PREFACE

Thin plates of different shapes frequently occur in many structures and study of bending properties of a plate is imperative to a design engineer. If deflections 'W' of a plate are small in comparison with its thickness 'h', a very satisfactory approximate theory of bending of the plate by lateral loads can be developed neglecting the strains in the middle plane of the plates. In this way one gets the simplest and most widely used plate theory known as the classical small - deflection theory (Linear theory) developed by Lagrange (1811) and this is based on the following assumptions :

(i) points which lie on a normal to the middle plane of the undeflected plate lie on a normal to the mid - plane of the deflected plate ;

(ii) the stresses normal to the mid - plane of the plate, arising from applied loading are negligible in comparison with the stresses in the plane of the plate ;

(iii) the slope of the deflected plate in any direction is small so that its square may be neglected in comparison with unity ;

(iv) the mid-plane of the plate is a 'neutral plane', i.e. any mid-plane stresses arising from the deflection of the plate into a non-developable surface may be ignored.

With the increase of size and velocity in modern mechanics, the analysis of vibration problems becomes more and more important in mechanical engineering design. It is well known that problems of great practical significance such as the balancing of machines, the vibration of turbine blades and turbine discs, the vibretion of railway tracks and bridges under the action of rolling loads and the vibration of foundation can be thoroughly understood only on the basis of the theory of vibration.

Based on the linear theory, vibrations of different isotropic elastic plates under different edge conditions and under different mechanical loadings have been determined by several authors. The major contributions in this field are due to Timoshenko S. (1961), Liessn A. W. (1969), Chopra I. & Durvasula S. (1972), Chandrasekaran K. & Kunukkasseril V. X. (1974), Anon (1975), Laura, Arias & Luisoni (1976), Sundararajan C. (1978), Irie T., Yamada G. & Narita Y. (1978), Kerstens J. G. M. (1979), Chan H. C. & Foo O. (1979), Leisea A. W. & Nerita Y. (1980), Prakash B. G. (1980), Groman D. J. (1981) and Laura P. A. A. (1981).

Problems of plates of orthotropic materials are always encountered in design and among the successful research workers whose works need special mention in this field are Mansfield E.H. (1964) , Timoshenko S. & Woinowsky - Krieger S. (1959) , Maurizi M.J. & Laura (1973), Soni S. R. & Amba Rao C. L. (1974) , Pratap G. & Varadan T. K. (1976) , Greenberg J. B. & Stavsky Y. (1978) , Sakata T. (1979), Kutbler J. R. & Sigillito V. G. (1980) , Laura P. A. A., Luisoni L. E. & Sanchez Sarmiento G. A. (1980) and Dobyns A. L. (1981). Mansfield and Timoshenko, Woinowsky - Krieger independently discussed various static problems of thin elastic plates of different shapes of orthotropic materials under different edge conditions and loadings. Their solutions are very interesting and accurate from the engineering point of view.

Thermally induced problems are of interest in aircraft and machine design, in nuclear engineering and in astronautical engineering. The formulation of elasticity problems including the effect of temperature variation was studied by Duhamel (1837) shortly after the basic formulation of the linear theory of vibration. But investigations of the effect of the inelastic behaviour of the materials have begun only in recent years. Among different workers whose works are noteworthy are Johns D.J. (1965) , Parkus H. (1976) , Nowinski J.L. (1978) , Boekens F. (1962) , Mansfield E. H.

(1962) , Newman M. & Forray M.J. (1962) , Sunakawa M. & Uemura M. (1960) , Sekiya T. , Sumi S. , Matsumoto E. , Katayana T. & Sugimoto I. (1976) , Misra J.C. (1973) , Boley A. & Barbar D. (1957) , Nowacki W. (1962) , Boley A. & Weiner H. J. (1960) , Boley A. (1972) , Kao W.T. & Pao Y.C. (1976) , Irie T. & Yamada G. (1978) , Buckens F. (1979) , Ganesan N. (1979) and Takeuti Y. & Furukawa T. (1981).

Johns in his book 'Thermal Stress Analysis' discussed many linear and non-linear problems of heated elastic plates and shells under different edge conditions. His analyses are accurate and interesting too. He has also discussed the effects of thermal buckling of different plates and shells.

Parkus in his book 'Thermoelasticity' gave a detailed analysis on different linear problems of heated rectangular and circular plates. He has also included some important discussions on thermal buckling and thermal vibrations of plates.

Boley and Barbar were the first to investigate thermal shock problems in rectanguler plates. Later on, Nowacki and Boley and Weiner merely included the above works of Boley and Barbar in thair respective monographs.

Viscoelastic pletes are interesting to design engineers for their time dependent properties. Based on linear theory many workers investigated the viscoelastic problems. Among them the works of Mase G. E. (1960), Sarkar S. (1964), Mase G. E. & Deleeuw S. L. (1963), Deleeuw S. L. (1961, 1965 & 1971), Mazumdar J. & Hewith J. (1974, 1976), Nagaya K. (1977), Kanovich M. Z. & Koltunov M. A. (1978), Negaya K. (1978) and Saito H. & Yamaguchi H. (1980) need special mention.

Mase investigated the problem of viscoelastic rectangular plate accepting the model offered by Biot (1955). He obtained the solution by the correspondence principle for a rectangular plate under uniformly distributed load over the whole area of the rectangle. Sarkar investigated the problem of a rectangular plate under a concentrated load accepting the same model of Biot. Later on Mazumdar and Hewitt jointly gave a general study of buckling of viscoelastic plates of arbitrary shape with full illustrations of the problems of a clamped rectangular and clamped semi-circular plates. Their analyses are very interesting and accurate from the engineering point of view but they require much computational labour.

Other than general viscoelastic property, materials at high temperature exhibit creep and thus the mechanical properties of the plates must be time dependent as well as temperature dependent. So the plates once again exhibit the property of viscoelasticity. A few works could be located in this field. These works are due to Misra J.C. & Samanta S. (1980), Mazumdar J., Hill D. & Clements D.L.(1980), Hill D., Mazumdar J. and Clements D.L.(1982).

The main bulk of the classical approach in applied mechanics rests on the assumption that the phenomena involved can be adequately described by linear mathematical models. With the advent of modern technology and systems exposed to oppressive operation conditions this hypothesis could no longer be retained. Whenever forces, deformations, velocities, temperatures and other factors become excessive, non-linear effects come into play and their influence can no longer be ignored. This situation occurs also in a particular field of applied mechanics involving plates and shallow shells. Thus when the deflections of a plate are no longer small in comparison with its thickness the supplementary stresses in the middle plane of the plate must be taken into account in deriving the differential equations governing the deflection of the plate. In this way one gets the non-linear differential equations in the classical non-linear theory of plates.

The coupled non-linear partial differential equations for large amplitude axisymmetric deformations were initially derived by Von Karman (1910). These equations in the coupled form are difficult to solve and different numerical methods have been employed by different authors for investigation of large deflections of plates.

The outstanding research workers who employed Von Karman's equations to analyse the non-linear character of thin plates are Chu H.N. and Hermann G. (1956), Weil N.A. & Newmark N.M. (1956), Nash W.A. & Cooley I.D. (1959), Nowinski J.L. (1963), Nowinski J.L. & Ismail I.A. (1965), Schmidt R. (1968), Bauer H.F. (1968) and Bolton R. (1972).

Chu and Hermann quite elegantly analysed the influence of large amplitudes on free flexural vibrations of rectangular elastic plates. The authors' investigations are based on Von Karman's equations which have been solved by numerical method.

Weil and Newmark investigated the large deflections of elliptic plates. The authors have solved Von Karman's equations numerically and the method of solution is laborious.

Nash and Cooley also investigated the large deflections of elliptic plates. In this paper Von Karman's equations have been solved by the method of perturbation.

Nowinski quite elegantly analysed non-linear vibrations of elastic circular plates exhibiting the rectilinear orthotropy. The author has excellently solved the differential equations for the stress functions with the help of a trial function involving fourteen unknown constants.

Nowinski and Ismail treated the non-linear dynamic behaviour of triangular plates. The authors have solved Von Karman's equations by semi-inverse method. This is an example of the application of Von Karman's equations to a plate geometry other than circular or rectangular.

Schmidt's work is concerned with the large deflection of a clamped circular plate under a concentrated load at the centre. The author has employed the method of iteration to obtain his solution. Schmidt's observation is noteworthy.

Bauer investigated quite elegantly the dynamic response of a circular and rectangular plate due to pulse excitations. The author has solved the corresponding differential equations for the stress functions completely.

Bolton's paper presents an interesting solution of stresses of circular plates on elastic foundations of the Winkler type. The author has solved Von Karman's equations with the help of error minimising technique. Bolton's numerical results are of practical interest in the modern design.

The other useful workers who treated Von Karman's equations to analyse the non-linear behaviour of different elastic plates are Sayed M.A. and Schmidt R. (1977) , Satyamoorthy M. (1979) , Karmaker B.M. (1979) , Banerjee B. and Dutta S. (1980) , Banerjee B. (1982) , Chowdhury S. (1983) and Sathyamoorthy M. (1981).

An approximate method for solving the large deflections of thin plates has been proposed by Berger H.M. (1955). Berger's method is based on the neglect of the second strain invariant of the middle surface strains in the expression corresponding to the total potential energy of the system. Although no complete explanation of this method is given, the results obtained so far for different problems are in good agreement with those obtained from precise analysis. Following this approximate method, different non-linear plate problems have been solved with remarkable ease by many investigators, among which the works of Iwinski T. & Nowinski J.L. (1957) , Nash W.A. & Modeer J. (1959) , Basuli S. (1961) , Sinha S.N. (1963) and Banerjee B. (1967) need special mention. It is to be noted that Berger's method is still an intriguing subject.

Iwinski and Nowinski succesefully applied Berger's method to analyse the large deflections of orthotropic circular and rectangular plates and achieved satisfactory results.

Nash and Modeer extended Berger's method to investigate the non-linear dynamic behaviour of elastic plates and obtained solutions which are in excellent agreement with those obtained from classical equations.

Basuli employed Berger's equations to obtain the solution of a clamped circular plate under the concentrated load at large deflections. The method of solution represented in the paper is very interesting.

Sinha obtained an interesting solution of the large deflection of circular and rectangular plates on elastic foundations of the Winkler type. The author employed Berger's equations and his numerical results are very interesting from the engineering point of view.

Banerjee discussed the large amplitude free vibrations of elliptic plates based on Berger's approximations. Mathieu functions have been used in this investigation.

Other successful workers who worked on Berger's equations are Dutta S. (1975, 1976) , Banerjee M.M. (1976) , Biswas P. (1975) , Karmakar B.M. (1977) , Mazumdar J. & Jones R. (1977).

From the above survey it is observed that almost all the investigations except a few are confined to a particular plate geometry of the simple type such as circular or rectangular. But in modern design , plates of different irregular shape frequently occur and these types of irregular shaped plate-problems are of much interest to design enginoers.

If the boundary of a plate is a curve natural to any of the common coordinato systems, the governing equations can be solved in terms of known functions. For more "exotic" boundaries, the natural co-ordinates must first be determined and after this is done, the solution would inovitably involve some unfamiliar functions. Therefore, a common co-ordinate system and its associated function are advantageous for plates having complicated boundaries. If the given domain can be conformally mapped on to a simpler one, i.e. the unit circle, the problem then reduces to the solution of the transformed differential system. This is known as the conformal mapping technique which is based on the complex variable theory.

The conformal mapping technique is more useful because :

(i) If the mapping function is known the static, thermal & dynamic behaviour of plates of any shape can be obtained from the solution of the same differential equation and thus minimises the computational labour.

(ii) The solutions of irregular shaped plates are attractive now-a-days in modern design. These solutions can only be obtained with ease and accuracy with the help of this method.

(iii) Problems of polygonal plates have taken little attention in the current literature. The study of polygonal plates is possible with the help of this technique.

(iv) A comparative study of the static, dynamic and thermal behaviours of different plate problems can be done using this conformal technique.

From the survey of literature on arbitrary shaped plate by the method of conformal mapping technique, only a few works could be located. These are mainly due to Gutierrez R.H., Laura P.A.A. & Steinberg D.S. (1977), Laura P.A.A., Luisoni L.E. and SanchezSarmiento (1980), Laura P.A.A. & Gutierrez R.H. (1981), Banerjee B. and Datta S. (1979) and Banerjee M. M. (1976). Thus investigations of different elastic plate problems using conformal technique are necessary in the interest of modern design.

The main purpose of the present thesis is to show the application of the conformal mapping technique in analysing the linear and non-linear behaviour of thin elastic plates of arbitrary shape. To do this the thesis has been divided into two chapters. The first chapter containing four problems is confined to simple plate geometry. The second chapter, the major portion of the thesis, is devoted to the investigation of different plate-problems of arbitrary shape using conformal mapping technique. Thus a comparative study has been done showing the merits of the conformal technique.

The first paper of this chapter deals with the vibrations of circular plates supported at several points. The corresponding static problem has been investigated by Timoshenko & Woinowsky - Krieger (1959) and has been experimentally verified by Nadai A. (1925). Dutta S. (1975) and Banerjee M. M. (1976) discussed the non-linear static behaviour of circular plates supported at several points based on controversial Berger's approximation. On the other hand this paper presents a complete analysis of the problem with any number of support. The corresponding frequency equations have been deduced and given in a tabular form.

The second paper is devoted to analyse the deflections of heated circular plate supported at several points. Determination of thermal stresses in thin elastic plates is of immense interest in modern design. A comprehensive bibliography of different research workers in this field is given in the book "Thermoelasticity" by Nowacki W. (1962). But as far as it is known no work has been devoted to investigation of thermal deflections of circular plates pinned at several points. The aim of the present paper is to study the deflections of heated circular plate supported at several points with a complete analysis of the particular cases where the number of support is two, three, four, five and infinite number. Results obtained have been plotted graphically and discussed.

The third paper deals with the non-linear analysis of a clamped circular plate under symmetrical loads. Von Karman's equations expressed in displacement components have been used in the investigation.

The behaviours of circular plates under symmetric transverse loading have been investigated by Dutta S. (1975) and Banerjee B. and Dutta S. (1980). The former author considered the problem based on Berger's approximation while Banerjee and Dutta jointly investigated the same problem following Von Karman's field equations and solving differential equations for the stress function completely. Their solution is accurate but requires much computations. In the present paper the same problem has been investigated following displacement formulations of Von Karman's field equations. The deflections are obtained in the form of an infinite series involving Bessel functions. Numerical results showing variation of maximum deflection with load function are given in tabular form and compared with known results.

The fourth paper discusses non-linear analysis of a rectangular plate with sides clamped and supported under concentrated load at the centre. The investigation is based on well known Berger's approximation. Various authors followed Berger's method for solutions of various large deflection plate problems involving static, dynamic as well as thermal loadings with ease and sufficient accuracy. But from survey of current literatures on large deflection problems it is found that no paper has apparently been devoted to study the non-linear characteristics of plates clamped as well as supported. In the present papar non-linear analysis of rectangular plate with sides clamped and simply supported under a concentrated load at the centre has been discussed. The corresponding small deflection problem has been studied by Wang Lei (1982). Numerical results have been obtained for different side ratios of the rectangle and discussed.

The second chapter, containing seven problems, is the major part of the thesis. It is devoted to study the static, dynamic and thermal behaviours of polygonal plates using conformal mapping technique. These analyses are based on the linear and non-linear theory.

The first paper of the second chapter deals with the deflections of clamped orthotropic polygonal plates under the action of concentrated load at the centre. The dynamic behaviours of different isotropic plates have been investigated by Laura P. A. A. (1967) and his co-workers. They have applied conformal mapping technique to get their solutions. The deflections of irregular shaped plates under uniform load have been studied by Mansfield E. H. (1964). But no paper has apparently been devoted to investigate the behaviours of orthotropic irregular shaped plates under concentrated loadings. In this paper the deflection of orthotropic polygonal plates under the action of concentrated load at the centre has been investigated. Conformal mapping technique has been applied to get the desired solutions. The solutions thus obtained are sufficiontly accurate for practical purposo. They are given in tabular form for both weaker and stronger orthotropy. Tho results for square and circular isotropic plates deduced from the present study are found to be in excellent agreement with those obtained by Timoshenko and Woinowsky - Krieger (1959).

The second paper of this chapter deals with the study of vibrations of clamped orthotropic polygonal plates. The conformal mapping technique has been used by various authors to study dynamic behaviours of polygonal isotropic plates. But the investigation of the vibrations of polygonal orthotropic plates has attracted little attention. The object of the present paper is to study the vibrations of polygonal plates of orthotropic materials with sides clamped. The well known Huber's approximation (1929) has been employed. The solution of the differential equation in this case has been obtained in terms of Bessel's functions. The frequency equations have been deduced with the help of conformal mapping technique. The solutions thus obtained for different polygonal plates are given in tabular form. The result for square orthotropic plate is in good agreement with the known result.

The third paper of the chapter deals with vibrations of polygonal pletes, due to sonic boom. The importance of this type of problems is day-by-day increasing, especially after the advent of supersonic aircrafts. The boom disturbance moves in form of shock waves and creates not only annoyence to the living beings but also has an adverse effect upon the building industry. The successful research workers in this field are Doak P.E. (1965) , Crocker and Hudson (1969) , Cheng and Benveniste (1968). In recent years Mazumdar and Coleby (1976) jointly studied the vibretions of arbitrary pletes due to sonic boom. They solved the problem with the help of constant deflection contour method. The results thus obtained are sufficiently accurate and interesting but they require much computations. In the present case the same problem has been studied using complex variable theory. The deflections of such vibrations for polygonal plates have been obtained by Galerkin's error minimising technique. The results obtained are in excellent agreement with those of Masumdar and Coleby.

The fourth paper of this chapter aims at the investigation of the deflections of regular polygonal viscoelastic plates with edges simply supported. Viscoelastic plates are interesting to design engineers for their time dependent properties. From the survey of literature it is observed that only a few workers have been attracted in this field. Mase G. E. (1960) and Sarker S. (1964) independently investigated the viscoelastic problem of rectangular plates under concentrated load accepting the model offered by Biot. Masumdar J. and Hewitt J. (1974, 1976) jointly investigated the buckling of viscoelastic plates of arbitrary shape following constant deflection contour method. In the present paper attempt has been made to investigate the problem of regular polygonal plates applying conformal mapping technique. Galerkin's error minimising method has also been used to obtain the final solution. Numerical results have been obtained for different plates and given in tabular form and discussed. The results for square plates have been compared with known results and found to be in excellent agreement with those of Mase G. E. (1960).

The fifth paper of this chapter deals with the vibrations of elastic plates due to thermal shock. In modern days the thermally induced vibrations are interesting in aircraft machine design and in astronautical engineering. All investigations in this field were based on the assumption that the non-stationary motion which results from the thermal shock can be written as the sum of a quasi-static displacement 'W_s' and a dynamic displacement 'W_d'. According to the above assumptions the following two differential equations result :

$$\nabla^4 W_s + K_s \nabla^2 \tau = 0 \qquad \ldots (a)$$

$$\nabla^4 W_d + K_d (\ddot{W}_s + \ddot{W}_d) = 0 \qquad \ldots (b)$$

It would seem however that the quasi-static assumption which resulted in the above two equations is not valid for thermal shock problems, not indeed it is valid in general for any type of impulsive or step loading. The quasi-static assumption is valid when the thermal loading is applied slowly so that the inertial forces are negligible. For accurate analysis of thermal shock problems there is no need for the quasi-static assumption and only one differential equation is to be solved for time dependent plate deflection. Here in the present problem the vibrations of polygonal plates due to thermal shock have been investigated without the usual quasi-static assumption. Complex variable theory has been applied in the investigation. The maximum displacement has been obtained using Galerkin's error minimising method. The results thus obtained are found to be in excellent agreement with the known results. The numerical results have been plotted graphically and discussed.

The sixth paper of this chapter aims at the investigation of the thermal deflection of regular polygonal viscoelastic plates. At high temperature materials exhibit creep and thus the mechanical properties of plates must be time dependent as well as temperature dependent. So the plates then become viscoelastic in character. In recent years some works have been done in this field. But no paper has apparently been devoted to investigate the deflection of thermally loaded viscoelastic plates of polygonal shape. In the present study of heated viscoelastic polygonal plates conformal mapping technique has been employed. The results obtained are simple and sufficiently accurate. Numerical results have been given for different polygons. Results for square plates have been compared with known results and found to be in excellent agreement.

The last paper of this chapter is a non-linear vibration - problem of polygonal orthotropic plates under the action of concentrated mass at the centre. The problems of vibrations of plates with a concentrated mass at any arbitrary location are of interest in aircraft and structural industries. As far as it is known the useful workers in this field are Chen Y. (1936), Amba Rao C. L. (1964), Laura P.A.A. and his co-workers (1967, 1974, 1976 and 1977). All these workers investigated their problems based on the linear theory. Regarding literature on the non-linear analysis of elastic plates carrying concentrated mass the works of Chiang D.C. and Chen S.S.H. (1972), Ramchandran J. (1973, 1976) and Karmakar B.M. (1978) need special mention. From the survey of literature it is found that no paper has apparently been devoted to the large amplitude vibrations of orthotropic polygonal plates with concentrated mass at the centre. In the present paper the behaviours of orthotropic polygonal plates under the simply supported and clamped edge conditions have been studied following Berger's approximation.

Complex variable theory has been used. The desired solutions have been obtained by Galerkin's error minimising method. Frequencies of free vibrations of polygonal plates under different edge conditions have been presented graphically and discussed. The results of a clamped square plate of side 'a' for different values of concentrated mass have been given in tabular form for comparison with other known results.