CHAPTER I

LINEAR AND NON-LINEAR ANALYSES OF CIRCULAR AND RECTANGULAR PLATES

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LINEAR AND NON-LINEAR ANALYSES OF CIRCULAR AND RECTANGULAR PLATES

ABSTRACT

This chapter containing four problems, is devoted to study the static, dynamic and thermal behaviours of circular and rectangular plates under different edge conditions. These analyses are based on the linear and non-linear theory and confined to simple plate geometry.

DIFFERENTIAL EQUATIONS

Following linear theory the well known differential equation for free vibration of plates is

$$\nabla^4 W + \frac{\rho h}{D} \frac{\partial^2 W}{\partial t^2} = 0 \qquad (I.1)$$

For heated plates the above differential equation takes the following form

$$\nabla^4 W + (1+\nu) \alpha_t \nabla^2 \tau = 0 \qquad (I.2)$$

Following non-linear theory the well known Von Karman's equations (in polar co-ordinates) expressed in term of displacement components are

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = \frac{\nu - 1}{2r}\left(\frac{dW}{dr}\right)^2 - \left(\frac{dW}{dr}\right).\left(\frac{d^2W}{dr^2}\right) \qquad (I.3)$$

and

$$\frac{1}{r}\frac{d}{dr}\left[\left\{ r\frac{d^3W}{dr^3} + \frac{d^2W}{dr^2} - \frac{1}{r}\frac{dW}{dr}\right\} - \frac{12}{h^2} r \frac{dW}{dr}\left\{\frac{du}{dr} + \nu\frac{u}{r} + \frac{1}{r}\left(\frac{dW}{dr}\right)^2\right\}\right] = \frac{q}{D} \qquad (I.4)$$

A. VIBRATIONS OF CIRCULAR PLATES SUPPORTED AT SEVERAL POINTS *

Let us consider a circular plate of radius 'a' supported at several points along the boundary. Let the centre of the plate be the origin and a diameter as the initial line $\theta = 0$ (Fig. I.1).

Let

$$W(r, \theta, t) = W_1(r, \theta)\, e^{i\omega t} \tag{I.5}$$

with the substitution of this value of W in equation (I.1) one gets

$$\nabla^4 W_1 - K^4 W_1 = 0 \tag{I.6}$$

where $$K^4 = \frac{\rho h \omega^2}{D}$$

From (I.6) we have,

$$(\nabla^2 - k^2)(\nabla^2 + k^2) W_1 = 0 \tag{I.7}$$

Now the solution W_1 for equation (I.7) can be put in the form

$$W_1 = A_0 J_0(kr) + B_0 I_0(kr) + \sum_{m=1}^{\infty} \Big[\left\{ A_m J_m(kr) + B_m I_m(kr) \right\} \cos m\theta + \left\{ C_m J_m(kr) + D_m I_m(kr) \right\} \sin m\theta \Big] \tag{I.8}$$

where A_0 , B_0 , A_m , B_m , ... etc. are constants.

* To be presented at the Seminar on "Wave Motion In Continuous Media" sponsored by University Grants Commission in September 1984 (Dept.of Mathematics, University of North Bengal).

Considering the number of points of support is 'i' and denoting the concentrated reactions at these points by $N_1, N_2 \ldots N_i$ the expression for each reaction N_i is

$$\frac{N_i}{\pi a}\left[\frac{1}{2}+\sum_{m=1}^{\infty}\cos m\theta_i\right] \qquad (I.9)$$

where $\theta_i = \theta - \gamma_i$ and γ_i is the anglo dofining the position of the support i.

The intensity of reactive forces et any point of the boundary is then given by the expression

$$\sum_{i=1}^{i}\frac{N_i}{\pi a}\left[\frac{1}{2}+\sum_{m=1}^{\infty}\cos m\theta_i\right] \qquad (I.10)$$

in which the summation is extended over all the concentrated reactions.

For determining the constants of equation (I.9) we have the following boundary conditions.

$$(W_1)_{r=a} = 0 \qquad (I.11)$$

$\theta =$ at the supports

$$\left[\frac{\partial^2 W_1}{\partial r^2}+\frac{\nu}{r}\frac{\partial W_1}{\partial r}+\frac{\nu}{r^2}\frac{\partial^2 W_1}{\partial \theta^2}\right]_{r=a} = 0 \qquad (I.12)$$

$$\left[Q_r - \frac{1}{r} \frac{\partial}{\partial \theta} M_{r,t} \right]_{r=a} = - \sum_{i=1}^{i} \frac{N_i}{\pi a} \left[\frac{1}{2} + \sum_{m=1}^{\infty} \cos m\theta_i \right] \quad (I.13)$$

where

$$Q_r = - D \frac{\partial}{\partial r} \left[\nabla^2 W_1 \right] \quad (I.14)$$

and

$$M_{r,t} = (1 - \nu) D \left[\frac{1}{r} \frac{\partial^2 W_1}{\partial r \partial \theta} - \frac{1}{r} \frac{\partial W_1}{\partial \theta} \right] \quad (I.15)$$

With these above boundary conditions we have

$$C_m = D_m = 0 \quad (I.16)$$

$$J_o(ka)A_o + I_o(ka)B_o + \left[\sum_{m=1}^{\infty} \left\{ J_m(ka)A_m + I_m(ka)B_m \right\} \cos m\theta \right]_{\theta = \text{ at the suppts.}} = 0 \quad (I.17)$$

$$\lambda_1 A_o + \lambda_2 B_o + \sum_{m=1}^{\infty} \left\{ \lambda_3 A_m + \lambda_4 B_m \right\} \cos m\theta = 0 \quad (I.18)$$

$$D \lambda_5 A_o + D \lambda_6 B_o + D \sum_{m=1}^{\infty} \left\{ \lambda_7 A_m + \lambda_8 B_m \right\} \cos m\theta$$

$$= - \sum_{i=1}^{i} \frac{N_i}{\pi a} \left[\frac{1}{2} + \sum_{m=1}^{\infty} \cos m\theta_i \right] \quad (I.19)$$

Eliminating the four constant A_o, B_o, A_m and B_m we get the frequency equation in the following form,

$$\begin{vmatrix} J_o(ka) & I_o(ka) & J_m(ka) & I_m(ka) \\ \lambda_1 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & \lambda_4 \\ 2\lambda_5 & 2\lambda_6 & -\lambda_7 & -\lambda_8 \end{vmatrix} = 0 \qquad (I.20)$$

where

$$\lambda_1 = \frac{k\nu}{a} J_o'(ka) + k^2 J_o''(ka)$$

$$\lambda_2 = \frac{k\nu}{a} I_o'(ka) + K^2 I_o''(ka)$$

$$\lambda_3 = K^2 J_m''(ka) + \frac{k\nu}{a} J_m'(ka) - \frac{\nu m^2}{a^2} J_m(ka)$$

$$\lambda_4 = K^2 I_m''(ka) + \frac{k\nu}{a} I_m'(ka) - \frac{\nu m^2}{a^2} I_m(ka)$$

$$\lambda_5 = \frac{k}{a^2} J_o'(ka) - \frac{K^2}{a} J_o''(ka) - K^3 J_o'''(ka)$$

$$\lambda_6 = \frac{k}{a^2} I_o'(ka) - \frac{K^2}{a} I_o''(ka) - K^3 I_o'''(ka)$$

$$\lambda_7 = -K^3 J_m'''(ka) - \frac{K^2}{a} J_m''(ka) + \left(k \frac{1+m^2}{a^2} + km^2 \frac{1-\nu}{a^2} \right) J_m'(ka) - \left(\frac{2m^2}{a^3} + m^2 \frac{1-\nu}{a^3} \right) J_m(ka)$$

$$\lambda_8 = -K^3 I_m'''(ka) - \frac{K^2}{a} I_m''(ka) + \left(k \frac{1+m^2}{a^2} + km^2 \frac{1-\nu}{a^2} \right) I_m'(ka) - \left(\frac{2m^2}{a^3} + m^2 \frac{1-\nu}{a^3} \right) I_m(ka)$$

NUMERICAL RESULTS

The frequency equation (I.20) has been solved approximately by expanding the determinant. The following table shows the frequencies for different number of supports.

TABLE I.1

No. of Supports	Supports at points	Fundamental frequency occurs for	Value of fundamental frequency (ka) present study	Other frequencies occur for
3	$\theta = 0$ $\theta = \frac{2\pi}{3}$ $\theta = \frac{4\pi}{3}$	$m = 3$	Ka = 1.189	$m = 6, 9, 12$ ...etc.
4	$\theta = 0$ $\theta = \frac{\pi}{2}$ $\theta = \pi$ $\theta = \frac{3}{2}$	$m = 4$	Ka = 1.722	$m = 8, 12, 16$ ...etc.
5	$\theta = 0$ $\theta = \frac{2\pi}{5}$ $\theta = \frac{4\pi}{5}$ $\theta = \frac{6\pi}{5}$ $\theta = \frac{8\pi}{5}$	$m = 5$	Ka = 2.229	$m = 10, 15, 20$ ... etc.
∞	The plate is entirely simply supported		Ka = 2.2361	

DISCUSSION

It is interesting to note that the fundamental frequencies occur for m = 3,4 and 5 i.e. when number of support is three, four and five respectively.

From the above analysis we see that the frequency of vibration of a circular plate increases with the increase of the number of supports. When the number of support is infinity i.e. the plate is entirely simply supported and the pleta vibrates with frequancy Ka = 2.2361. Thus it can be concluded that for this type of vibration the maximum frequency is ka = 2.2361.

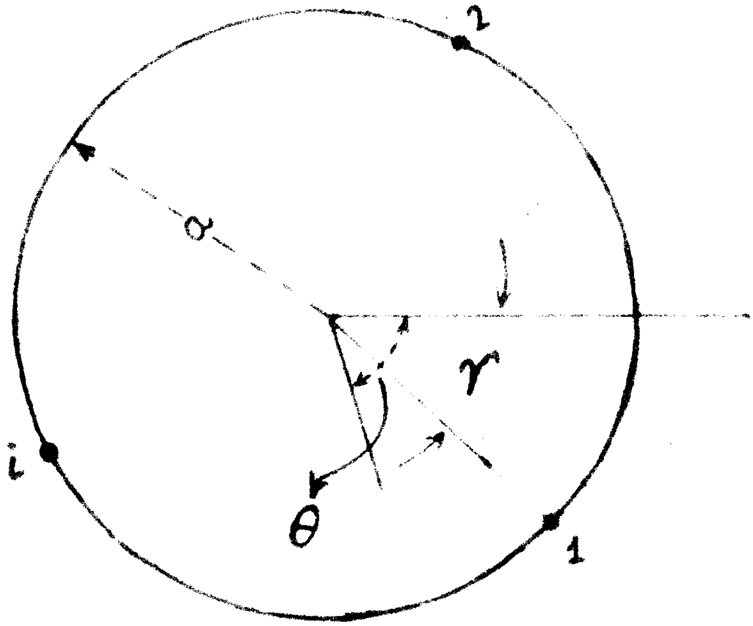

FIG. I.1

B. DEFLECTION OF A HEATED CIRCULAR PLATE SUPPORTED AT SEVERAL POINTS

Let us consider a circular plate of radius 'a' supported at several points along the boundary. Let the centre of the plate be the origin and a diameter as the initial line $\theta = 0$.

For simply supported plate it is sufficient to solve the following equation [Nowacki, Thermoelasticity, page 446]

$$\nabla^2 W + (1+\nu)\alpha_t \tau = 0 \qquad (I.21)$$

In polar co-ordinates the above equation reduces to

$$\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2} + (1+\nu)\alpha_t \tau = 0 \qquad (I.22)$$

We take the general expression for the deflection in the following form

$$W = W_0 + W_1 \qquad (I.23)$$

in which W_0 is the deflection of the plate simply supported along the entire boundary and W_1 satisfies

$$\nabla^2 W_1 = 0 \qquad (I.24)$$

Now solution for W_1 in the above equation can be put in the form

$$W_1 = A + \sum_{m=1}^{\infty} B_m r^m \cos m\theta \qquad (I.25)$$

where A and B_m are constants to be evaluated.

Considering the number of points of support is 'i' and denoting the concentrated reactions at these points by $N_1, N_2, \ldots N_i$, the expression for each reaction N_i is,

$$\frac{N_i}{\pi a} \left[\frac{1}{2} + \sum_{m=1}^{\infty} \cos m\theta_i \right] \qquad (I.26)$$

where $\theta_i = \theta - \gamma_i$ and γ_i is the angle defining the position of the support 'i'.

The intensity of reactive forces at any point of the boundary is then given by the expression

$$\sum_{i=1}^{i} \frac{N_i}{\pi a} \left[\frac{1}{2} + \sum_{m=1}^{\infty} \cos m\theta_i \right] \qquad (I.27)$$

in which the summation is extended over all the concentrated reactions. It is evident that the concentrated reactions in (I.27) are all functions of the thermal load τ.

For determining the constants of equation (I.25) we have the following boundary conditions

$$(W_1)_{\substack{r = a \\ \theta = \text{at the supports}}} = 0 \qquad (I.28)$$

and

$$\left[Q_r - \frac{1}{r} \frac{\partial}{\partial \theta} M_{r,t} \right] = -\sum_{i=1}^{i} \frac{N_i}{\pi a} \left[\frac{1}{2} + \sum_{m=1}^{\infty} \cos m\theta_i \right] \qquad (I.29)$$

where

$$Q_r = - D \frac{\partial}{\partial r} \left[\nabla^2 W_1 \right] \qquad (I.30)$$

and

$$M_{r,t} = -(1-\nu) D \left[\frac{1}{r} \frac{\partial^2 W_1}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial W_1}{\partial \theta} \right] \qquad (I.31)$$

With these boundary conditions we have

$$A = \frac{N}{\pi} \sum_{m=2}^{\infty} \frac{a^2}{(\nu-1) Dm (m-1)} \qquad (I.32)$$

and

$$B_m = \frac{N}{\pi} \cdot \frac{a^{2-m}}{(1-\nu) Dm (m-1)} \qquad (I.33)$$

Where $N_1 = N_2 = N_3 = \ldots\ldots\ldots\ldots = N_i = \frac{N}{i}$, i being the number of supports.

If τ = constant [Nowacki, Thermoelasticity, page 469] deflection W_0 is given by the expression :

$$W_0 = \frac{\alpha_t (1+\nu)\tau}{2} (a^2 - r^2) \qquad \text{(I.34)}$$

Thus for a thermally loaded plate the total deflection is given by

$$W = \frac{\alpha_t (1+\nu)\tau}{2} (a^2 - r^2) + \frac{N}{\pi} \sum_{m=2}^{\infty} \frac{a^2}{(\nu - 1)\, Dm\, (m-1)}$$

$$+ \frac{N}{\pi} \sum_{m=2}^{\infty} \frac{a^{2-m}\, r^m \cos m\theta}{(1-\nu)\, Dm\, (m-1)}$$

$$= \frac{1+}{2} K (a^2 - r^2) + \sum_{m=2}^{\infty} \left\{ \frac{1}{(\nu - 1)\pi m (m-1)} + \frac{a^{2-m} r^m}{\pi (1-\nu) m (m-1)} \cos m\theta \right\} Ka^2 \qquad \text{(I.35)}$$

where

$$K = \alpha_t \tau = \frac{N}{D}$$

NUMERICAL RESULTS AND DISCUSSIONS

Maximum deflection occurs at the centre of the plate. Thus

$$\left(\frac{W}{h}\right)_{max.} = \frac{1+\nu}{2}\cdot\left(\frac{Ka^2}{h}\right) + \sum_{m=2}^{\infty} \frac{1}{(\nu-1)\pi m(m-1)}\cdot\left(\frac{Ka^2}{h}\right) \qquad (I.36)$$

It is interesting to note that for two supports one must tak $m = 2, 4, 6, 8 \ldots$ etc., for three supports $m = 3, 6, 9, 12 \ldots$ etc., for four supports $m = 4, 8, 12, 16 \ldots$ etc., for five supports $m = 5, 10, 15, 20 \ldots$ etc.. For simply supported plate m is infinite.

Two graphs have been plotted. Fig. I.2 shows variation of maximum deflections with number of supports for the same load $\frac{Ka^2}{h} = 5$. Fig. I.3 shows different values of maximum deflections for different loads.

From the figures following observations can be made,

1) Fig. I.2 shows that deflection increases with the number of supports but after five or six supports the deflection attains almost the same as that of infinite no. of supports. Physically it is reasonable because when the number of support exceeds four the stability of the plate is almost same as that of a entirely simply supported plate.

2) From Fig. I.3 it is clear that as the load increases the deflection increases but this rate of increase diminishes after three supports.

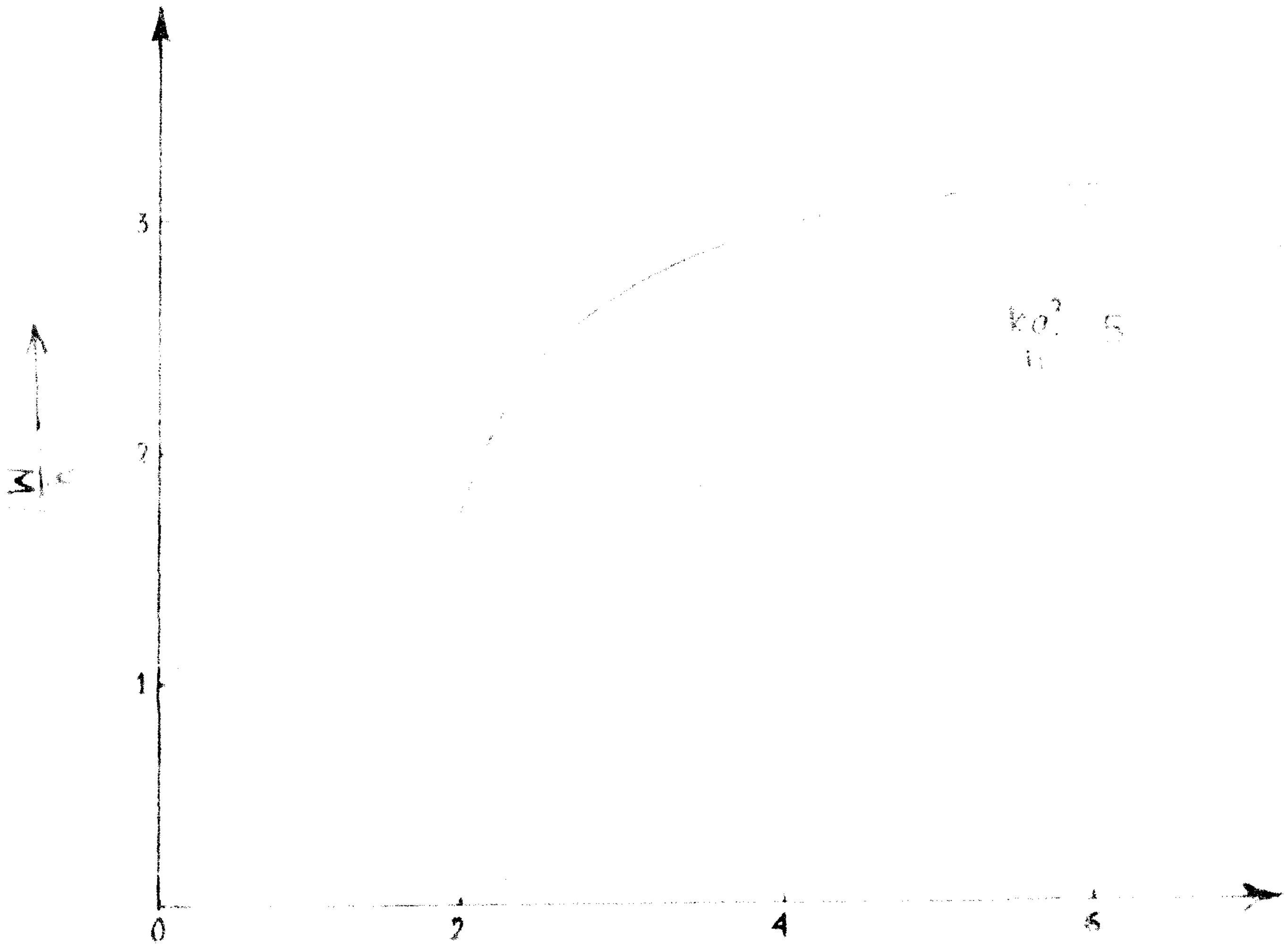

FIG. I . 2

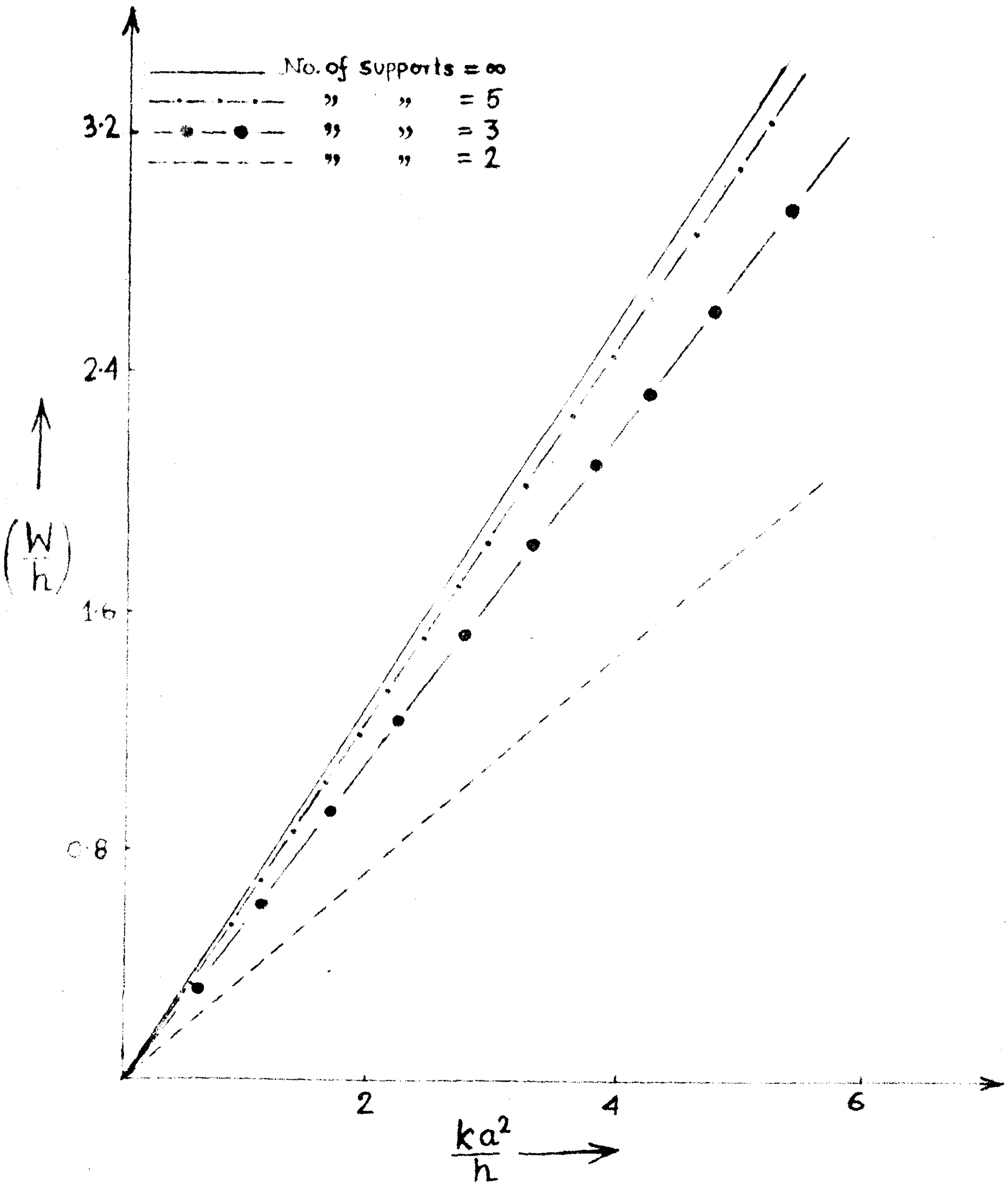

FIG. I.3

C. LARGE DEFLECTION OF A CIRCULAR PLATE UNDER SYMMETRICAL LOAD

Let us consider a clamped circular plate of radius 'a' with the centre of the plate taken as the origin.

Let there be a symmetrical distribution of transverse load varying as $(b^2 - r^2)^{1/2}$ over a concentric circular area of radius $b < a$. Hence

$$\frac{q}{D} = \begin{cases} f(r) = C\,(b^2 - r^2)^{1/2} & \text{for} \quad r < b < a \\ 0 & \text{for} \quad b < r < a \end{cases} \tag{I.37}$$

where C = a constant.

The equation (I.4) now becomes

$$\frac{1}{r}\frac{d}{dr}\left[\left\{ r\frac{d^3W}{dr^3} + \frac{d^2W}{dr^2} - \frac{1}{r}\frac{dW}{dr}\right\} - \frac{12}{h^2}r\frac{dW}{dr}\left\{\frac{du}{dr} + \nu\frac{u}{r} + \frac{1}{r}\left(\frac{dW}{dr}\right)^2\right\}\right] = f(r) \tag{I.38}$$

The boundary conditions for clamped edges are

$$(W)_{r=a} = 0 = \left(\frac{dW}{dr}\right)_{r=a} \tag{I.39}$$

Let us now assume the deflection W in the form

$$W = \sum_{S=1}^{\infty} A_s\left[J_0(P_s r) - J_0(P_s a)\right] \tag{I.40}$$

where J_o is the Bessel function of the first kind and zeroth order, P_s is the s th. root of $J_1(Pa) = 0$, J_1 being the Bessel function of the first kind and first order.

The equation (I.40) clearly satisfies the boundary conditions for clamped edges.

From equation (I.3) we get after substituting for W

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = \frac{\nu - 1}{2r} \sum_{s=1}^{\infty} \sum_{\substack{k=1 \\ (s \neq k)}}^{\infty} \left\{ A_s^2 P_s^2 J_1^2 (P_s r) + A_s A_k P_s P_k J_1 (P_s r) J_1 (P_k r) \right\}$$

$$- \sum_{s=1}^{\infty} \sum_{j=1}^{\infty} \left\{ A_s A_j P_s^2 P_j J_o (P_s r) J_1 (P_j r) - A_s A_j \frac{P_s P_j}{r} J_1 (P_s r) J_1 (P_j r) \right\} \quad \ldots \text{(I.41)}$$

Solving equation (I.41) one gets

$$u = Ar + \sum_{n=0}^{\infty} B_n r^{2n+3} \qquad \text{(I.42)}$$

where

$$B_n = \frac{B_1' + B_2' - B_3' + B_4'}{4(n+2)(n+1)} \qquad \text{(I.43)}$$

$$B_1' = (\nu - 1) \sum_{s=1}^{\infty} \frac{(-1)^n \Gamma(3+2n)}{n!\,\Gamma(3+n)\{\Gamma(2+n)\}^2} \left(\frac{1}{2}\right)^{2n+3} . P_s^{2n+4} . A_s^2 \qquad (I.44)$$

$$B_2' = (\nu - 1) \sum_{s=1}^{\infty} \sum_{\substack{k=1 \\ (s \neq k)}}^{\infty} \frac{(-1)^n {}_2F_1(-n, -1-n, 2; \frac{P_k^2}{P_s^2})}{n!\,\Gamma(2)\,\Gamma(n+2)} \left(\frac{1}{2}\right)^{2n+3} P_k^2 P_s^{2n+2} A_s A_k$$

... (I.45)

$$B_3' = \sum_{s=1}^{\infty} \sum_{j=1}^{\infty} \frac{(-1)^n {}_2F_1(-n, -n, 2; \frac{P_j^2}{P_s^2})}{n!\,\Gamma(n+1)\,\Gamma(2)} \left(\frac{1}{2}\right)^{2n+1} P_j^2\, P_s^{2n+2} A_s A_j$$

... (I.46)

$$B_4' = \sum_{s=1}^{\infty} \sum_{j=1}^{\infty} \frac{(-1)^n {}_2F_1(-n, -1-n, 2; \frac{P_j^2}{P_s^2})}{n!\,\Gamma(2)\,\Gamma(n+2)} \left(\frac{1}{2}\right)^{2n+2} P_j^2\, P_s^{2n+2} A_s A_j$$

... (I.47)

and A is a constant to be evaluated for the boundary condition imposed on u.

Now $u = o$ at $r = a$ for immovable edge

Hence $$A = - \sum_{n=o}^{\infty} B_n\, a^{2(n+1)} \qquad (I.48)$$

Thus u is completely known for clamped immovable circular plate.

Now from equation (I.38) we get after substituting the values of W and u from equations (I.40) and (I.42)

$$\begin{aligned}
&\sum_{s=1}^{\infty} A_s P_s^4 J_o(P_s r) + \frac{12}{h^2}\sum_{s=1}^{\infty} A_s P_s^2 J_o(P_s r)\Big[(1+\nu)A \\
&+ \sum_{n=o}^{\infty}(2n+3+\nu)B_n r^{2n+2} + \frac{1}{2}\sum_{k=1}^{\infty}\sum_{\substack{l=1\\(k\neq l)}}^{\infty}\Big\{A_k^2 P_k^2 J_1^2(P_k r) \\
&+ A_k A_l P_k P_l J_1(P_k r) J_1(P_l r)\Big\}\Big] + \frac{12}{h^2}\sum_{s=1}^{\infty} A_s P_s J_1(P_s r) \\
&\Big[\sum_{n=o}^{\infty}(2n+3+\nu)(2n+2)B_n r^{2n+1} + \frac{1}{2}\sum_{k=1}^{\infty}\sum_{\substack{l=1\\(k\neq l)}}^{\infty}\Big\{2A_k^2 P_k^3 J_o(P_k r) J_1(P_k r) \\
&- \frac{2}{r} P_k^2 A_k^2 J_1^2(P_k r) + P_k^2 P_l A_k A_l J_o(P_k r) J_1(P_l r) - \frac{2}{r} P_k P_l A_k A_l J_1(P_k r) J_1(P_l r) \\
&+ P_k P_l^2 A_k A_l J_o(P_l r) J_1(P_k r)\Big\}\Big] = f(r)
\end{aligned} \tag{I.49}$$

Now expanding f(r) in a series of Bessel functions one gets

$$\sum_{s=1}^{\infty} A_s \frac{a^2}{2} P_s^4 J_o^2(P_s a) + \frac{12}{h^2} \int_o^a \sum_{s=1}^{\infty} A_s P_s^2 \, r \, J_o^2(P_s r) \Big[(1+\nu) A +$$

$$\sum_{n=o}^{\infty} (2n+3+\nu) B_n r^{2n+2} + \frac{1}{2} \sum_{k=1}^{\infty} \sum_{\substack{l=1 \\ (k \neq l)}}^{\infty} \Big\{ A_k^2 \, P_k^2 \, J_1^2(P_k r) +$$

$$+ A_k A_l P_k P_l J_1 (P_k r) J_1 (P_l r) \Big\} \Big] dr$$

$$+ \frac{12}{h^2} \int_o^a \sum_{s=1}^{\infty} A_s P_s r \; J_1 (P_s r) J_o (P_s r) \Big[\sum_{n=o}^{\infty} (2n+3+\nu)(2n+2) B_n r^{2n+1} +$$

$$+ \frac{1}{2} \sum_{k=1}^{\infty} \sum_{\substack{l=1 \\ (k \neq l)}}^{\infty} \Big\{ 2 A_k^2 P_k^3 J_o (P_k r) J_1 (P_k r) - \frac{2}{r} P_k^2 A_k^2 J_1^2 (P_k r) +$$

$$+ P_k^2 P_l A_k A_l J_o (P_k r) J_1 (P_l r) - \frac{2}{r} P_k P_l A_k A_l J_1 (P_k r) \, J_1 (P_l r) +$$

$$+ P_k P_l^2 A_k A_l J_o (P_l r) J_1 (P_k r) \Big\} \Big] dr \; = \int_o^a f(r) r \; J_o (P_s r) \, dr \qquad (I.50)$$

The integration of the left hand side of equation (I.50) can be done numerically by well known Simpson's one-third rule. The integration of right hand side of equation (I.50) can be performed by setting $r = b \sin \theta$ in $f(r) = c(b^2 - r^2)^{1/2}$.

Thus

$$\int_0^a f(r)\, r J_o(P_s r)\, dr = \int_0^b cr\,(b^2 - r^2)^{1/2} J_o(P_s r)\, dr$$

$$= c\, b^3 \int_0^{\pi/2} \sin\theta \cos^2\theta J_o(P_s b \sin\theta)\, d\theta$$

$$= \frac{c b^3 J_{3/2}(P_s b)\,\Gamma(\tfrac{3}{2})}{(P_s b)^{3/2}} \qquad (I.51)$$

NUMERICAL RESULTS AND DISCUSSIONS

Evalueting the integrals of oquation (I.50) numerically as stated before, a cubic equation determining the deflection of the plate is obtained.

The following Table shows the maximum deflections of the plate against the different values of the load function $\frac{Cb^5}{h}$.

For convenience of the numerical calculation $s = 1, 2$; $j = 1, 2$; $k = 1, 2$; $L = 1, 2$; have been taken and these have yielded sufficient accurate results.

TABLE I.2

$\nu = 0.3$ $a = 2b$

$\frac{Cb^5}{h}$	$(W/h)_{max.}$	
	Present study	Results o[f] Dutta (19[…]
0	0	0
25	1.154	1.18
50	1.61	1.75
75	1.91	2.00
100	2.125	2.35

Thus it is seen that the present study has yielded results which are in excellent agreements with the known results. Also in the present study the numerical results have been obtained with less compatational labour than those obtained by Banerjee, B (1980) who used Von-Karman equation and solved the differential equation for the stress function completely.

D. NON-LINEAR ANALYSIS OF A RECTANGULAR PLATE WITH SIDES CLAMPED AND SUPPORTED UNDER A CONCENTRATED LOAD AT THE CENTRE

Following Berger's approximation, in cartesian co-ordinates, the strain energy V of a thin rectangular plate of sides 'a', 'b' and of thickness 'h' is given by

$$V = \frac{1}{2} \int_0^a \int_0^b D \left[\left\{ (\nabla^2 W + \frac{12}{h^2} e^2 \right\} - 2(1-\nu) \left\{ \frac{\partial^2 W}{\partial x^2} \cdot \frac{\partial^2 W}{\partial y^2} - \left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 \right\} \right] dx\, dy - \int_0^a \int_0^b qW\, dx\, dy \qquad (I.52)$$

where

$$e = \frac{1}{2} \left[\left(\frac{\partial W}{\partial x} \right)^2 + \left(\frac{\partial W}{\partial y} \right)^2 \right] \qquad (I.53)$$

and q is the load function, taken in the form

$$q = P \delta \left(x - \frac{a}{2} \right) \delta \left(y - \frac{b}{2} \right) \qquad (I.54)$$

P being the concentrated load at the centre of the plate. Let us now take W in the form [Wang Lei, 1982]

$$W = C \left[2 \frac{x^4}{a^4} - 5 \frac{x^3}{a^3} + 3 \frac{x^2}{a^2} \right] \left[2 \frac{y^4}{b^4} - 5 \frac{y^3}{b^3} + 3 \frac{y^2}{b^2} \right] \qquad (I.55)$$

This clearly satisfies the boundary condition given in figure I.4, namely

i) at $x = 0$, $y = 0$ $W = 0$ and $\frac{\partial W}{\partial x} = \frac{\partial W}{\partial y} = 0$

ii) at $x = a$, $y = b$

$$W = 0 \text{ and } \frac{\partial^2 W}{\partial x^2} = \frac{\partial^2 W}{\partial y^2} = 0.$$

Substituting (I.53), (I.54) and (I.55) in the strain energy relation (I.52) and then on integration we have

$$V = \psi_1 C^2 + \psi_2 C^4 - {}^{\cdot}0625 \; PC \qquad (I.56)$$

where

$$\psi_1 = \left[\frac{0.21743}{2} \left(\frac{a}{b^3} + \frac{b}{a^3} \right) + \frac{0.117551}{ab} \cdot \nu + (1 - \nu) \frac{0.117551}{ab} \right] \qquad (I.57)$$

and

$$\psi_2 = \frac{0.0006415}{ab} \qquad (I.58)$$

Now putting $\frac{\partial V}{\partial C} = 0$ one gets

$$\left(\frac{C}{h}\right) + \psi_3 \left(\frac{C}{h}\right)^3 = \psi_4 \qquad (I.59)$$

where $$\Psi_3 = \frac{2\Psi_2}{\Psi_1} \qquad (I.60)$$

and $$\Psi_4 = \frac{0.0625}{2\Psi_1}\left(\frac{P}{Dh}\right) \qquad (I.61)$$

NUMERICAL RESULTS

Numerical values of the coefficients Ψ_3 and Ψ_4 in equation (I.59) have been calculated for different side ratios of the rectangle. A graph (Fig. I.5) has been plotted showing the variation of the central deflection with load for side ratio $\frac{a}{b} = 1$ and $\frac{a}{b} = 2$. The results following linear theory have also been plotted side by side for comparison. It is interesting to note that the effect of non-linearity becomes prominent in the higher range of loading and this effect diminishes with the increase of the side ratio.

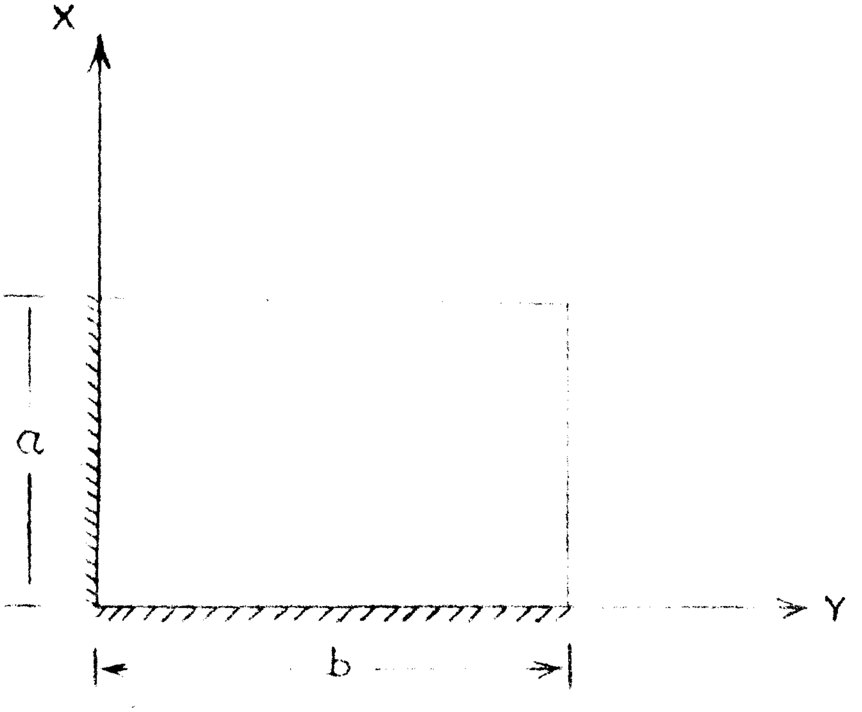

FIG. I.4

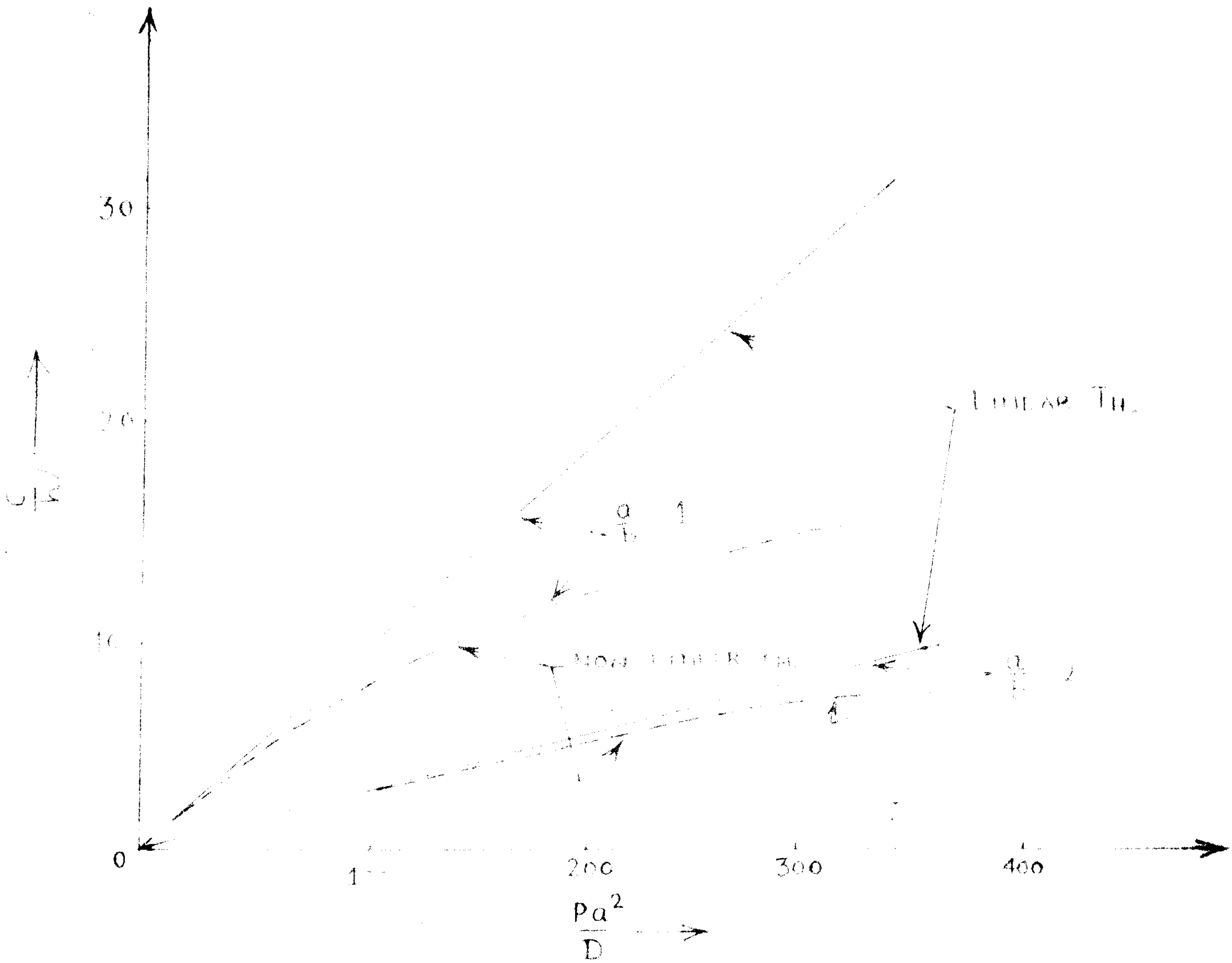

Fig. I.5