CHAPTER – VI

SUMMARY AND CONCLUSIONS

The nonlinear dynamic (free vibration) analysis of thin plate and shell structures of different geometrical shapes with or without thermal loading is presented. The structures are assumed to be made of homogeneous, isotropic and elastic materials. In some problems of plate and shell structures elastic properties of materials are assumed constant and temperature-independent. Thin isotropic circular plates, made of materials having temperature-dependent material properties, are also considered.

Continuum model has been used to derive basic governing differential equations as and whatever required, in the sense of von Karman classical large deflection theory or on the basis of Berger approximations i.e. the energy contribution due to second strain invariant of the middle surface is neglected in the total potential energy expression. The basic governing differential equations for the resulting system has been solved by Galerkin's error minimizing technique incorporating prescribed boundary conditions to obtain the dynamic characteristics, both linear as well as nonlinear.

Numerical results ·) study the effects of different parameters, as occurred in the analysis, on nonlinear dynamic behavior of such structures have been presented and compared with the available published results or observed behavior wherever possible in some cases. The present study reveals some interesting nonlinear dynamic behavior which may prove useful to the designers.

The Chapter I is introduction which includes scope, previous works, outline of present work and organization.

The Chapter II studies nonlinear free vibrations of axisymmetric thin isotropic circular plates with clamped immovable edges subjected to thermal loading characterized by constant surface temperatures T_u and T_b, measured from stress free temperature, for upper surface and lower surface respectively. Basic governing differential equations in the von Karman sense have been derived in terms of displacement components with the inclusion of thermal loading. These equations are solved by Galerkin error minimizing technique incorporating clamped immovable edge conditions. A parametric study of such structures is also presented.

It is observed that clamped circular plates exhibit strain-hardening type of nonlinearity for immovable edge conditions valid within the proportional limit of plate material. The additional stiffness due to large deflection i.e. stretching of the middle surface increases the nonlinear stiffness and hence, the nonlinear frequency of the plate with increase of amplitude. On the other hand, the linear frequency is independent of amplitude of vibration. This phenomenon explains the strain-hardening behavior of thin circular plate structures. This behavior is more pronounced in presence of thermal loading.

The average surface temperature $\left(\dfrac{T_u + T_b}{2}\right)$ part of thermal loading induces compressive stress in the circular plate with clamped immovable edges. As a result, increase of average surface temperature $\left(\frac{T_u + T_b}{2}\right)$ reduces stiffness and hence frequency of vibration; both linear as well as nonlinear. Due to the presence of the additional stiffness associated with large deflection the linear frequency decreases more than the nonlinear one proportionately and as a result $\frac{\omega_{NL}}{\omega_L}$ increases. At certain stage linear frequency becomes zero i.e. thermal instability of the structure occurs based on linear theory; but nonlinear frequency is still non-zero which makes $\frac{\omega_{NL}}{\omega_L}$ infinity. So, thermal instability is delayed due to additional stiffness associated with large deflection as per nonlinear theory. However, the constant linear thermal gradient part $\left(\frac{T_u - T_b}{H} z\right)$ along the thickness direction has no effects on nonlinear dynamic behavior of such structures for the assumed thermal loading characterized by unequal constant surface temperatures. This is due to the fact that the constant linear thermal gradient part does not contribute to the thermal stress resultants since the material properties are assumed independent of temperature. Further, the constant linear thermal gradient along the thickness direction through out the plate induce constant pure bending moment uniformly distributed along the edges of the plate and no shear will be induced in the thickness direction. As a result the linear thermal gradient part does not influence relative nonlinear frequency of vibrations in the transverse direction of such structures.

In presence of thermal loading, increase of slenderness parameter reduces stiffness and hence frequency of vibration; both linear as well as nonlinear; but the linear frequency decreases more than the nonlinear one proportionately due to the presence of the additional stiffness associated with large deflection and as a result $\frac{\omega_{NL}}{\omega_L}$ increases. At certain stage linear frequency becomes zero due to presence of thermal loading i.e. thermal instability occurs based on linear theory; but nonlinear frequency is still non-zero which makes $\frac{\omega_{NL}}{\omega_L}$ infinity.

In absence of thermal loading, besides amplitude (A) and plate thickness (H) natural frequencies, both linear and nonlinear, depend on the radius of the plate by the same proportion and increase of slenderness parameter $\left(\frac{R}{H}\right)$ may be viewed as increase of the radius at constant plate thickness (H). So, with increase of slendemess parameter natural frequencies, both linear and nonlinear, decrease in the same proportion and relative nonlinear frequency is independent of slenderness parameter $\left(\frac{R}{H}\right)$ at constant amplitude (A) of vibrations and constant plate thickness (H).

Chapter III presents nonlinear free vibrations of thin elastic plates of various geometric shapes with clamped immovable edges viz. triangular and parabolic plates in presence of thermal loading. The thermal loading is again characterized by constant surface temperatures T_u and T_b, measured from stress free temperature, for upper surface and lower surface respectively. Decoupled nonlinear governing differential equations on the basis of Berger approximation (i. e. neglecting second strain invariant e_2) have been used. These equations have been have been solved by Galerkin's error minimizing technique. This Chapter also include parametric studies.

The triangular plates and the parabolic plates with clamped immovable edges exhibit strain-hardening type of nonlinearity i.e. $\frac{\omega_{NL}}{\omega_L}$ increases with increase of nondimensional amplitude $\left(\frac{A}{H}\right)$ valid within the proportional limit of the plate material and such effects are greatly influence by plate geometry and thermal loading.

In case of triangular plates with clamped immovable edges, as the deviation from the geometric configuration of triangular plates having aspect ratio around 1.0 and skew angle around 30^0 increases, any two supports relatively come closer or one of the perpendicular from an apex to the opposite side becomes shorter and as a result the stiffness of the plate increases. In other words, as the deviation from the geometric configuration of triangular plates having aspect ratio around 1.0 and skew angle around 30^0 decreases, the stiffness of the plate and hence, the frequency decreases. However, the rate of decrease of linear frequency is more than nonlinear frequency because of the additional stiffness of the plate due to large deflection and the relative nonlinear frequency increases. As a consequence, a triangular plate with the geometric configuration having aspect ratio around 1.0 and skew angle around 30^0 is more prone to develop dynamic instability due to reasons discussed above. Such effects are more pronounced at higher values of average surface temperature $\frac{(T_u + T_b)}{2}$ and slenderness parameter $\left(\frac{ab}{H^2}\right)$ in presence of thermal loading.

Similarly, for parabolic plates with clamped immovable edges it is observed that with increase of aspect ratio relative nonlinear frequency increases up to a maximum around the value of aspect ratio 1.4 and then decreases. This is due to the fact that as the deviation from the geometric configuration of a parabolic plate having aspect ratio around 1.4 increases, opposite supports relatively come closer and as a result the stiffness of the plate in the transverse direction increases. In other words, as the deviation from the geometric configuration of a parabolic plate having aspect ratio around 1.4 decreases, the stiffness of the plate i.e. the frequency decreases. However, the rate of decrease of linear frequency is more than nonlinear frequency because of the additional stiffness associated with large deflection. The effect is more pronounced at higher values of average surface temperature $\frac{(T_u + T_b)}{2}$ and/or slenderness parameter $\left(\frac{ab}{H^2}\right)$ in presence of thermal loading. So, a parabolic plate having aspect ratio around 1.4 is more prone to develop dynamic instability with or without thermal loading due to the reasons discussed above.

In absence of thermal loading, besides amplitude (A) of vibrations, plate thickness (H), aspect ratio $\left(\frac{a}{b}\right)$ and skew angle (ϕ) natural frequencies, both linear and nonlinear, also depend on the plate dimensions in the same proportion and increase of slenderness ratio may be achieved by increasing plate dimensions at constant plate thickness (H). So, with increase of slenderness ratio natural frequencies, both linear and nonlinear, decrease in the same proportion and the relative nonlinear frequency remains constant at constant amplitude (A) of vibrations and plate thickness (H) in absence of thermal loading.

The thermal loading, characterized by unequal constant surface temperatures, and the slenderness parameter influence the free vibrations of triangular and parabolic plates in the same fashion as in case of circular plates.

The Chapter IV investigates the nonlinear dynamics of thin circular elastic plates under thermal loading considering nonhomogeneity arising due to temperature dependency of material properties such as thermal conductivity, modulus of elasticity, Poisson's ratio and coefficient of thermal deformations under thermal loading with temperature gradient along the thickness direction. The basic governing differential equations have been derived in the von Karman sense in terms of displacement components and solved with the help of Galerkin procedure. Further, to solve the steady-state heat conduction problem temperature dependency of thermal conductivity has been considered.

Parametric studies have been presented for thin circular elastic plates made of Titanium Alloy $(Ti-6Al-4V)$ to understand the influences of different parameters; temperature level is being kept below the temperature which causes thermal instability. The modulus of elasticity decreases with increase of temperature and hence, the stiffness of the plate reduces which results in reduction of nonlinear frequency of vibration. The coefficient of thermal deformation increases with increase of temperature in the temperature range under consideration. Since the presence of thermal loading reduces plate stiffness, the increase of coefficient of thermal deformation intensifies effects of thermal loading and hence reduces plate stiffness resulting in reduction of nonlinear frequency. The effect of temperature dependency of Poisson's ratio on nonlinear frequency is negligible. Finally, the combined effect of temperature dependency of modulus of elasticity, coefficient of thermal deformation and Poisson's ratio is to reduce the nonlinear frequency; but the reduction is not appreciable compared to the effects of large deflection and direct heating for such structures.

This study reveals that temperature dependency of thermal conductivity does not alter temperature distribution across the thickness significantly within the temperature level considered. However, the temperature level across the thickness increases marginally due to higher value of thermal conductivity at increased level of temperature and as a result temperature-dependent thermal conductivity does not influence the natural frequency significantly for such structures. Though numerical results for natural frequency are not presented, it decreases the natural frequency slightly due to marginal increased level of temperature across the thickness. Again, the nonlinear frequency decreases with increase of the temperature difference between the surfaces when the material properties including the thermal conductivity are temperature-dependent; although the magnitude of reduction is very small compared to the effects of large deflection as well as direct heating.

The Chapter V studies the nonlinear free vibrations of axisymmetric thin shallow spherical shell of variable thickness having tangentially clamped immovable edges without thermal loading. Analysis has been performed by using both (i) coupled governing differential equations derived in the von Karman sense in terms of displacement components as well as (ii) decoupled nonlinear governing differential equations on the basis of Berger approximation (i. e. neglecting second strain invariant e_2) derived from energy expression applying Hamilton's principle and Euler's variational equations. The governing differential equations are solved by Galerkin error minimizing technique incorporating clamped immovable edge conditions. Axisymmetric thin shallow spherical shells exhibit both strain-hardening as well as strain-softening types of behavior. As the value of geometrical parameter $\frac{R^2}{2R_0H_0}$ approaches very low value i.e. a very shallow thin spherical shell tends to behave like a circular plate and exhibit strain-hardening type of nonlinearity i.e. $\frac{\omega_{NL}}{\omega_L}$ increases with increase of $\frac{A}{H}$; linear frequency being independent of $\frac{A}{H}$. With increase of $\frac{R^2}{2R_0H_0}$, the behavior of shallow spherical shell changes from strain-hardening type to strain-softening type. This change of behavior of shallow spherical shell may be attributed to large deflection which facilitates snap buckling of shallow spherical shell. During compression phase of vibrations, shallow spherical shell primarily contracts in size without appreciable bending; bending deformations being localized near the built-in edges and try to buckle in a direction opposite to the initial curvature, called snap buckling. Large deflection facilitates snap buckling of shallow shell and the force required to maintain equilibrium reduces i.e. nonlinear stiffness decreases. On the other hand, linear stiffness, which is independent of amplitude of vibrations, increases with increase of $\frac{R^2}{2R_0H_0}$. As a result, relative nonlinear frequency decreases to yield the lowest peak and this peak corresponds to the geometrical configuration of thin shallow spherical shell which is most liable to undergo snap buckling. With further increase of $\frac{R^2}{2R_0H_0}$ the resistance of shallow spherical shell to snap buckling increases and the shell stiffness, both linear as well as nonlinear, increases; the rate of increase of the nonlinear stiffness being slightly greater than the linear one due to additional stiffness associated with large deflection and the relative nonlinear frequency increases and finally it approaches unity. As the nondimensional geometric parameter $\frac{R^2}{2R_0H_0}$ for thin shallow shell increases the shallow spherical shell primarily contracts in size and the bending deformations, which can be neglected, remain localized near the edges while the remaining part of the shallow spherical shell remains spherical. As a result the contribution of additional stiffness associated with large deflection i.e. stretching of the middle surface tends to become insignificant and the nonlinear stiffness approaches the linear stiffness; hence $\frac{\omega_{NL}}{\omega_L}$ approaches unity.

The shell thickness increases towards the edges as the value of τ decreases algebraically which in turn increases shell stiffness both linear as well as nonlinear. At very low value of $\frac{R^2}{2R_0H_0}$ when a shallow spherical shell tends to behave like a circular plate, $\frac{\omega_{NL}}{\omega_L}$ increases with increase of τ. As τ increases algebraically shell stiffness, both linear as well as nonlinear, decreases; but the linear stiffness, which is independent of amplitude of vibrations, decreases faster compared to the nonlinear stiffness due to additional stiffness associated with large deflection and $\frac{\omega_{NL}}{\omega_L}$ increases with increase of τ. At higher values of $\frac{R^2}{2R_0H_0}$ i.e. the strain-softening zone, as the value of τ decreases algebraically the shell thickness increases towards the edges which in turn increases stiffness and hence decreases bending deformations of shell as well as reduces the shell contraction by lowering meridional stress level. As a result resistance of shallow spherical shell to snap buckling increases with decrease τ. Also, nonlinear frequency approaches linear frequency with decrease of τ since additional stiffness due to large deflection become negligible with decrease of bending deformations.

Further, at very low value of nondimensional geometrical parameter $\frac{R^2}{2R_0H_0}$ (below the transition value) when a shallow spherical shell tends to behave like a circular plate, $\frac{\omega_{NL}}{\omega_L}$ increases with increase of $\frac{A}{H}$. The behavior is strain-hardening one as mentioned earlier and this behavior is valid within the proportional limit of the structure material. At higher values of $\frac{R^2}{2R_0H_0}$ i.e. the zone where large deflection facilitates buckling of shallow spherical the nonlinear stiffness decreases faster compared to the linear stiffness with increase of $\frac{A}{H}$ and $\frac{\omega_{NL}}{\omega_L}$ decreases with increase of $\frac{A}{H}$ i.e. the behavior is strain-softening one. This phenomenon explains the behavior of shallow spherical at higher values of $\frac{R^2}{2R_0H_0}$ (above the transition value).

Finally, it can be mentioned that this study reveals some interesting dynamic behavior of thin isotropic plate and shell structures which may be useful to the designers, For example, the knowledge of dynamic behavior of such structures will help to achieve desired level of frequency of vibrations by properly proportioning the geometric elements. This will also help to utilize the advantage of extra strength due to strain-hardening and to reduce the failure probability by taking proper precaution against structural instability.

It is suggested that in a future work it will be very interesting to study nonlinear free vibrations of thin shallow spherical shell under thermal loading. Theoretical investigations considering various factors like material nonlinearity, initial imperfections, complex geometry of structures etc. will be very good work in this field. As thin plate and shell structures made from fiber-reinforced composite materials are increasingly widely used as major structural components in industries, there is lot of scope of investigations in this field.

Experimental investigations on large deflection static and dynamic behavior of plate and shell structures with or without thermal loading will be very useful to strengthen and develop theoretical investigations further.