Chapter 1

INTRODUCTION

1.1 General:

For a machine represented by n number of coils there are n voltage equations and a torque equation. If the n applied voltages and the applied torque are known, as well as initial conditions, the (n+1) equations are sufficient to determine the n currents and the speed. Hence theoretically the performance of the machine can completely be determined[1],[61].

In general case, the equations containing the product terms in which the speed and the currents, i.e., the state variables of the machine are strongly coupled, are nonlinear differential equations and have to be solved by the numerical integrations. For particular condition of operation, however, considerable simplifications are often made, and the types of problems are accordingly classified[4].

Under steady state conditions, the speed is constant and the voltage equations are dealt independently of the torque equations. The voltage equations reduce under steady conditions to a set of n ordinary linear algebraic equations containing real variables for dc conditions and complex variables for ac conditions.

Under transient conditions, when voltages and currents may vary in any manner, the problem is greatly simplified by taking a constant value of speed, then the voltage equations are treated independently of the torque equations and they are linear differential equations with constant co-efficient.

When the voltages are known the equations can be solved, either by algebraic methods for steady state problems, or by operational methods for transient problems, and the current thus obtained can be substituted in the torque equation. If the speed is constant the externally applied torque is obtained directly because it is equal to the electrical torque.

The net stage in difficulty in study of the dynamics of the electrical machines arises if the speed varies but is a known function of time. It is then still possible to solve the voltage equations separately, but the coefficients are not constant as before. The equations with variable coefficients for which the simple operational method does not apply. Usually only numerical solutions are possible. Once the current is determined , torque can be calculated.

The most difficult problems are those for which the speed is an unknown variable. Most of the transient problems are of this type. These problems are solved by transfer Function Method, Eigen Vector Method, Transition Matrix Method, State Variable method etc. In all of those methods of solution, system is linearized and solved. Therefore, due to linearization, some nonlinear phenomenon are eliminated[7].

Since the 1970s, dynamic characteristics of various motors are widely studied, to deal with starting up, speed control, and oscillations of the motors. In the study of dynamic characteristics of motors, there are many problems that remain to be further addressed, such as their low-speed feature, known as the low-frequency oscillations of speed-controlled motors. These problems are closely related to the studies of chaos in nonlinear systems.

Bifurcation analysis and the study of chaos in nonlinear system is gradually increasing. However, its application in the field of electrical machine is till now very insignificant. Some of those works are reported in next section.

1.2 Background and Review of the previous works:

Most of the researchers have studied the nonlinear phenomenon for different electrical machines using their classical models. Some of them only identified the phenomenon by simulation, very few of them were experimentally verified. In some works, route of chaos, characteristics, different bifurcations are reported. Therefore, previous works are categorized for different machines.

1.2.1 Permanent Magnet Synchronous motor(PMSM):

A detailed model of Permanent Magnet Synchronous Motor was first presented by Consoli et al. in [25]. They developed the model and equivalent circuit using d-q variables for steady state analysis and experimental verification of the proposed model was reported.

The nonlinear dynamic characteristics of variable-speed permanent-magnet machines in the presence of reluctance variations was considered in [26]. A compact representation of the machines' dynamics was presented and in turn incorporated in investigating the steady-state characteristics of the machines, subject to constant input voltages and load torques. It was shown that the systems under investigation possess multiple equilibria which profoundly affect their global stability and dynamic characteristics. Furthermore, conditions leading to chaotic behavior and Hopf Bifurcation was pointed out for the first time and briefly discussed. Finally, numerical simulations were presented to help verify the results. However, the computed results were inadequate and no experimental result presented. Similar information was found in [28].
A brief design review of the Permanent Magnet Synchronous Motors was presented in [27]. A procedure was described to predict the steady state and dynamic performances of a brushless permanent magnet synchronous motor. The proposed techniques have been experimentally verified in a laboratory permanent magnet synchronous motor.

In [28], a complete model was developed to examine the dynamic behaviors of a high performance vector controlled permanent magnet synchronous motor (PMSM) drive. The d,q axis model of the PMSM was used to simulate and analyze the complete drive system. Based on the vector control principle and current feedback control, the nonlinear dynamic model of the PMSM drive can be simplified and linearized. The resultant linear model was used for studying the dynamic behaviors of the drive system. Moreover, it is used for the design of speed controller. The entire control scheme of the drive system was successfully implemented using a high speed DSP. The experimental results was validated the theoretical ones.

G Chen et al. studied the nonlinear phenomenon in Permanent Magnet Synchronous Motor [29]. The mathematical model of the PMSM was derived, which was a three-dimensional autonomous equation with only two quadratic terms, and this model is fit for carrying on bifurcation and chaos analysis. Secondly, the steady state characteristics of the motor, when subject to constant input voltage and external torque, were formulated. A third-order polynomial equation is also derived whose solutions correspond to the steady-state values of the motor angular velocity. Furthermore, based on the Hopf bifurcation condition, the parameters of the PMSM in three different cases, with which the system can exhibit such desired dynamic characteristics as limit cycles (LCs) and chaotic behaviors were determined. Finally, computer simulations were presented to verify the presence of strange attractors in the PMSM. Mathematical model of PMSM as adopted in [29] was:

$$\begin{aligned} \frac{di_d}{dt} &= (u_d - R_1 i_d + \omega L_q i_q)/L_d \\ \frac{di_q}{dt} &= (u_q - R_1 i_q - \omega L_d i_d - \omega \psi_r)/L_d \\ \frac{d\omega}{dt} &= [n_p \psi_r i_q + n_p (L_d - L_q) i_d i_q - T_L - \beta\omega]/J \end{aligned} \qquad [1.1]$$

where i_d, i_q and ω are the state variables, which represent currents and motor angular frequency, respectively, u_d and u_q the direct- and quadrature-axis stator voltage components, respectively, J, the polar moment of inertia, T_L the external load torque, β the viscous damping coefficient, R_1 the stator winding resistance, L_d and L_q the direct and quadrature-axis stator inductors, respectively, ψ_r the permanent magnet flux, and np the number of pole-pairs.Eqn.1.1 was normalized as follows:

$$\frac{d\tilde{i}_d}{d\tilde{t}} = -\tilde{i}_d + \tilde{\omega}.\tilde{i}_q + \tilde{u}_d$$
$$\frac{d\tilde{i}_q}{d\tilde{t}} = -\tilde{i}_q + \tilde{\omega}.\tilde{i}_d + \gamma\tilde{\omega} + \tilde{u}_q \qquad [1.2]$$
$$\frac{d\tilde{\omega}}{d\tilde{t}} = \sigma(\tilde{i}_q - \tilde{\omega}) - \tilde{T}_L$$

Where $b = \frac{L_q}{L_d}$ $\gamma = \frac{-\psi_r}{kL_q}$, k= $\frac{\beta}{n_p \tau \psi_r}$, $\tau = \frac{L_q}{R_1}$, $\sigma = \frac{\beta\tau}{J}$,

$i_d = bk.\tilde{i}_d$, $i_q = k\tilde{i}_q$, $\omega = \tilde{\omega}/\tau$, $t = \tau.\tilde{t}$. It was also assumed that Ld=Lq=L

Equilibrium of the system was calculated as follows:

$$\tilde{i}_d = \tilde{\omega}^2 + \frac{\tilde{T}_L}{\sigma}\tilde{\omega} + \tilde{u}_d$$
$$\tilde{i}_q = \tilde{\omega} + \frac{\tilde{T}_L}{\sigma} \qquad [1.3]$$
$$\tilde{\omega}^3 + \frac{\tilde{T}_L}{\sigma}\tilde{\omega}^2 + (\tilde{u}_d - \gamma + 1)\tilde{\omega} + \frac{\tilde{T}_L}{\sigma} - \tilde{u}_q = 0$$

It may be noted here that the existence or creation or disappearance of the equilibrium point depends on the value of $\tilde{\omega}$ .i.e., the roots of third equation of Eqn. 1.3. It may be further noted the equation was a cubic equation whose roots and their nature were not readily available. It was also reported that as parameter changes bifurcation occurs. Hopf bifurcation occurs when the corresponding Jacobian matrix has a pair of purely imaginary eigenvalues, with the remaining eigenvalues having nonzero real parts. For the PMSM system, its Hopf bifurcation and chaotic behavior are discussed under different sets of parameter values.

When $\tilde{u}_d = \tilde{u}_q = \tilde{T}_L = 0$, the system was identical to the Lorenz equation. This case can be thought of as that, after an operating period of the system, the external inputs were set to zero. Applying the equilibrium conditions, it was determined that the origin was an equilibrium state and that other two nontrivial equilibria exist, if $\gamma \prec 1$, which are defined by

$$\begin{bmatrix} \tilde{i}_d^{eq} \\ \tilde{i}_q^{eq} \\ \tilde{\omega}^{eq} \end{bmatrix} = \begin{bmatrix} \gamma - 1 \\ \pm\sqrt{\gamma - 1} \\ \pm\sqrt{\gamma - 1} \end{bmatrix}$$

A little linear analysis shows that the origin is stable if $0 < \gamma < 1$and loses stability in a pitchfork bifurcation at $\gamma = 1$, creating the two nontrivial equilibria which are initially stable. when evaluated at the nontrivial equilibria. Note that since the two nontrivial equilibria are symmetric, their stability must be the same. For the bifurcation of the two nontrivial equilibrium, Hopf bifurcation point was determined explicitely. It was given by

$$\gamma_h = \frac{\sigma(\sigma + 4)}{\sigma - 2}$$

Similarly, setting $\tilde{u}_q = \tilde{T}_L = 0$, Hopf bifurcation point was determined as

$$u_{dh} = \frac{\sigma^2 - \gamma\sigma + 4\sigma + 2\gamma}{2 - \sigma}$$

Some significant simulation results were reported for the parameters:

Ld=Lq=L=14.25mH, R1=0.9Ω, $\psi_r = 0.031 Nm/A$, n_p=1, J=4.7x10^{-5} Kgm^2

$\beta = 0.0162 N/rad/s$. However no experimental result was presented in its support.

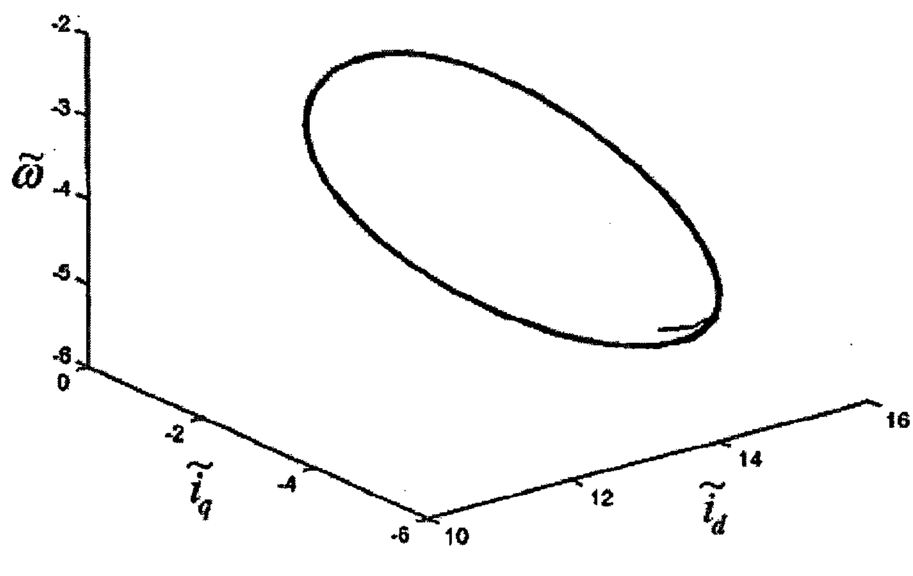

Fig.1.1: A limit cycle generated at $\gamma = 14.1$

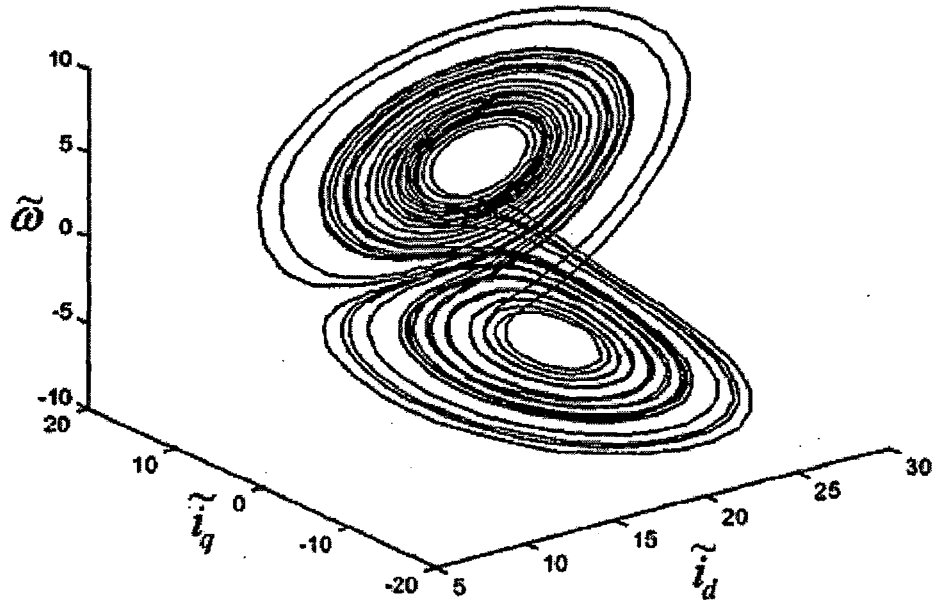

Fig.1.2:Chaotic Attractor generated at $\gamma = 20$

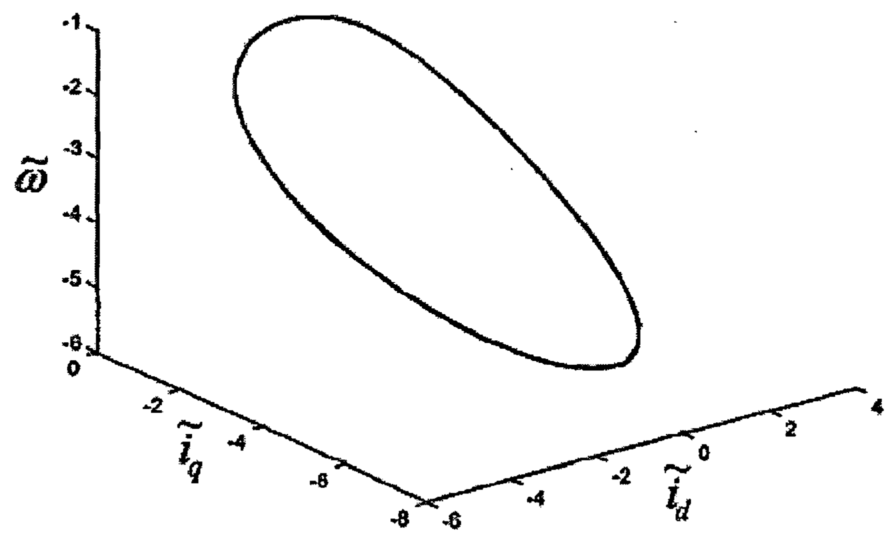

Fig.1.3: A limit cycle generated at $\tilde{u}_d = \tilde{u}_{dh} + 2.3432675$

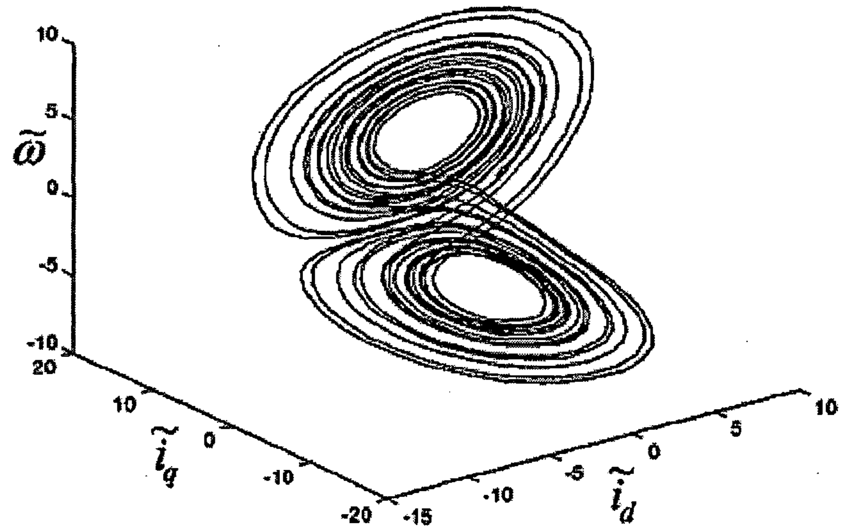

Fig.1.4: Chaotic Attractor generated at $\tilde{u}_d = \tilde{u}_{dh} - 3$

This paper [30] analyzed the effect of permanent magnets (PMs) on the occurrence of chaos in PM synchronous machines (PMSMs). Based on the derived nonlinear system equation, the bifurcation analysis revealed that the sizing of PMs significantly determines the stability of PMSMs. Hopf bifurcation and chaos may even occur in the PMSMs if the PMs are not properly sized. Experimental results of two practical PMSMs are provided to support the theoretical analysis. It may be noted here that for the first time, the experimental verification of occurrence of chaos was done in this paper.

This paper [31] is basically based on control of chaos. The performance of Permanent Magnet Synchronous Motor (PMSM) degrades due to chaos when its parameters fall into a certain area. Therefore, chaos should be suppressed or eliminated. The drawbacks of the existing control methods was analyzed. The nonlinear feedback principle was developed by using the direct-axis and the quadrature-axis stator voltage as manipulated variables. The control target attained unique asymptotically stable equilibrium under the nonlinear feedback principle. In this manner the control objective was implemented. The method investigated in this paper can be physically realized. The control forces put into force at any time. The target of the method may be any point in the strange attractor. The influences of the model error and measurement noise upon the control performance were studied by simulations. Simulation results established the effectiveness of the method in the presence of the model error and the measurement noise.

A nonlinear chaos controller based on the adaptive back-stepping approach for a chaotic PMSM drive was proposed in [32]. The controller was designed to prevent the PMSM drive from chaos and make it track the desired speed command. With the proposed controller, the PMSM drive could recover from chaotic behavior quickly and possesses good transient performance and robustness to parameter uncertainties. Finally, numerical simulations was carried out to validate the effectiveness of the proposed approach.

The drawbacks of existing chaos control methods Permanent magnet synchronous motor (PMSM) were analyzed, and a new nonlinear feedback control method was suggested to control the chaos in PMSM. The nonlinear feedback principle is developed using the direct axis and the quadrature axis stator voltage as manipulated variables. The control target will become a unique asymptotically stable equilibrium under the nonlinear feedback principle, by this way, the controlled states can reach the target and the control objective can be implemented. This method can be physically realized using nonlinear state feedback. The control forces can be put into effect at any time. The target of the method may be any point in the strange attractor. The influence of the model error and the measurement noise upon the control performance is studied via simulations. Simulation results show the effectiveness of this method under the presence of the model error and the measurement noise.

1.2.2 Brushless DC Motor(BLDCM):

Modeling of Brushless DC motor was reported in [10] . This paper addressed the modeling problem associated with brushless dc motors with non-uniform air gaps that operate in a range where magnetic saturation may exist. The mathematical model included the effects of reluctance variations as well as magnetic saturation to guarantee proper modeling of the system. An experimental procedure was also described and implemented in a laboratory environment to identify the electromagnetic characteristics of a BLDCM in the presence of magnetic saturation. It is demonstrated that the modeling problem associated with this class of BLDCM can be formulated in terms of mathematically modeling a set of multidimensional surfaces corresponding to the electromagnetic torque function and the flux linkages associated with the motor phase windings. The accuracy of the mathematical model constructed by the developed method was checked against experimental measurements.

[11] dealt with the open-loop dynamic characteristics of smooth-air-gap brushless dc motors. The steady-state characteristics of these systems, subject to constant input voltages and constant external torques, are formulated, whereby it is shown that the presence of viscous damping friction can cause the system to possess multiple physical equilibria. Furthermore, using an affine transformation, it was shown that the open-loop dynamics of smooth-air-gap brushless dc motors and the Lorem system, a system known to possess chaotic behavior, are equivalent. Finally, computer simulations were presented that verify the existence of strange attractors in the open-loop dynamics of brushless dc motors. It may be noted here that no experimental results was reported to establish the existence of the strange attractors.

1.2.3 Synchronous Reluctance Motor(SRM):

Nonlinear Phenomenon in Synchronous Reluctance Motor was reported in [38].This paper first presented the occurrence of Hopf bifurcation and chaos in a practical synchronous reluctance motor drive system. Based on the derived nonlinear system equation, the bifurcation analysis shows that the system loses stability via Hopf bifurcation when the -axis component of its three-phase motor voltages loses its control. Moreover, the corresponding Lyapunov exponent calculation further proved the existence of chaos. Finally, computer simulations and experimental results were presented to support the theoretical analysis.

1.2.4 Switched Reluctance Motor(SwRM):

Nonlinear phenomenon in Switched Reluctance Motor was first studied in [36]. In this paper, the investigation of the nonlinear dynamics of an adjustable-speed switched reluctance motor drive with voltage pulse width modulation (PWM) regulation was carried out. Nonlinear iterative mappings based on both nonlinear and approximately linear flux linkage models are derived, hence the corresponding sub-harmonic and chaotic behaviors are analyzed. Although both flux linkage models can produce similar results, the nonlinear one offers the merit of accuracy but with the sacrifice of computational time. Moreover, the bifurcation diagrams show that the system generally exhibits a period-doubling route to chaos.

In [17],[37], modeling, analysis, and experimentation of chaos in a switched reluctance drive system using voltage pulse width modulation were presented. Based on the proposed nonlinear flux linkage model of the SR drive system, the computation time to evaluate the Poincaré map and its Jacobian matrix can be significantly shortened. Moreover, the stability analysis of the fundamental operation was conducted, leading to determine the stable parameter ranges and hence to avoid the occurrence of chaos. Both computer simulation and experimental measurement were presented to verify the theoretical modeling and analysis.

1.2.5 Induction Motor(IM):

The reduced order modeling of Induction motor was described in [20].The generalized approach for simulation of the Induction Motor Model was presented in [21].

Variable frequency induction motor drives are known to become unstable at certain operating conditions, which causes unusual vibrations in the Systems. In the paper[22], the instability phenomena in power electronic induction motor drive systems were investigated from the point of view of bifurcation theory. A method to determine bifurcation values of system parameters is discussed. It was shown that some kinds of bifurcations were observed in power electronic induction motor drive systems. The proposed method made it possible not only determine instability regions of system parameters but also to investigate qualitative properties of the instability phenomena.

The nonlinear behaviour of Direct Torque Controlled (DTC) Induction Machine (IM) is studied in [23]. The nonlinearity was due to the dependence of the switching instants on the state variables. The aim of the paper was to derive analytical relations for the determination of the time evolution of state variables and on that basis to reveal the possible states of the system such as periodic, sub-harmonic and chaotic states.

In the paper [24], we explore further the occurrence of bifurcations in the indirect field oriented control of induction motors. New results reveal the occurrence of codimension-two bifurcation phenomena, such as a Bogdänov- Takens bifurcation.

1.3 Development in the field of theory of Bifurcation and Chaos:

Electrical Machines are inherently Lorenz like system with higher dimension, in general. Dimensions may vary depending on the no. of windings. However, their basic nature of nonlinear phenomenon does not vary significantly. Some developments are found in the theory of nonlinear dynamics in the systems inherently nonlinear. These are summarized categorically in this section.

In 1963, Lorenz[52] discovered chaos in a simple system of three autonomous ordinary differential equations that has only two quadratic nonlinearities, in order to describe the simplified Rayleigh-Benard problem. It is notable that the Lorenz system has seven terms on the right-hand side, two of which are nonlinear (xz and xy). In 1976, R¨ossler. found a three-dimensional quadratic autonomous chaotic system , which also has seven terms on the right-hand side, but with only one quadratic nonlinearity (xz). Obviously, the R¨ossler system has a simpler algebraic structure as compared to the Lorenz system. It was believed that the R¨ossler system[62] might be the simplest possible chaotic flow, where the simplicity refers to the algebraic representation rather than the physical process described by the equations nor the topological structure of the strange attractor. It is therefore interesting to ask whether or not there are three-dimensional autonomous chaotic systems with fewer than seven terms including only one or two quadratic nonlinearities? The fact is that R¨ossler actually had produced another even simpler chaotic system in 1979, which has only six terms with a single quadratic nonlinearity (y^2). Thus, the question becomes "How complicated a three-dimensional autonomous system must be in order to produce chaos?" The well-known Poincar′e-Bendixson theorem shows that chaos does not exist in a two-dimensional continuous-time autonomous system (or a second-order equation). Therefore, a necessary condition for a continuous-time autonomous system to be chaotic is to have three variables with at least one nonlinear term. As a side note, it is also known that there is a direct connection between three-dimensional quadratic chaotic systems and Lagrangian mixing. Lagrangian mixing poses some interesting questions about dynamical systems; however, since realistic models are mainly experimental and numerical, this subject is still in its early involving phase of development. Three-dimensional quadratic autonomous systems are very important for studying bifurcations, limit cycles, and chaotic flows. Recently, it is proved that three-dimensional dissipative quadratic systems of ordinary differential equations, with a total of four terms on the right-hand side, cannot exhibit chaos. Very recently, this result was extended to three-dimensional conservative quadratic systems. Later, it was known that autonomous chaotic flow could be produced by a three-dimensional quadratic autonomous system having five terms on the right-hand side, with at least one quadratic nonlinearity, or having six terms with a single quadratic nonlinearity. Lately, chaotic flow in an algebraically simplest three-dimensional quadratic autonomous system was found by using jerky functions, which has only five terms with a single quadratic nonlinearity (y^2). In fact, this system is simpler than any others previously found, in the sense of both its jerky representation and its representation as a dynamical system. However, it is noticed that the simplicity of a system can be measured in various ways. Algebraic simplicity of system's structure is one way, and topological simplicity of chaotic attractor is another. R¨ossler's attractor and most of Sprott's examples[65-67] are topologically simpler than the two-scroll Lorenz attractor. In fact, R¨ossler attractor has a single folded band structure. Furthermore, its one-scroll structure is the simplest topological structure for a three dimensional quadratic autonomous chaotic system. Thus, it is interesting to ask whether or not there are three-dimensional quadratic autonomous chaotic systems that can display attractors with more complex topological structures than the two-scroll Lorenz attractor. That is, "Is the two-scroll Lorenz attractor the most complex topological structure of this class of chaotic systems?" The answer is *no*. In fact, the recently discovered Chen's attractor and its associate transition attractor have more complex topological structures than the original Lorenz attractor. Nevertheless, these newly found attractors also have two-scrolls but not more than that. Therefore, in combining these two sides of the view on simplicity (or complexity) of a chaotic system, it would be truly interesting to seek for lower-dimensional chaotic systems that have a simple algebraic system structure but with a complex topological attractor structure. This is not just for theoretical interest; such chaotic systems would be useful in some engineering applications such as secure communications. In the endeavor of finding three-dimensional quadratic autonomous chaotic systems, other than luckily encountering chaos in unexpected simulations or experiments, there seems to be two sensible methods: one is Sprott's exhaustive searching via computer programming, and the other is Chen's theoretical approach via chaotification. For nearly 40 years, one of the classic icons of modern nonlinear dynamics has been the Lorenz attractor. In 2000, Smale described eighteen challenging mathematical problems for the twenty-first century, in which the fourteenth problem is about the Lorenz attractor. In this regard, one concerned problem has been: "Does it really exist?" Only very recently, the Lorenz attractor was mathematically confirmed to exist. Another interesting question regarding chaotic systems is: "How complex of the topological structure of the chaotic attractor, if it exists, of a three-dimensional quadratic autonomous system can be?" Here, it is noted that the complexity of the topological structure of a chaotic attractor may be measured in two aspects: the number of sub-attractors and the number of parts ("scrolls" or "wings") of the attractor. It has been well known that piecewise-linear function can generate n-scroll attractors in Chua's circuit, and in a circuit with the absolute value as the only nonlinearity, it can also create a complex n-scroll chaotic attractor. Recently, found a simple three-dimensional quadratic autonomous chaotic system, which can display a 2-scroll and also (visually) a 4-scroll attractor. Motivated by these works, this article introduces one more simple three-dimensional quadratic autonomous system, which can generate two 1-scroll chaotic attractors simultaneously, or two complex 2-scroll chaotic attractors simultaneously. It is believed that a three-dimensional quadratic autonomous chaotic system can have at most two chaotic attractors simultaneously, and the system to be discussed here is one such simple but interesting chaotic systems.

1.4 *Motivation of the present work:*

It has been noticed that nonlinear phenomenon in conventional electrical machines have been studied for each machines individually. No attempt so far have been made to study the same in generalized manner. On the other hand, the generalized approach of studying electrical machine is increasingly popular day by day as all conventional machines can be modeled using this approach. So instead, of studying the nonlinear phenomenon for each conventional machine as piece meal it can be studied for the generalized machine as a whole. Though many books and literature are available on Generalized theory of Electrical Machines no literature was found so far toward the study of nonlinear phenomenon in Generalized Electrical Machines related to Bifurcation and Chaos.

Secondly, in most cases, it was noticed that the nonlinear phenomenon was investigated through numerical simulation. Very few cases were found where the numerical results were verified using experimental results.

Therefore, keeping the above fact in the mind, following attempts can be made through some relevant works:

- an attempt may be made to study the nonlinear phenomenon in Generalized Electrical Machines and that approach has to be applied to other machines as much as practicable to study the same for them.
- The results obtained form numerical simulation may be verified using experimental results.