Chapter 5

Optimal Control and Feedback Rule:

Non-Linear Dynamic Optimization Model in Bangladesh Trawl Shrimp Fishery

-Continuous Time Analysis

5.1. Introduction:

In management of renewable resources, the sole target that has been tried to achieve is the control of the level of exploitation subject to attainment of maximum economic benefit. In our last chapter we tried to analyze Bangladesh trawl shrimp fishery both under static and dynamic situation. Using the bioeconomic parameters estimated in Chapter-3, we also tried to compute optimal values of the decision and control variables when the system is at steady state. But all these are done when the time is supposed to be discretized at yearly interval. Considering the present practice and policy of Bangladesh trawl shrimp fishery, this assumption grossly *underm'mes* the basic objective of optimal management of this renewable resource. In Bangladesh, trawlers are given annual license to operate in the zone and thus, there is no effective controlling and monitoring system on the extent of effort level being operationalised and the amount of fish actually harvested. If we sincerely desire to manage the resource optimally and to achieve the objective of operating at steady state when both the level of biomass stock and level of harvest are maximum, then continuous monitoring is required. It requires two essential managerial conditionalities to be satisfied. Firstly, there should be a mechanism to provide managers with an adaptive method of regulating resource use to achieve this defined target. Secondly, the mechanism simultaneously should be able to evaluate alternative harvesting strategies.

One of the mechanisms that has been suggested in recent literatures to fulfill these two conditionalities. simultaneously improving the management of renewable resources is to develop optimal feedback rules. This short of adaptive management policy which requires adequate and appropriate feedback rules as essential, necessitates to assume the time variable (t) as continuous. Clark and Munro (1975), Conard and Clark (1987) attempted to provide such feedback rules. Recommendations made by them, however, have not been proved feasible to be operationalised. Grafton et al. (2000) put forward a model to show how to operationalise a feedback rule which is generalization of one suggested by Sandal and Steinshamn (1997a) and tested for the data of Canada's Northern Cod Fishery. This chapter is an attempt to study the applicability of such feedback rules for management of renewable resources like trawl shrimp in Bangladesh.

The other important point *to* be noted here is that, in recent times, the resources in economics is, viewed from capita! theoretic approach. From this point of view, we can assume that shrimp fishery desires to maintain its total net revenue over some period of time. Depending on past, history of exploitation of resource, the fishery will have initiated a certain stock of capital at any time(t). With this stock of capital, x(t) in our case, at that particular time(t), the fishery is in a position to take'decisions. This decision at time(t) is h(t) in our case. Inherited stock of capital (biomass level) together with the specified current decision (harvest policy), fishery derives a net benefit per unit of time. This can be denoted by ri(x(t),h(t),t) when appropriately discounted. This function Tl(.), therefore, specifies the rate at which net benefits are being earned at timeft) over a stock x(t) and as a result of the decision h(t).

Now, for any terminal time T, starting with initial stock x_0 at time t=0, the total net discounted benefit will be given by a function, say dp(Xo, h), which is integral of n(.), discounted and added up for all instants over the time period (0,T). This implies that if fishery starts out with an initial amount of capital (biomass level x_0) and then follows the decision policy h, will obtain a sum total of net benefit, <*p,* which is the integral of the benefits obtained and discounted at every instant. These results, however, in turn are depending upon (i) the pertinent instant, i.e. time (ii) the capital stock at that time and (iii) the decision applicable to that moment. Thus h does represent the entire time path of the decision variable h from the initial time t=0 to t=T. The fishery managers are at liberty to choose the time path of h but cannot select independently the amount of capital at different instant, which is actually a consequence of x_0, the stock of capital at the initial time, and the time path chosen for decision variable. This phenomenon acts as a constraint upon the objective function. This constraint thus represents the rate of change of capital stock at any instant as a function of its present level, the time and the decision taken. This explicitly indicates that the decisions taken at any time have two consequences: (i) they vary the rate at which net benefits are realized at that time and (ii) they moderate the rate at which the biomass stock is changing and thereby the actual stock of biomass that will be available at subsequent times.

Representing the task of resource optimization under the above mentioned setting shows that the essence of the problem is of making decisions in a dynamic context, Moreover, it reveals that the problem is to select the time path, h, so as to make the total net benefit, dp, as high as possible taking into account the effect of the choice variable h on both the instantaneous rate of net benefit and the biomass level of shrimp to be carried into the future. The solution of this problem is not to assign the best possible variable (s) to a single variable or multiple variables but to identify an entire time path which is the best. It is, therefore, imperative that the time must be continuous. The advance mathematics which is applicable for the solution, is the maximum principle of Optimal Control Theory.

For the reason that the present day technique in managing resources from the capital theoretic approach gives the best understanding, which, in turn, demands application of the maximum principle of Optimal Control Theory, we take resort to continuous analysis. Moreover, the effective feedback rules for adaptive policy of resources management also implies time to be continuous. Hence, in this Chapter, we undertake to apply Optimal Control Theory which actually will provide us the feedback rules for Bangladesh trawl shrimp fishery. In the literature, a feedback rule means that the optimal control is a function of state variable.

In this Chapter we also try to provide a schema which can be best suited for the specific situation which differs significantly from developed country both in nature and complexity, of how an optimal feedback rule may be applied in exploiting and managing this particular resource in Bangladesh.

5.2. The General Model:

On the basis of the discussion in Section 5.1, our integral function, $ (x, h, t) (eliminating the argument t in x(t) and h(t) for convenience), is the objective function constrained by the growth function of the resource, depending upon biomass stock, x and harvest policy h. The resource utilization will be at optimum when it reaches at steady state given enough time {t tends to infinity). '

The problem of Bangladesh trawl shrimp fishery can thus be formulated as a general dynamic optimization problem as follows:

Maximize

$$O(x, h, t) = \int e^{-\delta t}\, \pi(x, h)\, dt$$

Subject to (5.1)

$$\frac{dx}{dt} = f(x, h), \quad \lim_{t \to \infty} x(t) = x^*$$

$$\text{where} \quad f(x, h) = f(x) - h;$$

$$\text{and} \quad f(x) = rx\left(1 - \frac{x}{K}\right)$$

n (.) is the net revenue function of state variable x and control variable h. Here, x, the state variable, denotes the size of the resources; h, the control variable, denotes the exploitation of the resource; x* denotes state variable at steady state; **t** denotes time; 8 is a constant discount rate; r denotes the intrinsic growth rate and K denotes the saturation level or carrying capacity. Since, we integrate our objective function over the time interval (0,°°). It is necessary that the functions fulfill the Mangasarian sufficiency theorem for infinite horizon.

5.2.1. Sandal-Stienshamn Solution Procedure of the Model:

Solution of the model may appear, at the first sight, to be very common in optimal control theory if maximum principle is applied on the Hamiltonian function (H) after introducing the fourth variable (m), known as co-state variable, along with already existing time(t), state(x) and control(h) variables in the function. Maximum principle provides three equations through (i) equation of motion for the state variable (x), (ii) equation of motion for the co-state variable (m) and (iii) transversality condition. Usually first order condition for the maximization of the Hamiltonian with respect to the control variable is also set equal to zero. These four relations are sufficient to solve the system of the differential equations sequentially to get the optimal trajectory or the decision path. But it is the transversality condition which is unknown both in terms of finite time or in terms of the value of the state variable at terminal point, makes the solution of the model difficult. Transversality condition m(T) is such a condition which only gives what should happen at the terminal time T, when T tends to infinity in our case. The finite time horizon does not facilititate the solution either. Moreover, for any renewable resource management it is always emphasized and discussed that the solution must be of infinite time horizon. Because, the complete finite time problem should imply to cut the time at a point beyond which the resource exploitation is undesirable, whatever may be the reasons. Moreover, the finite time does not help to get rid of the difficulties of the solution procedure, unless either the terminal time{T) or the terminal value of the state variable(x) is exogenously determined.

Conventional 'brute force' numerical methods, as suggested by Conard and Clark (1987), may be applied in some cases. Sandal and Steinshamn (1997a) is another general model to search the optimal stock for quadratic objective function. This is, however, not applicable in our case. We follow Sandal and Steinshamn (2001) to find the solution of the non-linear dynamic problem of the marine shrimp fisheries of the problem of Bangladesh formulated as above.

Sandal and Steinshamn (2001) provides a procedure by which co-state variable(m) can be expressed as a function of other two variables - state and control. Then the argument co-state variable in the Hamiltonian will be replaced by this function in order to make Hamiltonian as a function of two variables only. Combining this with the first order conditions of maximization of the Hamiltonian function, we get, in general, a highly non-linear ordinary differential equation (ODE). We get an exact solution of the ODE in the limiting case when there is no discounting. It thus facilititates to find very good approximative solutions using perturbation theory. Thus the procedure suggests solution sequentially in the following order:

(i) to find the exact solution when the discount factor 6=0. Under this situation the value of the Hamiltonian function becomes constant:

(ii) to find an approximate solution by the perturbation method when the discounting factor 5> 0. It has been shown that the approximation is a good one and relatively easy to obtain.

Common practice of the perturbation method is to formulate a general problem, find a particular solution and, using this solution as a starting point, find an approximate solution. Present procedure differs from the common perturbation method in a way that it uses solution with zero discounting as a starting point. Sandal and Steinshamn (2001) shows that the solution at zero discounting contains most of the complexities and non-linearities in the model and is such a nontrivial in character that the perturbation method does not require many correction terms.

The same approach will be adopted to solve our formulated problem here except the perturbation method which has been used in case of the situation 'with discounting'. In fact, there are two basic ways to continue with the non-zero discount cases: (a) Numerical method (b) Perturbational approximation. Numerical method again can follow any one of the two numerical ways. Sandal and Steinshamn follow the perturbation approximation. But we shall adopt one of the two numerical methods. All these three approaches will be discussed in 5.2.1 (b), where we will deal with the solution of non-zero discount case.

Mathematics of Solution Procedure: The current value Hamiltonian of the problem (5.1) is given by,

$$H(x,h,m)-II(x,h) + mf\ (x,h) \tag{5.2}$$

where m is the co-state variable.

Thus, the first order conditions for the maximization problem are

$H_h = 0$,

$m = 8\ m - H_x$

and $x = f\ (x,\ h)$

From time derivative of the current value Hamiltonian function together with these equations we get

$H = 8\ m\ x$, since $H\ h = 0$,

.". $IIh + m.\ f\ h = 0$

$^n\ h$

(5.3)

Now it shows that co-state variable is a function of state and control variable. Hence we can write

$$m = M(x,\ h) = \frac{n}{f\sim h} \tag{5-4}$$

Thus, if we substitute this M(x,h) in place of m in the current value Hamiltonian function H(x, h, m), then we get a new function which is always equal to the Hamiltonian in value along an optimal trajectory. Let the new function be denoted as P (x, h) = H (x, h, M (x, h)) (5.5)

Thus,

$$P = \frac{dp}{dx}$$

$$\frac{dp}{dx} \quad \frac{dp}{dh} \quad \frac{dh}{dx}$$

Since $P = H$ (by construction) and $H = 8\ m\ x$, We have

$$\frac{\wedge}{dx} = 5m = 5M(x,h) \tag{5.6}$$

For convenience, we like to write the equation in the following forms:

$$\left(\frac{dp}{0x} + \frac{dp}{dh} \cdot \frac{dh}{dx}\right) = 5\ M(x,\ h) \tag{5.6a}$$

$$P_x + p_h . h' = 5M(x, h) \qquad (5.6b)$$

where partial derivatives with respect to a variable are denoted by respective subscripts, like

$$\frac{3p}{dx} = P_{x}' \frac{d^2v}{3_x 9h} = P_{xh}$$

This (5.6) is a basic equation that will be utilized to find the optimal value of the control variable expressed as a function of state variable.

The solution may be sought from relation (5.6) for two different situations, with 5 = 0 and 5

(a) When 5 = 0 (without discounting): When 5 is set equal to zero, it implies that the discounting is not being done, i.e. a situation'without discounting. Thus, if 5 = 0 then from (5.6) we get dp/dx = 0. It implies that the value of the Hamiltonian P is constant. Let it be denoted by P_0. From the Hamiltonian function (5.2) and expression of the co-state variable given in (5.4) we get,

$$n(x, h) - T \frac{^n h}{n} . f(x, h) = P_0 \qquad (5.7)$$

In order to find out the value of this constant Po, the infinite time horizon limiting condition may be imposed on (5.7). With the objective of exploitation of surplus production, the resource would reach at the steady state when f (x, h) = 0. Under this situation, equation (5.7) gives that

$$Po = n (x, h) \qquad (5.8)$$

Since, (5.8) indicates net revenue, it must be maximized and therefore, it represents maximum sustainable economic rent. As harvest level (h) will reach at the maximum and will remain constant, therefore this sustainable economic rent will be a function of x alone. We denote this function as S (x) as follows,

$$S(x) = n(x,h)|_{f()(,h)=} o \qquad (5.9)$$

The terminal time T can be calculated to know the time horizon. Let the initial given level of biomass is x_0 and the steady state biomass level is x\ The time to be taken to reach the biomass stock from Xo to x* is given by the integral value of the rate at which the growth function grows when harvest itself is determined by the level of biomass stock maintained with zero discounting. The growth function is

$$f(x_; h) = f(x_J h(x; Po))$$

and the time T is

$$T = \int_{x_0}^{x^*} \frac{r}{f(x, h(x; p_0))} \qquad (5\text{-}10)$$

where, $h(x;P_0)$ is actually the solution of the equation (5.7).

Since S (x) = 0 is a necessary condition for an interior solution with respect to optimal steady state, the optimal steady state is given by x* = arg.[max S (x)] and the corresponding exploitation level can be found by solving f(x*,h)=0 with respect to h. Thus, both x* and h* can be obtained. Knowing the values of x* and h*, optimal effort (E*) and optimal shadow price (A*) can be obtained from the equation (4.13) and (4.14) when price (p) is supplied by the demand function.

(b) When 8 s* 0. (with discounting): Let us first describe the existing methods of solution of non-zero discounting cases. ,

(1) Numerical Procedure:

(i) Let us take the first order ODE in h(x) given by the equation (5.6b). Differentiating (5.6b) further with respect to x and inserting the equilibrium point x = x* and h = h*, we get a second order differential equation as $P_h = 0$ at (x*, h*). It then, gives us the slope of the feedback solution at the equilibrium point. Moving a little away from the equilibrium point along the tangent we get another starting point (with non-zero velocity). AH initial value solvers or straight forward discretizing will give the exact numerical solution. The equilibrium point of this kind models are saddle points. The correct slope through the saddle point is the one with positive slope. This is a standard numerical way to get the separatrices through the equilibrium points. .

(ii) For the second numerical procedure, integrating on both sides of equation (5.6b), we get,

$$P(x, h) = P(x^*, h^*) + 5.\ JM(x, h(x))dx$$

$$= R(x, h)$$

For each chosen x-value, we iterate on h in the following way:

$$R(k) = R(x, h[k]), \text{ and}$$

$$P(x,h[k+1]) = P(k+1),$$

then we solve for h[k+1]. We can get a sequence of functions (or numerical values) h(1), h(2), ...for each value of x. This iteration is extremely efficient (strong contraction) if one starts with h(1)=f (x) (the growth function).

(2) Perturbational approximation method suitable for such cases in discussed in details in Sandal and Steinshamn (2001).

We, however, follow the first numerical procedure. When 5 is not set equal to zero, the optimal steady state would change and can be obtained by solving the following equations with respect to x : ,

$$S'(x) = -6, \quad f_h \quad f(x.\ h) = 0 \qquad (5.11)$$

$$S/(x) = 8.M(x, h) \quad I(X,h)=0 \qquad (5.11a)$$

Now, an initial point in (x,h) plane is needed. The steady state point without discounting may be used as a starting point. But it may not appear to be very good starting point. For this, we can try to manipulate the equation (5.6b) further to have the slope of the control path and thus get the initial point in (x, h) plane.

Differentiating (5.6b) with respect to x, we get

$$P_{xx} + 2P_{xh}h' + P_{hh}h'^2 = \delta(M_x + M_h h') \qquad (5.12)$$

Arranging the equation as a quadratic function of h', we find

$$P_{hh}h'^2 + (2P_{xh} - \delta M_h)h' + (P_{xx} - \delta M_x) = 0 \qquad (5.13)$$

The solution of this quadratic function gives the slope of the control path as,

$$\frac{dh}{dx} = \frac{(\delta M_h - 2P_{xh}) \pm [(2P_{xh} - \delta M_h)^2 - 4P_{hh}(P_{xx} - \delta M_x)]^{1/2}}{2P_{hh}}$$

This h' is the general expression of the slope of the feedback solution at equilibrium point and would be applied in section 5.4 for each of the three different demand situations of Bangladesh trawl shrimp fishery for optimal control path.

5.2.2 Algorithms of the Solution:

Algorithms of the solution procedure that we have followed for both the situations of with and without discounting are given below:

Algorithms are written in an informal notation called pidgin algol[1].

(i) Without Discounting:

```
Procedure Solution without:
begin
      K = 11400:1=1
         begin
               Find x such Uiat S'(x) = 0
               X*: = x
               Find the value of S(x) at x = x* to get Max S(X)
               Constant Po: = S(x*) = Max S(x)
               Find the value of h setting Po = P( x*, h)
               h*:=h
               (x*, h*) gives steady state
         end
         begin
               Take any value xfl]
               if xp] < K then find h for Po = P( x[i], h)
               if h< >0 then
               begin
                          h[i]:=h
                             i: = i+1
               end
         end
end.
```

[1] "The teem pidgin algol appears to have been introduced in A. V. Aho, J. E. Hopcroft and J. A. Ullman, The Design and Analysis of Computer Algorithms (Reading, Mass. : Addison-Weseley Publishing Co., Inc., 1974" - *Papadimitrioa & Steiglitz, Combinatorial Optimization: Algorithm &Complexity*, Prentice-Hall, India, 1997

(ii) With Discounting:

```
Procedure Solution with:
begin
    Limit = an assigned value : i=1
    [Comment: depending on the required precision the assigned value
    is a very small number representing zero]
    Find x setting S'(x) - 5 m = 0
    x*[i] := x
    begin
            Find h = f(x*[i])
            h*[i]:*h
          . Findh'at(xmh*[i])
            If —7 > limit then
               h
            begin
                    x*[i+1] :=x*p]+Ax
                    h*[i+1]:=h'[i]+Ax.  h'
            end
    end
end.
```

Algorithm of without discounting will provide steady state (x*, h*) at the beginning and then for different values of biomass levels, higher and lower than x*, would give corresponding values of harvest levels. This series of biomass levels and corresponding levels of harvest will give the complete trajectory of the control path when there is no discounting. Levels of harvest (h) corresponding to any given value of biomass levels (x) physically signify the optimal level of harvest at that level of biomass.

5.3 Optima] Control Path of Bangladesh Trawl Shrimp Fishery without Discounting:

Using the net profit function given in 4.2 and the values of other parameters i.e. c_2, q, u, r and K, the numerical solution of the non-linear dynamic system that we have formulated, now can be obtained.

With the willing to pay (WTP) function of harvest h described in section 3.3.1, the exact net benefit function of our model becomes,

$$n(h,x) = [(p_1 + (p_2 - p_1).e^{-bh}].h - \frac{\wedge}{qx} \quad \{5.15)$$

given that, pi = 6083.97729, pa= 13,977.93881 and b = 0.0004682.

Sustainable economic rent, therefore,

S(x) = n (x, h)|

$$= [(P_1+(P_2-p_1).e^{-bh}].h - \frac{c_2 h}{qx}$$

Since, at the steady state harvest equals to growth, we find

$$S(x)=[(p_1+(p_2.p_1).e^{-brx(1-\frac{x}{K})}].[rx(1-\frac{x}{K})]-\frac{c_2.rx(1-\frac{x}{K})}{qx}$$

and differentiating with respect to x, we have

$$S'(x)=p_1 r - \frac{\wedge}{K} + \frac{\wedge}{Kq} + (p_2-p_1).$$

$$[(rx-\frac{rx^2}{K}).(-\frac{2brx}{K}-br).e^{-brx(1-\frac{x}{K})}+(r-\frac{2rx}{K}).e^{-brx(1-\frac{x}{K})}] \quad (5.16)$$

$$\text{and } m=\wedge=p_1+(p_2-p_1).(1-bh).e^{-bh}-\wedge \quad (5.17)$$

The Hamiltonian equation of our model, given by (5.5), becomes

$$P_0=[\{p_1+(p_2-p_1).e^{bh}\}.h-\frac{\wedge}{qx}]+$$

$$[fe+fe \bullet!>!).(1-bh)e^{bh}\}-\wedge-].[fC»)-h)] \quad (5.18)$$

The Hamiltonian equation 5.18 defines the optimal harvest as a feedback control rule for non-linear exponential demand situation. Setting S'(x) = 0, we calculate x which implies optimal biomass x*. Again putting the value of x = x* in S (x), we get S (x*) = P_0 =Max S (x). Po remaining constant, each of the different values of x* when substituted in the Hamiltonian equation 5.18 gives corresponding different values of h*., These computed values, presented in Table 5.1, are optimal values of biomass stock and optimal harvest of marine shrimp of Bangladesh along the optima! control path. The optimal control path which implies optimal values of state variable (x*) a function of optimal control variable (h*) is shown graphically in Figure 5.1. This optimal control path shows the steady state situation.

5.4. Optimal Control Path of Bangladesh Trawl Shrimp Fishery With Discounting:

Steady state solution with discount of the marine shrimp of Bangladesh are computed in this section. We have already discussed the complexities involved in such type of solution in section 5.2. The steady state solution has been provided here in accordance with the assumed exponential demand function. Net revenue, function and the values of the parameters are same as in the case of without discounting.

The Hamiltonian equation will be same as 5.18 but it would not be equal to a constant Po. The Hamiltonian would be,

$$P=[\{p_1+(p\wedge-p_1).e^{h}M.h-\frac{\wedge}{qx}]+[\{p_1+(p_2-p_1).(1-bh).e^{bh}\}-\frac{\wedge}{qx}].Crx-\frac{\wedge}{K}-h) \quad (5.19)$$

where P is not given as constant. Partial differentiation of (5.19) with respect to x and h gives,

$$P_{x} := p_1 r - - \wedge^{+} \wedge^{+r} - (p_2 - p_1) - (1 - bh) d - - \wedge). e^{-bh} \quad \frac{}{K} \quad \frac{}{Kq} \quad \frac{}{K} \tag{5.20}$$

$$P_h = b.(p_2 - p_1).(bh.-2).(rx - \frac{rx^2}{K} - h).e^{-bh} \tag{5.21}$$

Differentiating partially (5.20) further with respect to x and h, we get 5.22 and 5.23 respectively as follows:

$$P_{xx} = \frac{-2p_1 r}{K} - \frac{2r(p_2 - p_1)(1-bh).e^{-bh}}{K} \tag{5.22}$$

$$P_{xh} = br(p_2 - p_1).(bh-2).(1 - \wedge).e^{-bh} \tag{5.23}$$

Similarly, differentiating partially 5.21 further with respect to h we get 5.24 as follows:

$$P_{hh} = b(p_2 - p_1) e^{-bh} [- b^2 hrx + \frac{b^2 hrx^2}{K} + b^2 h^2 + 3brx - \frac{3brx^2}{K} - 4bh + 2] ; \tag{5.24}$$

The co-state variable 'm' is given by the equation 5.17 as before. Partial differentiation of 5.17 with respect to x and h gives the relations 5.25 and 5.26 respectively.

$$m = - \frac{}{T} \tag{5.25}$$

$$m_h = b(p_2 - p_1).(bh-2).e^{-bh} \tag{5.26}$$

Following the algorithm that we have discussed in Section 5.2.2, we calculated the value of x* when S'(x) - 8 m = 0. This value of x* gives the corresponding value of harvest h* from the growth function and thus provide a point (x*, h*) in (x, h) plane. Equations 5.20 to 5.26 provide all the values of the arguments of equation 5.14, and thus, the slope or the tangent of the trajectory dh/dx, is calculated. Moving a little away from the equilibrium point along this tangent we get another starting point. With this as an initial point, we get a new equilibrium (x*, h*). Repetitive process gives us the separatrices of the optimal control path. Computed values of optimal biomass (x*) and optimal harvest (h*) are presented in Table 5.1 for discount rate 5 = 0.05 and 5 = 0.10. Graphical representation of the control paths for two different discount rates are shown in Figure 5.1. The graph shows the steady state where control path intersects with growth function and the minimum viable biomass level when control path intersects with horizontal axis.

Table 5.1
Optimal Harvest Levels of Bangladesh Trawl Shrimp at Different Discount Rates

Stock (tons)	Optimal Harvest (tons) (0% Discount Rate)	Optimal Harvest (tons) *(5%* Discount Rate)	Optimal Harvest (tons) (10% Discount Rate)
2000	-187.36	-108.34	-42.29
2500	167.12	295.96	363.76
3000	• 501.46	668.74	738.47
3500	823.09	1017.49	1089.82
4000	1139.28	1350.23	1426.35
4500	1457.84	1676.90	1757.71
5000	1788.34	2005.40	2096.01
5500	2144.53	2354.27	2459.43
6000	2550.48	2750.73	2883.95
6500	3063.12	3274.55	3500.13
7000	4468.40	—	~

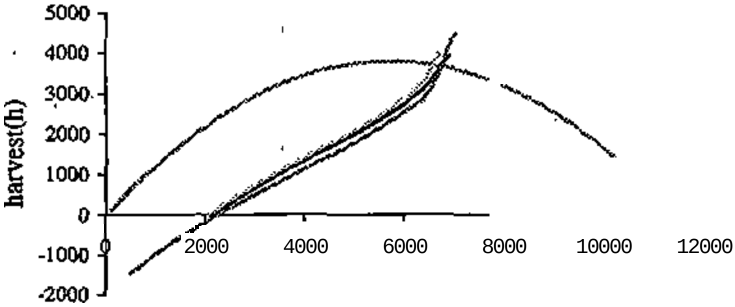

Figure 5.1: Optimal control paths (non-linear exponential demand function) with discount (5% &10%) and without discount (0%).

5.5. Results and Discussion:

Steady state solution of Bangladesh trawl shrimp fishery have been derived under 'with' or 'without' discounting condition. At different level of stock, optimal harvest levels are shown in Tables 5.1. Since, harvest represents control variable and stock represents state variable, these values when plotted along with growth function give us the optimal control path or optimal harvest path as shown in Figure 5.1. The point at which this control path intersects with growth function implies the steady state optimal situation, thus, we get optimal biomass stock (x*) and corresponding optimal harvest levels (h*). Table 5.2 is provided with such steady state optimal stock (x*) and optimal harvest (h*) under 'without' discounting and 'with' discounting condition.

Two different discount rates, 5 percent and 10 percent, are considered. Results presented in Table 5.2 reveal that optimal stocks or biomass levels are relatively lower for higher discount rate. Similarly, the optimal harvest levels appear relatively higher for the higher discount rate than the system under without discount. Both the results substantiate the expected notion regarding the direction of change. It is known that the biomass stock would

Table 5.2
Optimal Stocks and Optimal Harvests under Exponential Demand Situations for Different Discount Rates

	Optimal Stock (x*)			Optimal Harvest (h*)		
Demand function	8 = 0	6 = 0.05	5 = 0.10	5 = 0	5 = 0.05	5 = 0.10
Non-linear exponential	6866.35 (-)	6728.13 (-2.01)	6592.81 (-3.98)	3634.02 C-)	3669.43 (0.97)	3699.78 (1.81)

Note: Figures in the parenthesis indicate percentage change with the increase in discount rate.

be lesser with the increase in discount rate as the harvest level would be higher for higher discount rate. But the extent of variation due to discounting, shown by the percentage changes over the values of the parameters at the steady state (without discount) given in the parenthesis in Table 5.2, is very small and insignificant. Optimal control path along with growth function shown by the Figure 5.1 also indicates this insignificant variations due to discount factor. However, when we tried for the non-linear inverse demand function we observe that it shows relatively greater variation due to increase in discounting factor than the non-linear exponential demand function (Results of inverse demand function is not shown here).

These optimal control paths (or optimal harvest paths) along with growth function also provide minimum viable stock point. The point at which the optimal harvest path intersects with x-axis is called limit reference point. The physical significance of this point is that the exploitable biomass is at minimum viable stock Xmin. It implies that the exploitable biomass is at such a level that a harvesting moratorium is bioeconomically optimal at that point of stock. These values of Bangladesh marine shrimp and at different discount rates are given in the Table 5.3.

Table 5.3
Minimum Viable Levels (X^n)
for Different Discount Rates

X min			
Demand function	Discount Rate		
	5 = 0	8 = 0.05	5 = 0.10
Non-linear exponential	2260.05 (-)	2129.52 (-5.78)	2050.08 (-9.29)

Note: Figures in the parenthesis indicate percentage change with the increase in discount rate.

Results shown in the Table exhibit that $x_m$in values are 2260.05, 2129.52 and 2050.08 at zero, 5 and 10 percent discount rates. It shows that 5% and 10% increase in discount rate leads to 5.78% and 9.29% decrease in limit reference point respectively. It implies that the increase of discount rate upto 10%, decreases the limit reference point by 9.29 percent and, in absolute terms, it lies between twenty two hundred to twenty hundred metric tonns.

If optimal stocks (x*) and optimal harvests (h*) are known, then optimal values of other parameters can be obtained. By using relations 4.13 and 4.14 we calculated optimal effort levels (E*) and shadow prices (A*). These calculated optimal values are shown in Table 5.4.

Table 5.4

Optimal Efforts and Shadow Prices at Different Discount Rates

	Optimal Effort (E*)			Optimal Shadow Price (X*)		
Demand function		Discount Rate			discount Rate	
	5 = 0	5 = 0.05	5 = 0.10	5 - 0	5 = 0.05	5 = 0.10
Non-linear exponential	5415.27 (-)	5580.37 (3.05)	5742.01 (6.03)	5800.23 (-)	6028.20 (3.93)	6254.12 (7,83)

Note: Figures in the parenthesis indicate percentage change with the increase in discount rate.

Results show that higher the discount rate, higher is the optimal level for both effort and incremental cost or shadow price. Results clearly establish that the present level of actual effort is much higher than the optimal level of effort. As we find in the'last chapter, when the system was studied under discrete-time, that higher level of effort is employed at present to harvest lesser amount of catch - exactly identical situation is revealed by this continuous time analysis. It, therefore, causes loss due to the same reason that we have explain in the previous chapter {section 4.3). Following the same procedure, we calculated the notional losses for three different discount rates on the basis of optimal harvest and stock on 1997-98. As optimal harvest (x*) and optimal effort (E*) are different for different discount rates, the estimated losses would also be different. These estimated losses are given in Table 5.5, However, it is to be mentioned that causes of limitations in the estimation, explained earlier in Section 4.3, equally applicable here. Results presented in Table 5.5 show that the annual estimated loss is nearly $32 million. The result is almost identical to the losses under static and dynamic discrete analysis for the year 1997-98, As the cumulative loss of the nation over the whole period of our study starting from 1981-82 to 1997-98 compounded with appropriate rate upto present year would be same as before, we prefer to avoid repetition, and therefore, did not mention separately in a table here in this chapter again.

Table 5.5

Annual Losses Calculated at Different Discount Rates
(for the year 1997-98)

Demand function	5 = 0	5 = 0.05	8 = 0.10
Non-linear exponential	31814841.33	31832201.53	31767154.09

5.5.1. An Optimal Feedback Rule for the Bangladesh Trawl Shrimp fishery:

Many fisheries operate under some type of suitable feedback rule. Bangladesh marine shrimp fishery operates under an ad hoc policy at present. Different parameters, such as biomass stock, effort level etc., subject to constant change, are seldom taken into account as feedback to decide on the catch- quota at present. Naturally, the present practice of allotting trawling permission to different trawlers is not aimed at to optimize the resource utilization keeping the long-run sustainability in mind. Moreover, policy makers have to consider various type of socio-economic factors while allotting quota, such as, employment of people, investment made in advance of catching season by owners of a trawler, political

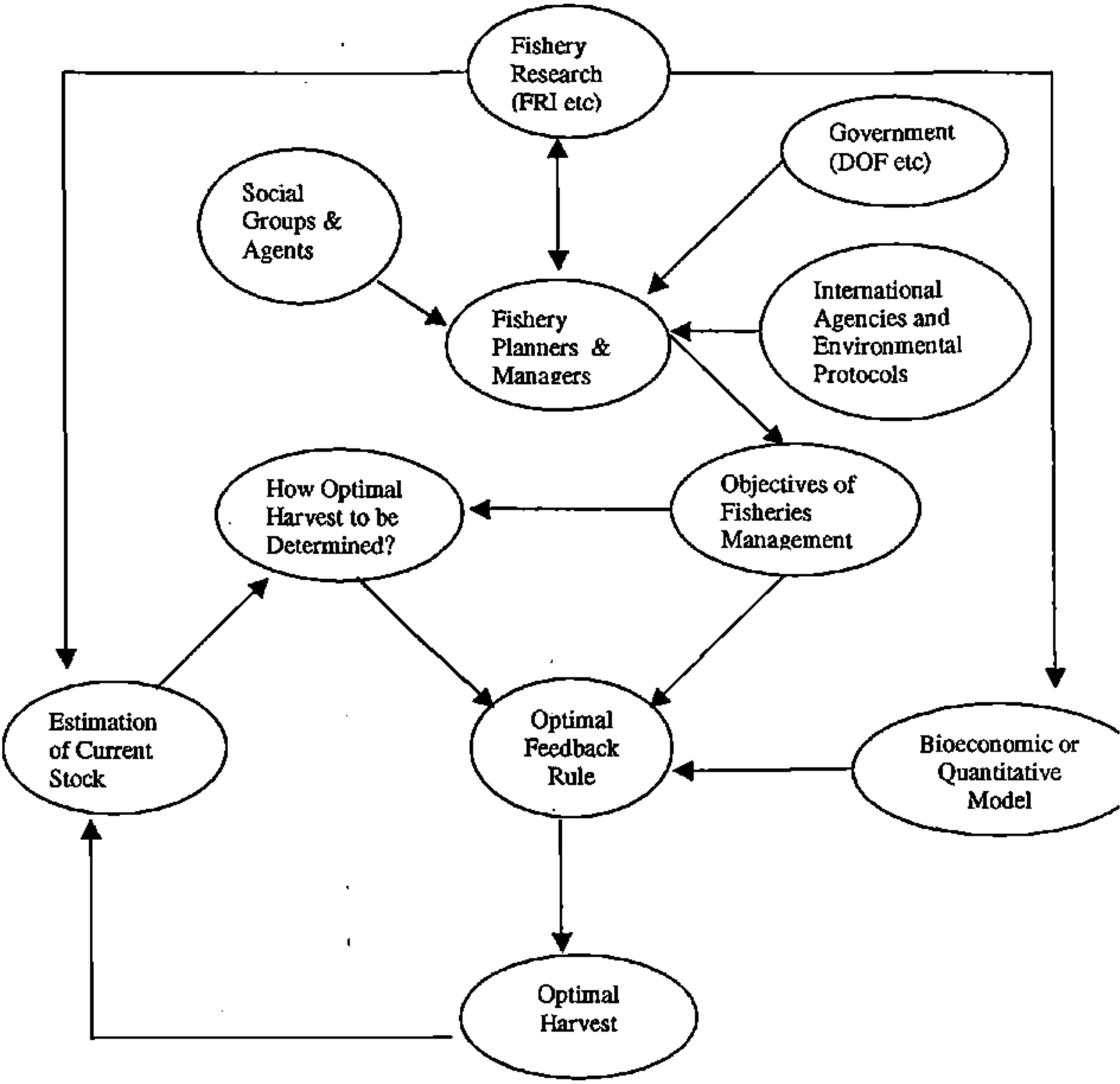

Figure 5.2: A Schema of Implementation of the Feedback Rule for Optimal Management of Trawl Shrimp Fishery in Bangladesh.

compulsion of avoiding social unrest among the people involved in the industry etc. All these factors combined together along with the lack of optimization model, gives birth an ad hoc management system which is in any sense be termed as optimal feedback rule.Our study perhaps be able to provide an optimization model which can be implemented as an optimal feedback rule for Bangladesh trawl shrimp fishery if proper system can be developed. A feedback rule means that the optimal control is a function of the state variable. This feedback rule may be used in a sense of passively adaptive policies (PAP) (Walters, 1986). It is adaptive for the reason that the control variable (harvest) changes immediately when new knowledge about the state variable (stock) is available. This feedback rule is PAP as sporadic shocks and disturbances can be handled within certain limits not beyond catastrophic discontinuities. Moreover, the adaptive management can be able to make the system gradually perfect as the data series grow. Results of our study already expressed how an optimal control path can be obtained.

This optimization model can determine the optimal harvest if the existing stock can be estimated. A simple method of stock assessment has been provided in Chapter-3. Given the values of other parameters - either determined by the appropriate govt, or non-govt agencies, the present model can be suitably used in ease. Following Grafton et. al, we provide here a simple schema in order to give an idea that how different factors can be considered simultaneously and,, at the same time, resource can be managed optimally as practical as possible. A set of objectives of trawl shrimp management would be laid down in consultation with several agencies by the fishery planners and managers. This set of objectives actually provide the specific values of different parameters of the model. For an example to illustrate, the policy decision by the planners that how the resource revenue would be discounted, would give us the value of the factor 5. Estimation of current stock, relevant data from the set of objectives and new information provided by the bioeconomic or quantitative research etc. would be used as an input of optimal feedback rule. For example, if a new study shows change in cost parameter or demand'curve, then those changes would be appropriately incorporated in the specific function of the model. Optimal harvest thus can be determined continuously and trawlers movements, trawling time, net size, gear size etc. can be monitored accordingly at any point of time of season. There is no need in fixing the quota prior to the season started and issuing license to the trawlers in an ad hoc basis.

However, we do not claim that this optimal feedback rule is able to capture all the complexities of the ecological and fish population dynamics, which is perhaps simply not possible, but it certainly provide a useful and easy-to-handle tool in determining optimal harvests and thereby to optimize the resource utilization with a long-run sustainability objective.

5.6. Concluding Remarks:

Results of this study presented in this chapter have the following findings and possible conclusions.

(a) We have found the optimal control path of Bangladesh marine shrimp fishery which is observed to be not managed and utilized optimally in this present study of non-linear dynamic (continuous) analysis in conformity with the findings obtained in previous chapter of discontinuous situation. We also obtained optimal steady state solution of the system. Thus it follows that following the optimal control path the present sub-optimal level of operation, level of exploitation of resource can be attained. This will also ensure to restore the optimal stock of resource froni the present sub-optimal level which appears to pose a genuine threat of depletion.

(b) The study also reveals that the optimal control path varies insignificantly with the social discount rate and thereby substantiate the theoretical findings of Farzin (1984), Hannesson (1983/1987) and Sandal and Steinshamn (1997).

(c) The study has been able to find out the minimum viable level of Bangladesh shrimp. The result, however, appears to be consistent with the expected theoretical value. However, the present practice of harvesting, the calculated minimum viable level and present estimated biomass stock together appears to be indicative that there is a possible threat of extinction if the present practice of harvesting continues.

(d) The study also show that due to non-optimal management system, the nation suffers loss both in terms of revenue and conservation of resource. Unlike cumulative revenue loss over the years calculated in the previous chapter, the loss in a particular year has been shown and the amount appears to be of serious proportion.

(e) The study has been able to formulate a feedback rule which gives control variable as a function of state variable. The study, therefore, suggest a scheme, through which the feedback rule can be implemented for effective management of the resource.

(f) The study is implicitly indicative in the sense that it seems to assert that a serious and in-depth study is required to understand the real dynamics of equilibrium situation under chaos. Dynamics of the system arising out of fluctuation in price and biomass population may help to formulate appropriate policy of the management of this resource.