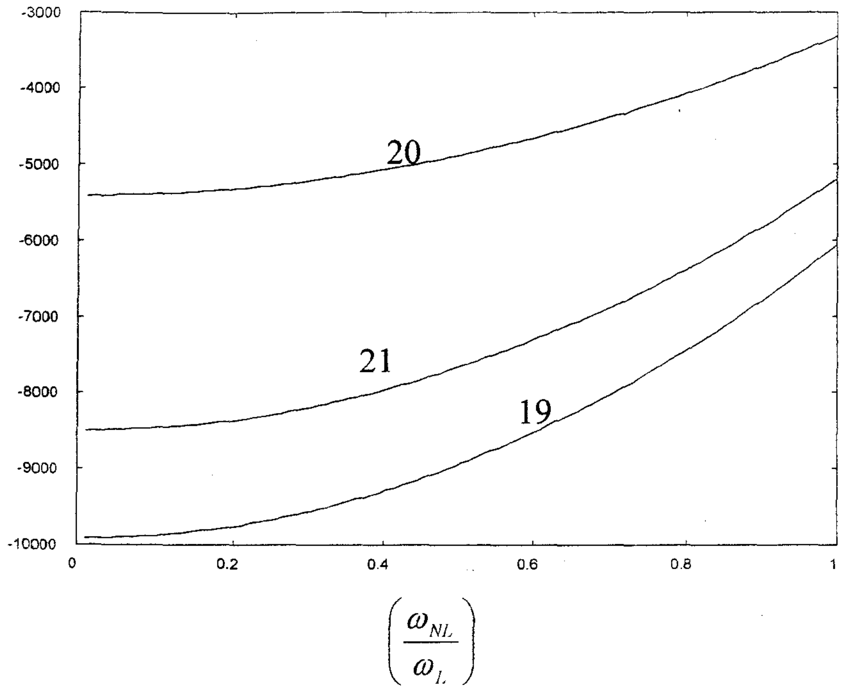


Fig.5.7 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for a=10, k_1=0, k_3=0

Here 19,20 and 21 represent circle, equilateral triangle and square respectively.

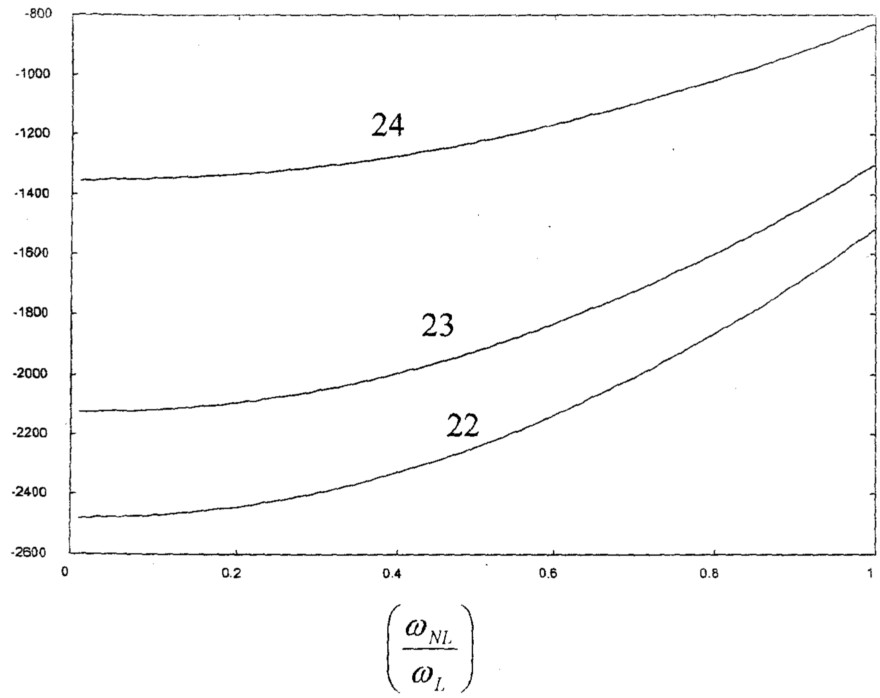
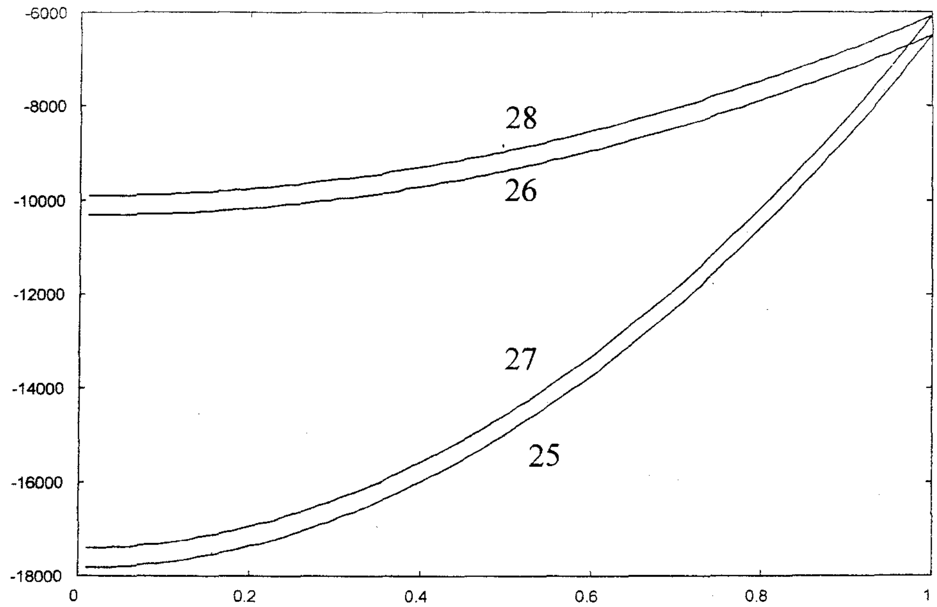


Fig.5.8 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for a=20, k_1=0, k_3=0

Here 22,23 and 24 represent circle, square and equilateral triangle respectively.



$$\left(\frac{\omega_{NL}}{\omega_L}\right)$$

Fig.5.9 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for circle (a=10)

Here 25,26,27 and 28 represent circle for $k_1 = 500$, $k_3 = 50$;

$k_1 = 0$, $k_3 = 50$; $k_1 = 500$, $k_3 = 0$; $k_1 = 0$, $k_3 = 0$ respectively.

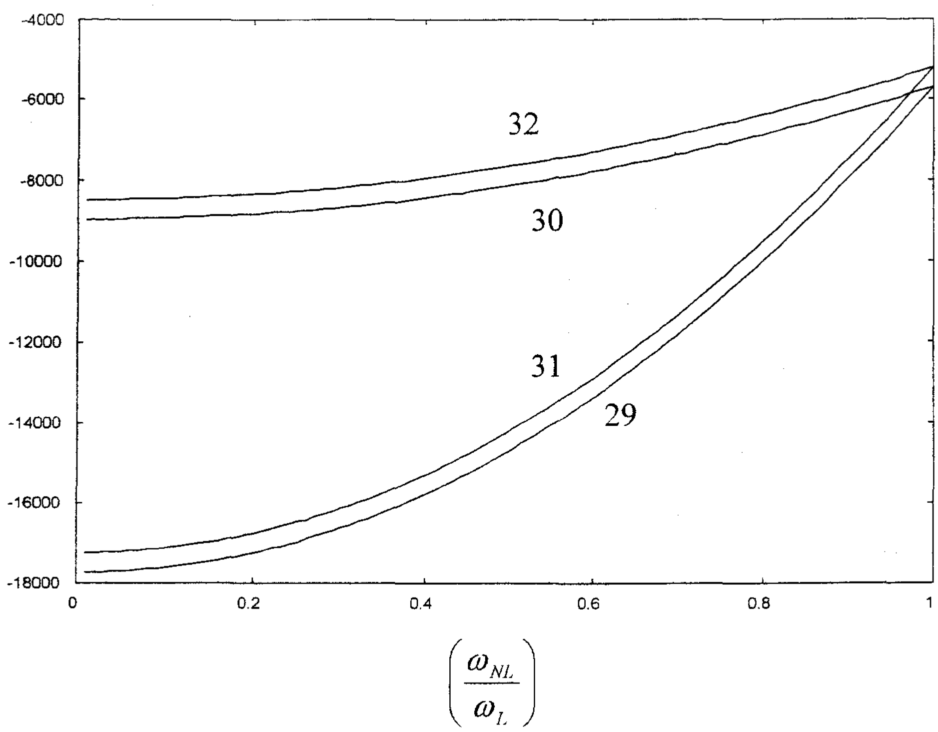
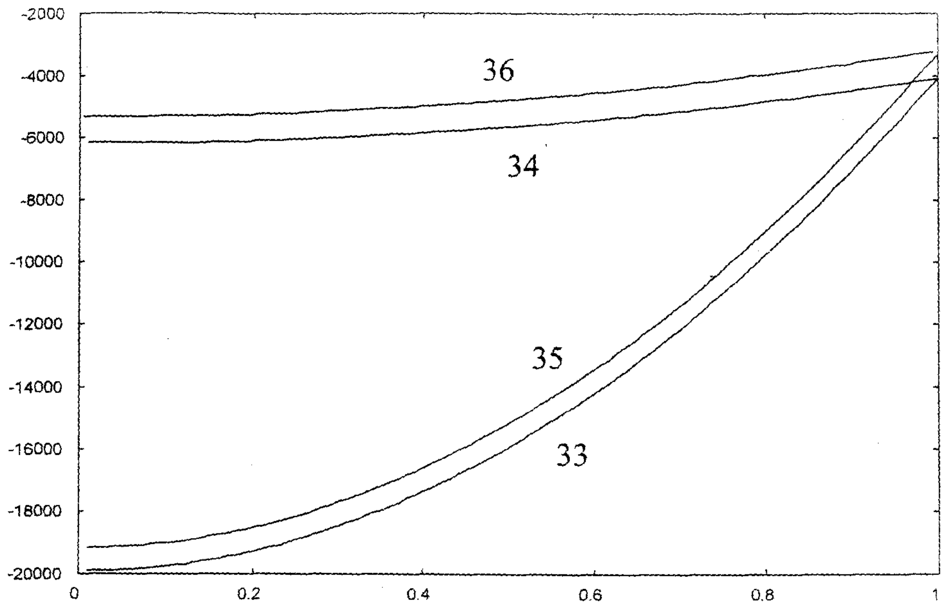


Fig.5.10 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for a square (a=10)

Here 29,30,31 and 32 represent the square for $k_1 = 500$, $k_3 = 50$;

$k_1 = 0$, $k_3 = 50$; $k_1 = 500$, $k_3 = 0$; $k_1 = 0$, $k_3 = 0$ respectively.



$$\left(\frac{\omega_{NL}}{\omega_L}\right)$$

Fig.5.11 Shows comparative variations of $\left(\frac{\omega_{NL}}{\omega_L}\right)$ vs. P for an equilateral triangle

Here 33,34,35 and 36 represent an equilateral triangle for

$k_1 = 500$,$k_3 = 50$; $k_1 = 0$, $k_3 = 50$; $k_1 = 500$, $k_3 = 0$;

$k_1 = 0$, $k_3 = 0$ respectively.

5.7 OBSERVATIONS AND DISCUSSIONS:

i) From the Figures (Fig.5.1-Fig.5.8) general observations are noted that with the increase of the load parameter P ; ratios of the nonlinear vs. linear frequencies $\left(\frac{\omega_{NL}}{\omega_L}\right)$ also increase.

ii) Keeping the values of the foundation parameters k_1 and k_3 fixed, the load-bearing capacities are seen to be different when dimensions of the plates are changed to attain the same level of frequency-ratios $\left(\frac{\omega_{NL}}{\omega_L}\right)$ as described below:-

a) From Fig.5.1,it is seen that for a square plate the compressive load capacities are enormously greater than those for the cases of a circular and an equilateral triangular plates for $a = 10$, $k_1 = 500$,k_3=50. It is further noted that after a certain level the results for the case of a triangular plate become higher than those for the cases of a circular and a square plate.

b) From Fig.5.2 it is observed that for the case of a circular plate the load parameters are greater than those of a square and an equilateral triangular plate when a=20,k_1= 500,k_3= 50.Similar behavior occurs as in the previous case.

c) From Fig.5.3 it is seen that for a triangular plate the load bearing capacities are greater than those for the cases of a square and a circular plate when a=10,k_1= 0,k_3= 50.

d) From Fig.5.4 it is seen that for a square plate the load capacities are higher than those of the cases of a circular plate and an equilateral triangular plate for a=20,k_1=0, k_3= 50.

Here also after a certain stage, results for the case of a circular plate become higher than those for the cases of a square and an equilateral triangular plate.

e) From Fig.5.5, it is seen that for a square plate the load capacities are higher than those for a circular and an equilateral triangular plates for a = 10, k_1= 500, k_3=0.

f) From Fig.5.6, it is seen that for a circular plate load capacities are higher than those for the cases of a square and an equilateral triangular plate for a=20, k_1=500, k_3= 0.

g) From Fig.5.7, it is seen that for a equilateral triangular plate load capacities are higher than those for the cases of an square and circular plate for a=10,k =0, k_3= 0.

h)From Fig.5.8, it is seen that for an equilateral triangular plate load capacities are higher than those for the cases of a square and a circular plate for a=20,k_1=0,k_3 = 0.

Since frequencies have been considered from 0 to 1 in all the above cases nature of variations of parameters in the Fig.5.3,Fig.5.7 and Fig.5.8 will be the same as observed in (a) or (d) above.

iii) From Fig.5.9,Fig5.10 and Fig.5.11 for each of three cases [circle, square and an equilateral triangle],it is seen that with the decrease of the foundation parameters (k_1 , and k_3) values of load parameters increase to attain the same level of ratios of nonlinear and linear frequencies$\left(\frac{\omega_{NL}}{\omega_L}\right)$.

6.1 BRIEF DISCUSSION OF RESULTS AND INTERPRETATIONS.

At the end of numerical computations in graphical forms of each paper detailed discussions and interpretations of results have been presented and as such repetition of such discussion is omitted.

6.2 CONCLUSION:

In the revised thesis the papers have been modified in the light of the valuable comments and observations of the honorable Examiner and in place of numerical results in tabular forms, graphical presentations have been made. The numerical results have been fully discussed and results interpreted as far as possible.

The Introduction of the Thesis has also been re-written in concise form with citation of previous works and mentioning the gaps required to be filled up by further research works.

Objectives and scopes of research activities related to the present Thesis have also been included. A separate section has been added on the Summary of Literature Survey. Summary of Formulation of problems considered in the Thesis have also been added in concise form.

6.3 RECOMMENDATION OF FUTURE WORKS:

Thermal stresses, deformations, buckling and vibrations of plates and shells have enormous applications in structural mechanics, aeronautics, high-speed spacecrafts, missiles, off-shore engineering, ship-building structures, nuclear power reactors and in the components of many engineering structure.

Nonlinear Analysis of such Plate and Shell Structures have been investigated by many researchers using different methods, namely

(i) By von Karmam's coupled nonlinear partial differential equations
(ii) Berger's quasilinear equations
(iii) Modified Berger's Method
(iv) Numerical and computational methods as well as by complex variable and conformal mapping approach.

Although intensive research have been carried out by earlier researchers, still a careful survey of literature reveals the fact that there is still enough scope for further research in many of the following class of problems:-

(i) Plates and shells of different polygonal shapes as well as complex geometrical shapes.
(ii) Plates of variable thickness under thermal loading ,. The main difficulty may be to solve heat conduction equations associated with boundary value problems for plates of variable thickness.
(iii) Composite plates under thermal loading
(iv) Plates and shells under non-stationary temperature field.
(v) Thermal post-buckling analysis of plates and shells.
(vi) Skew plate and shell structures exposed to severe thermal loading and under plates and shells under thermal loading
(vii) Skew plate and shell structures exposed to severe thermal loading and under non-stationary temperature fields.
(viii) Sandwich plates and shells under thermal loading
(ix) Vibration analysis of shells under thermal shock.
(x) Nonlinear analysis of plates and shells under general thermal boundary conditions

In addition to above, there may arise many other classes of problems which should be considered by the future researchers.

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