CHAPTER - 3
DATA AND METHODOLOGY

3.1 Nature and Period of Dataset

The study is based on secondary dataset. In this study datasets of Rupee/Nepalese Rupee exchange rate and relative price level in India and Nepal have been used. We have used historical datasets on exchange rate and CPIs of both the currencies in our study. The dataset is quarterly by nature. The period covers 1976:1 to 2006:1. The base period is 2000. Specified variables are e_t and p_t where

e_t = Rupee/Nepalese Rupee exchange Rate

p_t = Relative Price Level

3.2 Source of the Dataset

The time series data for Rupee/Nepalese Rupee exchange rate and consumer price index (CPI) for India and Nepal have been used. These datasets have been collected from the different issues of International Financial Statistics (IFS), published by the International Monetary Fund (IMF).

3.3 Rationale Behind the Choice of the Period of Study

Reserve Bank of India adopted the '*Basket Peg System*' in September, 1975. Since then until 1991, value of rupee was being pegged to a '*basket of currencies*'. The adoption of the 'Basket Peg system' virtually marked the initiation of the '*Flexible Exchange rate System*' in India.

The Nepal Rasta Bank, on the other hand, in Nepal resorted to '*Crawling Peg System*' on regular basis since 1976. This marked the era of '*flexible exchange rate system*' in Nepal.

Thus in the year 1976 *'flexible exchange rate system'* became operative in both India and Nepal. This accounts for the choice of 1976 as the starting year of the dataset on Rupee/Nepalese Rupee exchange rates in our present study.

3.3.1 Rationale Behind the Choice of the Consumer Price Index(CPI)

Price Indices in case of Multiple Goods:

In the simplest form of PPP, for a single homogeneous good, the exchange rate would eventually be equal to the ratio of domestic price and foreign price when both are expressed in the same currency units.

However, in practical life there are a large number of goods with different prices. In such case PPP theory involves the comparison of the domestic and foreign price level through the use of an aggregate index number of the many prices in each country. Thus in case of multiple goods, PPP states that the nominal exchange rate (e_t) should be equal to the ratio of domestic price index (p_t) and the foreign price index (p^*_t) such that

$$E_t = A(\frac{P_t}{P^*_t}) \qquad (3.1)$$

With A=1, we have the Absolute version of PPP. If $A \neq 1$, then we have the relative version of PPP.

3.3.2 Price Indices Rationale Behind the Use of Tradable Goods only:

The scope of arbitrage in commodity underlies the PPP theory. According to PPP differences in the prices in the same goods in different countries, when converted to a common currency unit, open up the prospect of profits to be made by buying the good in one country and selling it in other. Thus deviations from PPP represent profitable commodity arbitrage opportunities. Such opportunities exist only for commodities which are traded internationally. However, there are many goods which are not traded internationally. For these goods, there exists no international market in which these can be bought and sold. So these goods do not contribute to the demand for and supply of foreign currency. Consequently, it is argued that the price indices which are used to measure PPP should be constructed from the prices of traded goods. In such case, wholesale price indices for these goods should be chosen.

3.3.3 Arguments Behind the Use of CPI

The alternative view is that exchange rate represents the relative price of national currencies. Currency is held as an asset. It can be converted, like any other form of wealth, into the purchasing power over tradable and non-tradable goods. A Consumer Price Index includes the prices of both traded and non-traded goods. Consequently, CPI can comfortably and reasonably be used as a measure of purchasing power of the currencies concerned.

3.4 Use of CPI in this Study

It, therefore, follows that the ratio of two consumer price indices (CPIs) measures the relative price of domestic currency to foreign currencies. Alternatively, the ratio of two consumer price indices measures the relative purchasing power of currencies concerned. This provides the rationale behind the use of CPI in our study. The ratios of the CPI of India to that of Nepal in any quarter over the period 1976:1-2006:1 have been used to measure the relative price level in the corresponding quarter.

3.5 Tests for PPP

3.5.1 Testability of Absolute Purchasing Power Parity (APPP)

Let e_t be the normal exchange rate between the currencies for the domestic and foreign countries at time t.

Let P_t and p^*_t be the domestic and foreign price indices. Then in case of multiple goods PPP states that the exchange rate at time t should equal the ratio of domestic price index (P_t) and the foreign price index (p^*_t) such that

$$e_t = A.(\frac{P_t}{P^*_t}) \tag{3.2}$$

With A = 1, then Absolute Purchasing Power Parity (APPP) holds.

However, differences exist in the construction of index numbers in the domestic and foreign countries because the conventions of setting the price index to some commonly member such as 1 or 100 in a particular base are usually different in different countries. Thus in estimations, the finding that $A \neq 1$ simply because of different statistical conventions would have no bearing on the validity of PPP. *APPP, therefore, is not in general a testable proposition especially because P_t and p^*_t are price indices rather than the price of a single good.*

3.5.2 Testability of Relative Purchasing Power Parity (RPPP)

From (3.2) we have

$$RE_t = E_t\left(\frac{P_t}{P^*_t}\right) = A \tag{3.3}$$

A is not necessarily equal to unity if P_t and p^*_t are indices.

Taking logarithm on (3.3)

$$In\ RE_t = In\ E_t + In\ p^*_t - In\ P_t = InA$$

$$or, re_t = e_t + p^*_t - p_t = a \tag{3.4}$$

Where $In\ RE_t = re_t$

$In\ E_t = e_t$

$In\ P_t = p_t$

$In\ p^*_t = p^*_t$

$InA = a$

If APPP holds, then In(A) = a = In 1 = 0.

If RPPP holds, then $re_t \neq 0$

Now taking first differencing of re_t we get

$$\Delta re_t = \Delta e_t + \Delta p^*_t - \Delta p_t \tag{3.5}$$

Then (3.5) states that the rate of change of the real exchange rate is equal to the rate of change of nominal exchange rate plus the inflation in the foreign country minus the inflation in the domestic country.

Again from (3.4) and (3.5)

$$\Delta e_t + \Delta p^*_t - \Delta p_t = 0 \quad \text{[since, } \Delta a] \tag{3.6}$$

Therefore,

$$\Delta e_t = \Delta p_t - \Delta p^*_t \tag{3.7}$$

(3.7) indicates that nominal exchange rate moves to exactly compensate the relative growth in foreign and domestic price indices. If (3.7) holds, then $\Delta re_t = 0$.

It, therefore, follows that once RPPP holds such that nominal exchange rate moves to reflect exactly the inflation differences in both the trading countries, then there will be no change in real exchange rate over time. This indicates, on the other hand, that RPPP indirectly establishes the '*Constancy*' of real exchange rate. *Thus the testing of PPP relates to testing the constancy of real exchange rate. Such testing of PPP, therefore, constitutes an exercise for testing the RPPP in practice.*

3.6 Basic Format for the Test of RPPP

Let e_t = log of the nominal exchange rate

p_t = log of domestic CPI

p^*_t = log of foreign CPI

Then in RPPP the regression equation is

$$e_t = \alpha + \beta(p_t - p^*_t) + u_t \tag{3.8}$$

where $u_t \sim iidN(0, \sigma^2_u)$

Consequently, $u_t \sim I(0)$.

From (3.8) we have

$$e_t - \alpha - \beta(p_t - p_t^*) = u_t \tag{3.9}$$

Since $u_t \sim I(0)$, then

$[e_t - \alpha - \beta(p_t - p_t^*)]$ must be I(0).

Therefore, (3.9) indicates that

e_t and $(p_t - p_t^*)$ must be *cointegrated* **at level** if RPPP holds. Consequently, RPPP holds iff exchange rate and the ratio of relative price level are *cointegrated at level. Thus test of PPP or RPPP relates to examining if exchange rate and relative price level datasets are cointegrated at level.*

3.7 Stationarity:

The study involves the extensive use of time series techniques. *Box-Jenkins techniques* along with the latest developments in theoretical and empirical analysis have been adopted in our study. The time series for exchange rate and relative price level have been subject to tests for stationarity.

In this study the *Dickey-Fuller method* has been adopted for the test of the presence of unit roots in the time series concerned. The detection of the unit root in a time series is undertaken to examine if the time series exhibit random walk processes, i.e. non-stationarity.

Non-stationarity has further been verified through the estimation of the *Autocorrelation Function (ACF)* and *Partial Autocorrelation Function (PACF). Box- Ljung* values, along with the relevant probabilities for significance, have been reported along with the estimated autocorrelation coefficients at different lags. The *ACF* and *PACF* plots showing the estimated coefficients for different lags along with the upper and lower critical values for the confidence limits have been derived.

The detection of non-stationarity in the time series has been followed up through appropriate transformation of the time series concerned for ensuring stationarity. This has mainly been accomplished through first differencing. The first differenced series have then been subject to *Dickey-Fuller tests* in order to examine if stationarity were really obtained in the series concerned. This has further been confirmed through the examination of the relevant *ACF* and *PACF* plots.

3.7.1 Properties of Stationarity Dataset and Unit Root Test

In case of time series analysis unit root tests are important since these tests detect stationarity and non- stationarity of the time series data used for the study. A stationary time series data set has three basic properties.

First, it has a finite mean, which implies that a stationary series fluctuates around a constant long run mean.

Second, a stationary time series has a finite variance. This implies that variance is *time-invariant.*

Third, a stationary time series dataset has finite auto-covariances. This reflects that theoretical autocorrelation co-efficient decay fast as lag length increases.

Regression runs on non- stationary time series produce a spurious relationship. In order to avoid a spurious relationship, it becomes necessary to perform a unit root test on variables. The *Dickey-Fuller (DF)* and *Augmented Dickey-Fuller (ADF)* tests are widely used for performing unit root tests. The ADF test involves the autoregressive AR(1) process. For this we consider the following equation.

$$Y_t = \alpha + \rho Y_{t-1} + \zeta_t$$

In case ρ carries the value $-1<\rho<1$, the variable Y is stationary. If the value of ρ is one, the variable Y is non- stationary. Hence, the unit root test null hypothesis is:

$$H_0 : \rho = 1$$

While testing the null hypothesis of unit root, the following equation is used.

$$\Delta Y_t = \alpha + \gamma Y_{t-1} + \zeta_t \qquad (3.10)$$

where, $\gamma = \rho-1$ and ΔY is the first difference of the series Y. Here the unit root null hypothesis is:

$$H_0 : \gamma = 0$$

3.7.2 Unit Root Test: The Methodology

Let us consider the data generating process

$$Y_t = \varphi Y_{t-1} + \varepsilon_t$$

The associated question is whether $\varphi = 1$. Subtracting Y_{t-1} from both sides we get,

$$\Delta Y_t = (\varphi - 1)Y_{t-1} + \varepsilon_t$$
$$= \gamma Y_{t-1} + \varepsilon_t$$

$\gamma=0$ implies that $\varphi = 1$ which indicates the presence of a unit root in $\{Y_t\}$.

A drift is allowed by including an intercept

$$\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \varepsilon_t$$

Allowing for linear trend with a drift gives us

$$\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \alpha_1 t + \varepsilon_t$$

In any event, the test hypothesis is

H_0: $\gamma = 0$ (Y_t has a unit root)

against

H_1: $\gamma \neq 0$ (Y_t is stationary)

The test statistic $\frac{\hat{\gamma}}{\sqrt{\operatorname{var}\hat{\gamma}}}$ is a statistic. The critical values come from a set of tables prepared by Dickey and Fuller. This test is known *'Dickey-Fuller Test'*.

The immense literature and diversity of unit root tests can at times be confusing and present a truly daunting prospect for a researcher. The unit root theory has been examined with an emphasis on testing principles. The summary of the finding is given below:
When time series data are used in econometric analyses, the preliminary statistical step is to test the stationary of each individual series. Unit root tests provide information about

stationarity of the data. Non-stationary data contain unit roots. The main objective of unit root tests is to determine the degree of integration of each individual time series. Various methods for unit root tests have been applied in the study. Some of them are being explained below.

3.7.3 Augmented Dickey Fuller Unit Root Test

In order to test for the existence of unit roots, and to determine the degree of differencing necessary to induce stationarity, the *Augmented Dickey-Fuller test* is used. Dickey and Fuller (1976, 1979), Said and Dickey (1984), Phillips (1987), Phillips and Perron (1988), and others developed modifications of the Dickey-Fuller tests when ε_t is not white noise. The results of the *Augmented Dickey-Fuller test* (ADF) determine the form in which the data should be applied in any econometric analyses. The test is based on the following equations:

$$\Delta y_t = \gamma + \alpha y_{t-1} + \sum_{j=2}^{k} \theta_j \Delta y_{t-j+1} + e_t \qquad (3.11)$$

$$\Delta y_t = \gamma + \delta t + \alpha y_{t-1} + \sum_{j=2}^{k} \theta_j \Delta y_{t-j-1} + e_t \qquad (3.12)$$

$$\Delta y_t = \alpha y_{t-1} + \sum_{j=2}^{k} \theta_j \Delta y_{t-j+1} + e_t \qquad (3.13)$$

where, y_t = Modeled Variables,

Δy_t = First differenced series of y_t.

Δy_{t-j+1} = First differenced series of y_t at $(t-j+1)^{th}$ lags. ($j = 2$ ----- k)

The equation (3.11) is related to ADF test with constant as exogenous. Equation (3.12) is based on constant and linear trend as exogenous and ADF test with no exogenous is presented in equation (3.13).

3.7.4 The D-F GLS Unit Root Test

The DF-GLS test developed by *Elliott, Rothenberg and Stock (1996),* which has greater power than standard ADF test is also employed in the study. The DF-GLS t-test is performed by testing the hypothesis $a_0=0$ in the regression

$$\Delta y_t^d = a_0 y_t^d + a_1 \Delta y^d_{t-1} + \text{-----} + a_p \Delta y^d_{t-p} + \text{error} \quad (3.14)$$

where y_t^d is the locally de-trended series y_t .The local de-trending depends on whether we consider a model with drift only or a linear trend.

(i) The model for DF-GLS unit root test without time trends i.e., a model with drift only is

$$y_t^\mu = \alpha y^\mu_{t-1} + \sum_{i=1}^{k} \Psi i \, \Delta y^\mu_{t-i} + u_t \quad (3.15)$$

(ii) The model for DF-GLS unit root test with time trends i.e. a model with linear trend is

$$y_t^{\tau} = \alpha y^\mu_{t-1} + \sum_{i=1}^{k} \Psi i \, \Delta y^\tau_{t-i} + u_t \quad (3.16)$$

3.7.5 Phillips –Perron Unit Root Test

Phillips (1987), Phillips and Perron (1988) have generalized the DF tests to situations where disturbance processes ε_t are serially correlated, without augmenting the initial regression with lagged dependent variables. The PP is intended to add a 'correction factor' to the DF test statistic and the test is designed for examining the presence of any 'structural shift' in the dataset.

Let the AR (1) model be

$$Y_t = \mu + \varphi_1 Y_{t-1} + \varepsilon_t, \; [t = 1, \text{---------}, T] \quad (3.17)$$

with $\quad Var(\varepsilon_t) \equiv \sigma_\varepsilon^2$.

If ε_t is serially correlated, the ADF approach is to add lagged ΔY_t to 'whiten' the residuals. To illustrate the alternative approach, the test statistic $T(\varphi_1\text{-}1)$ has been considered which is distributed as ρ_μ from the maintained regression with an intercept but no time trend. The PP modified version is

$$Z\rho_\mu = T(\varphi_1 - 1) - CF \quad (3.18)$$

where the correction factor CF is

$$CF = 0.5(s^2_{Tl} - s_\varepsilon^2) / (\sum_{t=2}^{T} (Y_{t-1} - \bar{Y}_{-1})^2 / T^2) \tag{3.19}$$

and,
$$s_\varepsilon^2 = T^{-1} \sum_{t=1}^{T} \varepsilon_t^2 \tag{3.20}$$

$$s^2_{Tl} = s_\varepsilon^2 + 2\sum_{s=1}^{l} W_{sl} \sum_{t=s+1}^{T} \varepsilon_t \varepsilon_{t-s} / T \tag{3.21}$$

$W_{sl} = 1 - s/(l+1)$ and $\varepsilon_t = Y_t - \mu - \varphi_1 Y_{t-1}$

$$\bar{Y}_{-1} = \sum_{t=2}^{T} \frac{Y_t}{T-1} \tag{3.22}$$

3.8 Cointegration:

Macro-Economic variables, which are used in this study, are of time series by nature. These series are not deterministic variables. On the contrary, these are considered to be generated by some underlying stochastic processes. In any time series (Y_t), each value of Y_1, Y_2..............., Y_t is assumed to be drawn randomly from a probability distribution. To be completely general, the observed series Y_1, Y_2,...................., Y_t is assumed to be drawn from a set of jointly distributed random variables. Thus if the underlying probability distribution function of the series could be specified, then one could determine the probability of one or another future values of the variable concerned.

The complete specification of the probability distribution function for any time series is usually impossible. However, it is possible to construct a simplified model for the time series, which explains its randomness in a manner that is useful for econometric studies. This simple model may be a reasonable approximation of the actual and more complicated underlying stochastic process. The usefulness of such a model depends on how closely it captures the true probability distribution and the true random behavior of the series. Consequently, the validity and usefulness of macroeconomic studies with time series like money supply, price level etc. depends upon the nature of underlying stochastic process and upon approximation of the process.

Specification of the underlying stochastic process is preceded by the identification of the nature of the stochastic process. More specifically, it is necessary to know whether the underlying stochastic process is invariant with time or whether it describes a random walk. If the process is non-stationary, it will be difficult to represent time series over past and future intervals of time by an algebraic model. By contrast, if the stochastic process is fixed in time i.e., if it is 'stationary', then one can model the process via an equation with fixed coefficients that can be estimated from the past data. It is analogous to the single equation regression model in which one variable is related to another variable with coefficients that are estimated under the assumption that the structural relationship described by the equation is 'invariant' over time. The probability of a given fluctuation in the process from the mean level is assumed to be the same at any point of time. In other wards, the stochastic properties of the stationary process are assumed to be invariant with respect of time. For a stationary process both the joint probability distribution and conditional probability distribution are invariant with respect of time.

Cointegration between the time series has been studied for estimating a stable long-run equilibrium relationship between the variables concerned. This concept is very useful in empirical analysis because it allows the research to describe the nature of an equilibrium or stationarity relationship between two time series each of which is individually non-stationary.

For the study of cointegration between the variables concerned, the following procedures have been adopted.

i. The cointegrating equation has been estimated with the OLS Method.
ii. The residuals of the estimated equation have been obtained.
iii. The residuals are subject to Augmented Dickey-Fuller (ADF) test to examine if random walk exists or if the residuals are white noise.
iv. The ADF test results have been further confirmed through the examination of the ACF and PACF plots of the residuals.

v. If the residuals exhibit random walk, the time series are subject to first differencing.
vi. The cointegrating equation has been re-estimated through the use of the differenced dataset.
vii. The residuals of the estimated equation have been obtained.
viii. The residuals again are subject to Augmented Dickey-Fuller (ADF) test and we examine if the residuals are white noise.
ix. The ADF test results have further been confirmed through the examination of the ACF and PACF plots of the residuals.

The procedures have been repeated until the residuals of the estimated co-integrating equation are free from random walk. Thus the order of co-integration has been ascertained.

a. The Basic Problem:

In a provocative study *Charles R. Nelson and C.I. Plosser (1982)* found evidence that macro-economic variables like GNP, exchange rate, interest rate, employment, money supply, price level etc. behave like random walks. As these series follow 'Random Walks', these are not 'trend reverting'. Consequently, these economic variables do not tend to revert back to a long run trend after a shock.

These findings of *Nelson and Plosser (1982)* posed serious problems for econometric studies for macroeconomic variables. The studies so for carried out with the macroeconomic variables were based on the idea that these variables were 'deterministic non-stationary' series. Stationarites in these series were ensured through 'Filtering' like differencing of the series and identifying appropriate *Auto-Regressive Moving Average (ARMA)* processes. Findings of Nelson and Plosser (1982) hit the basic idea underlying these studies and the relevance of the studies was threatened consequently.

b. The Nature of the Problem:

The reason why Nelson-Plosser findings would threaten the basic approach behind the econometric studies with macroeconometric time series and why random walk process for the time series would limit the use of this series in econometric studies need serious consideration.

First, variances of the random walk processes in the joint probability distribution are no longer constant. Instead, the variances expand out with time and the random walk errors are no longer '*Homoscedastic'*. Consequently, the *Gauss- Markov Theorem* would not hold, and *Ordinary Least squares (OLS)* method would not yield consistent estimates of the parameters concerned.

Second, random walk processes fail to posses finite variance. In such case, the regression analysis fails and econometric studies with these series become irrelevant.

Third, detrending the variable before running the regression will not help because even the detrended series still remains non-stationary. Consequently, the random walk process becomes non-deterministic, non-stationary process. In such case detrending fails to ensure stationarity.

Fourth, if a variable follows a random walk, the effects of a temporary shock will not dissipate after several years but instead will be permanent. This occurs because the auto-correlation functions for such variables are 'uniform' by nature and it declines geometrically over time. The random walk process in such case, has an infinite memory. The current value of the process depends on all past values and the magnitude of the effect remains unaltered with time. As a result, the effect of a temporary shock will not dissipate after several years but will remain permanent. This further indicates that, in case of the presence of non-stationarity in the series for the variable describing random walks, the series does not revert back to a long run trend after a shock.

3.8.1 Cointegration: Engel Granger Method

Cointegration is the study concerning the existence of long run equilibrium relationships among variables. The study allows the researcher to describe the existence of an equilibrium or stationary relationship among two or more time series, each of which is individually non-stationary. According to *Engle and Granger (1987)* the variables will be cointegrated when the linear combination of non-stationary variables is stationary.

Cointegration provides a method for elimination of the cost of differencing by rationalizing terms in levels but only in linear combinations, which are stationary.

For example, if there are two variables X (Relative Price Level) and Y (Exchange Rate), the following equation may be considered for cointegrating test.

$$\varepsilon_t = Y_t - \alpha - \beta X_t \tag{3.23}$$

where ε_t is the residual. If ε_t, the residuals are stationary, the series Y_t and X_t are said to be cointegrated.

Here the cointegration study may be based on the estimation of any of the models given below:

$$e_t = \alpha_1 + \beta p_t + \mu_{1t} \tag{3.24}$$

$$p_t = \alpha_2 + \beta e_t + \mu_{2t} \tag{3.25}$$

where, e_t = Exchange Rate,

p_t = Relative Price Level

μ_{1t} & μ_{2t} are residual terms.

3.8.2 Johansen Cointegration Test:

Both the *Johansen (1988) and the Stock and Watson(1988)* methodologies rely heavily on the relationship between the rank of the matrix and the characteristic roots. The *Johansen cointegration* test equation is presented below:

$$\Delta y_t = \gamma + \pi y_{t-1} + \sum_{i=1}^{\rho-1} \pi_i \Delta y_{t-i} + \mu_t \tag{3.26}$$

where, γ is the vector of constants, y_t is the m dimensional vector of variables, (i.e., e_t, p_t in our analysis), ρ is the number of lags, μ_t is the error vector, which is multivariate normal and independent across observations.

$$\pi = -(1 - \sum_{i=1}^{p} A_i) \text{ and } \pi = -\sum_{j=i+1}^{p} A_i$$

Here, the rank of the matrix π is equal to the number of independent cointegrating vectors. Specifically,

If $\pi = 0$, the matrix is null and is the usual VAR model in first differences.

If π is of rank n, the vector process is stationary.

If $\pi = 1$, there is a single cointegrating vector.

If $1 < \pi < n$, there are multiple cointegrating vector.

Let the matrix be π and ordered the n characteristics roots be λ_1, λ_2, ……λ_k such that $\lambda_1 > \lambda_2 > \ldots\ldots\ldots \lambda_n$. If the variables in y_t are not cointegrated, the rank of π is zero and all of these characteristic roots will be zero. Since Log (1) = 0 each of the expressions Log(1- λ_i) will equal to zero if the variables are not cointegrated. Similarly, if the rank of π is unity, $0<\lambda_1<1$, so the expression Log(1- λ_1) will be negative and the other $\lambda_i = 0$, so that Log(1- λ_2) = log (1- λ_3) = …….= Log (1- λ_n) = 0.

Here the number of distinct cointegrating vectors can be determined by checking the significance of the characteristic roots of π. The test for the number of characteristic roots that are significantly different from the unity can be obtained by using the following two test statistics:

i. *the Trace Statistic,*
ii. *the Max-Eigen Statistic.*

The Trace Statistic can be calculated in terms of the following expression:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{n} Log(1 - \hat{\lambda}_i)$$

On the other hand, the Max-Eigen Statistic can be calculated as

$$\lambda_{trace}(r, r+1) = -TLog(1 - \hat{\lambda}_{r+1})$$

where, $\hat{\lambda}_i$ = the estimated values of the characteristic roots (i.e., Eigen values) obtained from the estimated π matrix, T= the number of unstable observations.

The '*Trace Statistic*' is used to test the null hypothesis that the number of distinct cointegrating vector is less than or equal to 'r' against the general alternative. The '*Max-Eigen Statistic*' test the null hypothesis that the number of cointegrating vector is 'r' against the alternative of (r+1) cointegrating vectors. The *critical values* of the λ_{trace} and the λ_{max} statistic are calculated using the *Monte Carlo approach.*

3.9 Correlogram:

One of the simple, intuitive and interesting methods of testing '*stationarity*' is running a *correlogram.* Correlogram is nothing but a graphical representation of *Autocorrelation Function* (ACF) and *Partial Autocorrelation Function* (PACF). The nature of stationarity can also be found almost accurately in most of the cases with the help of *Correlogram.*

3.10 Vector Error Correction Modeling:

Vector Error Correction modeling provides important information on the short-run relationship (short-run dynamics) between any two cointegrated variables. *Vector Error Correction* test has provided empirical evidence on the short run causality among variables concerned.

In the present study the *vector error correction* estimates have been specified using by the following model. The models have been used in both cases i.e. involving exchange rate and relative price level.

$$\Delta e_t = \alpha_1 + \rho_1 z_{t-1} + \beta_{1i} \sum_{i=1}^{m} \Delta e_{t-i} + \gamma_{1i} \sum_{i=1}^{m} \Delta p_{t-i} + \omega_t \qquad (3.27)$$

$$\Delta p_t = \alpha_2 + \rho_2 z_{t-1} + \beta_{2i} \sum_{i=1}^{m} \Delta e_{t-i} + \gamma_{2i} \sum_{i=1}^{m} \Delta p_{t-i} + v_t \qquad (3.28)$$

Δe_{t-i}=First Differenced series of Exchange Rate at time t-i; i=1,2,............,m

Δp_{t-i}=First Differenced series of Relative Price Level at time t-i; i=1,2,........,m

Z_{t-1} is the *error correction term* since the *Johansen Cointegration Tests* confirm the existence of *only one Cointegration Equation* between e_t and p_t. The lag length (m), in the estimation, is determined through the *Akaike Information Criterion* (AIC) and *Schwartz Information Criterion* (SIC) etc. ω_{1t} and v_{2t} are white noise errors; β_{1i} and β_{2i}

are the coefficients of lagged exchange rates and γ_{1i} and γ_{2i} are the coefficients of relative price levels.

The focus of the *vector error correction* analysis is on the lagged z_t terms. These lagged terms are the residuals from the previously estimated cointegrating equations. In the present case the residuals from two lag specifications of the cointegrating equations have been used in the *vector error correction* estimates. Lagged z_t terms provide an explanation of short run deviations from the long run equilibrium for the test equations above. Lagging these terms means that disturbance of the last period impacts upon the current time period. Statistical significance tests are conducted on each of the lagged z_t term in equations (3.27) and (3.28). In general, finding a statistically insignificant coefficient of the z_t term implies that the system under investigation is in the short run equilibrium as there are no disturbances present. If the coefficient of the z_t term is found to be statistically significant, then the system is in the state of the short run disequilibrium. In such a case the sign of z_t term gives an indication of the causality direction between the two test variables.

3.11 Vector Autoregressive Model

Economic theories sometimes suggest a relationship between two variables, y_t and z_t. In that case modeling each series involves an autoregression of y_t on lagged values of y_t and an autoregression of z_t on lagged values of z_t. However such a separate approach would not capture any interactions between the variables concerned.

However, such interactions between the variable are captured through a *Vector Autoregression (VAR)* model where the time path of $\{y_t\}$ is affected by the current and past realizations of $\{z_t\}$ sequence and the time path of $\{z_t\}$ sequence is allowed to be affected by current and past realizations of $\{y_t\}$ sequence. In a VAR model y_t is related not just to its own lagged values but also those of z_t and similarly, z_t is related to its own lagged values and those of y_t, such that

$$y_t = b_1 + b_{11} y_{t-1} + b_{12} z_{t-1} + \varepsilon_{1t} \tag{3.29}$$

$$z_t = b_2 + b_{21} y_{t-1} + b_{22} z_{t-1} + \varepsilon_{2t} \tag{3.30}$$

The VAR model, consisting of the equations (3.29) and (3.30), can be written as

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$\text{or } x_t = b + \pi_1 x_{t-1} + \varepsilon_t \qquad (3.31)$$

where, , $b' = (b_1 \quad b_2)$ is the vector of constants usually known as drift

$\varepsilon_t' = (\varepsilon_{1t} \quad \varepsilon_{2t})$ are innovations relative to information

set $x'_{t-1} = (z_{t-1}, z_{t-1})$

The equation (3.31) defines a VAR (1, 2) Model where order (p) = 1 and k (number of variables) = 2.

This form of the VAR is a '*Reduced Form*' system in the sense that no current dated values of y_t and z_t appear in any of the equations. The genesis of the '*Reduced From*' VAR could serve as the solution in a dynamic simultaneous equation model. For example, let us consider a VAR system with contemporaneous relationship between two variables such that

$$y_t = b_{10} + b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt} \qquad (3.32)$$

$$z_t = b_{20} + b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt} \qquad (3.33)$$

where it is assumed that

i. both y_t and z_t are stationary, and
ii. ε_{yt} *and* ε_{zt} are white noise disturbances

such that $\varepsilon_{yt} \sim$ iid N (0, σ_y^2) , $\varepsilon_{zt} \sim$ iid N (0, σ_z^2)

Now the VAR system consisting of equations (3.32) and (3.33) can be written as

$$y_t + b_{12} z_t = b_{10} + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}$$

$$z_t + b_{21} y_t = b_{20} + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}$$

$$\text{or} \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

$$\text{or} \quad Bx_t = \tau_0 + \tau_1 x_{t-1} + \varepsilon_t \tag{3.34}$$

$$\text{where} \quad B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$$

$$x_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$$

$$\tau_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$$

$$\tau_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

Pre-multiplying equation (3.34) by B^{-1} we have

$$X_0 = A_0 + A_1 x_{t-1} + e_t \tag{3.35}$$

where $A_0 = B^{-1} \tau_0$

$A_1 = B^{-1} \tau_1$

$e_t = B^{-1} \varepsilon_t$

Now '*equivalent form*' of (3.35) is

$$y_t = a_{10} + a_{11} y_{t-1} + a_{12} z_{t-1} + e_{1t} \tag{3.36}$$

$$z_t = a_{20} + a_{21} y_{t-1} + a_{22} z_{t-1} + e_{1t} \tag{3.37}$$

Thus the '*Structural VAR*' constituted by equations (3.32) and (3.33) is converted to the '*Standard Form*' constituted by equation (3.36) and (3.37)

Here

$$e_{1t} = (\varepsilon_{yt} - b_{12}\varepsilon_{zt})/(1 - b_{12}b_{21})$$

$$e_{2t} = (\varepsilon_{zt} - b_{21}\varepsilon_{yt})/(1 - b_{12}b_{21})$$

Thus e_{1t} and e_{2t} are the composites of the two shocks ε_{yt} and ε_{zt}.

3.11.1 Stability of the VAR Model

The first order VAR model of (3.35) defines a first order difference equation which can be iterated backward to obtain

$$X_t = A_0 + A_1(A_0 + A_1 X_{t-1} + e_{t-1}) + e_t$$

$$= (I + A_1)A_0 + A_1^2 X_{t-2} + A_1 e_{t-1} + e_t$$

where I = 2*2 identity matrix

After n iterations, we have

$$X_t = (1 + A_1 + ... + A_1^n)A_0 + \sum_{i=0}^{n} A_1^i e_{t-1} + A_1^{n+1} X_{t-n-1} \qquad (3.38)$$

It is observed that the convergence requires that the expression A_1^n vanish as n→∞. Consequently, the stability of the VAR model requires that the roots of (1- a_{11}L) (1-a_{22}L) - ($a_{12}a_{21}L^2$) lie outside the unit circle. This stability conditions holds if

i. the $\{y_t\}$ and $\{z_t\}$ sequences are jointly covariance stationary
ii. each sequence has a finite and time-invariant mean and a finite time-invariant variance

3.12 Granger Causality: Methodology

3.12.1 Introduction:-

The study of *cointegration* of variables examines if the variables are related or not. The *cointegration* procedure stresses upon estimating distributed lag relationship along with the *error correction* structure. However, the *autoregressive structure* does not play any significant role in the study of *cointegration* between variables concerned.

This particular feature of the *cointegration equations* accounts for the inability of the equation to explain if the variables concerned are '*exogenous*' or '*endogenous*'. *Engel, Hendry and Richard (1983)* define a set of a variable X_t in a parameterized model to be '*weakly exogenous'* if the full model can be written in terms of a marginal probability distribution of X_t and a conditional distribution of Y_t/X_t such that estimation of the parameters of the conditional distribution is no less efficient than estimation of the full set of parameters of the joint distribution. With reference to time series applications variable X_t is said to be predetermined in the model if X_t is independent of all subsequent structural disturbances ε_{t-s} for s>0. Variables that are predetermined in the model can be treated, at least asymptotically, as if they were *exogenous* in the sense that consistent estimates can be obtained when they appear as regressors.

Cointegrating equations cannot establish if any of the variables is *exogenous*. Consequently, *cointegrating equations* cannot be used for forecasting purposes. These equations, therefore, cannot explain if one of the variables could be used for the effective prediction for variation in another variable. This explains why *cointegrating relation* fails to establish 'Granger' causal relationship between variables concerned.

3.12.2 The Methodology

Let us consider a jointly covariance stationary stochastic process y_t, x_t with $E(x_t) = E(y_t) = 0$ and with a *covariance generating function* $g_x(z)$, $g_y(z)$ and $g_{xy}(z)$. It is assumed that x possesses an *autoregressive representation* and that both y and x are linearly indeterministic. Then the projection of x_t on past values of x and past values of y_t is given by

$$X_t = \sum_{j=1}^{\infty} h_j X_{t-j} + \sum_{j=1}^{\infty} v_{t-j} y_{t-j} + u_t \tag{3.39}$$

where the least square residual u_t obeys the *orthogonality condition*

$$E(u_t X_{t-\beta}) = E(u_t y_{t-\beta}) = 0 \text{ for } \beta = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots$$

Solving (3.39) for u_t permits the *orthogonality condition* to assume the form of normal equations

$$E\{(X_t - \sum_{j=1}^{\infty} h_j X_{t-j} - \sum_{j=1}^{\infty} v_j y_{t-j}) X_{t-\beta}\} = 0, \tag{3.40}$$

$\beta = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$E\{(X_t - \sum_{j=1}^{\infty} h_j X_{t-j} - \sum_{j=1}^{\infty} v_j y_{t-j}) y_{t-\beta}\} = 0, \tag{3.41}$$

$\beta = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

These equations can be written as

$$c_x(\beta) = \sum_{j=1}^{\infty} h_j c_x(\beta - j) + \sum_{j=1}^{\infty} v_j c_{yx}(\beta - j) \tag{3.42}$$

$$c_x(\beta) = \sum_{j=1}^{\infty} h_j c_{xy}(\beta - j) + \sum_{j=1}^{\infty} v_j c_x(\beta - j) \tag{3.43}$$

These equations hold only for positive integer $\beta = 1$

Multiplying both sides of (3.42) and (3.43) by z^{β} and summing over all β, we get the following equations in terms of z transformation

$$g_x(z) + m(z) = h(z) g_x(z) + v(z) g_{yx}(z) \tag{3.44}$$

$$g_{xy}(z) + n(z) = h(z) g_{xy}(z) + v(z) g_y(z) \tag{3.45}$$

where m(z) and n(z) are each unknown series in non-positive power of z only. That m(z) and n(z) series are non-positive powers of z is equivalent with equations (3.42) and (3.43) holding only for $\beta > 1$. Equations (3.44) and (3.45) are the *normal equations* for h(z) and v(z).

Following *Weiner, Granger (1969)* has proposed that *"y causes x" whenever v(z) ≠ 0 That is, y is said to cause x if, given all past values of x, past values of y help predict x.*

The conditions under which v(z) does or does not equal to zero turn out to be of substantial interest to econometrician and macro- economists.

Let us consider the projection of y_t on the entire x process

$$y_t = \sum_{j=1}^{\infty} b_j X_{t-j} + \varepsilon_t \qquad (3.46)$$

where $E(\varepsilon_t X_{t-j}) = 0$ for all j

Let X_t have the ' *Wold Moving Average* ' presentation such that

$$X_t = d(L)\eta_t$$

$$\eta_t = X_t - P[X_t / X_{t-1}, X_{t-2}, \ldots\ldots\ldots]$$

$$\sum_{j=1}^{\infty} d_j^2 < \infty \qquad (3.47)$$

Then $g_X(z) = \sigma_n^2 d(z) d(z^{-1})$

It is assumed that x possesses an *autoregressive representation* so that [d(z^{-1})] is *one sided square summable* in non-negative power of z. It is always possible to uniquely factor the cross covariance generating function as

$$g_{yX}(z) = \alpha(z)\phi(z^{-1}) \qquad (3.48)$$

where $\alpha(z)$ and $\phi(z)$ are one sided in non-negative power of z .

Substituting (3.46) and (3.47) into the usual relation

$$b(z) = g_{yX}(z) / g_X(z) \qquad (3.49)$$

we have

$$b(z) = \alpha(z)\phi(z^{-1}) / \sigma_n^2 d(z) d(z^{-1}) \qquad (3.50)$$

Evidently, b(z) is one sided in non-negative powers of z if and only if $\phi(z^{-1}) = kd(z^{-1})$, where k is a constant . Under this condition (3.49) becomes

$$b(z) = k\alpha(z) / \sigma_\eta^2 d(z) \qquad (3.51)$$

Here $\alpha(z)$ has an inverse that is one sided in non-negative power of z.

Now if b(z) is one sided in non-negative power of z, the equation (3.44) and (3.45) are both satisfied with v(z) = 0 and

$$h(z) = z[d(z)/z] + d(z^{-1}) \tag{3.52}$$

Consequently, equation (3.45) becomes

$$\phi(z)\alpha(z^{-1})n(z) = z[d(z)/z] + 1/d(z)\alpha(z^{-1})\phi(z) \tag{3.53}$$

Dividing both sides of equation (3.53) by $z\alpha(z^{-1})$ gives

$$\phi(z)/z + n(z)/z\alpha(z^{-1}) = [d(z)/z] + \phi(z)/d(z) \tag{3.54}$$

where $n(z)/z\alpha(z^{-1})$ involves only negative powers of z. Since the right hand side involves only non-negative power of z, (3.54) implies

$$d(z)[\phi(z)/z] = [d(z)/z] + \phi(z) \tag{3.55}$$

This equation (3.55) can be satisfied if $\phi(z) = kd(z)$ where k is a constant.

Now let (x_t, y_t) be a *jointly covariance stationary, strictly indeterministic process* with zero mean. Then $\{y_t\}$ fails to *Granger cause* $\{x_t\}$ if and only if there exists a vector moving average representation

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} C(L)_{11} & 0 \\ C(L)_{21} & c(L)_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix} \tag{3.56}$$

where ε_t and u_t are serially uncorrelated processes with zero means and $E(\varepsilon_t u_s)=0$ for all t and s, and where the one-step-ahead prediction errors $[x_t - p(x_t / x_{t-1},........., x_{t-1},.......)]$ and $[y_t - p(y_t / y_{t-1},........., y_{t-1},.......)]$ are each linear combination of ε_t and u_t.

Under these situations Sims (1972) explains the concept of causality through the following theorem:

'*Y_t can be expressed as a distributed lag of current and past x's (with no further x's) with a disturbances process that is orthogonal to past, present and future x's if and only if y does not Granger cause x*'.

Consequently, the test involves estimating the following regressions:

$$y_t = \sum_{i=1}^{n} \alpha_i X_{t-j} + \sum_{j=1}^{n} \beta_j y_{t-j} + u_{1t} \tag{3.57}$$

$$X_t = \sum_{i=1}^{n} \lambda_i y_{t-j} + \sum_{j=1}^{n} \delta_j X_{t-j} + u_{2t} \tag{3.58}$$

where it is assumed that the disturbances u_{1t} and u_{2t} are uncorrelated.

Equation (3.57) postulates that current y_t is related to past values of y_t itself as well as of x_t and (3.58) postulates a similar behavior for x_t. Four cases then can be distinguished.

1 Unidirectional causality from x to y

It is indicated if the estimated coefficients on the lagged x in (3.57) are statistically different from zero as a group (i.e. $\Sigma\alpha_i \neq 0$) and the set of estimated coefficients on the lagged y in (3.58) is not statistically different from zero (i.e, $\Sigma\delta_j = 0$).

2 Unidirectional causality from y to x

It exists if the set of lagged x coefficients in (3.57) is not statistically different from zero(i.e, $\Sigma\alpha_i = 0$) and the set of the lagged y coefficient in (3.58) is statistically different from zero (i.e. $\Sigma\delta_j \neq 0$) .

3 Feedback or Bilateral Causal

It is suggested when the sets of x and y coefficient are statistically significantly different from zero in both regressions.

4 Independence

It is suggested when the sets of x and y coefficients are not statistically significant in both the regressions.

3.13 Intervention Analysis: Impulse Response Functions

If the stability condition is met, then the particular solution for x_t in (3.38) can be written as

$$X_t = \mu + \sum_{i=0}^{n} A_1^i e_{t-i} \tag{3.59}$$

where $\mu = [\overline{y, z}]'$ and

$$\bar{y} = [a_{10}(1 - a_{22}) + a_{12}a_{20}]/\Delta$$
$$\bar{z} = [a_{20}(1 - a_{11}) + a_{21}a_{10}]/\Delta$$
$$\Delta = (1 - a_{11})(1 - a_{22}) - a_{12}a_{21}$$

Equation (3.59) is *Vector Moving Average (VMA)* representation of (3.35) in that the variables, y_t and z_t are represented in terms of the current and past values of two types of shocks (i.e e_{1t} and e_{2t}). Again equation (3.59) can further be simplified as

$$X_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i} \tag{3.60}$$

where
$$\phi_i = \frac{A_1^i}{(1 - b_{12}b_{21})} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}$$

Consequently, we have for (3.60)

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum \begin{bmatrix} \phi_{11(i)} & \phi_{12(i)} \\ \phi_{21(i)} & \phi_{22(i)} \end{bmatrix} \begin{bmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{bmatrix}$$

The four sets of coefficients $\phi_{11(i)}, \phi_{12(i)}, \phi_{21(i)} \, and \, \phi_{22(i)}$ are called the 'Impulse Response Functions'. Plotting the coefficients of $\phi_{jk(i)}$ against i is a practical way to visually represent the behaviours of the $\{y_t\}$ and $\{z_t\}$ series in response to various shocks. Further elaboration follows in Chapter 9.

3.14 Intervention Analysis: Variance Decomposition of Forecast Errors:

Given the equation (3.60), we have for n^{th} period

$$X_{t+n} = \mu + \sum_{i=0}^{\infty} \varepsilon_{t+n-i} \tag{3.61}$$

where $E(x_{t+n}) = \mu$. Then the unconditions n period ahead forecast error is

$$X_{t+n} - E(X_{t+n}) = \sum_{i=0}^{n-1} \varepsilon_{t+n-1} \tag{3.62}$$

Using (3.62) we can find one-period ahead, two period ahead and thus n period ahead forecast errors. Each of the forecast errors would have variances. It is possible to decompose the n-step ahead forecast error variance owing to shocks in $\{y_t\}$ and $\{z_t\}$ sequences.

Thus the '*Forecast Error Variance Decomposition*' indicates the proportion of variation in a sequence owing to its '*own shock*' versus *shocks to other variables.*

3.15 'Window Finding' of Structural Changes:

The choice of sub-periods objectively involves the identification of structural changes through '*Window Finding*'. The basic procedure is described below.

3.15.1 Methodology

Sometimes researcher seeks to investigate into the stability of the coefficient estimates as the sample size increases. Sometimes researcher also wants to find out whether the estimates will be different in enlarged samples and whether these will remain stable over time. Working with a sample, a researcher may produce a regression which is too closely tailored to his sample by experimenting with too many formulations of his model. In this case, he is not contained that the estimated function will perform equally well outside the sample of data which has been used for the estimation of coefficients. Furthermore, there may have occurred events which change the structure of the relationship like changes in taxation law, introduction of birth control measures and so on. If such structural changes occur, the coefficient may not be stable. They may be sensitive to the changes in the sample compositions.

Testing for structural stability calls for the use of additional observations besides the sample that are used to estimate a given model. Procedures for testing structural stability are given by Rao (1960) and Chow (1952).

The econometric method which involves '*Window Finding*' uses *Chow test* to identify the sub-periods. Here equality between two regression coefficients concerning the relationship over two different periods is tested. This is done by F-test. Let us consider two samples with n_1 and n_2 observations respectively and the general model for data set is

$$Y = X\beta + u \tag{3.63}$$

where
$Y \rightarrow n \times 1$
$X \rightarrow n \times k$
$\beta \rightarrow k \times 1$
$n \rightarrow n_1 + n_2$

Let us rewrite the model for these two individual samples such as

$$Y_1 = (z_1 \quad w_1) \begin{pmatrix} \gamma_1 \\ \delta_1 \end{pmatrix} + u_1 \tag{3.64}$$

$$Y_2 = (z_2 \quad w_2) \begin{pmatrix} \gamma_2 \\ \delta_2 \end{pmatrix} + u_2 \tag{3.65}$$

where
$Y_1 \rightarrow n_1 \times 1$
$Y_2 \rightarrow n_2 \times 1$
$Z_1 \rightarrow n_1 \times 1$
$Z_2 \rightarrow n_2 \times 1$
$W_1 \rightarrow n_1 \times m$
$W_2 \rightarrow n_2 \times m$
$\gamma_1 \rightarrow 1 \times 1$

$\gamma_1 \rightarrow 1 \times 1$

$\delta_1 \rightarrow m \times 1$

$\delta_2 \rightarrow m \times 1$

By combining (3.64) and (3.65) we have

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} z_1 & 0 & w_1 & 0 \\ 0 & z_2 & 0 & w_2 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \delta_1 \\ \delta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{3.66}$$

and the null hypothesis of interest is

$H_0: \gamma_1 = \gamma_2 (= \beta \text{ say})$

Under the null hypothesis, the model is

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} z_1 & w_1 & 0 \\ z_2 & 0 & w_2 \end{pmatrix} \begin{pmatrix} \beta \\ \delta_1 \\ \delta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{3.67}$$

The L.S estimate of the efficient vector in (3.67) is

If we fit (3.64) and (3.65) individually, their LS estimates of the coefficients will be

$$\begin{pmatrix} \hat{\beta} \\ \hat{\delta}_1 \\ \hat{\delta}_2 \end{pmatrix} = \left[\begin{pmatrix} z_1 & w_1 & 0 \\ z_2 & 0 & w_2 \end{pmatrix}' \begin{pmatrix} z_1 & w_1 & 0 \\ z_2 & 0 & w_2 \end{pmatrix} \right]^{-1} \left[\begin{pmatrix} z_1 & w_1 & 0 \\ z_2 & 0 & w_2 \end{pmatrix}' \right] \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{3.68}$$

$$\begin{pmatrix} c_1 \\ d_1 \end{pmatrix} = \left[(z_1 \quad w_1)' \ (z_1 \quad w_1) \right]^{-1} (z_1 \quad w_1)' y_1 \tag{3.69}$$

$$\begin{pmatrix} c_1 \\ d_2 \end{pmatrix} = \left[(z_1 \quad w_1)' \ (z_1 \quad w_1) \right]^{-1} (z_1 \quad w_1)' y_1 \tag{3.70}$$

where c_i is the estimate of γ_i. The sum of squares necessary for computing test statistics can then be obtained by using the results in (3.68), (3.69) and (3.70). The sum of squares measures the distance of individual observations from the common regression plane is

$$Q_1 = \left[\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} z_1 & w_1 & 0 \\ z_2 & 0 & w_2 \end{pmatrix} \begin{pmatrix} b \\ \delta_1 \\ \delta_2 \end{pmatrix}\right]' \left[\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} z_1 & w_1 & 0 \\ z_2 & 0 & w_2 \end{pmatrix} \begin{pmatrix} b \\ \delta_1 \\ \delta_2 \end{pmatrix}\right] \quad (3.71)$$

Here Q_1 / δ_2 has χ^2 distribution with (n-2m-1) degrees of freedom where we assume that u_1 and u_2 have a common variance δ_2. Now Q_1 can be decomposed into two sum squares Q_2 and Q_3. Q_2 will measure the distances of observations from the individual estimated regression planes, and Q_3 measures the distance of the individual estimated plane from the common regression plane. Thus,

$$Q_2 = \left[y_1 - \begin{pmatrix} z_1 & w_1 \end{pmatrix} \begin{pmatrix} c_1 \\ \delta_1 \end{pmatrix}\right]' \left[y_1 - \begin{pmatrix} z_1 & w_1 \end{pmatrix} \begin{bmatrix} c_1 \\ \delta_1 \end{bmatrix}\right] + \left[y_2 - \begin{pmatrix} z_2 & w_2 \end{pmatrix} \begin{pmatrix} c_2 \\ \delta_2 \end{pmatrix}\right]'$$

$$\left[y_2 - \begin{pmatrix} z_2 & w_2 \end{pmatrix} \begin{bmatrix} c_2 \\ \delta_2 \end{bmatrix}\right] \quad (3.72)$$

and $Q_3 = Q_1 - Q_2$. Here Q_2/δ_2 has a χ^2 distribution with (n-2m -1) degrees of freedom. Again,

$$Q_3 = \left[\begin{pmatrix} z_1 & w_1 \end{pmatrix} \begin{pmatrix} c_1 \\ \delta_1 \end{pmatrix} - \begin{pmatrix} z_1 & w_1 \end{pmatrix} \begin{pmatrix} b \\ \delta_1 \end{pmatrix}\right]' \left[\begin{pmatrix} z_1 & w_1 \end{pmatrix} \begin{pmatrix} c_1 \\ \delta_1 \end{pmatrix} - \begin{pmatrix} z_1 & w_1 \end{pmatrix} \begin{pmatrix} b \\ \delta_1 \end{pmatrix}\right] +$$

$$\left[\begin{pmatrix} z_2 & w_2 \end{pmatrix} \begin{pmatrix} c_2 \\ d_2 \end{pmatrix} - \begin{pmatrix} z_2 & w_2 \end{pmatrix} \begin{pmatrix} b \\ d_2 \end{pmatrix}\right]' \left[\begin{pmatrix} z_2 & w_2 \end{pmatrix} \begin{pmatrix} c_2 \\ d_2 \end{pmatrix} - \begin{pmatrix} z_2 & w_2 \end{pmatrix} \begin{pmatrix} b \\ d_2 \end{pmatrix}\right] \quad (3.73)$$

It may be noted that c_1 is the estimate of γ_1 obtained from the first regression and that d_2 is the estimate of δ_2, obtained from pooled regression plane. So the ratio is

$$F = \frac{\frac{Q_3}{l}}{\frac{Q_2}{(n-2m-2l)}} \tag{3.74}$$

So, we have an F-distribution with (l, n-2m-2l) degress of freedom. Here Q_3 is the restricted sum of squares and that Q_2 is the unrestricted sum of squares.

If, however, the new observations n_2 are fewer than the number of parameters in the function we may proceed as follows. First, from the augmented sample we obtain the regression equation.

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 + \ldots\ldots\ldots\ldots + \hat{\beta}_k \hat{X}_k \tag{3.75}$$

From which we calculate the residual sum of squares

$$\Sigma e^2 = \Sigma y^2 - \Sigma \hat{y}^2 \tag{3.76}$$

with ($n_1 + n_2 - k$) degrees of freedom. Second from the original sample of size n_1 we have

$$\hat{Y}_1 = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_k \hat{X}_k \tag{3.77}$$

from which the unexplained sum of squares is

$$\Sigma e_1^2 = \Sigma y_1^2 - \Sigma \hat{y}_1^2 \tag{3.78}$$

with n_1 - k degrees of freedom.

Third, subtracting the two sums of residuals we find

$$\Sigma e^2 = \Sigma e_1^2 \tag{3.79}$$

with ($n_1 + n_2 - k$) - (n − k) = n_2 degrees of freedom, where n_2 are the additional observations. Further, we form F* ratio where

$$F^* = \frac{\Sigma e^2 - \Sigma e_{12} / n_2}{\Sigma e_{12} / (n_1 - k)} \tag{3.80}$$

The null hypotheses are

$$H_0 : b_i = \beta_i (i = 0,1,2,...........k)$$
$$H_0 : b_i \neq \beta_i$$

The F* ratio is compared with the theoretical value of F obtained from the F- table with $v_1 = n_2$ and $v_2 = (n - k)$ degrees of freedom.

If F* ratio exceeds the table value of F, we reject the null hypothesis i.e, we accept that the structural coefficients are unstable. This indicates that their values are changing in extended sample period.