CHAPTER VI

STATES ESTIMATION OF ELECTRICAL POWER SYSTEM BY A TRACKING ALGORITHM

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STATE ESTIMATION OF ELECTRICAL POWER SYSTEM BY A TRACKING ALGORITHM

6.0 Introduction

The reliable operation of a power network depends on a real time data base for monitoring, security and control of power system. State estimation programmes can enhance the data base for on-line real time operation of power networks. The basic function of an estimation programme is to convert the telemetered raw measurement data into a reliable information base. The information base contains all complex bus voltages, power and current flows as well as injections along with network status and parameters errors. For a successful operation of a state estimator it is essential that the measurement system must have a degree of redundancy greater than unity (i.e., (number of measurements)/(number of state variables)>1). Because the redundancy in the measurement set improves state estimation accuracy.

6.1 State Estimation Tracking Algorithm

The System Equation is

$Z = f(x) + \eta$ for the k-th iteration

$$Z(k) = f(x(k)) + \eta_k \qquad (6.1.1)$$

Let

$$\Theta = \mathcal{E}\{\eta\eta^T\} \qquad (6.1.2)$$

$f(x)$ is a non-linear function of x, and S is defined as a positive, definite weighting matrix as

$$S = \mathcal{E}\{(x - \bar{x})(x - \bar{x})^T\} \qquad (6.1.3)$$

The objective function is formulated as for the k-th iteration

$$J_k(x(k)) = \sum_{j=1}^{k} [Z(k) - f(x_k)] \Theta^{-1}(k) [Z(k) - f(x_k)]$$

$$+ [x - x(o)]^T S^{-1}(o) [x - x(o)] \qquad (6.1.4)$$

$x(o)$ is a priori estimate x and $S(o)$ is the initialised positive definite weighting matrix.

By Taylor's series expansion, neglecting higher order terms

$$Z(k) - f(x_k) = \Delta Z(k) - \Delta x^T F(x_{k-1}) \qquad (6.1.5)$$

$$\therefore\; J_k(x_k) = \sum_{j=1}^{k} \left[\Delta z(j) - \Delta x^T F(x_{j-1}) \right]^T \theta^{-1}$$

$$\left[\Delta z(j) - \Delta x^T F(x_{j-1}) \right] + \Delta x^T S^{-1}(o)\, \Delta x \qquad (6.1.6)$$

For minimisation

$$\frac{\delta J_k(x_k)}{\delta(\Delta x)} = 0 = -2 \sum_{j=1}^{k} F(x_{j-1})\, \theta^{-1} \left[\Delta z(j) - \Delta x^T F(x_{j-1}) \right]$$

$$+\; 2\, S^{-1}(o)\, \Delta x \qquad (6.1.7)$$

or,

$$\sum_{j=1}^{k} F(x_{j-1})\, \theta^{-1} \Delta z(j) + S^{-1}(o)\Delta x = \sum_{j=1}^{k} F(x_{j-1})\, \theta^{-1} F^T(x_{j-1})\, \Delta x$$

$$\ldots \quad (6.1.8)$$

Let

$$S^{-1}(k) = \sum_{j=1}^{k} F(j-1)\, \theta^{-1} F^T(j-1) \qquad (6.1.9)$$

and

$$d(k) = \sum_{j=1}^{k} F(j-1)\, \theta^{-1} \Delta z(j) + S^{-1}(o)\, \Delta x \qquad (6.1.10)$$

So,

$$d(k+1) = d(k) + F(k)\, \theta^{-1}\, \Delta z(k+1) \qquad (6.1.11)$$

Let the estimate of Δx at the k-th iteration is $\Delta x(k)$

$$\therefore \; S^{-1}(k)\,\Delta x(k) = d(k) \tag{6.1.12}$$

From equation (6.1.9)

$$S^{-1}(k+1) = S^{-1}(k) + F(k)\,\theta^{-1}\,F^{T}(k) \tag{6.1.13}$$

By Matrix inversion lemma,

Let

$$M = S(k) - \frac{S(k)\;F(k)\,\theta^{-1}\,F^{T}(k)\;S(k)}{1+F^{T}(k)S(k)F(k)\,\theta^{-1}} \tag{6.1.14}$$

Multiplying equation (6.1.14) by equation (6.1.13)

$$M\,S^{-1}(k+1) = I+S(k)F(k)\,\theta^{-1}\,F^{T}(k) - \frac{S(k)F(k)\,\theta^{-1}F^{T}(k)+S(k)F(k)\,\theta^{-1}F^{T}(k)S(k)F(k)\,\theta^{-1}F^{T}(k)}{1+F^{T}(k)S(k)F(k)\,\theta^{-1}} \quad \ldots (6.1.15)$$

Since θ^{-1} and $F^{T}(k)S(k)F(k)$ are scaler

$$S(k)F(k)\,\theta^{-1}F^{T}(k)+S(k)F(k)\,\theta^{-1}F^{T}(k)S(k)F(k)\,\theta^{-1}F^{T}(k)$$

$$M\,S^{-1}(k+1) - I = \frac{-S(k)F(k)\theta^{-1}F^{T}(k)-S(k)F(k)\theta^{-1}F^{T}(k)S(k)F(k)\theta^{-1}F^{T}(k)}{1+F^{T}(k)S(k)F(k)\,\theta^{-1}} \quad \ldots (6.1.16)$$

So,

$$M\, S^{-1}(k+1) = I \qquad (6.1.17)$$

$$\therefore \quad M = S(k+1) \qquad (6.1.18)$$

$$\therefore \quad S(k+1) = S(k) - \frac{S(k).\,F(k)\, e^{-1} F^{T}(k) S(k)}{1 + F^{T}(k) S(k) F(k)\, e^{-1}} \qquad (6.1.19)$$

$$\Delta x(k+1) = S(k+1)\, d(k+1) \qquad (6.1.20)$$

From equation (6.1.19) and (6.1.11),

$$\Delta x(k+1) = \left[S(k) - \frac{S(k) F(k)\, e^{-1} F^{T}(k)\, S(k)}{1 + F^{T}(k) S(k) F(k)\, e^{-1}} \right]$$

$$\left[d(k) + F(k)\, \Delta z(k+1)\, e^{-1} \right] \qquad (6.1.21)$$

$$\Delta x(k+1) = \Delta x(k) + S(k) F(k) \Delta z(k+1)\, e^{-1}$$

$$- \frac{S(k) F(k)\, e^{-1} F^{T}(k) \Delta x(k) + S(k) F(k)\, e^{-1} F^{T}(k) S(k) F(k)\, e^{-1} \Delta z(k+1)}{1 + F^{T}(k) S(k) F(k)\, e^{-1}}$$

$$\Delta x(k+1) = \Delta x(k) + S(k) F(k)\, e^{-1} \left[\Delta z(k+1) - \Delta x^{T}(k) F(k) \right]$$

$$\times \left[1 + F^{T}(k) S(k) F(k)\, e^{-1} \right]^{-1} \qquad (6.1.22)$$

So the algorithms for the recursive estimation of states of the power system are

$$\Delta x(k+1) = \Delta x(k)+S(k)F(k)\, \theta^{-1} \left[\Delta Z(k+1) - \Delta x^T(k)F(k) \right]$$

$$\times \left[1+ F^T(k)S(k)F(k)\, \theta^{-1} \right]^{-1} \qquad (6.1.23a)$$

$$S(k+1) = S(k) - S(k)F(k)\, \theta^{-1} F^T(k)S(k)$$

$$\times \left[1+ F^T(k)S(k)F(k)\, \theta^{-1} \right]^{-1} \qquad (6.1.23b)$$

6.2 Implementation of the Algorithm

Step 1

Individual measurements are considered one after another as a scaler quantity, e.g. z_1, z_2, z_3, ... and so on. θ^{-1} is the variance of measurements of individual quantity and it is assumed to be constant for all the measured variables and its value is taken as 2500. S - is initialized as a positive definite weighting matrix of diagonals as 0.0004 and all non-diagonals as zero. Initial values of the state vectors are taken as those values obtained from an off-line load flow analysis with reactive component of the voltage of one bus as zero. $F(.)$ is the corresponding column of the Jacobian.

Step 2

Deviations of P and Q of the measured from the calculated ones are obtained for all the buses such as

$\Delta P_1(k) = P_1 - P_1(k)$ calculated, $\Delta Q_1(k) = Q_1 - Q_1(k)$ calculated.

For a 5 bus system we have $\Delta Z(k)$ as

$$[\Delta P_1(k), \Delta P_2(k), \Delta P_3(k), \Delta P_4(k), \Delta P_5(k), \Delta Q_1(k), \Delta Q_2(k), \Delta Q_3(k), \Delta Q_5(k)]^T$$

where k- is the iteration count.

The state vectors at the k-th iteration are denoted as

$\Delta e_1(k), \Delta e_2(k), \ldots, \Delta e_5(k), \Delta f_2(k), \ldots, \Delta f_5(k) \ldots;$ since

$\Delta f_1(k) = 0$ for all k.

Elements of the Jacobian matrix (9 x 9) are computed for the k-th iteration as $F(k)$.

To introduce recursiveness in the algorithm and for possible on-line application P and Q are initialised as the measured quantities and as the iteration proceeds P and Q become one step back af the calculated values of P and Q as stated as

$$\Delta P_1(k) = P_1(k-1) - P_1(k) \text{ and } \Delta Q_1(k) = Q_1(k-1) - Q(k).$$

Step 3

Iteration proceeds with the algorithms for the state vectors as

$$\begin{bmatrix} s_{11}(k+1), s_{12}(k+1), \ldots, s_{19}(k+1) \\ \vdots \\ s_{91}(k+1), s_{92}(k+1), \ldots, s_{99}(k+1) \end{bmatrix}$$

$$= \begin{bmatrix} s_{11}(k), s_{12}(k), \ldots, s_{19}(k) \\ \vdots \\ s_{91}(k), s_{92}(k), \ldots, s_{99}(k) \end{bmatrix} - \begin{bmatrix} s_{11}(k), s_{12}(k), \ldots, s_{19}(k) \\ \vdots \\ s_{91}(k), s_{92}(k), \ldots, s_{99}(k) \end{bmatrix} \times$$

$$\times \begin{bmatrix} F_{11}(k) \\ F_{21}(k) \\ \vdots \\ F_{91}(k) \end{bmatrix} e^{-1} \begin{bmatrix} F_{11}(k), F_{21}(k), \ldots, F_{91}(k) \end{bmatrix} \begin{bmatrix} s_{11}(k), s_{12}(k), \ldots, s_{19}(k) \\ \vdots \qquad\qquad \vdots \\ s_{91}(k), s_{92}(k), \ldots, s_{99}(k) \end{bmatrix} \times$$

$$\left\{ 1 + \left[F_{11}(k), F_{21}(k), \ldots, F_{91}(k) \right] \begin{bmatrix} s_{11}(k), s_{12}(k), \ldots, s_{19}(k) \\ \vdots \\ s_{91}(k), s_{92}(k), \ldots, s_{99}(k) \end{bmatrix} \begin{bmatrix} F_{11}(k) \\ F_{21}(k) \\ \vdots \\ F_{91}(k) \end{bmatrix} e^{-1} \right\}$$

... (6.2.1a)

$$
\begin{bmatrix} \Delta e_1(k+1) \\ \Delta e_2(k+1) \\ \Delta e_3(k+1) \\ \Delta e_4(k+1) \\ \Delta e_5(k+1) \\ \Delta f_2(k+1) \\ \Delta f_3(k+1) \\ \Delta f_4(k+1) \\ \Delta f_5(k+1) \end{bmatrix}
=
\begin{bmatrix} \Delta e_1(k) \\ \cdot \\ \cdot \\ \cdot \\ \Delta e_5(k) \\ \Delta f_2(k) \\ \\ \\ \Delta f_5(k) \end{bmatrix}
+
\begin{bmatrix} S_{11}(k), & \ldots, & S_{19}(k) \\ S_{21}(k), & \ldots, & S_{29}(k) \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \\ \\ \\ S_{91}(k), & \ldots, & S_{99}(k) \end{bmatrix}
\begin{bmatrix} F_{11}(k) \\ F_{21}(k) \\ \cdot \\ \cdot \\ \cdot \\ \\ \\ \\ F_{91}(k) \end{bmatrix}
e^{-1} \; x
$$

$$
\Big[F_1(k+1) - \left[\Delta e_1(k), \ldots, \Delta e_5(k), \Delta f_2(k), \ldots, \Delta f_5(k) \right]
\times \left[F_{11}(k), F_{21}(k), \ldots, F_{91}(k) \right]^T \Big] \; x
$$

$$
\left\{ 1 + \left[F_{11}(k), F_{21}(k), \ldots, F_{91}(k) \right]
\begin{bmatrix} S_{11}(k), \ldots, & S_{19}(k) \\ \cdot & \\ \cdot & \\ \cdot & \\ S_{91}(k), \ldots, & S_{99}(k) \end{bmatrix}
\begin{bmatrix} F_{11}(k) \\ \cdot \\ \cdot \\ \cdot \\ F_{91}(k) \end{bmatrix}
e^{-1} \right\}
$$

... (6.2.1b)

Now, $e_1(k+1) = e_1(k) + \Delta e_1(k+1)$ (6.2.2)

$$\vdots$$

$$e_5(k+1) = e_5(k) + \Delta e_5(k+1)$$

$$f_2(k+1) = f_2(k) + \Delta f_2(k+1)$$

$$\vdots$$

$$f_5(k+1) = f_5(k) + \Delta f_5(k+1)$$

Step 4

Go to step 2, if max $\Delta P_i(k+1) <=$ tolerance and if max $\Delta Q_i(k+1) <=$ tolerance, then go to step 5
Else GO TO Step 2.

Step 5

Print results.

A 5-Bus 7 lines network [67] was considered. For tolerance of 0.01 state vectors were found to converge after 3 iterations [91]. The power network under consideration has been shown in Fig.6.2.1. Line parameters, line Admittance, Bus Admittance Matrix, Initial values of the States Vectors, Active and Reactive Bus Power Measurements are shown in Tables 6.2.1, 6.2.2, 6.2.3a and 6.2.3b,

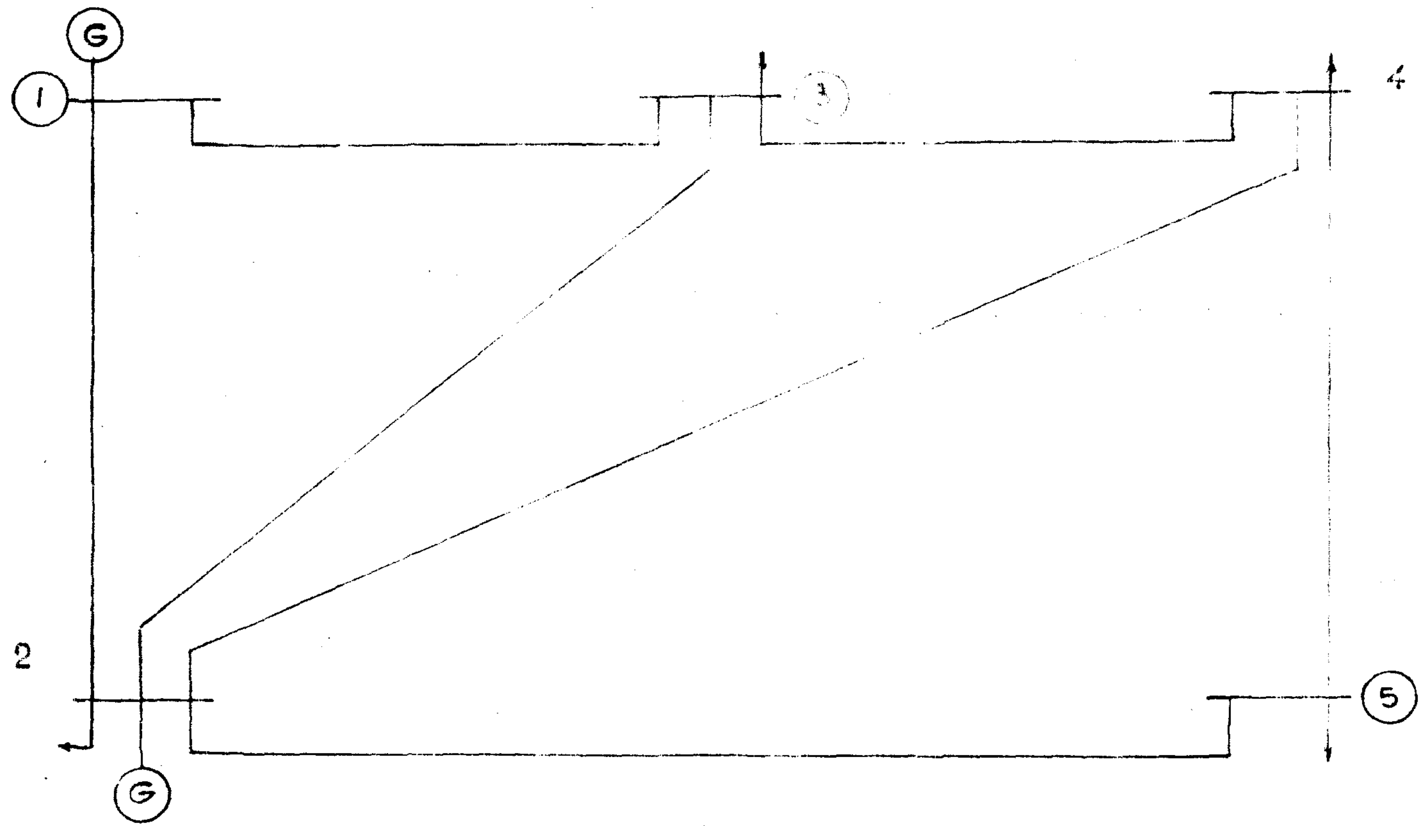

FIG. 6.2.1 : SAMPLE POWER NETWORK.

6.2.4. For iteration 1 elements of the Jacobian, the weighting Matrix S(I,J), the States Vectors e(.) and f(.) are shown in Table 6.2.5, 6.2.6 and 6.2.7 respectively.

For iteration 2, the elements of the Jacobian, the weighting matrix S (I,J), the States Vectors are shown in Tables 6.2.8, 6.2.9, and 6.2.10 respectively.

For iteration 3 , the weighting matrix S (I, J) and the States Vectors are shown in Tables 6.2.11 and 6.2.12 respectively. The software developed in BASIC is given in the Appendix as A4.1.

TABLE 6.2.1

LINE PARAMETERS

Line No.	Between Buses	Line impedance	Half line charging admittance
1	1 - 2	0.02 + j 0.06	0.0 + j 0.03
2	1 - 3	0.08 + j 0.24	0.0 + j 0.025
3	2 - 3	0.06 + j 0.18	0.0 + j 0.02
4	2 - 4	0.06 + j 0.18	0.0 + j 0.02
5	2 - 5	0.04 + j 0.12	0.0 + j 0.015
6	3 - 4	0.01 + j 0.03	0.0 + j 0.01
7	4 - 5	0.08 + j 0.24	0.0 + j 0.025

TABLE 6.2.2

LINE ADMITTANCE AND LINE CHARGING

Line No.	Between Buses	Line admittance	Line Half charging
1	1 - 2	5.00 -j 15.00	0.0 + j 0.03
2	1 - 3	1.25 -j 3.75	0.0 + j 0.025
3	2 - 3	1.66 -j 5.00	0.0 + j 0.02
4	2 - 4	1.66 -j 5.00	0.0 + j 0.02
5	2 - 5	2.50 -j 7.50	0.0 + j 0.015
6	3 - 4	10.00 -j 30.00	0.0 + j 0.01
7	4 - 5	1.25 -j 3.75	0.0 + j 0.025

TABLE 6.2.3a

BUS ADMITTANCE

Column / Row	1	2	3	4	5
1	6.25-j18.69	-5.00+j15.00	-1.25+j3.75	0	0
2	-5.00+j15.00	10.83-j32.41	-1.66+j5.00	-1.66+j5.00	-2.50+j7.50
3	-1.25+j3.75	-1.66+j5.00	12.91-j38.69	-10.00+j30.00	0
4	0	-1.66+j5.00	-10.00+j30.00	12.91-j38.69	-1.25+j3.75
5	0	-2.50+j7.50	0	-1.25+j3.75	3.75-j11.21

TABLE 6.2.3b

INITIAL VALUES OF THE STATE VECTOR

Bus No.	Active Voltage	Reactive Voltage
1	1.060	-0.000
2	1.0463	-0.0512
3	1.0204	-0.0888
4	1.0193	-0.0961
5	1.0123	-0.1091

TABLE 6.2.4

ACTIVE AND REACTIVE BUS POWER MEASUREMENT

Bus No.	Generation		Load	
	Megawatts	Megavars	Megawatts	Megavars
1	129.5	0	0	7.5
2	40	30	20	10
3	0	0	45	15
4	0	0	40	5
5	0	0	60	10

TABLE 6.2.5

ELEMENTS OF THE JACOBIAN MATRIX FOR FIRST ITERATION

Column / Row	1	2	3	4	5
1	7.847	- 5.3	- 1.325	0	0
2	- 6.001	13.1793	- 2.0003	- 2.0003	- 3.0005
3	- 1.61	- 2.1466	16.208	-12.88	0
4	0	- 2.1743	-13.046	16.4603	- 1.6307
5	0	- 3.349	0	- 1.6745	4.4437
6	19.7444	-15.9	- 3.975	0	0
7	-15.438	33.5609	- 5.146	- 5.146	- 7.719
8	- 3.715	- 4.9533	38.1467	-29.72	0
9	0	- 4.938	-29.628	38.1201	- 3.7036
10	0	- 7.3195	0	- 3.6597	10.7790

TABLE 6.2.3 (Continued)

Column / Row	6	7	8	9	10
1	19.889	-15.6	- 3.975	0	0
2	-15.438	32.6034	- 5.146	- 5.146	7.719
3	- 3.175	- 4.9533	37.3655	-29.72	0
4	0	- 4.938	-29.628	37.0683	-3.7035
5	0	- 7.3195	0	- 3.6597	10.6893
6	- 5.403	5.30	1.325	0	0
7	6.001	-12.8162	2.0003	2.0003	3.0005
8	1.61	6.1466	-17.0555	12.88	0
9	0	2.1743	13.046	-17.2313	1.6307
10	0	3.349	0	1.6745	-5.5945

TABLE 6.2.6

WEIGHTING MATRIX S(I,J) FOR FIRST ITERATION

Column / Row	1	2	3	4	5
1	7.3842E-05	7.3721E-05	7.5213E-05	7.5217E-05	7.5479E-05
2	7.3721E-05	7.4968E-05	7.6450E-05	7.6530E-05	7.7119E-05
3	7.5213E-05	7.6450E-05	7.9358E-05	7.9366E-05	7.8686E-05
4	7.5217E-05	7.6530E-05	7.9366E-05	7.9536E-05	7.8942E-05
5	7.5479E-05	7.7119E-05	7.8686E-05	7.8942E-05	8.1667E-05
6	3.4676E-06	3.5096E-06	3.6074E-06	3.6125E-06	3.6374E-06
7	6.1504E-06	6.2260E-06	6.4621E-06	6.4726E-06	6.3929E-06
8	6.5610E-06	6.6442E-06	6.8964E-06	6.9036E-06	6.8360E-06
9	7.1706E-06	7.2881E-06	7.4521E-06	7.4725E-06	7.6729E-06

TABLE 6.2.6 (Continued)

Column / Row	6	7	8	9
1	3.4676E-06	6.1504E-06	6.5610E-06	7.1706E-06
2	3.5096E-06	6.2260E-06	6.6442E-06	7.2881E-06
3	3.6074E-06	6.4621E-06	6.8964E-06	7.4521E-06
4	3.6125E-06	6.4726E-06	6.9036E-06	7.4725E-06
5	3.6374E-06	6.3929E-06	6.8360E-06	7.6729E-06
6	8.4216E-07	1.0593E-06	1.1560E-06	1.4291E-06
7	1.0593E-06	2.7378E-06	2.7828E-06	1.7990E-06
8	1.1560E-06	2.7828E-06	2.9945E-06	2.0210E-06
9	1.4291E-06	1.7990E-06	2.0210E-06	4.3106E-06

TABLE 6.2.7

STATES VECTORS e(.), f(.) FOR THIRD ITERATION

Bus No.	Active voltage	Reactive voltage	Voltage Magnitude	Angle
1	1.0606	0	1.0606	0
2	1.0389	-0.0527	1.0403	-2.9062
3	1.0104	-0.0901	1.0144	-5.0963
4	1.0101	-0.0964	1.0147	-5.4516
5	1.0160	-0.1160	1.0226	-6.5127

TABLE 6.2.8

ELEMENTS OF THE JACOBIAN MATRIX FOR THE SECOND ITERATION

Column / Row	1	2	3	4	5
1	7.8049	- 5.2532	- 1.3133	0	0
2	- 5.9867	13.1530	- 1.9966	- 1.9955	- 2.9933
3	- 1.6010	- 2.1347	16.1144	-12.8087	0
4	0	- 2.1655	-12.9936	16.3909	- 1.6241
5	0	- 3.4103	0	- 1.7061	4.5180
6	19.5330	-15.7597	- 3.9399	0	0
7	-15.3208	33.2731	- 5.1069	- 5.1068	- 7.6604
8	- 3.6766	- 4.9082	37.7110	-29.4132	0
9	0	- 4.8898	-29.3398	37.7185	- 3.6674
10	0	- 7.3306	0	- 3.6653	10.9478

TABLE 6.2.8 (Continued)

Column / Row	6	7	8	9	10
1	19.7507	-15.7597	- 3.9399	0	0
2	-15.3208	32.3686	- 5.1069	- 5.1069	- 7.6604
3	- 3.6766	- 4.9082	36.9977	-29.4132	0
4	0	- 4.8898	-29.3398	36.7180	- 3.6674
5	0	- 7.3306	0	- 3.6653	10.5876
6	- 5.3281	5.2532	1.3133	0	0
7	5.9867	-12.7806	1.9966	1.9955	2.9933
8	1.6010	2.1347	-16.9649	12.8087	0
9	0	2.1655	12.9936	-17.1651	1.6241
10	0	3.4103	0	1.7061	- 5.7036

TABLE 6.2.9

WEIGHTING MATRIX S(I,J) FOR SECOND ITERATION

Column / Row	1	2	3	4	5
1	7.0319E-05	7.1027E-05	7.2205E-05	7.2332E-05	7.2774E-0
2	7.1027E-05	7.2079E-05	7.3012E-05	7.3084E-05	7.4084E-0
3	7.2205E-05	7.3012E-05	7.6458E-05	7.6489E-05	7.5946E-05
4	7.2332E-05	7.3084E-05	7.6489E-05	7.6602E-05	7.6100E-05
5	7.2774E-05	7.4084E-05	7.5946E-05	7.6100E-05	7.7253E-05
6	3.4369E-06	3.4839E-06	3.5844E-06	3.5884E-06	3.5945E-06
7	6.0264E-06	6.1056E-06	6.3167E-06	6.3213E-06	6.2674E-06
8	6.4453E-06	6.5313E-06	6.7534E-06	6.7601E-06	6.7113E-06
9	7.2996E-06	7.4121E-06	7.6053E-06	7.6151E-06	7.7107E-06

TABLE 6.2.9 (Continued)

Column / Row	6	7	8	9
1	3.4369E-06	6.0264E-06	6.4453E-06	7.2996E-06
2	3.4839E-06	6.1056E-06	6.5313E-06	7.4121E-06
3	3.5844E-06	6.3167E-06	6.7534E-06	7.6053E-06
4	3.5884E-06	6.3213E-06	6.7601E-06	7.6151E-06
5	3.5945E-06	6.2674E-06	6.7113E-06	7.7107E-06
6	5.1246E-07	6.2672E-07	7.4565E-07	9.1252E-07
7	6.2527E-07	1.6427E-06	1.6845E-06	1.2067E-06
8	7.4565E-07	1.6845E-06	1.8118E-06	1.3754E-06
9	9.1252E-07	1.2067E-06	1.3754E-06	2.5959E-06

TABLE 6.2.10

STATES VECTORS e(.), f(.) FOR SECOND ITERATION

Bus No.	Active Voltage	Reactive Voltage	Voltage Magnitude	Angle
1	1.0545	0	1.0546	0
2	1.0420	-0.0521	1.0433	-2.8653
3	1.0146	-0.0897	1.0186	-5.0538
4	1.0139	-0.0958	1.0185	-5.3996
5	1.0146	-0.1121	1.0209	-6.3614

TABLE 6.2.11

WEIGHTING MATRIX S(I,J) FOR THIRD ITERATION

Column / Row	1	2	3	4	5
1	7.0219E-06	7.1087E-06	7.2806E-06	7.2832E-06	7.2774E-06
2	7.1087E-06	7.2079E-06	7.3918E-06	7.3984E-06	7.4084E-06
3	7.2806E-06	7.3918E-06	7.6458E-06	7.6429E-06	7.5946E-06
4	7.2832E-06	7.3984E-06	7.6429E-06	7.6602E-06	7.6100E-06
5	7.2774E-06	7.4084E-06	7.5946E-06	7.6100E-06	7.7253E-06
6	3.4388E-06	3.4839E-06	3.5844E-06	3.5884E-06	3.5945E-06
7	6.0864E-06	6.1056E-06	6.3167E-06	6.3213E-06	6.2674E-06
8	6.4453E-06	6.5313E-06	6.7534E-06	6.7601E-06	6.7113E-06
9	7.2996E-06	7.4121E-06	7.6055E-06	7.6191E-06	7.7107E-06

TABLE 6.2.11 (Continued)

Column / Row	6	7	8	9
1	3.4388E-06	6.0864E-06	6.4453E-06	7.2996E-06
2	3.4839E-06	6.1056E-06	6.5313E-06	7.4121E-06
3	3.5844E-06	6.3167E-06	6.7534E-06	7.6055E-06
4	3.5884E-06	6.3213E-06	6.7601E-06	7.6191E-06
5	3.5945E-06	6.2674E-06	6.7113E-06	7.7107E-06
6	5.1246E-07	6.8572E-07	7.4565E-07	9.1852E-07
7	6.8572E-07	1.6427E-06	1.6845E-06	1.2087E-06
8	7.4565E-07	1.6845E-06	1.8118E-06	1.3754E-06
9	9.1852E-07	1.2087E-06	1.3754E-06	2.5959E-06

TABLE 6.2.12

STATES VECTORS e(.), f(.) FOR THIRD ITERATION

Bus No	Active Voltage	Reactive Voltage	Voltage Magnitude	Angle
1	1.0545	0	1.0545	0
2	1.0420	-0.0521	1.0433	-2.8653
3	1.0146	-0.0897	1.0186	-5.0522
4	1.0139	-0.0958	1.0185	-5.3996
5	1.0146	-0.1131	1.0209	-6.3614