NONLINEAR PHENOMENON

IN

ELECTRICAL MACHINES

A

Thesis

Submitted for the Degree of

Doctor of Philosophy (Engineering)

by

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CERTIFICATE

This is to certify that the thesis entitled **"NONLINEAR PHENOMENON IN ELECTRICAL MACHINES "** submitted by Sri Goutam Kumar Panda who got his name registered on 07.10.2002 for the award of PhD (Engineering) degree of the University of North Bengal , absolutely based upon his own work under my supervision and that neither the thesis nor any part of it has been submitted for any degree or diploma or any other academic award anywhere before.

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ABSTRACT

The state-space model of an electrical machine having n no. of windings is represented by n no. of first order differential equations. Some of these equations are inherently nonlinear. These equations alongwith the torque/power equation are used to study the dynamical behaviors of the machines. However, as some equations are inherently nonlinear different techniques are adopted to solve these equations, e.g., speed is assumed to be constant or a known function of time, Doherty's Constant flux-linkage Theorem is applied, nonlinear terms in the state equations are ignored etc. Using these techniques, equations are actually linearized and solved. Therefore, the nonlinear phenomenon which occurs in a practical machine, is completely or partially omitted in the conventional studies and the dynamical behaviors of the machines are not being properly explained or still remain unexplained which causes a lot of inconvenience in understanding and problem-solving of the electrical machines. Furthermore, now electrical machines find some of it's sophisticated application like in space craft, remote sensing etc. So, the detailed study of the dynamical behaviors of the electrical machines which include bifurcation, chaos etc., have to be done to cater the need of the modern age.

In this work, an attempt will be made to study the dynamical behaviors of the electrical machines. In stead of classical approach of studying the machines individually, here generalized approach will be adopted to study the dynamical behaviors so that the same can be exploited for individual conventional machines. State-space model of a Generalized Machine will be developed and State equations will be normalized and using these equations dynamical behaviors of the machine will be rigorously studied. In this work, entire parameter space of the machine may be explored and the route of chaos through bifurcation may be identified. This idea about the parameters may be used in designing the electrical machines and thus desired dynamical behavior will be obtained from those machines.

PREFACE

It is well known that the existing mathematical models of motors are multivariable, nonlinear, and strongly coupled, therefore these systems can exhibit complex behaviors It is now a common belief that understanding and utilizing the rich dynamics, such as bifurcations and chaos, of nonlinear systems have an important impact on the modern technology. In this work , an attempt has been made to study the nonlinear phenomenon in Generalized Electrical Machines and that approach is applied to other machines to study its effectiveness.

In Chapter 1, the study of Nonlinear Phenomenon in the field of electrical machines done so far outlined. The literature survey is furnished. The scope of the present work is described and the relevance of the work is established.

In Chapter 2, the basic idea about the nonlinear phenomenon , bifurcation and chaos has described in brief so that it can be used as a ready reference.

In Chapter 3, the bifurcation and chaos in Lorenz like systems have been studied. as the dynamic equations of conventional electrical electrical machines and generalized machine are similar to those of Lorenz like systems with higher dimensions. Therefore, the idea about the bifurcation and chaos of Lorenz like systems will in turn be very helpful for the present work.

In Chapter 4, the generalized theory of electrical machines alongwith the recent development are outlined so that it can be applied to model the conventional electrical machines effectively.

In chapter 5, the nonlinear phenomenon in Generalized Electrical Machines have been studied in details. Initially, it has been studied for exact model of Generalized Electrical Machine. Then, it has been studied for the approximate model. Both models of the Generalized Electrical Machines have been simulated and the results of the simulation have been furnished. The results show that the machine may operate with periodicity one, two or more or become chaotic with different sets of parameters. On the basis of the observations, the nonlinear phenomenon in other conventional machines have been outlined.

In chapter 6, the approach followed in studying the nonlinear phenomenon in Generalized Electrical Machines has been applied to conventional machines and the effectiveness has been studied.

Finally the conclusion and future works are outlined in chapter 7.

CONTENTS

Chapter 1

INTRODUCTION

1.1 General:

For a machine represented by n number of coils there are n voltage equations and a torque equation. If the n applied voltages and the applied torque are known, as well as initial conditions, the (n+1) equations are sufficient to determine the n currents and the speed. Hence theoretically the performance of the machine can completely be determined[1],[61].

In general case, the equations containing the product terms in which the speed and the currents, i.e., the state variables of the machine are strongly coupled, are nonlinear differential equations and have to be solved by the numerical integrations. For particular condition of operation, however, considerable simplifications are often made, and the types of problems are accordingly classified[4].

Under steady state conditions, the speed is constant and the voltage equations are dealt independently of the torque equations. The voltage equations reduce under steady conditions to a set of n ordinary linear algebraic equations containing real variables for dc conditions and complex variables for ac conditions.

Under transient conditions, when voltages and currents may vary in any manner, the problem is greatly simplified by taking a constant value of speed, then the voltage equations are treated independently of the torque equations and they are linear differential equations with constant co-efficient.

When the voltages are known the equations can be solved, either by algebraic methods for steady state problems, or by operational methods for transient problems, and the current thus obtained can be substituted in the torque equation. If the speed is constant the externally applied torque is obtained directly because it is equal to the electrical torque.

The net stage in difficulty in study of the dynamics of the electrical machines arises if the speed varies but is a known function of time. It is then still possible to solve the voltage equations separately, but the coefficients are not constant as before. The equations with variable coefficients for which the simple operational method does not apply. Usually only numerical solutions are possible. Once the current is determined , torque can be calculated.

The most difficult problems are those for which the speed is an unknown variable. Most of the transient problems are of this type. These problems are solved by transfer Function Method, Eigen Vector Method, Transition Matrix Method, State Variable method etc. In all of those methods of solution, system is linearized and solved. Therefore, due to linearization, some nonlinear phenomenon are eliminated[7].

Since the 1970s, dynamic characteristics of various motors are widely studied, to deal with starting up, speed control, and oscillations of the motors. In the study of dynamic characteristics of motors, there are many problems that remain to be further addressed, such as their low-speed feature, known as the low-frequency oscillations of speed-controlled motors. These problems are closely related to the studies of chaos in nonlinear systems.

Bifurcation analysis and the study of chaos in nonlinear system is gradually increasing. However, its application in the field of electrical machine is till now very insignificant. Some of those works are reported in next section.

1.2 Background and Review of the previous works:

Most of the researchers have studied the nonlinear phenomenon for different electrical machines using their classical models. Some of them only identified the phenomenon by simulation, very few of them were experimentally verified. In some works, route of chaos, characteristics, different bifurcations are reported. Therefore, previous works are categorized for different machines.

1.2.1 Permanent Magnet Synchronous motor(PMSM):

A detailed model of Permanent Magnet Synchronous Motor was first presented by Consoli et al. in [25]. They developed the model and equivalent circuit using d-q variables for steady state analysis and experimental verification of the proposed model was reported.

The nonlinear dynamic characteristics of variable-speed permanent-magnet machines in the presence of reluctance variations was considered in [26]. A compact representation of the machines' dynamics was presented and in turn incorporated in investigating the steady-state characteristics of the machines, subject to constant input voltages and load torques. It was shown that the systems under investigation possess multiple equilibria which profoundly affect their global stability and dynamic characteristics. Furthermore, conditions leading to chaotic behavior and Hopf Bifurcation was pointed out for the first time and briefly discussed. Finally, numerical simulations were presented to help verify the results. However, the computed results were inadequate and no experimental result presented. Similar information was found in [28].
A brief design review of the Permanent Magnet Synchronous Motors was presented in [27]. A procedure was described to predict the steady state and dynamic performances of a brushless permanent magnet synchronous motor. The proposed techniques have been experimentally verified in a laboratory permanent magnet synchronous motor.

In [28], a complete model was developed to examine the dynamic behaviors of a high performance vector controlled permanent magnet synchronous motor (PMSM) drive. The d,q axis model of the PMSM was used to simulate and analyze the complete drive system. Based on the vector control principle and current feedback control, the nonlinear dynamic model of the PMSM drive can be simplified and linearized. The resultant linear model was used for studying the dynamic behaviors of the drive system. Moreover, it is used for the design of speed controller. The entire control scheme of the drive system was successfully implemented using a high speed DSP. The experimental results was validated the theoretical ones.

G Chen et al. studied the nonlinear phenomenon in Permanent Magnet Synchronous Motor [29]. The mathematical model of the PMSM was derived, which was a three-dimensional autonomous equation with only two quadratic terms, and this model is fit for carrying on bifurcation and chaos analysis. Secondly, the steady state characteristics of the motor, when subject to constant input voltage and external torque, were formulated. A third-order polynomial equation is also derived whose solutions correspond to the steady-state values of the motor angular velocity. Furthermore, based on the Hopf bifurcation condition, the parameters of the PMSM in three different cases, with which the system can exhibit such desired dynamic characteristics as limit cycles (LCs) and chaotic behaviors were determined. Finally, computer simulations were presented to verify the presence of strange attractors in the PMSM. Mathematical model of PMSM as adopted in [29] was:

$$\begin{aligned}
\frac{di_d}{dt} &= (u_d - R_1 i_d + \omega L_q i_q)/L_d \\
\frac{di_q}{dt} &= (u_q - R_1 i_q - \omega L_d i_d - \omega \psi_r)/L_d \\
\frac{d\omega}{dt} &= [n_p \psi_r i_q + n_p (L_d - L_q) i_d i_q - T_L - \beta\omega]/J
\end{aligned} \qquad [1.1]$$

where i_d, i_q and ω are the state variables, which represent currents and motor angular frequency, respectively, u_d and u_q the direct- and quadrature-axis stator voltage components, respectively, J, the polar moment of inertia, T_L the external load torque, β the viscous damping coefficient, R_1 the stator winding resistance, L_d and L_q the direct and quadrature-axis stator inductors, respectively, ψ_r the permanent magnet flux, and np the number of pole-pairs.Eqn.1.1 was normalized as follows:

$$\frac{d\tilde{i}_d}{d\tilde{t}} = -\tilde{i}_d + \tilde{\omega}.\tilde{i}_q + \tilde{u}_d$$
$$\frac{d\tilde{i}_q}{d\tilde{t}} = -\tilde{i}_q + \tilde{\omega}.\tilde{i}_d + \gamma\tilde{\omega} + \tilde{u}_q \qquad [1.2]$$
$$\frac{d\tilde{\omega}}{d\tilde{t}} = \sigma(\tilde{i}_q - \tilde{\omega}) - \tilde{T}_L$$

Where $b = \frac{L_q}{L_d}$ $\gamma = \frac{-\psi_r}{kL_q}$, k= $\frac{\beta}{n_p \tau \psi_r}$, $\tau = \frac{L_q}{R_1}$, $\sigma = \frac{\beta\tau}{J}$,

$i_d = bk.\tilde{i}_d$, $i_q = k\tilde{i}_q$, $\omega = \tilde{\omega}/\tau$, $t = \tau.\tilde{t}$. It was also assumed that Ld=Lq=L

Equilibrium of the system was calculated as follows:

$$\tilde{i}_d = \tilde{\omega}^2 + \frac{\tilde{T}_L}{\sigma}\tilde{\omega} + \tilde{u}_d$$
$$\tilde{i}_q = \tilde{\omega} + \frac{\tilde{T}_L}{\sigma} \qquad [1.3]$$
$$\tilde{\omega}^3 + \frac{\tilde{T}_L}{\sigma}\tilde{\omega}^2 + (\tilde{u}_d - \gamma + 1)\tilde{\omega} + \frac{\tilde{T}_L}{\sigma} - \tilde{u}_q = 0$$

It may be noted here that the existence or creation or disappearance of the equilibrium point depends on the value of $\tilde{\omega}$ .i.e., the roots of third equation of Eqn. 1.3. It may be further noted the equation was a cubic equation whose roots and their nature were not readily available. It was also reported that as parameter changes bifurcation occurs. Hopf bifurcation occurs when the corresponding Jacobian matrix has a pair of purely imaginary eigenvalues, with the remaining eigenvalues having nonzero real parts. For the PMSM system, its Hopf bifurcation and chaotic behavior are discussed under different sets of parameter values.

When $\tilde{u}_d = \tilde{u}_q = \tilde{T}_L = 0$, the system was identical to the Lorenz equation. This case can be thought of as that, after an operating period of the system, the external inputs were set to zero. Applying the equilibrium conditions, it was determined that the origin was an equilibrium state and that other two nontrivial equilibria exist, if $\gamma \prec 1$, which are defined by

$$\begin{bmatrix} \tilde{i}_d^{eq} \\ \tilde{i}_q^{eq} \\ \tilde{\omega}^{eq} \end{bmatrix} = \begin{bmatrix} \gamma - 1 \\ \pm\sqrt{\gamma - 1} \\ \pm\sqrt{\gamma - 1} \end{bmatrix}$$

A little linear analysis shows that the origin is stable if $0 < \gamma < 1$and loses stability in a pitchfork bifurcation at $\gamma = 1$, creating the two nontrivial equilibria which are initially stable. when evaluated at the nontrivial equilibria. Note that since the two nontrivial equilibria are symmetric, their stability must be the same. For the bifurcation of the two nontrivial equilibrium, Hopf bifurcation point was determined explicitely. It was given by

$$\gamma_h = \frac{\sigma(\sigma + 4)}{\sigma - 2}$$

Similarly, setting $\tilde{u}_q = \tilde{T}_L = 0$, Hopf bifurcation point was determined as

$$u_{dh} = \frac{\sigma^2 - \gamma\sigma + 4\sigma + 2\gamma}{2 - \sigma}$$

Some significant simulation results were reported for the parameters:

Ld=Lq=L=14.25mH, R1=0.9Ω, $\psi_r = 0.031 Nm/A$, n_p=1, J=4.7x10^{-5} Kgm^2

$\beta = 0.0162 N/rad/s$. However no experimental result was presented in its support.

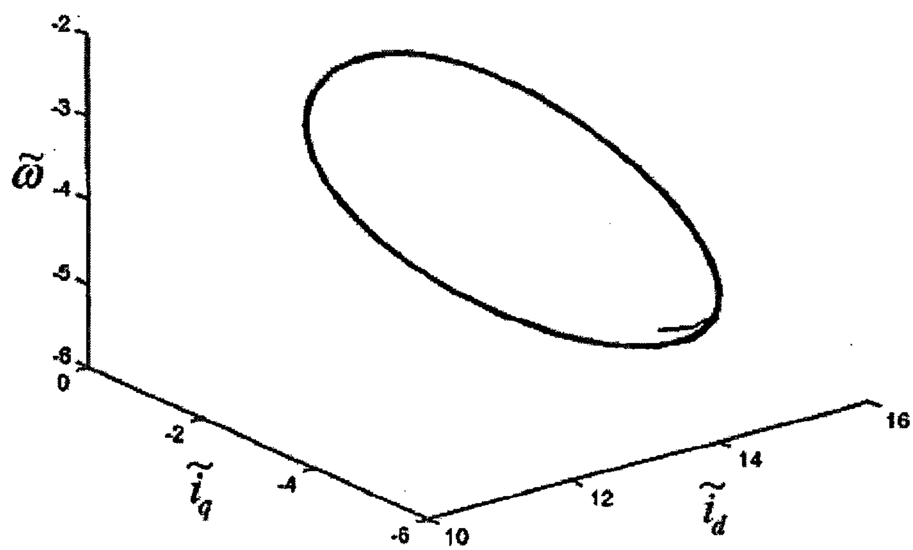


Fig.1.1: A limit cycle generated at $\gamma = 14.1$

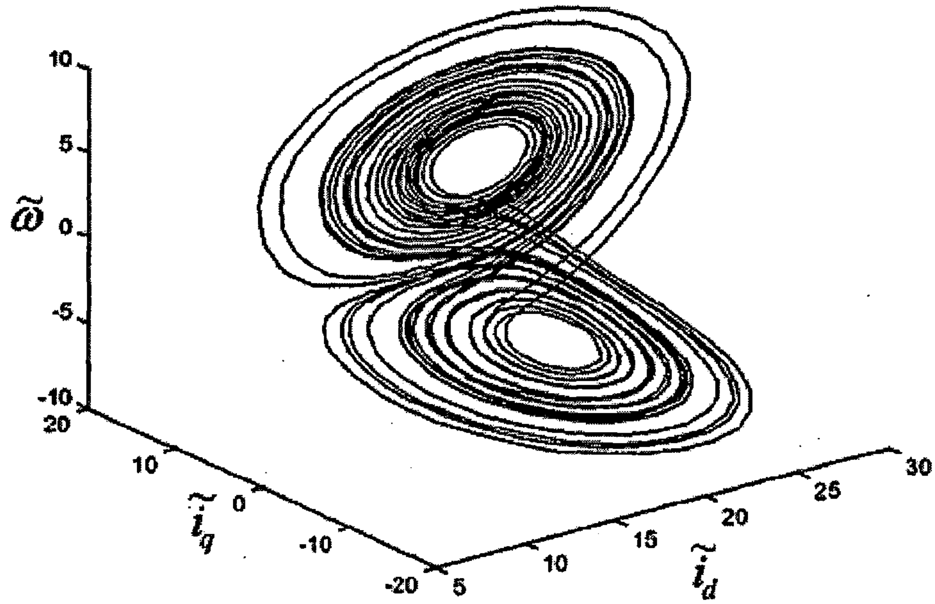


Fig.1.2:Chaotic Attractor generated at $\gamma = 20$

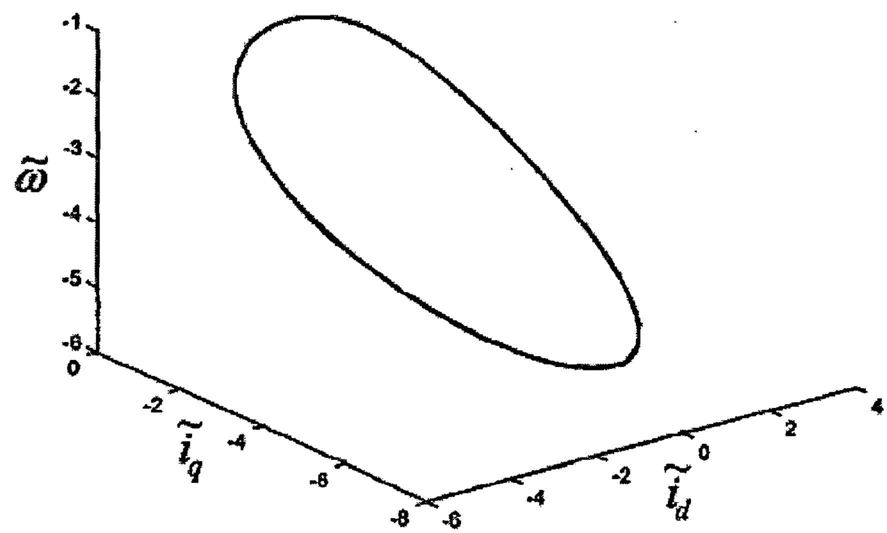


Fig.1.3: A limit cycle generated at $\tilde{u}_d = \tilde{u}_{dh} + 2.3432675$

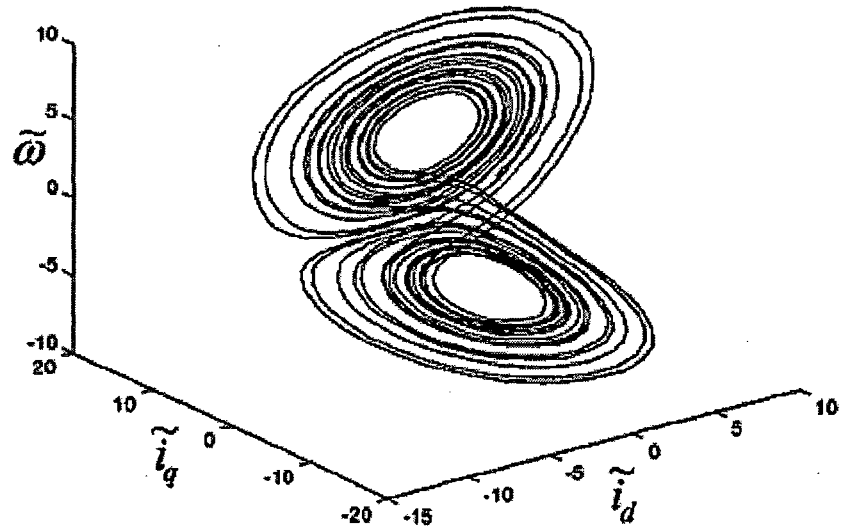


Fig.1.4: Chaotic Attractor generated at $\tilde{u}_d = \tilde{u}_{dh} - 3$

This paper [30] analyzed the effect of permanent magnets (PMs) on the occurrence of chaos in PM synchronous machines (PMSMs). Based on the derived nonlinear system equation, the bifurcation analysis revealed that the sizing of PMs significantly determines the stability of PMSMs. Hopf bifurcation and chaos may even occur in the PMSMs if the PMs are not properly sized. Experimental results of two practical PMSMs are provided to support the theoretical analysis. It may be noted here that for the first time, the experimental verification of occurrence of chaos was done in this paper.

This paper [31] is basically based on control of chaos. The performance of Permanent Magnet Synchronous Motor (PMSM) degrades due to chaos when its parameters fall into a certain area. Therefore, chaos should be suppressed or eliminated. The drawbacks of the existing control methods was analyzed. The nonlinear feedback principle was developed by using the direct-axis and the quadrature-axis stator voltage as manipulated variables. The control target attained unique asymptotically stable equilibrium under the nonlinear feedback principle. In this manner the control objective was implemented. The method investigated in this paper can be physically realized. The control forces put into force at any time. The target of the method may be any point in the strange attractor. The influences of the model error and measurement noise upon the control performance were studied by simulations. Simulation results established the effectiveness of the method in the presence of the model error and the measurement noise.

A nonlinear chaos controller based on the adaptive back-stepping approach for a chaotic PMSM drive was proposed in [32]. The controller was designed to prevent the PMSM drive from chaos and make it track the desired speed command. With the proposed controller, the PMSM drive could recover from chaotic behavior quickly and possesses good transient performance and robustness to parameter uncertainties. Finally, numerical simulations was carried out to validate the effectiveness of the proposed approach.

The drawbacks of existing chaos control methods Permanent magnet synchronous motor (PMSM) were analyzed, and a new nonlinear feedback control method was suggested to control the chaos in PMSM. The nonlinear feedback principle is developed using the direct axis and the quadrature axis stator voltage as manipulated variables. The control target will become a unique asymptotically stable equilibrium under the nonlinear feedback principle, by this way, the controlled states can reach the target and the control objective can be implemented. This method can be physically realized using nonlinear state feedback. The control forces can be put into effect at any time. The target of the method may be any point in the strange attractor. The influence of the model error and the measurement noise upon the control performance is studied via simulations. Simulation results show the effectiveness of this method under the presence of the model error and the measurement noise.

1.2.2 Brushless DC Motor(BLDCM):

Modeling of Brushless DC motor was reported in [10] . This paper addressed the modeling problem associated with brushless dc motors with non-uniform air gaps that operate in a range where magnetic saturation may exist. The mathematical model included the effects of reluctance variations as well as magnetic saturation to guarantee proper modeling of the system. An experimental procedure was also described and implemented in a laboratory environment to identify the electromagnetic characteristics of a BLDCM in the presence of magnetic saturation. It is demonstrated that the modeling problem associated with this class of BLDCM can be formulated in terms of mathematically modeling a set of multidimensional surfaces corresponding to the electromagnetic torque function and the flux linkages associated with the motor phase windings. The accuracy of the mathematical model constructed by the developed method was checked against experimental measurements.

[11] dealt with the open-loop dynamic characteristics of smooth-air-gap brushless dc motors. The steady-state characteristics of these systems, subject to constant input voltages and constant external torques, are formulated, whereby it is shown that the presence of viscous damping friction can cause the system to possess multiple physical equilibria. Furthermore, using an affine transformation, it was shown that the open-loop dynamics of smooth-air-gap brushless dc motors and the Lorem system, a system known to possess chaotic behavior, are equivalent. Finally, computer simulations were presented that verify the existence of strange attractors in the open-loop dynamics of brushless dc motors. It may be noted here that no experimental results was reported to establish the existence of the strange attractors.

1.2.3 Synchronous Reluctance Motor(SRM):

Nonlinear Phenomenon in Synchronous Reluctance Motor was reported in [38].This paper first presented the occurrence of Hopf bifurcation and chaos in a practical synchronous reluctance motor drive system. Based on the derived nonlinear system equation, the bifurcation analysis shows that the system loses stability via Hopf bifurcation when the -axis component of its three-phase motor voltages loses its control. Moreover, the corresponding Lyapunov exponent calculation further proved the existence of chaos. Finally, computer simulations and experimental results were presented to support the theoretical analysis.

1.2.4 Switched Reluctance Motor(SwRM):

Nonlinear phenomenon in Switched Reluctance Motor was first studied in [36]. In this paper, the investigation of the nonlinear dynamics of an adjustable-speed switched reluctance motor drive with voltage pulse width modulation (PWM) regulation was carried out. Nonlinear iterative mappings based on both nonlinear and approximately linear flux linkage models are derived, hence the corresponding sub-harmonic and chaotic behaviors are analyzed. Although both flux linkage models can produce similar results, the nonlinear one offers the merit of accuracy but with the sacrifice of computational time. Moreover, the bifurcation diagrams show that the system generally exhibits a period-doubling route to chaos.

In [17],[37], modeling, analysis, and experimentation of chaos in a switched reluctance drive system using voltage pulse width modulation were presented. Based on the proposed nonlinear flux linkage model of the SR drive system, the computation time to evaluate the Poincaré map and its Jacobian matrix can be significantly shortened. Moreover, the stability analysis of the fundamental operation was conducted, leading to determine the stable parameter ranges and hence to avoid the occurrence of chaos. Both computer simulation and experimental measurement were presented to verify the theoretical modeling and analysis.

1.2.5 Induction Motor(IM):

The reduced order modeling of Induction motor was described in [20].The generalized approach for simulation of the Induction Motor Model was presented in [21].

Variable frequency induction motor drives are known to become unstable at certain operating conditions, which causes unusual vibrations in the Systems. In the paper[22], the instability phenomena in power electronic induction motor drive systems were investigated from the point of view of bifurcation theory. A method to determine bifurcation values of system parameters is discussed. It was shown that some kinds of bifurcations were observed in power electronic induction motor drive systems. The proposed method made it possible not only determine instability regions of system parameters but also to investigate qualitative properties of the instability phenomena.

The nonlinear behaviour of Direct Torque Controlled (DTC) Induction Machine (IM) is studied in [23]. The nonlinearity was due to the dependence of the switching instants on the state variables. The aim of the paper was to derive analytical relations for the determination of the time evolution of state variables and on that basis to reveal the possible states of the system such as periodic, sub-harmonic and chaotic states.

In the paper [24], we explore further the occurrence of bifurcations in the indirect field oriented control of induction motors. New results reveal the occurrence of codimension-two bifurcation phenomena, such as a Bogdänov- Takens bifurcation.

1.3 Development in the field of theory of Bifurcation and Chaos:

Electrical Machines are inherently Lorenz like system with higher dimension, in general. Dimensions may vary depending on the no. of windings. However, their basic nature of nonlinear phenomenon does not vary significantly. Some developments are found in the theory of nonlinear dynamics in the systems inherently nonlinear. These are summarized categorically in this section.

In 1963, Lorenz[52] discovered chaos in a simple system of three autonomous ordinary differential equations that has only two quadratic nonlinearities, in order to describe the simplified Rayleigh-Benard problem. It is notable that the Lorenz system has seven terms on the right-hand side, two of which are nonlinear (xz and xy). In 1976, R¨ossler. found a three-dimensional quadratic autonomous chaotic system , which also has seven terms on the right-hand side, but with only one quadratic nonlinearity (xz). Obviously, the R¨ossler system has a simpler algebraic structure as compared to the Lorenz system. It was believed that the R¨ossler system[62] might be the simplest possible chaotic flow, where the simplicity refers to the algebraic representation rather than the physical process described by the equations nor the topological structure of the strange attractor. It is therefore interesting to ask whether or not there are three-dimensional autonomous chaotic systems with fewer than seven terms including only one or two quadratic nonlinearities? The fact is that R¨ossler actually had produced another even simpler chaotic system in 1979, which has only six terms with a single quadratic nonlinearity (y^2). Thus, the question becomes "How complicated a three-dimensional autonomous system must be in order to produce chaos?" The well-known Poincar′e-Bendixson theorem shows that chaos does not exist in a two-dimensional continuous-time autonomous system (or a second-order equation). Therefore, a necessary condition for a continuous-time autonomous system to be chaotic is to have three variables with at least one nonlinear term. As a side note, it is also known that there is a direct connection between three-dimensional quadratic chaotic systems and Lagrangian mixing. Lagrangian mixing poses some interesting questions about dynamical systems; however, since realistic models are mainly experimental and numerical, this subject is still in its early involving phase of development. Three-dimensional quadratic autonomous systems are very important for studying bifurcations, limit cycles, and chaotic flows. Recently, it is proved that three-dimensional dissipative quadratic systems of ordinary differential equations, with a total of four terms on the right-hand side, cannot exhibit chaos. Very recently, this result was extended to three-dimensional conservative quadratic systems. Later, it was known that autonomous chaotic flow could be produced by a three-dimensional quadratic autonomous system having five terms on the right-hand side, with at least one quadratic nonlinearity, or having six terms with a single quadratic nonlinearity. Lately, chaotic flow in an algebraically simplest three-dimensional quadratic autonomous system was found by using jerky functions, which has only five terms with a single quadratic nonlinearity (y^2). In fact, this system is simpler than any others previously found, in the sense of both its jerky representation and its representation as a dynamical system. However, it is noticed that the simplicity of a system can be measured in various ways. Algebraic simplicity of system's structure is one way, and topological simplicity of chaotic attractor is another. R¨ossler's attractor and most of Sprott's examples[65-67] are topologically simpler than the two-scroll Lorenz attractor. In fact, R¨ossler attractor has a single folded band structure. Furthermore, its one-scroll structure is the simplest topological structure for a three dimensional quadratic autonomous chaotic system. Thus, it is interesting to ask whether or not there are three-dimensional quadratic autonomous chaotic systems that can display attractors with more complex topological structures than the two-scroll Lorenz attractor. That is, "Is the two-scroll Lorenz attractor the most complex topological structure of this class of chaotic systems?" The answer is *no*. In fact, the recently discovered Chen's attractor and its associate transition attractor have more complex topological structures than the original Lorenz attractor. Nevertheless, these newly found attractors also have two-scrolls but not more than that. Therefore, in combining these two sides of the view on simplicity (or complexity) of a chaotic system, it would be truly interesting to seek for lower-dimensional chaotic systems that have a simple algebraic system structure but with a complex topological attractor structure. This is not just for theoretical interest; such chaotic systems would be useful in some engineering applications such as secure communications. In the endeavor of finding three-dimensional quadratic autonomous chaotic systems, other than luckily encountering chaos in unexpected simulations or experiments, there seems to be two sensible methods: one is Sprott's exhaustive searching via computer programming, and the other is Chen's theoretical approach via chaotification. For nearly 40 years, one of the classic icons of modern nonlinear dynamics has been the Lorenz attractor. In 2000, Smale described eighteen challenging mathematical problems for the twenty-first century, in which the fourteenth problem is about the Lorenz attractor. In this regard, one concerned problem has been: "Does it really exist?" Only very recently, the Lorenz attractor was mathematically confirmed to exist. Another interesting question regarding chaotic systems is: "How complex of the topological structure of the chaotic attractor, if it exists, of a three-dimensional quadratic autonomous system can be?" Here, it is noted that the complexity of the topological structure of a chaotic attractor may be measured in two aspects: the number of sub-attractors and the number of parts ("scrolls" or "wings") of the attractor. It has been well known that piecewise-linear function can generate *n*-scroll attractors in Chua's circuit, and in a circuit with the absolute value as the only nonlinearity, it can also create a complex *n*-scroll chaotic attractor. Recently, found a simple three-dimensional quadratic autonomous chaotic system, which can display a 2-scroll and also (visually) a 4-scroll attractor. Motivated by these works, this article introduces one more simple three-dimensional quadratic autonomous system, which can generate two 1-scroll chaotic attractors simultaneously, or two complex 2-scroll chaotic attractors simultaneously. It is believed that a three-dimensional quadratic autonomous chaotic system can have at most two chaotic attractors simultaneously, and the system to be discussed here is one such simple but interesting chaotic systems.

1.4 *Motivation of the present work:*

It has been noticed that nonlinear phenomenon in conventional electrical machines have been studied for each machines individually. No attempt so far have been made to study the same in generalized manner. On the other hand, the generalized approach of studying electrical machine is increasingly popular day by day as all conventional machines can be modeled using this approach. So instead, of studying the nonlinear phenomenon for each conventional machine as piece meal it can be studied for the generalized machine as a whole. Though many books and literature are available on Generalized theory of Electrical Machines no literature was found so far toward the study of nonlinear phenomenon in Generalized Electrical Machines related to Bifurcation and Chaos.

Secondly, in most cases, it was noticed that the nonlinear phenomenon was investigated through numerical simulation. Very few cases were found where the numerical results were verified using experimental results.

Therefore, keeping the above fact in the mind, following attempts can be made through some relevant works:

- an attempt may be made to study the nonlinear phenomenon in Generalized Electrical Machines and that approach has to be applied to other machines as much as practicable to study the same for them.
- The results obtained form numerical simulation may be verified using experimental results.

Chapter **2**

INTRODUCTION TO NONLINEAR DYNAMICS AND CHAOS

2.1 Introduction:

Tell me, O Muses who dwell on Olympos, and observe proper order
for each thing as it first came into being.
Chaos was born first and after her came Gaia
the broad-breasted, the firm seat of all
the immortals who hold the peaks of snowy Olympos,...
- *Hesiod,* Theogony, *lines 114-118*

This chapter covers the basic concepts of chaos.

2.2 Dynamic System, State and State-space:

Everything in this world exists in motion. There is nothing static or unchangeable. Some matters of this material world may appear to be static but those are also changing. Ever since this fact is recognized the study of dynamics has been a major pursuit. At first, all investigations were piecemeal. Newtonian scientists were studying the dynamics of the moving bodies , biologists were studying the changes in living organisms, chemists were studying the chemical properties of the materials etc. Gradually, it has been recognized that though the objective of these studies are different, there are common elements in all changes. Therefore, a body of knowledge is gradually emerged which is Dynamical System in general. A system whose status changes with time is called a *Dynamical System.*[1].

The status of a Dynamical System at any instant and the change in status of the system with time is uniquely expressed by a minimum number of properly identified variables known as *State Variables*. The study of the dynamics of a dynamical system is essentially an investigation of how these state variables change with respect to time. Mathematically, this is expressed as the rate of change of these state variables to their current values in terms of a system of first order differential equations. Thus, if state variables are given by $\{x_i, i = 1,2,.........,n\}$ then the state-space model of the system is expressed as in the form of a set of first order differential equations as follows:

$$\dot{x}_1 = \frac{dx_1}{dt} = f(x_1, x_2, ...x_n)$$

$$\dot{x}_2 = \frac{dx_2}{dt} = f(x_1, x_2, ...x_n)$$

.

.

$$\dot{x}_n = \frac{dx_n}{dt} = f(x_1, x_2, ...x_n)$$

In general, $\dot{x}_i = f_i(x_1, x_2, ..., x_n)$ (2.1)

or in vector form, $\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X})$ (2.2)

The ability to express $\dot{\mathbf{X}}$ purely as a function of $\mathbf{X}$ is what identifies x_i as state variables.

Some systems change discretely. Such a situation may arise when a system is actually changing continuously but is observed only at certain intervals. Most power electronic circuits can be modeled this way. There can be inherently discrete systems as in digital electronic systems or populations of various species. In such cases the state variables at the (n+1)-th instant are expressed as a function of those at the n-th instant:

$$\mathbf{x_{n+1}} = f(\mathbf{x_n}) \tag{2.3}$$

The equations of forms 2.2 or 2.3 with a given set of initial conditions can be solved either analytically or numerically and the solutions give the future states of the system as functions of time.

The dynamics of a system can be visualized by constructing a space with the state variables as coordinates. This is called the state space or phase space. The state of the system at any instant is represented by appoint in the space. Starting from any given initial condition, the state-point moves in the state space and this movement is completely determined by the state equations. The path of the state-point is called the orbit or the trajectory of the system that starts from the given initial conditions. The trajectories are obtained as the solutions of the differential equations(2.2) or iterates of the map(2.3).

2.3 Autonomous and non-autonomous system:

If the system equations do not have any externally applied time-varying input or other time variations, the system is said to be ***autonomous***. In such systems the right hand side of (2.2) does not contain any time-dependent term. A typical example is the simplified model of atmospheric convection, known as the Lorenz system:

$$\begin{aligned} \dot{x} &= -3(x-y) \\ \dot{y} &= -xz + rx - y \\ \dot{z} &= xy - z \end{aligned} \tag{2.4}$$

where r is a parameter. Systems with external inputs or forcing functions or time variations in their definition, are called ***non-autonomous*** systems. In such systems, right hand side of (2.2) contains time dependent terms. As a typical example one can consider a pendulum with an oscillating support, the equations of which are

$$\begin{aligned} \dot{x} &= y + 5 \\ \dot{y} &= -y - y\sin x + r\sin \omega t \end{aligned} \tag{2.5}$$

Likewise, power electronic circuits with clock-driven control logic are non-autonomous systems.

2.4 Vector fields:

In studying the dynamical behavior of a given system, one has to compute the trajectory starting from a given initial condition. We have seen that this can be done numerically. However, it is generally not necessary to compute all possible trajectories (which may be a cumbersome exercise) in order to study a given system. It may be noted that the left hand side of (2.2) gives the rate of change of the state variables. This is a vector, which is expressed as a function of the state variables. The equation (2.2) thus defines a vector at every point of the state space. This is called the *vector field*. A solution starting from any initial condition follows the direction of the vectors, i.e., the vectors are tangent to the solutions. The properties of a system can be studied by studying this vector field. To give an example, the vector field for the system $\ddot{x}-(1-x^2)\dot{x}+x=0$ is shown in Fig. 2.1.

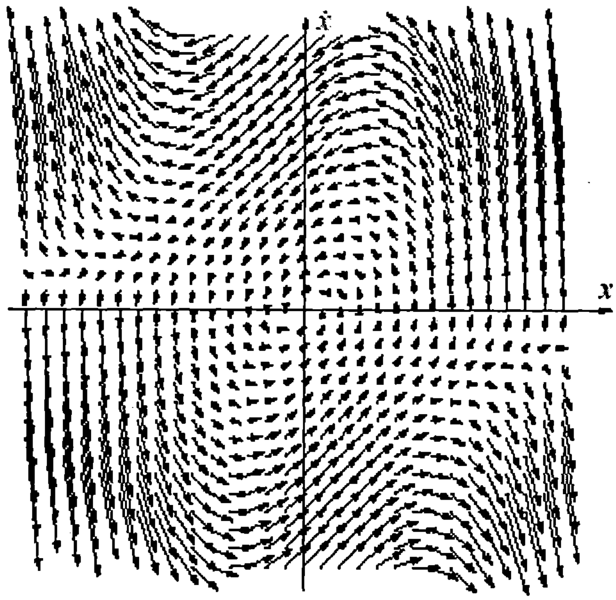


Fig.2.1: vector field for the system $\ddot{x}-(1-x^2)\dot{x}+x=0$

2.5 Local Behavior of Vector Fields Around Equilibrium Points:

The points where the $\dot{\mathbf{x}}$ vector has zero magnitude, i.e., where $\dot{\mathbf{x}}$ =f(x)=0, are called the *equilibrium points*. Since the velocity vector at the equilibrium point has magnitude zero, if an initial condition is placed there, the state-point will forever remain there. However, this does not guarantee that the equilibrium state will be stable, i.e., any deviation from it will die down. It is therefore important to study the *local* behavior of the system in the neighborhood of an equilibrium point. Since it is straightforward to obtain the solutions of a set of *linear* differential equations, the local properties of the state space in the neighborhood of an equilibrium point can be studied by locally linearizing the differential equations at that point. Indeed, most tools for the design and analysis of engineering systems concentrate only on the local behavior — because in general, the nominal operating point of any system is located at an equilibrium point, and if perturbations are small then the linear approximation gives a simple workable model of the dynamical system[2].

The local linearization is done by using the Jacobian matrix of the functional form at an equilibrium point. For example, if the state space is two dimensional, given by

$$\begin{aligned} \dot{x} &= f_1(x,y) \\ \dot{y} &= f_2(x,y) \end{aligned} \tag{2.6}$$

then the local linearization at an equilibrium point (x^*, y^*) is given by

$$\begin{bmatrix} \delta\dot{x} \\ \\ \delta\dot{y} \\ \\ \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \tag{2.7}$$

where $\delta x = x - x^*, \delta y = y - y^*$. The matrix containing the partial derivatives is called the Jacobian matrix and the numerical values of the partial derivatives are calculated at the equilibrium point. This is really just a (multivariate) Taylor series expanded to first order. Notice that in the linearized state space, the state variables are the *deviations* from the equilibrium point (x^*, y^*). To avoid notational complexity, we'll drop the δ and will proceed with the equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \tag{2.8}$$

with the understanding that the origin is shifted to the equilibrium point. If the original system is non-autonomous, there will be time-dependent terms in the Jacobian matrix. In engineering literature it is customary to separate out the time-dependent and time-independent terms in the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{2.9}$$

where **A** and **B** are time-independent matrices, and the components of the vector **u** are the externally imposed inputs of the system. From (2.9) it is evident that the term **Bu** has influence on the location of the equilibrium point, while stability of the equilibrium point is given by the matrix . Therefore, while studying the stability of the equilibrium point, one considers the unforced system (2.8).

2.6 Eigenvalues and eigenvectors:

Notice that in (2.8), **A** *operates* on the vector **x** to give the vector $\dot{\mathbf{x}}$. This is basic function of any matrix-mapping one vector into another vector. Generally the derived vector is different from the source vector, both in magnitude and direction. But there may be some special directions in the state space such that if the vector **x** is in that direction, the resultant vector $\dot{\mathbf{x}}$ also lies along the same direction. It only gets stretched or squeezed. Any vector along these special directions are called *eigenvectors* and the factor by which any eigenvector expands or contracts when it is operated on by the matrix **A** , is called the *eigenvalue*.[3]

To find the eigenvectors, we need to find their eigenvalues first. When the matrix **A** operates on the vector **x** , and if **x** happens to be an eigenvector, then we can write

$$\mathbf{Ax}=\lambda\mathbf{x}$$

where λ is the eigenvalue. This yields

$$(\mathbf{A}-\lambda\mathbf{I})\mathbf{x}=\mathbf{0}$$

where **I** is the identity matrix of the same dimension as **A** . This condition would be satisfied if the determinant $|\mathbf{A}-\lambda\mathbf{I}|=\mathbf{0}$.Thus

$$\left|\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A22 \end{bmatrix} \cdot \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right| = 0 \qquad (2.10)$$

$$\Rightarrow \lambda^2 - (A_{11} + A_{22})\lambda + (A_{11}A_{22} - A_{12}A_{21}) = 0$$

This is called the *characteristic equation,* whose roots are the eigenvalues. Thus, for a 2x2 matrix, one gets a quadratic equation — which in general yields two eigenvalues. For each eigenvalue there is one *direction* of eigenvector, and any vector in that direction is an eigenvector. The direction of the eigenvector is determinate but the magnitude is indeterminate.

If the eigenvalues are real and negative, the system is stable in the sense that any perturbation from an equilibrium point decays exponentially and the system settles back to the equilibrium point. Such a stable equilibrium point is called a *node.* If the real parts of the eigenvalues are positive, any deviation from the equilibrium point grows exponentially, and the system is unstable.

If one eigenvalue is real and negative while the other is real and positive, the system is stable along the eigenvector associated with the negative eigenvalue, and is unstable away from this. Such an equilibrium point is called a *saddle*, and a system with a saddle equilibrium point is globally unstable. The vector fields of the three types of systems are shown in Fig. 2.2.

Complex eigenvalues always occur as complex conjugate pairs. If $\lambda=\sigma+j\omega$ is an eigenvalue, $\lambda=\sigma-j\omega$ is also an eigenvalue. Let **v** be an eigenvector corresponding to the eigenvalue $\lambda=\sigma+j\omega$. This is a complex-valued vector. It is easy to check that $\bar{\mathbf{v}}$, the conjugate of the vector **v** , is associated with the eigenvalue, $\lambda=\sigma-j\omega$.

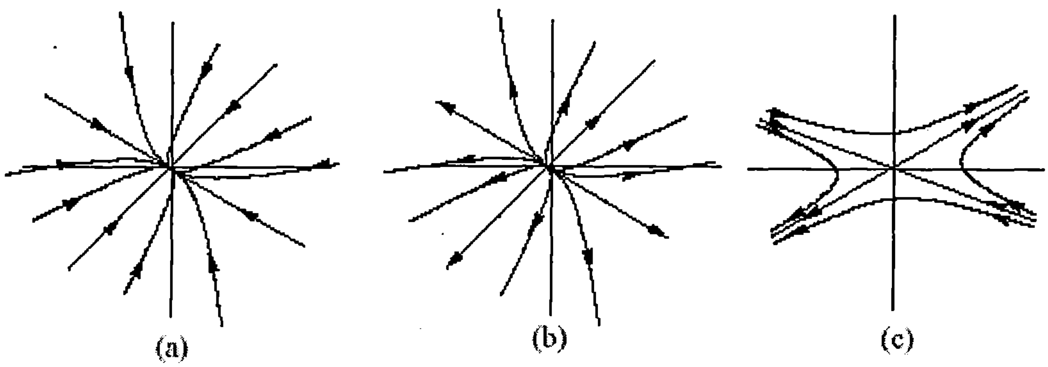


Fig.2.2: vector Field of the Linear Systems with Real Eigenvalues- (a) both eigenvalues negative, (b) both eigenvalues positive, (c) one eigen values negative and other positive.

In general, if the eigenvalues are purely imaginary, the orbits are elliptical. For initial conditions at different distances from the equilibrium point, the orbits form a family of geometrically similar ellipses which are inclined at a constant angle to the axes, but having the same cyclic frequency. When the eigenvalues are complex, with σ nonzero, the sinusoidal variation of the state variables will be multipled by an exponential term $e^{\sigma t}$

If σ is negative, this term will decay as time progesses. Therefore the waveform in time-domain will be a damped sinusoid, and in the state space the state will spiral in towards the equilibrium point. If σ is positive, the term $e^{\sigma t}$ will increase with time, and so in the state space the behavior will be an outgoing spiral as shown in Fig 2.3.

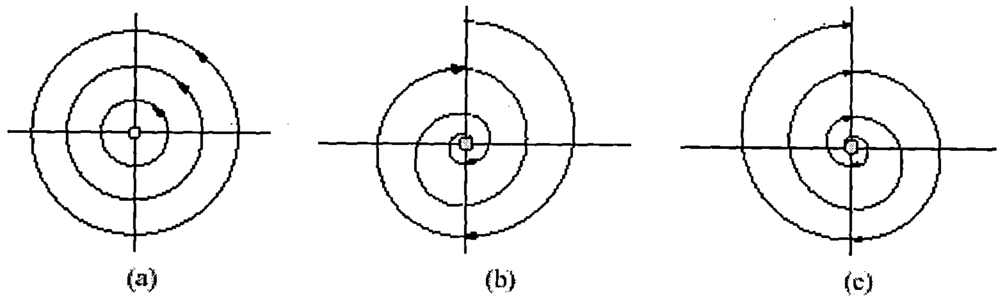


Fig.2.3: The structure of the vector field in the state space for (a) imaginary eigenvalues, (b) complex eigenvalues with negative real part, and (c) complex eigenvalues with positive real part.

2.7 Attractors in nonlinear systems:

To illustrate some typical features of nonlinear systems, we take the system given by $\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$ known as the *van der Pol* equation. Fig. 6.13 shows the vector field of this system with state variables x and y=$\dot{x}$. If the parameter μ is varied from a negative value to a positive value, a fundamental change in the property of the vector field occurs. The stable equilibrium point becomes unstable and the field lines spiral outwards. But it does not become globally unstable as the field lines at a distance from the equilibrium point still point inwards. Where the two types of field lines meet,

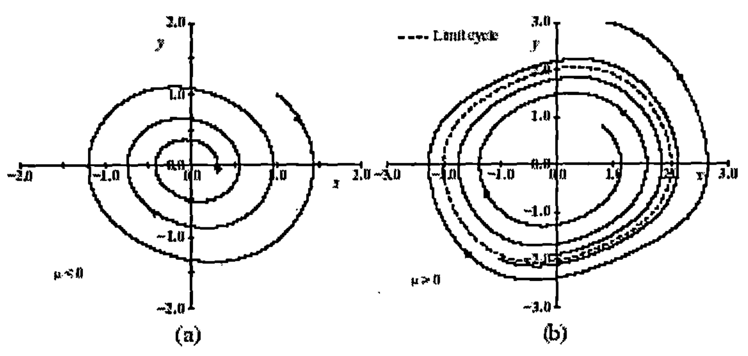


Fig.2.4: The vector fields for $\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$, (a) for μ<0, (b) for μ>0
The dashed line shows the limit cycle.

there develops a stable periodic behavior. This is called a limit cycle. It is a *global* behavior whose existence can never be predicted from linear system theory. One point is to be noted here. There is a fundamental difference between the periodic behaviors in a linear system with purely imaginary eigenvalues and Fig.2.4. In the first case a different periodic orbit (though of constant period) is attained for initial conditions at different radii, while in case of the limit cycle, trajectories starting from different initial conditions converge on to the same periodic behavior. The limit cycle appears to attract points of the state space. This is an example of an *attractor*. Thus in a two-dimensional nonlinear system one can come across periodic attractors as in Fig.2.4. If the state space is of higher dimension, say three, there can be more intricate attractors. To understand this point, suppose a third-order dynamical system is going through oscillations and when we plot one of the variables against time, it has a periodic waveform as shown in Fig.2.5, corresponding to a state-space trajectory that shows a single loop.

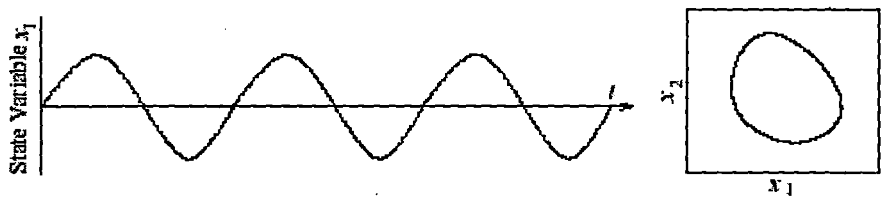


Fig2.5: The time plot (left) and the state space trajectory (right) for a period-1 attractor.

When some parameter is varied, the waveform can change to the type shown in Fig. 2.6, which has twice the period of the earlier periodic waveform. In order for such orbits to exist, the state must three dimensions. (Note that the figure actually shows a projection of a 3-D state space onto two dimensions — a real state-space orbit cannot cross itself because there is

a unique velocity vector $\dot{\mathbf{x}}$ associated with every point in the state space.)

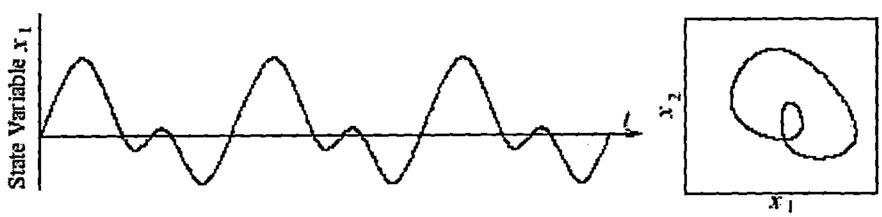


Fig.2.6: The appearance of period-2 waveform in the time domain and in state space.

Sometimes the orbit has one periodicity superimposed on another, and we have a torus-shaped attractor in the state space. This is called a quasiperiodic attractor. Fig.2.7 gives a graphic illustration.

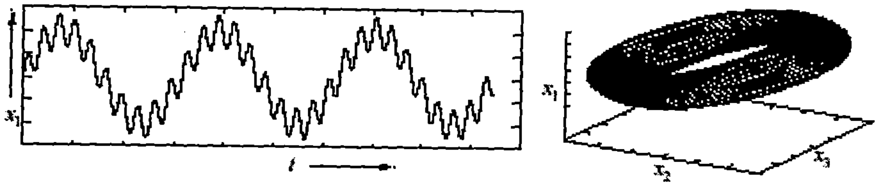


Fig. 2.7: The appearance of a quasiperiodic attractor in time domain and in state space.

One interesting possibility opens up in systems of order 3 or greater : bounded aperiodic orbits, as shown in Fig. 2.7. In such a case the system state remains bounded — within a definite volume in the state space, but the same state never repeats. In every loop through the state space the state traverses a new trajectory. This situation is called *chaos* and the resulting attractor is called a *strange attractor*. When such a situation occurs in an electrical circuit or a mechanical systems, the system undergoes apparently random oscillations.

2.8 Bifurcation:

A ***bifurcation*** is defined as a point where the flow is unstable.Aqualitative change in the dynamics which occurs as a system parameter is changed is called *bifurcation.* Conceptually, it is when there is a change in dynamic behavior, i.e., when a fixed point branches into two fixed points, or when a system changes from a sink to a saddle. This change does not happen over the course of time, but due to a change in parameters. Studying the bifurcations is helpful in determining whether a system is purely random, or an actual chaotic system. Bifurcations happen at regular intervals, which is the determinism inherent in an otherwise random system[8]. There are several kinds of bifurcations: the Hopf bifurcation, pitchfork bifurcation, explosive bifurcation and fold bifurcation. A pitchfork bifurcation branches from one fixed point into two fixed points and one unstable point. Using μ as the parameter that changes, we see a pitchfork bifurcation in the Fig2.8:

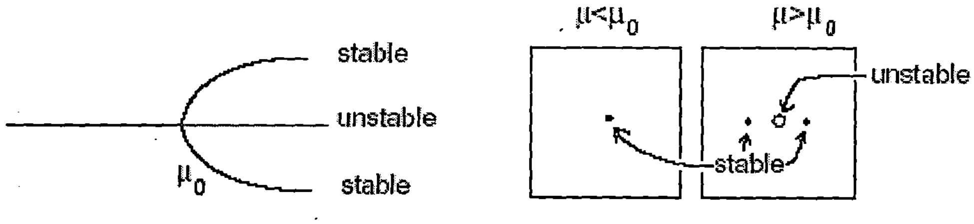


Fig. 2.8: Pitchfork bifurcation.

The phenomenon of a system evolving into a limit cycle from a fixed point is called a Hopf bifurcation, Fig. 2.9. As the system approaches the critical value μ_0, the trajectories take longer and longer to enter the equilibrium of the final state, until it takes an infinite amount of time to the equilibrium state. A secondary Hopf bifurcation when a system branches from a limit cycle to a torus. All other bifurcations transform the system from an (n- 1) dimensional torus to an n-dimensional torus.

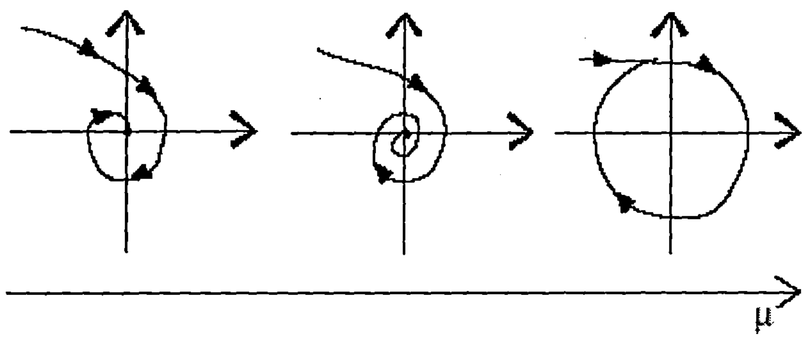


Fig. 2.9: Hopf bifurcation.

A Hopf bifurcation breeds a new limit cycle, whereas a flip bifurcation turns one limit cycle into two. Successive bifurcations give birth to more limit cycles.

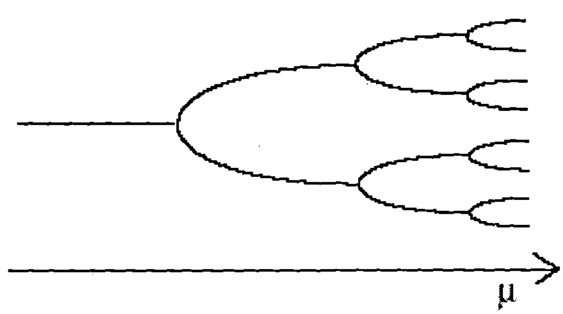


Fig. 2.10: Flip bifurcation.

Bifurcation occurs when a fixed point loses stability.Condition of stability of a fixed point, i.e., Eigenvalues should remain inside the unit circle. The classification of bifurcations depends on where an eigenvalue crosses the unit circle. Smooth systems can lose stability in three possible ways.

(a) A period doubling bifurcation: eigenvalue crosses the unit circle on the negative real line,

(b) A saddle-node or fold bifurcation: an eigenvalue touches the unit circle on the positive real line,

(c) A Hopf or Naimark bifurcation: a complex conjugate pair of eigenvalues cross the unit circle.

Explosive bifurcations are when bifurcations lead to chaotic attractors, like μ = 4:0 in the logistic map, which is an example of period doubling as well. The logistic map is as follows[79]:

$$x_{n+1} = \mu x_n(1 - x_n) \tag{2.10}$$

The system has one fixed point until μ = 2:98, then the system undergoes bifurcation, having two periods, rather than just one. When a system bifurcates and doubles the amount of stable points, the system undergoes ***period doubling***. The system bifurcates again around μ = 3:445. As μ is increased, the intervals between period doubling become shorter and shorter until at ' = 4:0, when the system becomes completely chaotic. At this point, the system is non-periodic, where it has an infinite amount of periods. This evolution of periodic doubling is a ***route to chaos*** as shown in Fig. 2.8.

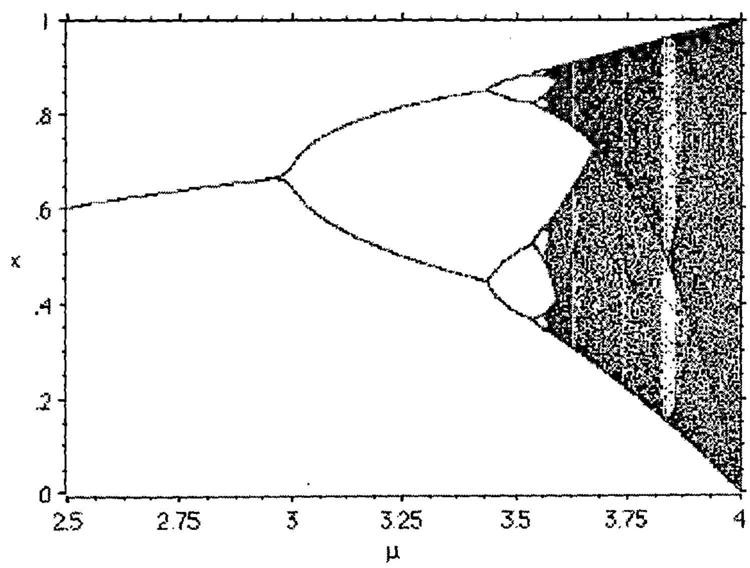


Fig. 2.10: Bifurcation diagram of the logistic mapping.

2.9 Chaos:

The word ***chaos*** is defined three ways. The word originates from the ancient Greek word ***χάος***. According to Hesiod's *Theogony,* Chaos was the first god to come into existence. She was Void, what the universe was before order and logic were laid down. The second definition of chaos is the vernacular one: a condition of great disorder or confusion, which implies randomness. Lastly, chaos may be defined as complex behavior that displays randomness, yet arises deterministically. This contrasts the original Greek meaning of the word. The ancient Greek definition of chaos conveys indescribability and incomprehension. There is no way of knowing or predicting the outcome of behavior, even in a probabilistic sense. There is no order in this description, it is the antithesis of logic. This is the primary difference between the ancient Greek usage and the vernacular definition. The American Heritage Dictionary defines vernacular chaos as, \a condition or place of great disorder or confusion." This use describes the inability to correctly predict future behavior, which is a half of the definition of scientific chaos. The difference between this definition and the ancient Greek one is that the behavior is somewhat ordered. This means that the behavior is not completely disordered: there are probabilities for future behavior, rather than a complete lack of information. Quantum mechanics is an example of this kind of chaos. There is a finite amount of accuracy in measuring the system due to Heisenberg's Uncertainty Principle, and the wave function deals with probabilities. So statistics make the behavior more logical, but complete determinism of the behavior is impossible. This is where the new definition of chaos is different. The new science of chaos is defined as stochastic behavior occurring in a deterministic system. Picking this definition apart, ***stochastic behavior*** is random behavior due to random external forces. For example, a spinning top that is randomly forced exhibits stochastic behavior. Thus, stochastic behavior is behavior that has random attributes due to *indeterminate* factors. Conversely, chaotic behavior has random attributes due to *determinant* factors. A ***system***, then, is a group of elements that form a complex whole. These elements may take the form of differential equations, or the factors that produce weather patterns. A system is chaotic if it is non-periodic, deterministic and exhibits sensitivity towards initial conditions. ***Non-periodic*** behavior does not follow a set pattern. If there is periodic behavior in a system, then future behavior may be determined. Non-periodicity is an outcome of randomness, a sign of a chaotic system. The second aspect to chaos is sensitivity towards initial conditions. A system is ***sensitive towards initial conditions*** when a slight difference in initial conditions exponentially grows over time. For example, let us drop a ball on a nail head: small differences in initial conditions result in vastly different behavior. To put it more mathematically, a system is chaotic if an initial difference of $\Delta f = x_0$ between two systems exponentially grows in the form of $\Delta f = x_0 e^{\lambda t}$ with time. This basic definition of chaos lead to the discovery of chaos in 1961, by Edward Lorenz. While working on a system of equations that is now called the Lorenz system, he ran the computer modeling program twice with the same initial conditions, except that one was to accurate to six digits, while the other was accurate to three digits. At first the two behaved identically, but after a short while they acted drastically different. He published his results in a meteorological journal in 1963, but was not recognized for his work for almost ten years until people asked questions about random, deterministic behavior[42],[43].

2.10 Poincáre Section:

While the phase plotting shows the general behavior of a system, it does not show whether a system is repetitious in a messy way, or truly chaotic. The *Poincáre Section* plot is a way to discern these two phenomena. This is done by taking an n-1 dimensional slice from an n dimensional system. See the fig below for a graphical representation of a Poincáre Section in a three dimensional chaotic system with two periods.

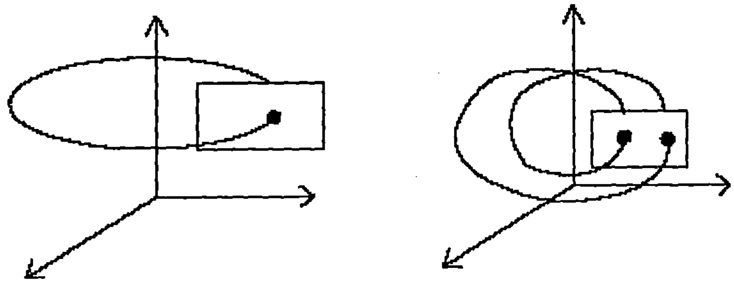


Fig. 2.11: Poincáre Section plot

2.11 Lyapunov Exponents:

A *Lyapunov exponent* measures the exponential rate of growth (or decay) of one variable in a system. A chaotic system exhibits sensitivity towards initial conditions, where the difference grows exponentially. Thus, if a system has a positive Lyapunov exponent, it is chaotic. The Lyapunov exponent of a function $f(x)$ may be found by taking two trajectories, x and x', where $x_0' = x_0 + \varepsilon$, and ε is some small amount. Let the difference d $=|x - x'|$. Looking at the rate of expansion over N iterations, we find:

$$d_N = \varepsilon^{N\lambda}$$

where λ is the Lyapunov exponent., may be found by:

$$\lambda = \underset{N\to\infty}{Lt} \frac{1}{N}\sum_{i=1}^{N-1} \ln\left|\frac{f(x_i) - f(x_{i-1})}{dt}\right| \tag{2.11}$$

In order to determine chaos, all the exponents must be looked at. If one of the Lyapunov exponents is positive, the system is chaotic. In phase space, this represents the system's volume growing over time. It takes more information to determine its original state. If the sum is zero, then the system is stable. There is no loss of information over time. If the sum is less than zero then the system is merely dissipative. Another tool for verifying chaos is to look at the frequency distribution of the data[78].

2.12 Frequency Distribution and Power Spectrum:

The frequency distribution of a system shows whether a system is periodic or not. A finite number of peaks corresponds to a number of periods, so if there are no distinguishable peaks, the system is non-periodic. So a chaotic system will have no distinguishable peaks. To find the frequency distribution, we must use Fourier analysis. Named after Joseph Fourier in the 1820's, Fourier analysis dictates that any signal may be represented by a series of sines and cosines. Given a set of $\{x\} = x_1, x_2, \dots\dots\dots, x_{N-1}$, it is possible to take the Fourier transform and get another set of data, $\{X\} = X_1, X_2, \dots\dots\dots, X_{N-1}$ where X is the Fourier transform. For any $0 < k < N$-1,

$$X_k = \sum_{j=0}^{N-1} x_j e^{-2\pi ijk/N} \tag{2.12}$$

which puts {X} on the complex plane. To see all the information, we look at the ***power spectrum*** *P*, where $P_i = |X_i|^2$. For a simple system of a 1Hz wave, the power spectrum is given in Fig. 2.12.

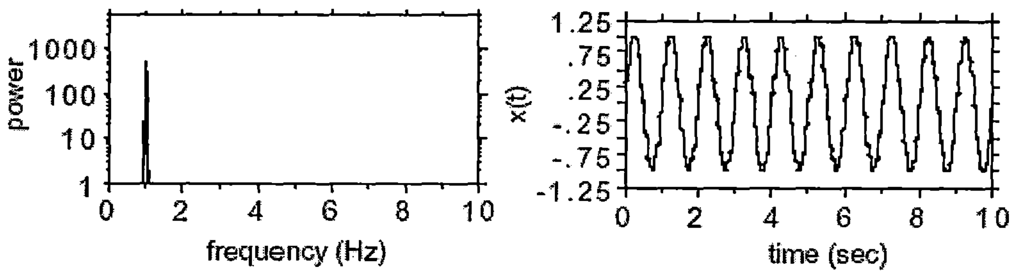


Fig. 2.12: Sinusoidal waveform power spectrum (left), or signal (right).

2.13 Dimension:

There are three kinds of dimensions: topological, Euclidean and fractal. Topological dimension is the continuity of the points, and Euclidean dimension is the dimension that the system is embedded in. Take the object below for example (Fig. 2.13). It is a disfigured plane, like a sheet of aluminum that has been struck several times with a hammer. A plane is a two dimensional object, giving it a topological dimension of $DT = 2$. However, this is embedded in three dimensions $(x; y; z)$, thus $DE = 3$. Therefore, the

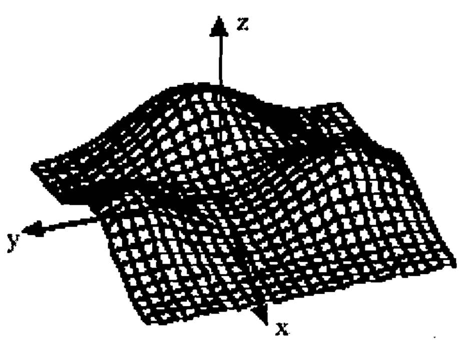


Fig.13: DT = 2 while DE = 3, thus 2 · D · 3

dimension of the object, D, must be between two and three dimensions, which is the ***fractal dimension***. The conventional method of determining dimension of a set of points is the Hausdorff- Besicovitch method, or ***box counting***. The H-B dimension is found by minimally covering the set of points with hypercubes y of different size. The difference comes from the dimension having identically sized cubes while the H-B dimension cubes may be differently sized. First we find a hypercube with side length L that encompasses that contains all points in the set. We may choose any length L as long as it binds the system. Numerically, it is better to choose the smallest possible one due to memory restrictions. Next, we fill the hypercube with hypercubes of side length $l = L/2$. $N(l)$ is the count of how many boxes contain a point inside. Doing this for $l_n = L/2^n$ for increasing n gives a dimension defined as follows:

$$D = \underset{n\to\infty}{Lt} \frac{\log(N(l_n))}{\log(l_n)} \tag{2.13}$$

Another way of finding dimension is using the Lyapunov exponents.

$$D = j + \sum_{i=1}^{j} \lambda_i / |\lambda_{j+1}| \tag{2.14}$$

where j is defined by the condition that:

$$\sum_{i=1}^{j} \lambda_i > 0 \text{ and } \sum_{i=1}^{j+1} \lambda_i < 0 \tag{2.15}$$

Chapter 3

INTRODUCTION TO NONLINEAR PHENOMENON IN LORENZ LIKE SYSTEMS

3.1 Introduction

It has already been revealed that a electrical machines are inherently Lorenz like system. Therefore, the knowledge about the Lorenz like system and their bifurcations amy help to get a quick idea about the bifurcation and chaos of the conventional machine. In this appendix an attempt is made to summarize the nonlinear phenomenon of Lorenz like system[44],[45],[47].

3.2 Normal Form of Three Dimensional Quadratic Autonomous Systems:

In this section, some known three-dimensional quadratic autonomous chaotic systems are reviewed, followed by a classification and their normal forms[68],[69].

- **System-1:** Lorenz system -

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - xz - y \\ \dot{z} &= xy - bz \end{aligned} \tag{3.1}$$

When a = 10, b = 8/3 and c=28 system (3.1) has a 2-scroll chaotic attractor.

Corresponding Equilibrium points are (0, 0, 0) , ($\pm 6\sqrt{2}$, $\pm 6\sqrt{2}$,27)

Eigen values are: (-22.8277, 2.6667,11.83277)
(-13.8546, 0.0940 $\pm$ 0.19445i)

- **System-2:** The Rössler system :

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + xz - cz \end{aligned} \tag{3.2}$$

When a = b =0.2 and c=5.7 system (3.2) has a single folded-band chaotic attractor 2-scroll chaotic attractor.

Corresponding Equilibrium points are (0.0070, -0.0351, 0.0351),

(5.6930, -28.4649, 28.4649)

Eigen values are: -5.6870, 0.0970 $\pm$ 0.99562i

- **System-3:** Rucklidge's system :

$$\begin{aligned} \dot{x} &= ax - ly - yz \\ \dot{y} &= x \\ \dot{z} &= -z + y^2 \end{aligned} \qquad \text{(B.3)}$$

When a = -2, l=-6.7 system (3.3) has a 2-scroll chaotic attractor.

Corresponding Equilibrium points are (0, 0, 0), (0, $\pm$2.5884, 6.7)

Eigen values are: (-3.7749, -1, 1.7749)

(-3.5154, 0.2577 $\pm$1.93553i)

- **System-4:** The Chen system, a *dual* system of the Lorenz system:

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= (c - a)x - xz + cy \\ \dot{z} &= xy - bz \end{aligned} \qquad (3.4)$$

When a = 35, b = 3 and c=28 system (3.4) has a complex 2-scroll chaotic attractor.

Corresponding Equilibrium points are (0, 0, 0), ($\pm 3\sqrt{7}$, $\pm 3\sqrt{7}$, 21)

Eigen values are: (-30.8359, -3, 23.8359)
(-18.4288,4.2140 $\pm$14.88846i)

- **System-5:** The transition system (coined by Lü and Chen):

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= -xz + cy \\ \dot{z} &= xy - bz \end{aligned} \qquad (3.5)$$

This system displays a 2-scroll chaotic attractor when $a = 36;\ b = 3;\ c = 20$

Corresponding Equilibrium points are (0, 0, 0), ($\pm 2\sqrt{15}$, $\pm 2\sqrt{15}$, 20)

Eigen values are: (-36, -3, 20)
(-22.6516,1.8258 $\pm$ 13.68857i)

- **System-6:** Lü and Chen introduced the following simple system:

$$\begin{aligned} \dot{x} &= ax + d_1 yz \\ \dot{y} &= by + d_2 xz \\ \dot{z} &= cz + d_3 xy \end{aligned} \qquad (3.6)$$

where $ab + ac + bc \neq 0$. It can create a complex 2- and 4-scroll attractors for the parameters:

$$d_1 = -1, d_2 = 1, d_3 = 1, a = 5, b = -10, c = -3.4$$

$$d_1 = 1, d_2 = -1, d_3 = -1, a = 0.5, b = -10, c = -4 \quad \text{respectively.}$$

Corresponding Equilibrium points are (0, 0, 0) , ($\sqrt{34}$, $\pm\sqrt{17}$, $\pm 5\sqrt{2}$) , (-$\sqrt{34}$, $\pm\sqrt{17}$, $\mp 5\sqrt{2}$)

Eigen values are: (-10, -3.4, 5)
(-12.6496, 2.1248 $\pm$ 7.0172i)

- **System-7:** Also, the new chaotic system introduced above in this article is

$$\begin{aligned}
\dot{x} &= -\frac{ab}{a+b}x - yz + c \\
\dot{y} &= ay + xz \\
\dot{z} &= bz + xy
\end{aligned} \tag{3.7}$$

which can display two 1-scroll chaotic attractors simultaneously for a = -10; b = -4; c = 18.1 and two Corresponding Equilibrium points are (-6.335, 0, 0)
(2$\sqrt{10}$, $\pm$4.7829, $\pm$7.5624)

Eigen values are: (-14.0094, 0.0094, 2.857711)
(-13.3021, 1.0796 $\pm$ 8.2233i)

Sprott found 19 algebraically simple chaotic systems by exhaustive computer searching as follows

- **System-8:** The first one is

$$\begin{aligned}
\dot{x} &= y \\
\dot{y} &= -x + yz \\
\dot{z} &= 1 - y^2
\end{aligned} \tag{3.8}$$

But Hoover pointed out that system is a special case of the Nos′e-Hoover thermostated dynamic system that had earlier been shown to exhibit time-reversible Hamiltonian chaos[76]. And the other 18 are:

- **System-9:**

$$\begin{aligned}
\dot{x} &= yz \\
\dot{y} &= x - y \\
\dot{z} &= 1 - xy
\end{aligned} \tag{3.9}$$

- **System-10**

$$\dot{x} = yz$$
$$\dot{y} = x - y \tag{3.10}$$
$$\dot{z} = 1 - x^2$$

- **System-11**

$$\dot{x} = -y$$
$$\dot{y} = x + z \tag{3.11}$$
$$\dot{z} = xz + 3y^2$$

- **System-12**

$$\dot{x} = yz$$
$$\dot{y} = x^2 - y \tag{3.12}$$
$$\dot{z} = 1 - 4x$$

- **System-13**

$$\dot{x} = y + z$$
$$\dot{y} = -x + 0.5y \tag{3.13}$$
$$\dot{z} = x^2 - z$$

- **System-14**

$$\dot{x} = 0.4x + z$$
$$\dot{y} = xz - y \tag{3.14}$$
$$\dot{z} = -x + y$$

- **System-15**

$$\dot{x} = -y + z^2$$
$$\dot{y} = x + 0.5y \tag{3.15}$$
$$\dot{z} = x - z$$

- **System-16**

$$\dot{x} = -0.2y$$
$$\dot{y} = x + z \tag{3.16}$$
$$\dot{z} = x + y^2 - z$$

- **System-17**

$$\begin{aligned} \dot{x} &= 2x \\ \dot{y} &= -2y + z \\ \dot{z} &= -x + y + y^2 \end{aligned} \tag{3.17}$$

- **System-18**

$$\begin{aligned} \dot{x} &= xy - z \\ \dot{y} &= x - y \\ \dot{z} &= x + 0.3z \end{aligned} \tag{3.18}$$

- **System-19**

$$\begin{aligned} \dot{x} &= -z \\ \dot{y} &= -x^2 - y \\ \dot{z} &= 1.7 + 1.7x + y \end{aligned} \tag{3.19}$$

- **System-20**

$$\begin{aligned} \dot{x} &= -2y \\ \dot{y} &= x + z^2 \\ \dot{z} &= 1 + y - 2z \end{aligned} \tag{3.20}$$

- **System-21**

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= x - z \\ \dot{z} &= x + xz + 2.7y \end{aligned} \tag{3.21}$$

- **System-22**

$$\begin{aligned} \dot{x} &= 2.7y + z \\ \dot{y} &= -x + y^2 \\ \dot{z} &= x + y \end{aligned} \tag{3.22}$$

- **System-23**

$$\begin{aligned} \dot{x} &= 0.9 - y - z \\ \dot{y} &= 0.4 + z \\ \dot{z} &= xy - z \end{aligned} \tag{3.23}$$

- **System-24**

$$\begin{aligned} \dot{x} &= -x - 4y \\ \dot{y} &= x + z^2 \\ \dot{z} &= 1 + x \end{aligned} \tag{3.24}$$

It is noticed that the R¨ossler attractor and Sprott's attractors. are all topologically simpler than the 2-scroll Lorenz attractor. Furthermore, Sprott's attractors behave similarly in that they all tend to resemble the single folded-band structure of the R¨ossler attractor[49]-[51].

- **System-25:** Genesio and Tesi discussed the following system:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -cx - by - az + x^2 \end{aligned} \tag{3.25}$$

where the parameters $a; b; c$ satisfy $ab < c$. When $a = 0.44; b = 1.1; c = 1$, system displays a typical chaotic attractor.

- **System-26:** The dynamical equations of the Shimizu-Morioka model are

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= x - ay - xz \\ \dot{z} &= -bz + x^2 \end{aligned} \tag{3.26}$$

where the parameters $a > 0; b > 0$. System shows a chaotic attractor for $a = 0.85; b = 0.5$.

- **System-27:** A low-order atmospheric circulation model is described by.

$$\begin{aligned} \dot{x} &= -ax - y^2 - z^2 + aF \\ \dot{y} &= -y + xy - bxz + G \\ \dot{z} &= -z + bxy + xz \end{aligned} \tag{3.27}$$

where x represents the strength of the globally averaged westerly current, and $y; z$ are the strength of the cosine and sine phases of a chain of superposed waves. The unit of the variable t is equal to the damping time of the waves, estimated to be five days. The terms in F and G represent thermal forcing terms, and the parameter b stands for the strength of the advection of the waves by the westerly current. Here, $F; G$ are treated as control parameters, with $a = 1\ 4\ ; b = 4$. This System is also called a new Lorenz model.

Given all the above models, the three-dimensional quadratic autonomous chaotic systems are now classified as follows[54]-[60].

3.3 Classification:

Case A: System properties

- Dissipative chaotic systems, such as systems

- Conservative chaotic systems

Case B: Topological structure of attractor

- One 1-scroll or single folded-band structure, such as the R¨ossler attractor and Sprott's attractors.
- One 2-scroll structure, such as the Lorenz attractor, Chen attractor, transition system attractor, and Rucklidge's attractor.
- One 4-scroll structure,
- Two 1-scroll structure
- Two 2-scroll structure

3.4 Dynamical Behavior of Lorenz System:

Lorenz System is given by

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - xz - y \\ \dot{z} &= xy - bz \end{aligned} \tag{3.28}$$

- Equations show that non-linearity exists in the system[46].
- Putting x=-x, y=-y and z=-z , same system of equations are obtained.
- The system is dissipative in nature.

The equilibrium points of the system can be determined from the state equations.

$$\begin{aligned} 0 &= a(y-x) \\ 0 &= cx - xz - y \\ 0 &= xy - bz \end{aligned}$$

Solving these equations, three equilibrium points are:

$$\begin{aligned} O &\equiv (0,0,0) \\ C_- &\equiv (-\sqrt{b(c-1)},-\sqrt{b(c-1)},(c-1)) \\ C_+ &\equiv (+\sqrt{b(c-1)},+\sqrt{b(c-1)},(c-1)) \end{aligned} \tag{3.29}$$

It is also clear that the equilibrium points C_- & C_+ exists when c>1.

Stability of the system at the origin ($O \equiv (0,0,0)$):

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - y \\ \dot{z} &= -bz = 0 \end{aligned} \tag{3.30}$$

i.e., the system does not evolve in the direction of z. therefore, at origin the dynamics of the system may precisely described by the equations:

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - y \end{aligned} \tag{3.31}$$

For eigen value of the system at origin:

$$\begin{vmatrix} -a-\lambda & a \\ c & -1-\lambda \end{vmatrix} = 0 \tag{3.32}$$

$$\text{or,}\ \ \lambda_{1,2} = \frac{-(1+a) \pm \sqrt{(1+a)^2 - 4a(1-c)}}{2} \tag{3.33}$$

λ is negative when c<1 and , then origin O becomes a stable node.

Further, as c<1, the equilibrium points $C_- \,\&\, C_+$ does not exist at that time.

When c>1, λ_1 becomes positive and λ_2 becomes negative. So, origin O then becomes a saddle node and $C_- \,\&\, C_+$ appear[53].

Stability at Equilibrium Points, $C_- \,\&\, C_+$**:**

$C_- \,\&\, C_+$ appears when c>1.

and $C_- \,\&\, C_+$ does not exist when c<1.

For stability at those equilibrium points, the Jacobian matrix of the system has to be determined.

$$f'(u) = J = \begin{bmatrix} -a & a & 0 \\ c-z & -1 & -x \\ y & x & -b \end{bmatrix} \tag{3.34}$$

Eigen Values may be determined at different equilibrium points from J.

At origin:

$$\begin{vmatrix} -a-\lambda & a & 0 \\ c & -1-\lambda & 0 \\ 0 & 0 & -b-\lambda \end{vmatrix} = 0 \tag{3.35}$$

Solving the equations same results for the eigen values are obtained as mentioned in (3.35)

Now for the eigen values of the system at C_-:

$$\begin{vmatrix} -a-\lambda & a & 0 \\ c-z^* & -1-\lambda & -x^* \\ y^* & x^* & -b-\lambda \end{vmatrix} = 0 \quad (3.36)$$

$$\Rightarrow \lambda^3 + (b+a+1)\lambda^2 + (b+ab+a-ac+az^*+z^{*2})\lambda + ab(1-c) + a\{b(c-1)+2b(c-1)\} = 0$$
$$\Rightarrow \lambda^3 + (b+a+1)\lambda^2 + b(a+c)\lambda + 2ab(c-1) = 0$$

For a trivial Solution,

$\lambda_1 = 0$

$\lambda_{2,3} = \pm j\omega$

$\Rightarrow -\omega^3 j + (b+a+1)(-\omega^2) + b(a+c)j\omega + 2ab(c-1) = 0$

Equating real and imaginary parts:

$$-\omega^3 + b(a+c)\omega = 0$$
$$-(b+a+1)\omega^2 + 2ab(c-1) = 0$$
$$\Rightarrow \omega^2 = b(a+c) = \frac{2ab(c-1)}{b+a+1}$$
$$\Rightarrow c_H = \frac{a+b+3}{a-b-1} \quad (3.37)$$

and $\qquad a_H = b+1 \qquad (3.38)$

Again, $\omega^2 = b(a+c)$

$= \dfrac{2ab(a+1)}{a-b-1} > 0$ as a>b+1

$$\omega = \pm\sqrt{\frac{2ab(a+1)}{a-b-1}} \quad (3.39)$$

For other eigen value,

$$\lambda^3 + (b+a+1)\lambda^2 + b(a+c)\lambda + 2ab(c-1) = 0$$
$$\Rightarrow \lambda^3 + b(a+c)\lambda + (b+a+1)\lambda^2 + 2ab(a-1) = 0$$
$$\Rightarrow (\lambda^2 + \omega^2)[\lambda + b + a + 1] = 0$$

So, $\qquad \lambda_1 = -b-a-1$ negative as a and bare positive

$\lambda_2 = +j\omega$

$\lambda_3 = -j\omega$

Therefore, at C_- Limit Cycle appears[63],[64].

The bifurcation diagrams and the phase portraits are shown in Fig.3.1, Fig 3.2 and 3.3 respectively.

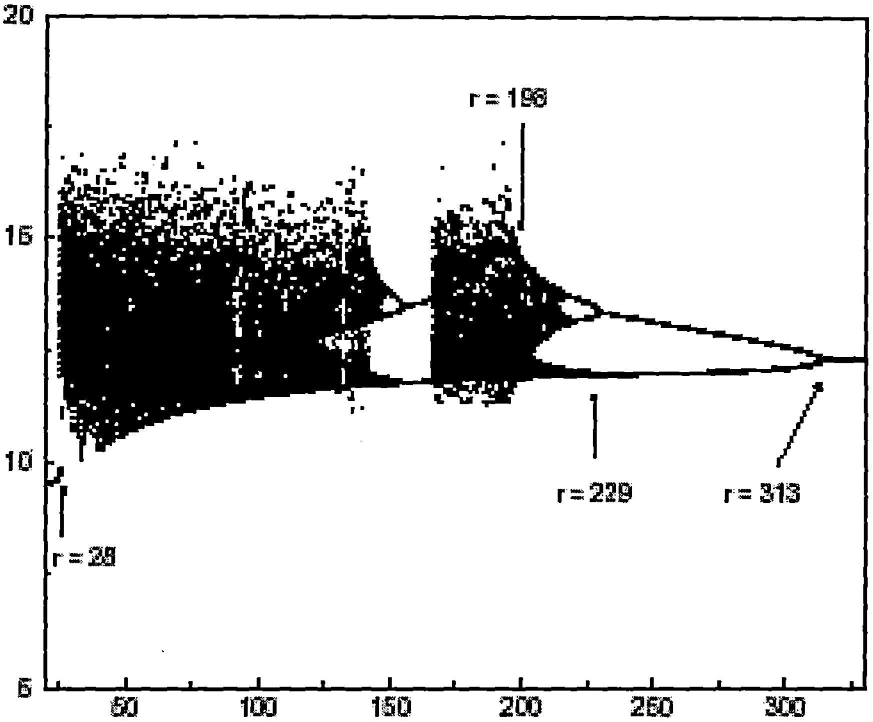


Fig.3.1 The Lorenz Equations :Bifurcation Diagram

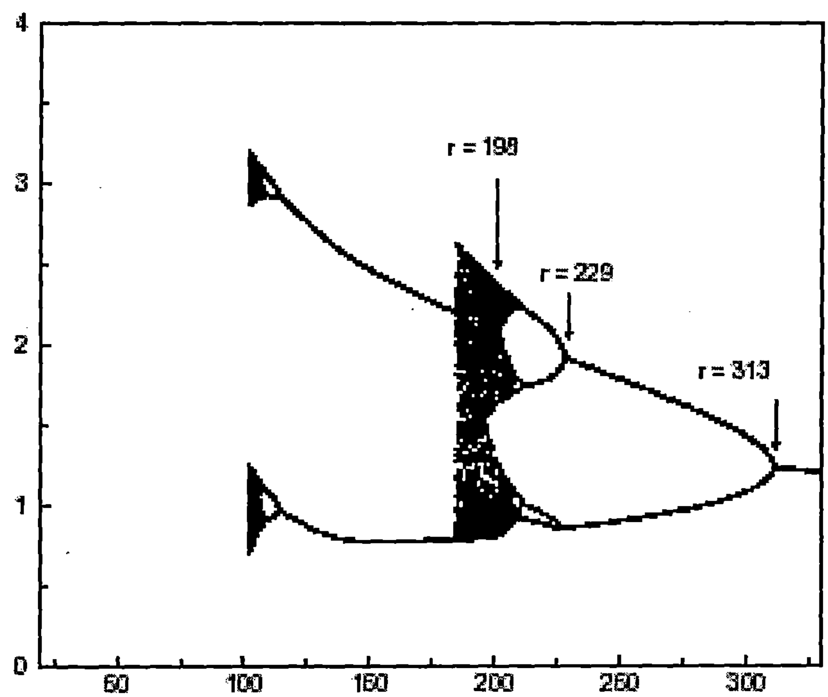


Fig.3.2 The Lorenz Equations :Bifurcation Diagram

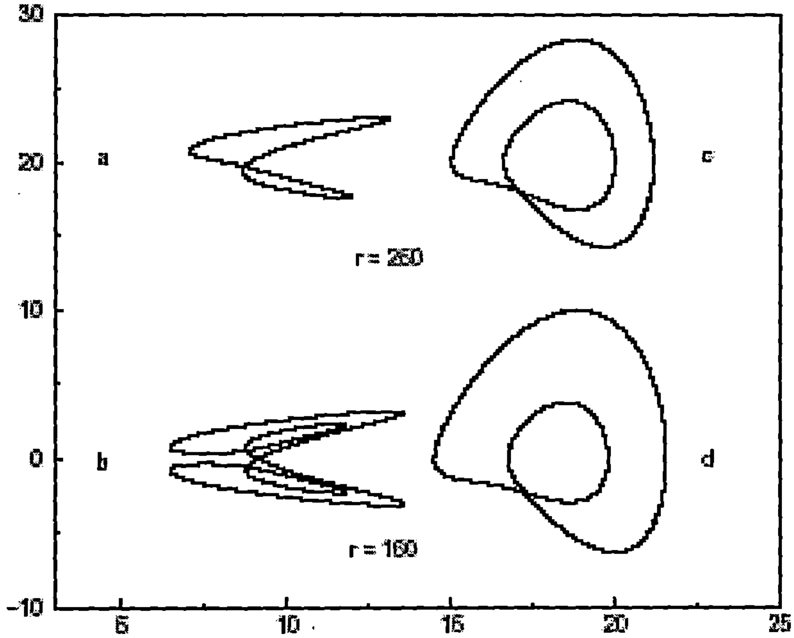


Fig.3.3 The Lorenz Equations :Phase portrait

Chapter 4

MATHEMATICAL MODEL OF GENERALIZED ELECTRICAL MACHINES

4.1 Introduction:

The generalized theory of Electrical Machines is used to cover a wide range of electrical machines in a unified manner. A very important of this generalization is the application of the two axis theory in which, by means of appropriate transformations, any machine can be represented by the coils on the axes. Kron called it the two axis idealized machine, from which many other can be derived, a primitive machine with one coil on each axis on each element. Any machine can be shown to be equivalent to a primitive machine with an appropriate number of coils on each fixed axis. If the coils of the practical machine are permanently located on the axes, they correspond exactly to those of the primitive machine, but, if they are not, it is necessary to make a conversion from the variables of the practical machine to the equivalent axis variables of the corresponding primitive machine, or vice versa. Any particular system of description of a machine is known as a reference frame and a conversion from one reference frame to another is known as transformation.

In this chapter the concept of different reference frames, their transformation , generalized machine and its mathematical model, the dynamic model to describe the dynamics of the generalized machine etc. are discussed[5],[7].

4.2 Reference Frame:

From the voltage and current equations that describe the performance of conventional machines, it is found that some of the machine inductances are the functions of the rotor speed, whereupon the coefficients of the governing differential equations that describe the behavior of these machines are time varying except when the rotors are stalled. A change of variables is often used to reduce the complexity of those differential equations. There are several changes of variables that are used and it was originally thought that each change of variable was different and therefore they were treated separately. It was later learnt that all changes of variables used to transform real variables are contained in one. This general transformation refers machine variables to a frame of reference that rotates at an arbitrary angular velocity. All known real transformations are obtained from this transformation by simply assigning the speed of rotation of the reference frame.

In late 1920s, R. H. Park introduced the new approach to electric machine analysis. He formulated a change of variables which, in effect, replaced the variables (voltages, currents and flux linkages) associated with the stator windings of a synchronous machine with variables associated with fictitious windings rotating with the rotor. In other words, he transformed, or referred, the stator variables to a frame of reference fixed in the rotor. Park's transformation, which revolutionized the electrical machine analysis, has the unique property of eliminating all time-varying inductances from the voltage equations of the synchronous machine which occur due to electric circuit in relative motion and electric circuit with varying magnetic reluctance.

In late 1930s, H. C. Stanley employed a change of variables in the analysis of induction machines. He showed that the time varying inductances in the voltage equations of an induction machine due to electric circuit in relative motion could be eliminated by transforming the variables associated with the rotor windings (rotor variables) to variables associated with fictitious stationary windings. In this case, the rotor variables are transformed to a reference frame fixed in the stator.

G. Kron introduced a change of variables that eliminated the position or time varying mutual inductances of a symmetrical induction machine by transforming both the stator variables and the rotor variables to a reference frame rotating in synchronism with rotating magnetic field. This reference frame is commonly referred to as the synchronously rotating reference frame.

D. S. Brereton et al. employed a change of variables that also eliminated the time-varying inductance of asymmetrical induction machine by transforming the stator variables to a reference frame fixed in the rotor. This is essentially Park's transformation applied to induction machine.

Park, Stanley, Kron and Brereton et al. developed change of variables each of which appeared to be uniquely suited for a particular application. Consequently, each transformation was derived and treated separately in literature until it was noted in[1965] that all known real transformations used in induction machine analysis are contained in one general transformation that eliminates all time varying inductances by referring the stator and rotor variables to a frame of reference that may rotate any arbitrary angular velocity or remain stationary. All known real transformations may then be obtained by simply assigning the appropriate speed of rotation, which may in fact be zero, to this so called arbitrary reference frame. It also may e noted that this transformation is sometimes referred to as " generalized rotating real transformation." Later it was noted that the stator variables of a synchronous machine could also be referred to the arbitrary reference frame[6]. It also may be noted that the time-varying inductances of a synchronous machine are eliminated only if the reference frame is fixed in the rotor (Park's Transformation), consequently the arbitrary reference frame does not offer the advantages in the analysis of the synchronous machines that it does in the case of induction machines.

A change of variables that formulates a transformation of 3-phase variables of stationary circuit elements to the arbitrary reference frame may be expressed as

$$\mathbf{f}_{qd0s} = \mathbf{K}_s \mathbf{f}_{abcs}$$

Where **f** =voltage, current, flux, flux linkage or charge

$$(\mathbf{f}_{qd0s})^T = \begin{bmatrix} f_{qs} & f_{ds} & f_{os} \end{bmatrix} \tag{4.1}$$

$$(\mathbf{f}_{abcs})^T = \begin{bmatrix} f_{as} & f_{bs} & f_{cs} \end{bmatrix}$$

$$\mathbf{K}_s = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{4.2}$$

$$\text{and } (\mathbf{K}_s)^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \qquad \omega = \frac{d\theta}{dt} \tag{4.3}$$

Power is given by

$$P_{abcs} = v_{as}i_{as} + v_{bs}i_{bs} + v_{cs}i_{cs} \tag{4.4}$$

$$P_{dq0s} = \tfrac{3}{2}(v_{ds}i_{ds} + v_{qs}i_{qs} + v_{0s}i_{0s}) = P_{abcs} \tag{4.5}$$

Using this transformations, stationary circuit variables can be transformed to the arbitrary reference frame.

For 3-phase resistive circuit,

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} \tag{4.6}$$

$$\mathbf{v}_{qd0s} = \mathbf{K}_s \mathbf{r}_s (\mathbf{K}_s)^{-1} \mathbf{i}_{dq0s} \tag{4.7}$$

$\mathbf{r}_s$ = resistance matrix of the machine.

When the machine is balanced or symmetrical and $\mathbf{r}_s$ is a diagonal matrix and its nonzero elements are equal then,

$$\mathbf{K}_s \mathbf{r}_s (\mathbf{K}_s)^{-1} = \mathbf{r}_s \tag{4.8}$$

When the machine is unbalanced or unsymmetrical and pase resistances are unequal then, arbitrary reference frame variables contain sinusoidal functions of θ except when $\omega = 0$

For inductive elements,

$$\mathbf{v}_{abcs} = p\lambda_{abcs} \qquad p \equiv \tfrac{d}{dt} \tag{4.9}$$

$$\mathbf{v}_{dq0s} = \mathbf{K}_s p[(\mathbf{K}_s)^{-1} \lambda_{dq0s}]$$

Or it can be written as

$$\mathbf{v}_{dq0s} = \mathbf{K}_s p[(\mathbf{K}_s)^{-1} \lambda_{dq0s}] + \mathbf{K}_s (\mathbf{K}_s)^{-1} p\lambda_{dq0s}$$

$$p[(\mathbf{K}_s)^{-1}] = \omega \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & 0 \\ -\sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & 0 \end{bmatrix} \tag{4.10}$$

$$\text{So, } \mathbf{K}_s p[(\mathbf{K}_s)^{-1}] = \omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.11}$$

Therefore, $\mathbf{v}_{dq0s} = \omega\lambda_{dq0s} + p\lambda_{qd0s}$

This can be expanded as

$$v_{qs} = \omega\lambda_{ds} + p\lambda_{qs}$$
$$v_{ds} = -\omega\lambda_{qs} + p\lambda_{ds} \tag{4.12}$$
$$v_{0s} = p\lambda_{0s}$$

Reference Frame	**Reference Frame speed**	**Interpretation**	**variables**	**transformations**
Arbitrary reference frame	ω (unspecified)	stationary circuit variables referred to the arbitrary reference frame	$(\mathbf{f}_{qd0s})^T = \lfloor f_{qs} \quad f_{ds} \quad f_{os} \rfloor$	$\mathbf{K}_s$
Stationary reference frame	0	Stationary circuit variables referred to the stationary reference frame	$(\mathbf{f}^{\mathbf{s}}_{qd0s})^T = \lfloor f^s{}_{qs} \quad f^s{}_{ds} \quad f_{os} \rfloor$	$\mathbf{K}^{\mathbf{s}}{}_s$
Rotor reference frame	ω_r	Stationary circuit variables referred to a reference frame fixed in the rotor	$(\mathbf{f}^{\mathbf{r}}{}_{qd0s})^T = \lfloor f^r{}_{qs} \quad f^r{}_{ds} \quad f_{os} \rfloor$	$\mathbf{K}^{\mathbf{r}}{}_s$
synchronously rotating reference frame	ω_e	Stationary circuit variables referred to the synchronously rotating reference frame	$(\mathbf{f}^{\mathbf{e}}{}_{qd0s})^T = \lfloor f^e{}_{qs} \quad f^e{}_{ds} \quad f_{os} \rfloor$	$\mathbf{K}^{\mathbf{e}}{}_s$

The 's' subscript denotes variables and transformations associated with circuits that are stationary in 'real life.' as opposed to rotor circuits that are free to rotate. Subscript 'r' is used to denote the variables and the transformation associated with rotor circuits. The raised index denotes the ds and qs variables and transformation associated with a specific reference frame except in case of the arbitrary reference frame that carries no raised index. Because the 0s variables are independent of ω and therefore not associated with a reference frame, a raised index is not assigned to f_{0s}. The transformation of variables associated with stationary circuits to a stationary reference frame was developed by E. Clarke, who used the notations $\lfloor f_\alpha \quad f_\beta \quad f_0 \rfloor$ rather than $\lfloor f^s{}_{qs} \quad f^s{}_{ds} \quad f_{os} \rfloor$. In Park's transformations to the rotor reference, he demonstrated the variables $\lfloor f_q \quad f_d \quad f_0 \rfloor$ rather than $\lfloor f^r{}_{qs} \quad f^r{}_{ds} \quad f_{os} \rfloor$. There appears to be no established notation for the variables in the synchronously rotating reference frame. The voltage equation for all reference frames may be obtained from those in the arbitrary reference frame. The transformations for a specific reference frame is obtained by substituting the appropriate reference frame speed for ω. In most cases the initial or time-zero displacement can be selected equal to zero; however, there are situations where the initial displacement of the reference frame to which the variables are being transformed will not be zero.

In some derivations and analysis it is convenient to relate variables of one reference frame to variables to another reference frame directly, without involving the abc variables in the transformation. In order to establish this transformation between any two frames of reference, let x denote the reference frame from which the variables are being transformed and let y denote the reference frame to which the variables are being transformed; then

$$\mathbf{f}^y_{qd0s} = {}^x\mathbf{K}^y \mathbf{f}^x_{qd0s}$$

$$\mathbf{f}^x_{qd0s} = \mathbf{K}^{\mathbf{x}}_{\mathbf{s}} \mathbf{f}_{abcs}$$

$$\mathbf{f}^y_{qd0s} = {}^x\mathbf{K}^y \mathbf{K}^{\mathbf{x}}_{\mathbf{s}} \mathbf{f}_{abcs}$$

$$\mathbf{f}^y_{qd0s} = \mathbf{K}^{\mathbf{y}}_{\mathbf{s}} \mathbf{f}_{abcs} \tag{4.13}$$

Therefore, ${}^x\mathbf{K}^y \mathbf{K}^{\mathbf{x}}_{\mathbf{s}} = \mathbf{K}^{\mathbf{y}}_{\mathbf{s}}$

from which ${}^x\mathbf{K}^y = \mathbf{K}^{\mathbf{y}}_{\mathbf{s}} (\mathbf{K}^{\mathbf{x}}_{\mathbf{s}})^{-1}$

$$ {}^{x}\mathbf{K}^{y} = \begin{bmatrix} \cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) & 0 \\ \sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.14} $$

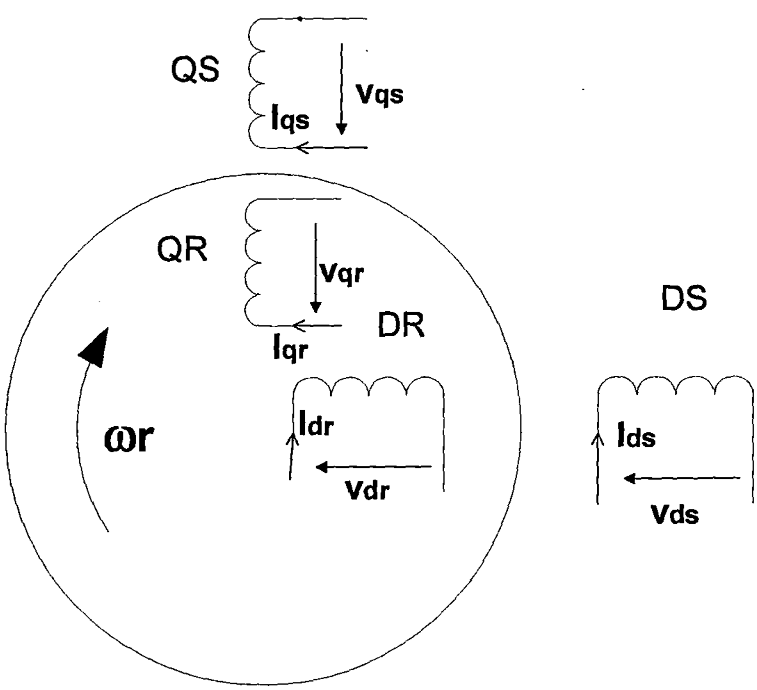
4.3 Generalized Machine:

For great majority of ac machines, including synchronous machine and induction machines, a great simplification is obtained by expressing the equations in a new reference frame and introducing certain fictitious currents and voltages which are different from but are related to the actual ones. The fictitious currents can have a physical meaning in that they can be considered to flow in fictitious windings acting along two axes at right angles called the direct and quadrature axes. In this way the two Reaction Theory of the ac machines has been developed, and the equations so obtained are found to correspond very closely to those of dc machines. The first step in its development was Blondel's 'Two Reaction Theory'of the steady state operation of salient pole synchronous machine. The method was examined in details by Doherty and Nickle, who published a series of five important papers on the same.. A paper by West on 'The Cross Field Theory of Alternating Current Machines' assumed without proof that a rotating cage winding is equivalent to a dc armature winding with two short circuited pair of brushes. A very valuable contribution to the subject was made by Park in asset of three papers. These papers not only develops the general two-axis equations of the synchronous machine, but they indicate how the equations can be applied to the practical problems. Park's transformations provide the most important fundamental concept in the development of Kron's Generalized Theory.

All Electrical Machines can be represented by a generalized model as proposed by Kron. This generalized model is basically a hypothetical machine and it's idea is based on some assumptions. These are :

1. As the distribution of current along the air gap periphery and flux is repeative in nature after every pole pair, a generalized machine is assumed to have only one pole pair in order to avoid the complicacy of understanding and conversion of electrical and mechanical angle. However, for torque and speed calculation, relevant quantities are to be suitably modified.

2. Each winding of the practical machine may have several parts. For generalized machine, it is assumed to have only a single coil. It makes the analysis simple.
3. The axis of the poles around which the field is wound, is called the direct axis and the axis 90^0 away from it, is called the quadrature axis. As per convention, direct axis is taken in horizontal direction and quadrature axis in vertical direction.
4. The positive direction of the current in any coil is towards the coil in the lead nearer to the center of the diagram. The positive direction of the flux linking a coil is radially outward along the axis of the coil.
5. The lower case v represents the externally impressed voltage of a coil and lower case i indicates the direction of current flowing as per the direction of the voltage.
6. The clockwise direction of the rotation of the machine is assumed as the positive direction of rotation.
7. The torque with clockwise sense is the is assumed as the positive direction.
8. The subscript d indicates direct axis, q as quadrature axis, s as stator and r indicates rotor.



Generalized Model of Electrical Machines

Fig.4.1

On the basis of the above assumptions the following voltage equations of the generalized machine, as shown in Fig.4.1, can be written.

$$v_{ds} = (r_{ds} + L_{ds}p)i_{ds} + M_d p i_{dr}$$

$$v_{qs} = (r_{qs} + L_{qs}p)i_{qs} + M_q p i_{qr}$$

$$v_{dr} = M_d p i_{ds} - M_q \omega_r i_{qs} + (r_{dr} + L_{dr}p)i_{dr} - \omega_r L_{qr} i_{qr}$$

$$v_{qr} = M_d \omega_r i_{ds} + M_q p i_{qs} + \omega_r L_{dr} i_{dr} + (r_{qr} + L_{qr}p)i_{qr} \tag{4.15}$$

Here,

$v_{ds}, v_{qs}, v_{dr}, v_{qr}$ = direct and quadrature axis input voltages as shown in fig.4.1

In a realistic machine we may not come across the voltages. However, these may obtained from available stator and rotor variables using suitable transformations as described in (4.1) –(4.3)

$i_{ds}, i_{qs}, i_{dr}, i_{qr}$ = direct and quadrature axis input currents as shown in fig.4.1. These are also the transformed variables used for the analysis of generalized machine. In practical machine, we may get i_a, i_b, i_c which may be transformed using (4.1) –(4.3) to obtain these variables.

$r_{ds}, r_{qs}, r_{dr}, r_{qr}$ = resistances of the windings as shown in fig.4.1

$L_{ds}, L_{qs}, L_{dr}, L_{qr}$ = resistances of the windings as shown in fig.4.1

M_d, M_q = direct and quadrature axis mutual inductances as shown in fig.4.1

The values of the parameters depends on the machine and the shape of the rotor. However, for a wide range of motors they remain within a range. For generalized machine, it is also assumed that they are linear. In practical machines these are to some extent nonlinear due to saturation, temperature rise and other factors.

In the matrix form,

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{bmatrix} r_{ds} + L_{ds}p & 0 & M_d p & 0 \\ 0 & r_{qs} + L_{qs}p & 0 & M_q p \\ M_d p & -M_q\omega_r & r_{dr} + L_{dr}p & -\omega_r L_{qr} \\ \omega_r M_d & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr}p \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} \tag{4.16}$$

Or $[v] = [Z][i]$

$[Z] = [R] + [L]p + [G]\omega_r$

Where $$[Z] = \begin{bmatrix} r_{ds} + L_{ds}p & 0 & M_d p & 0 \\ 0 & r_{qs} + L_{qs}p & 0 & M_q p \\ M_d p & -M_q\omega_r & r_{dr} + L_{dr}p & -\omega_r L_{qr} \\ \omega_r M_d & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr}p \end{bmatrix} \tag{4.17}$$

$$[R] = \begin{bmatrix} r_{ds} & 0 & 0 & 0 \\ 0 & r_{qs} & 0 & 0 \\ 0 & 0 & r_{dr} & 0 \\ 0 & 0 & 0 & r_{qr} \end{bmatrix}$$

$$[L] = \begin{bmatrix} L_{ds} & 0 & M_d & 0 \\ 0 & L_{qs} & 0 & M_q \\ M_d & 0 & L_{dr} & 0 \\ 0 & M_q & 0 & L_{qr} \end{bmatrix}$$

$$[G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -M_q & 0 & -L_{qr} \\ M_d & 0 & L_{dr} & 0 \end{bmatrix}$$

$P_i = [i]^T[v] = v_{ds}i_{ds} + v_{qs}i_{qs} + v_{dr}i_{dr} + v_{qr}i_{qr}$

Or, $P_i = [i]^T[R][i] + [i]^T[L]p[i] + \omega_r[i]^T[G][i]$ (4.18)

$[i]^T[R][i] = i_{ds}^2 r_{ds} + i_{qs}^2 r_{qs} + i_{dr}^2 r_{dr} + i_{qr}^2 r_{qr}$ it represents the copper loss

$[i]^T[L]p[i]$ It actually represents the change in stored energy .

The third term $\omega_r[i]^T[G][i]$ represents Power being converted to mechanical form.

T_e=Electrical torque developed $= \frac{P}{\omega_r} = [i]^T [G][i]$

$$T_e = i_{qr} M_d i_{ds} - M_q i_{dr} i_{qs} + (L_{dr} - L_{qr}) i_{qr} i_{dr} \qquad (4.19)$$

When $(L_{dr} = L_{qr})$ third term of the torque becomes zero. This is actually the reluctance torque. In absence of saliency this term becomes zero.

The dynamic equation of the drives is also applicable at all conditions:

$$T_e - T_L = J \frac{d\omega}{dt}$$

Therefore the dynamics of any machine can be studied using the voltage equation, torque equation and dynamic equation all together. The nature of load torque plays an important role in the dynamics.

Chapter 5

STUDY OF NONLINEAR PHENOMENON OF GENERALISED MACHINE

5.1 Introduction:

Any conventional machine can be represented using the model of the Generalized Electrical Machine. Therefore, the study of the nonlinear phenomenon of Generalized Machine may become a general way to study the nonlinear phenomenon of conventional electrical machines. In this chapter, a general method has been proposed to study the nonlinear phenomenon of the Generalized Machine. As the system is dissipative in nature due to the presence of resistances and viscous damping, as per convention focus of the study is to explore the long term behavior of the system[87],[88]. The long term dynamical behavior of the Generalized Machine due to the nonlinearity has been investigated and the nonlinearity due to product of state variables are considered for study. An approximate model is also considered for study. Discussing the results, the nonlinear phenomenon in conventional machines have been predicted so that the applicability of the proposed method on conventional machines can be verified and validated in the next chapter.

5.2 Dynamic Model:

The dynamic equation of the generalized electrical machine is given by [4.15-4.19]:

$$v_{ds} = (R_{ds} + L_{ds}p)i_{ds} + M_d p i_{dr}$$
$$v_{qs} = (R_{qs} + L_{qs}p)i_{qs} + M_q p i_{qr}$$
$$v_{dr} = M_d p i_{ds} - M_q \omega_r i_{qs} + (R_{dr} + L_{dr}p)i_{dr} - \omega_r L_{qr} i_{qr}$$
$$v_{qr} = M_d \omega_r i_{ds} + M_q p i_{qs} + \omega_r L_{dr} i_{dr} + (R_{qr} + L_{qr}p)i_{qr}$$
$$T_e = i_{qr} M_d i_{ds} - M_q i_{dr} i_{qs} + (L_{dr} - L_{qr}) i_{qr} i_{dr}$$
$$T_e - T_L = J\frac{d\omega}{dt}$$

where

$v_{ds}, v_{qs}, v_{dr}, v_{qr}$ = direct and quadrature axis input voltages as shown in fig.4.1

In a realistic machine we may not come across the voltages. However, these may obtained from available stator and rotor variables using suitable transformations as described in (4.1) –(4.3).

$i_{ds}, i_{qs}, i_{dr,} i_{qr}$ = direct and quadrature axis input currents as shown in fig.4.1. These are also the transformed variables used for the analysis of generalized machine. In practical machine, we may get i_a, i_b, i_c which may be transformed using(4.1) –(4.3) to obtain these variables.

$r_{ds}, r_{qs}, r_{dr,} r_{qr}$ = resistances of the windings as shown in fig.4.1

$L_{ds}, L_{qs}, L_{dr,} L_{qr}$ = resistances of the windings as shown in fig.4.1

M_d, M_q = direct and quadrature axis mutual inductances as shown in fig.4.1

The values of the parameters depends on the machine and the shape of the rotor. However, for a wide range of motors they remain within a range. For generalized machine, the assumptions made in chapter 4 are also holding good in this chapter i.e., the parameters are linear. In practical machines these are to some extent nonlinear due to saturation, temperature rise and other factors.

In matrix form[18],

$$[v] = \begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}, \quad [i] = \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$[v] = [Z][i]$$

$$[Z] = [R] + [L]p + [G]\omega_r$$

Therefore,

$$[Z] = \begin{bmatrix} r_{ds} + L_{ds}p & 0 & M_d p & 0 \\ 0 & r_{qs} + L_{qs}p & 0 & M_q p \\ M_d p & -M_q\omega_r & r_{dr} + L_{dr}p & -\omega_r L_{qr} \\ \omega_r M_d & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr}p \end{bmatrix}$$

$$[R]=\begin{bmatrix} r_{ds} & 0 & 0 & 0 \\ 0 & r_{qs} & 0 & 0 \\ 0 & 0 & r_{dr} & 0 \\ 0 & 0 & 0 & r_{qr} \end{bmatrix}$$

$$[L]=\begin{bmatrix} L_{ds} & 0 & M_d & 0 \\ 0 & L_{qs} & 0 & M_q \\ M_d & 0 & L_{dr} & 0 \\ 0 & M_q & 0 & L_{qr} \end{bmatrix}$$

$$[G]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -M_q & 0 & -L_{qr} \\ M_d & 0 & L_{dr} & 0 \end{bmatrix}$$

$$[v]=[Z][i]$$

$$[Z]=[R]+[L]p+[G]\omega_r$$

All Parameters and variables of the above mathematical model are obtained after transformation as mentioned in previous chapter in (4.1)-(4.3). In practical machines these might not be obtained readily. Still it is preferred to use these variables as they make the analysis simple and generalize. However, using inverse transformation, as mentioned in [4.1], real and practical machine variables can easily be available.

The no. of state variables in the equation is 5. These equations may be rearranged to express them in the form state equations[48].

$$p[i]=[L]^{-1}([v]-[R][i]-\omega_r[G][i]) \tag{5.1}$$

Putting

$$[i]=\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \text{ and } \omega_r = x_5 \text{ we get } [x]\equiv\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

From [5.1], the state equations for rearranged generalized equations may be written as,

$$
\begin{aligned}
\frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3x_5 + a_{14}x_4x_5 + a_{15} \\
\frac{dx_2}{dt} &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3x_5 + a_{24}x_4x_5 + a_{25} \\
\frac{dx_3}{dt} &= a_{31}x_3 + a_{32}x_4 + a_{33}x_1x_5 + a_{34}x_2x_5 + a_{35} \\
\frac{dx_4}{dt} &= a_{41}x_3 + a_{42}x_4 + a_{43}x_1x_5 + a_{44}x_2x_5 + a_{45} \\
\frac{dx_5}{dt} &= a_{51}x_1x_4 + a_{52}x_2x_3 + a_{53}x_2x_4 + a_{54}x_5 + a_{55}
\end{aligned}
\tag{5.2}
$$

The normalized parameters used in the above equation are given by,

$$a_{11} = \frac{-R_{ds}L_{dr}}{(L_{ds}L_{dr} - M_d^2)}$$

$$a_{12} = \frac{M_d R_{dr}}{(L_{ds}L_{dr} - M_d^2)}$$

$$a_{13} = \frac{-M_d M_q}{(L_{ds}L_{dr} - M_d^2)}$$

$$a_{14} = \frac{-L_{qr}M_d}{(L_{ds}L_{dr} - M_d^2)}$$

$$a_{15} = \frac{(L_{dr}V_{ds} - M_d V_{dr})}{(L_{ds}L_{dr} - M_d^2)}$$

$$a_{21} = \frac{R_{ds}M_d}{(L_{ds}L_{dr} - M_d^{\,2})}$$

$$a_{22} = \frac{-L_{ds}R_{dr}}{(L_{ds}L_{dr} - M_d^{\,2})}$$

$$a_{23} = \frac{M_q L_{ds}}{(L_{ds}L_{dr} - M_d^{\,2})}$$

$$a_{24} = \frac{L_{ds}L_{qr}}{(L_{ds}L_{dr} - M_d^{\,2})}$$

$$a_{25} = \frac{(L_{ds}V_{dr} - M_d V_{ds})}{(L_{ds}L_{dr} - M_d^{\,2})}$$

$$a_{31} = \frac{-R_{qs}L_{qr}}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{32} = \frac{MqRqr}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{33} = \frac{MqM_d}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{34} = \frac{LdrMq}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{35} = \frac{(L_{qr}V_{qs} - M_q V_{qr})}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{41} = \frac{R_{qs}M_q}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{42} = \frac{-L_{qs}R_{qr}}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{43} = \frac{-M_d L_{qs}}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{44} = \frac{-L_{qs}L_{dr}}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{45} = \frac{(L_{qs}V_{qr} - M_q V_{qs})}{(L_{qs}L_{qr} - M_q^{\,2})}$$

$$a_{51} = \frac{Md}{J}$$
$$a_{52} = \frac{-M_q}{J}$$
$$a_{53} = \frac{(Ldr - Lqr)}{J} \quad (5.3)$$
$$a_{54} = \frac{B}{J}$$
$$a_{55} = -\frac{T}{J}$$

5.3 Dynamics of the System:

The system represented by (5.1) and (5.2) are Lorenz like systems with higher dimensions. No. of the dimension of the system is 5. No. of parameters is twenty five. No. of nonlinear terms is 11. The possibility of occurrence nonlinear phenomenon like bifurcation and chaos cannot be predicted readily. However, Chapter 3 may give some idea about the same. Only numerical investigation can ensure about the occurrence of the nonlinear phenomenon. For analytical study, we can proceed as much as practicable[89]. The Equilibrium points of the system described by sets of equation (5.1) and (5.2) can be obtained from set of equations-

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3x_5 + a_{14}x_4x_5 + a_{15} &= 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3x_5 + a_{24}x_4x_5 + a_{25} &= 0 \\ a_{31}x_3 + a_{32}x_4 + a_{33}x_1x_5 + a_{34}x_2x_5 + a_{35} &= 0 \\ a_{41}x_3 + a_{42}x_4 + a_{43}x_1x_5 + a_{44}x_2x_5 + a_{45} &= 0 \\ a_{51}x_1x_4 + a_{52}x_2x_3 + a_{53}x_2x_4 + a_{54}x_5 + a_{55} &= 0 \end{aligned} \quad (5.4)$$

The roots of the equations are calculated directly.

$$\begin{aligned} X_1 &= \frac{v_{ds}}{R_{ds}} = I_{ds} \\ X_3 &= \frac{v_{qs}}{R_{qs}} = I_{qs} \end{aligned} \quad (5.5)$$

However, other roots are not easily available as they give a cubic equation.

$$\omega_r^{\,3} + c_2\omega_r^{\,2} + c_1\omega_r + c_0 = 0$$

where

$$c_2 = \frac{M_d M_q I_{ds} I_{qs} L_{dr} - M_d M_q I_{ds} I_{qs} L_{qr} + T L_{dr} L_{qr}}{L_{dr} L_{qr} B}$$

$$c_1 = \frac{(B R_{dr} R_{qr} + M_d^2 I_{ds}^2 R_{dr} + M_d I_{ds} L_{dr} V_{dr} + M_q^2 I_{qs}^2 R_{qr} + M_q I_{qs} L_{qr} V_{qr}}{L_{dr} L_{qr} B}$$

$$c_0 = \frac{- M_d I_{ds} R_{dr} V_{qr} - M_q I_{qs} R_{qr} V_{dr} + T R_{dr} R_{qr}}{L_{dr} L_{qr} B} \tag{5.6}$$

The Equation gives three roots for ω_r and accordingly solving the equation three equilibrium points will be obtained. The nature of the roots greatly depends on the coefficients of the equations. However, as the roots are associated with the equilibrium points of the system, the existence of those equilibrium points of the system actually depend on the co-efficient and thereby on the parameters. Therefore, for physical existence of the equilibrium points of the system, the roots of the equation are to be real. Roots of the equation can be calculated using cardano's Formula.

A cubic equation is a Polynomial equation of degree three. Given a general cubic equation

$$z^3 + a_2 z^2 + a_1 z + a_0 = 0 \tag{5.7}$$

(the Coefficient a_3 of z^3 may be taken as 1 without loss of generality by dividing the entire equation through by a_3), first attempt to eliminate the a_2 term by making a substitution of the form $z \equiv x - \lambda$

Defining

$$z = x - \frac{a_2}{3}$$

$$p \equiv \frac{3a_1 - a_2^2}{3}$$

$$q \equiv \frac{9a_1 a_2 - 27a_0 - 2a_2^3}{27} \tag{5.8}$$

then allows (12) to be written in the standard form

$$x^3 + px = q$$

The intermediate variables are defined as

$$Q \equiv \frac{3a_1 - a_2^2}{9}$$

$$R \equiv \frac{9a_1a_2 - 27a_0 - 2a_2^3}{54}$$

(which are identical to p and q up to a constant factor). The general cubic equation then becomes

$$x^3 + 3Qx - 2R = 0$$

Defining

$$D \equiv Q^3 + R^2$$

$$S \equiv \sqrt[3]{R + \sqrt{D}}$$

$$T \equiv \sqrt[3]{R - \sqrt{D}}$$

where D is the Discriminant (which is defined slightly differently, including the opposite Sign. At last, the Roots of the original equation in z are then given by

$$z_1 = -\frac{a_2}{3} + (S + T)$$

$$z_2 = -\frac{a_2}{3} + (S + T) + \tfrac{1}{2}i\sqrt{3}(S - T)$$

$$z_3 = -\frac{a_2}{3} + (S + T) - \tfrac{1}{2}i\sqrt{3}(S - T) \qquad (5.9)$$

with a_2 the Coefficient of z^2 in the original equation, and S and T as defined above. These three equations giving the three Roots of the cubic equation are sometimes known as Cardano's Formula. Note that if the equation is in the standard form of Vieta

$$x^3 + px = q$$

in the variable x , then $a_2 = 0$, $a_1 = p$, and $a_0 = -q$, and the intermediate variables have the simple form

$$Q = \frac{p}{3}$$

$$R = \frac{q}{2}$$

$$=\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2 \qquad (5.10)$$

The equation for z_1 in Cardano's Formula does not have an i appearing in it explicitly while z_2 and z_3 do, but this does not say anything about the number of Real and Complex Roots (since S and T are themselves, in general, Complex). However, determining which Roots are Real and which are Complex can be accomplished by noting that if the Discriminant D>0, one Root is Real and two are Complex Conjugates; if D=0 , all Roots are Real and at least two are equal; and if D<0 , all Roots are Real and unequal. If D<0, define

$$\theta \equiv \cos^{-1}\left(\frac{R}{\sqrt{-Q^3}}\right)$$

Then the Real solutions are of the form

$$z_1 = 2\sqrt{-Q}\cos\left(\frac{\theta}{3}\right)-\frac{a_2}{3}$$

$$z_2 = 2\sqrt{-Q}\cos\left(\frac{\theta+2\pi}{3}\right)-\frac{a_2}{3}$$

$$z_3 = 2\sqrt{-Q}\cos\left(\frac{\theta+4\pi}{3}\right)-\frac{a_2}{3} \qquad (5.11)$$

Now Cardano's Formula as demonstrated above will be applied to find the roots of the cubic equation given by equation no.

$$p_1 = \frac{3c_1 - c_2^2}{3}$$

$$p_2 = \frac{2c_2^3}{27} - \frac{c_1c_2}{3} + c_0$$

$$D = \left(\frac{p_1}{3}\right)^3 + \left(\frac{p_2}{2}\right)^2 \qquad (5.12)$$

D is the determinant which determines the nature of the roots of the cubic equation developed for the system. for physical existence, the roots are to be real and that is possible when D<= 0

Thus the conditions for real roots and existence of the equilibrium points will be obtained.

$$u_1 = \sqrt[3]{\left(\left(-\frac{p_1}{2}\right) + \sqrt{D}\right)}$$

$$u_2 = \frac{-p1}{3u_1}$$

Two roots of the equation are

$$W_1 = u_1 + u_2 - (c_2/3)$$

$$W_2 = u_1 - u_2 - (c_2/3) \tag{5.13}$$

The directly obtained roots of the dynamic equations of Generalized machine:

$$X_1 = \frac{v_{ds}}{R_{ds}} = I_{ds}$$

$$X_3 = \frac{v_{qs}}{R_{qs}} = I_{qs}$$

Other values of the state variables describing the equilibrium points are

$$X_{21} = I_{dr1} = \frac{R_{qr}(V_{dr} + M_q W_1 I_{qs}) + L_{qr} W_1 (V_{qr} - a_{14} W_1 I_{ds})}{R_{dr} R_{qr} + L_{dr} L_{qr} W_1^2}$$

$$X_{41} = I_{qr_1} = \frac{R_{dr}(V_{qr} - M_d W_1 I_{ds}) - L_{dr} W_1 (V_{dr} + M_q W_1 I_{qs}))}{(R_{dr} R_{qr} + L_{dr} L_{qr} W_1{}^2} \tag{5.14}$$

Dynamic Behavior of the system at this equilibrium point can be studied by determining the eigen values of the corresponding Jacobian matrix at that equilibrium point. Jacobian Matrix for the equilibrium point

$$(I_{ds}, I_{dr1}, I_{qs}, I_{qr1}, W_1) \equiv (X_1, X_{21}, X_3, X_{41}, W_1):$$

$$J1 = \begin{bmatrix} a_{11} & a_{12} & a_{13}W_1 & a_{14}W_1 & a_{14}I_{qr1} + a_{13}I_{qs} \\ a_{21} & a_{22} & a_{23}W_1 & a_{24}W_1 & a_{24}I_{qr1} + a_{23}I_{qs} \\ a_{33}W_1 & a_{34}W_1 & a_{31} & a_{32} & a_{33}I_{ds} + a_{34}I_{dr1} \\ a_{43}W_1 & a_{44}W_1 & a_{41} & a_{42} & a_{43}I_{ds} + a_{44}I_{dr1} \\ a_{51}I_{qr1} & a_{52}I_{qs} + a_{53}I_{qr1} & a_{52}I_{dr1} & a_{51}I_{ds} + a_{53}I_{dr1} & a_{54} \end{bmatrix} \tag{5.15}$$

The Eigen Values of J_1 is given by

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13}W_1 & a_{14}W_1 & a_{14}I_{qr1}+a_{13}I_{qs} \\ a_{21} & a_{22}-\lambda & a_{23}W_1 & a_{24}W_1 & a_{24}I_{qr1}+a_{23}I_{qs} \\ a_{33}W_1 & a_{34}W_1 & a_{31}-\lambda & a_{32} & a_{33}I_{ds}+a_{34}I_{dr1} \\ a_{43}W_1 & a_{44}W_1 & a_{41} & a_{42}-\lambda & a_{43}I_{ds}+a_{44}I_{dr1} \\ a_{51}I_{qr1} & a_{52}I_{qs}+a_{53}I_{qr1} & a_{52}I_{dr1} & a_{51}I_{ds}+a_{53}I_{dr1} & a_{54}-\lambda \end{vmatrix} = 0 \quad (5.16)$$

This will give five roots of the system which will provide the necessary information about the dynamic behavior of the system.

5.4 Case studies:

A Generalized machine is taken here for a case study. The parameters of the machine are as follows:

$$\mathrm{Vds} = 10\ \mathrm{V}, \mathrm{Vqs} = 10\mathrm{V}, \mathrm{Vqr} = 0\mathrm{V},$$
$$\mathrm{Rds} = 0.9\Omega, \mathrm{Rdr} = 0.9\Omega, \mathrm{Rqs} = 0.9\Omega, \mathrm{Rqr} = 0.9\Omega$$
$$\mathrm{Lds} = 0.01425\mathrm{H}, \mathrm{Ldr} = 0.01425\mathrm{H}, \mathrm{Lqs} = 0.01425\mathrm{H}, \mathrm{Lqr} = 0.01425\mathrm{H}$$
$$\mathrm{J} = 0.000047, \mathrm{B} = 0.0162, \mathrm{Md} = 0.01\mathrm{H}, \mathrm{Mq} = 0.01\mathrm{H}, \mathrm{T} = 0$$

Vdr is chosen as variable parameter while all other parameters will remain at the value as mentioned above during the ongoing study.

As Generalized machine is a hypothetical one, it can only be simulated. The machine has been simulated using Matlab 6.5. and Fortran. For some specific requirements like Bifurcation points, route to chaos continuation methods had to be adopted and therefore free version of MATCONT was used for the same. The values of the parameters adopted for the simulation are realistic and are in line with the previous researchers worked in the field. As per normal practice, the transformed state variables are also used in the simulation, however, using inverse transformation real states will be obtained. However, it may be mentioned here that due to such mathematical transformation or inverse transformation, no nonlinear phenomenon is suppressed or newly created, the nature of the dynamics remain unaffected.

For the said value of parameters, calculated Jacobian is as follows:

J1 = 1.0e+003 *

-0.1244	0.0873	0.2612	0.3722	0.0449
0.0873	-0.1244	-0.3722	-0.5303	-0.0639
-0.2612	-0.3722	-0.1244	0.0873	-0.0131
0.3722	0.5303	0.0873	-0.1244	0.0186
-8.5641	-2.3641	3.6685	2.3641	0.3447

Corresponding Eigen Values are

1.0e+002 *

-0.0535 + 4.8066i

-0.0535 - 4.8066i

-1.9758

1.1039

-0.5519

One eigen value is real and positive but the other eigen values are either real and negative or complex conjugate with negative real part which indicates that with those parameters, the system behavior would be periodic with periodicity one. The corresponding phase plane and time response are shown in Fig.5.1, Fig.5.2 and Fig.5.3.

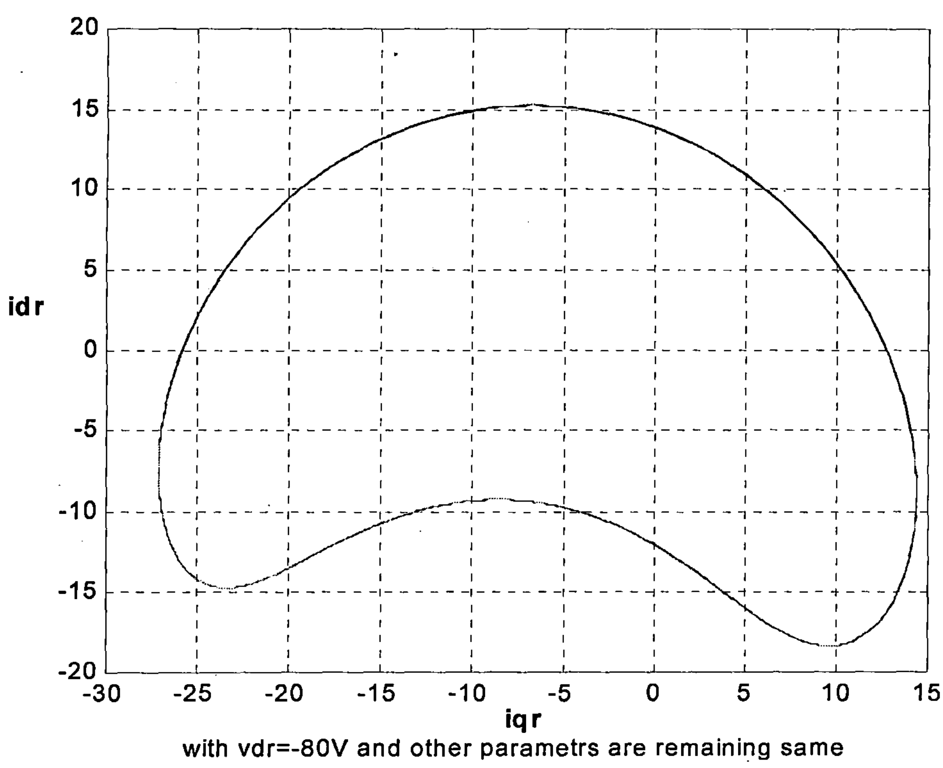


Fig.5.1:Phase Portrait with vdr=-80V

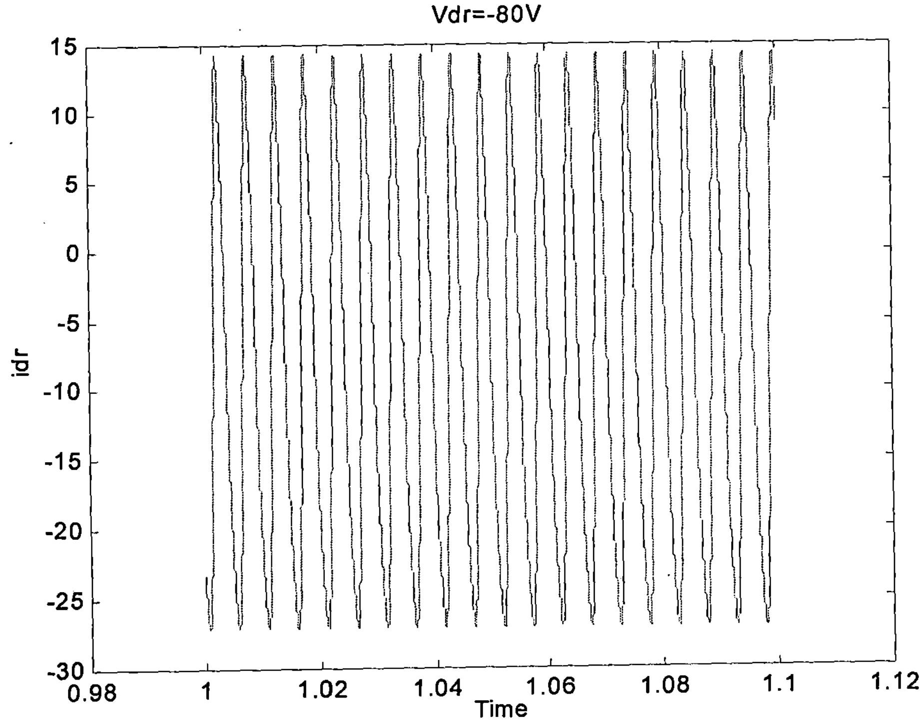


Fig.5.2:Time domain analysis for vdr=-80V

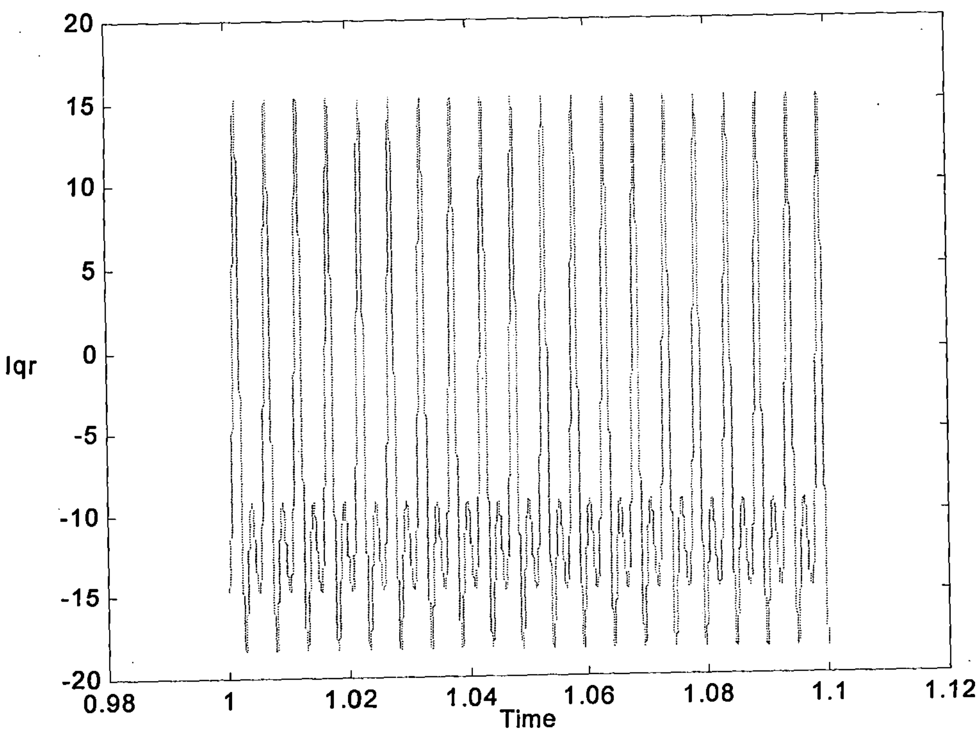


Fig.5.3:Time domain analysis for vdr=-80V

The parameter chosen for variation is Vdr. As Vdr is further varied to –140V bifurcation occurs. Periodicity of the system becomes 2. The same may be verified from the Phase portrait shown in Fig. 5.4 Further, corresponding time response are given in Fig. 5.5 and Fig.5.6

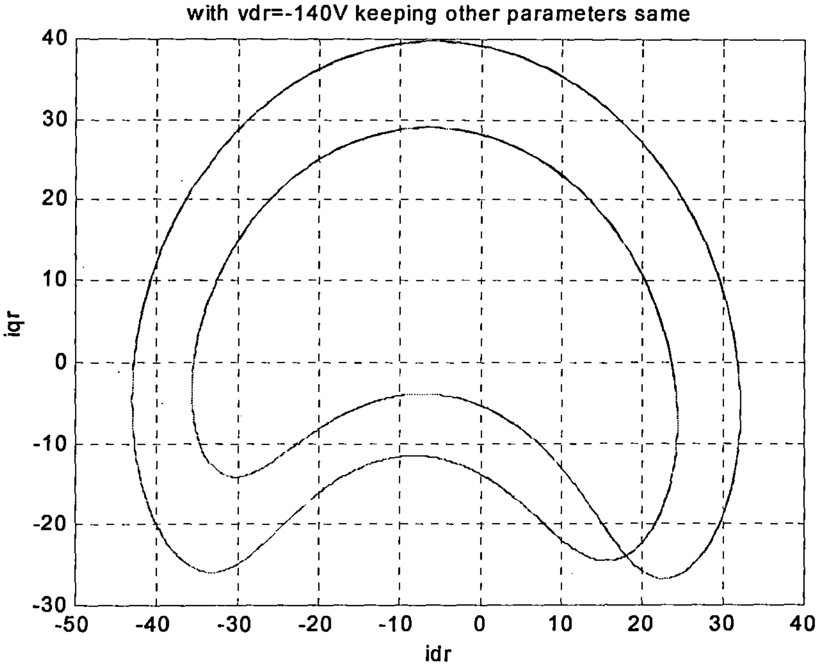


Fig.5.4:Phase Portrait with vdr=-140V

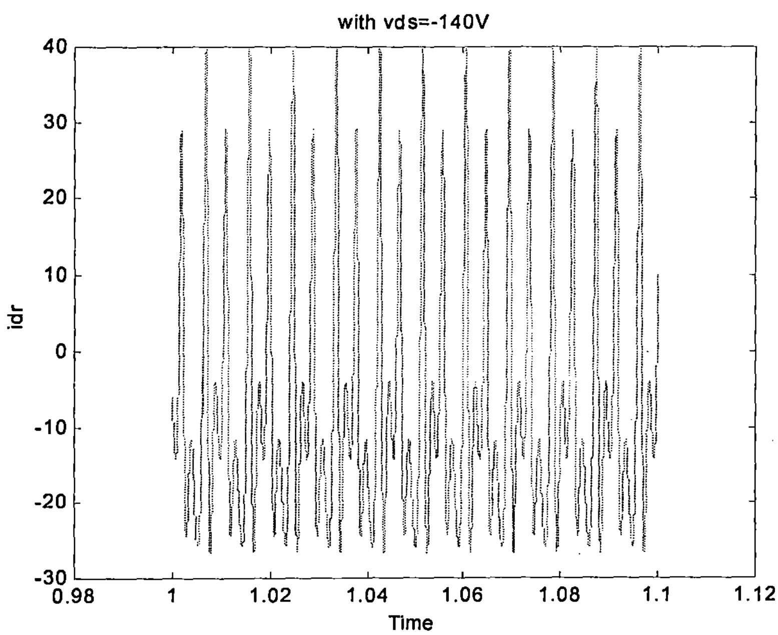


Fig.5.5:Time domain plot with vdr=-140V

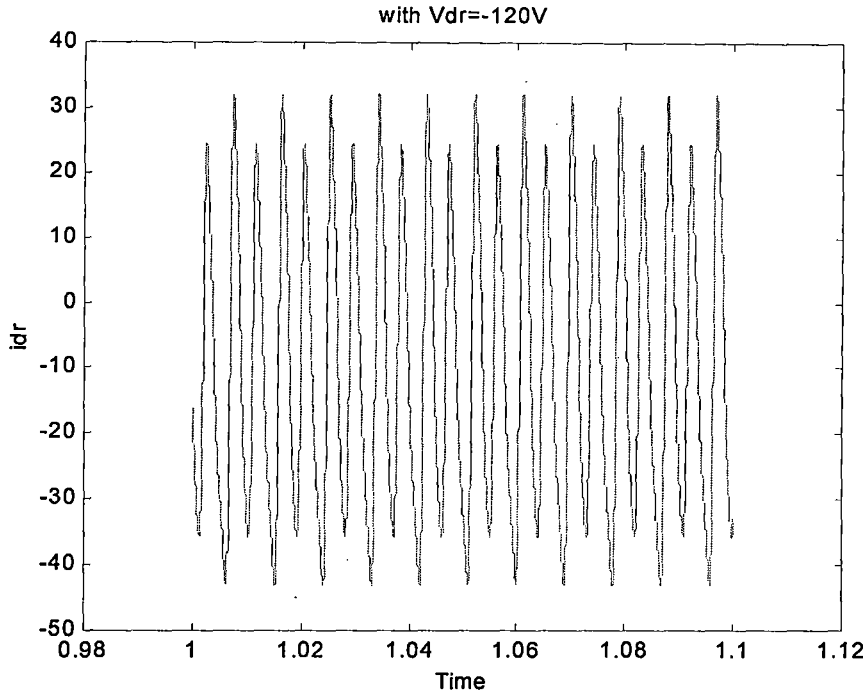


Fig.5.6:Time domain plot with vdr=-140V

As Vdr is made –160V periodicity is further increased to 4. The same be observed in Fig.5.7.The following Fig5.8 and 5.9 shows that the periodicity of the system increases with the variation of the parameters and finally the system becomes chaotic.

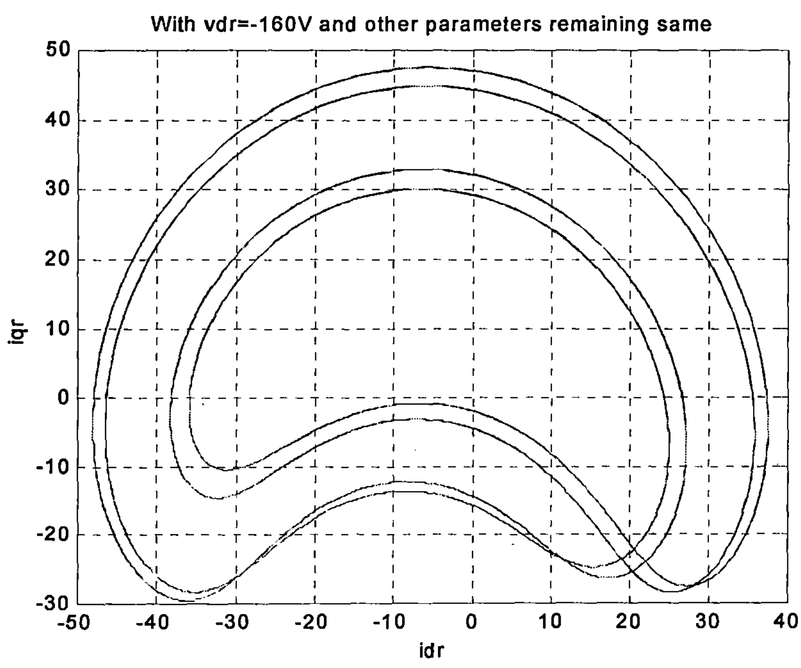


Fig.5.7:Phase Portrait with vdr=-160V

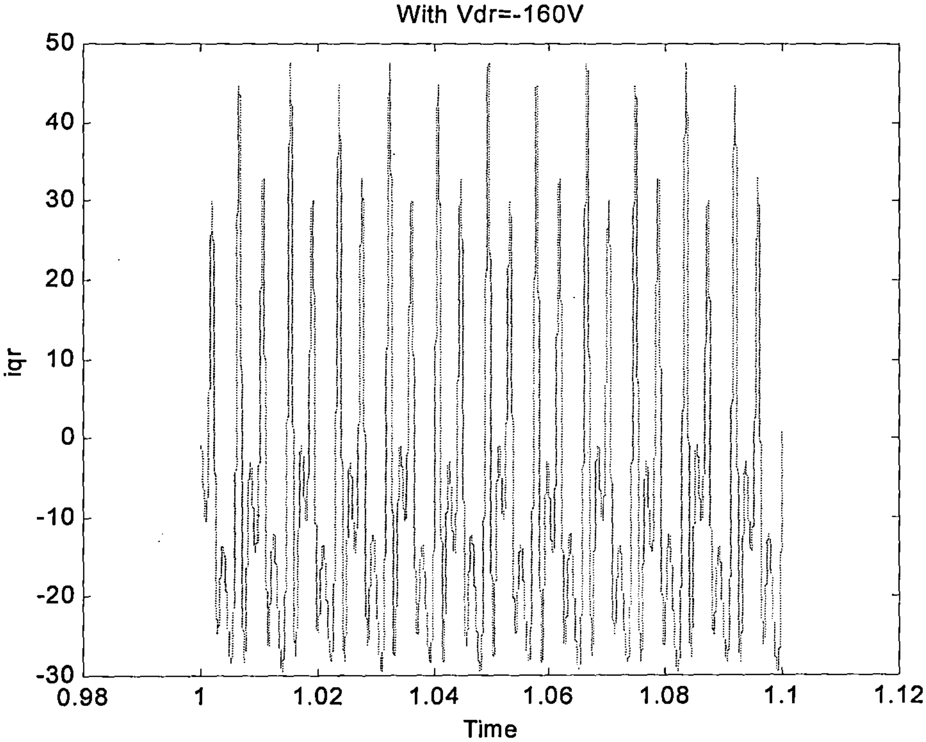


Fig.5.8:Time domain plot with vdr=-160V

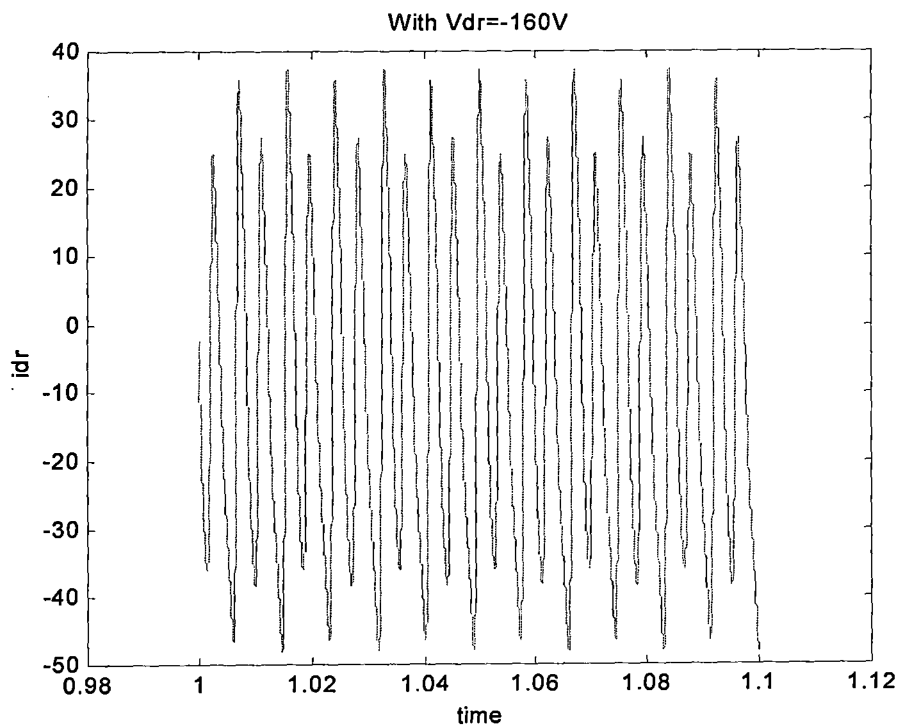


Fig.5.9:Time domain plot with vdr=-160V

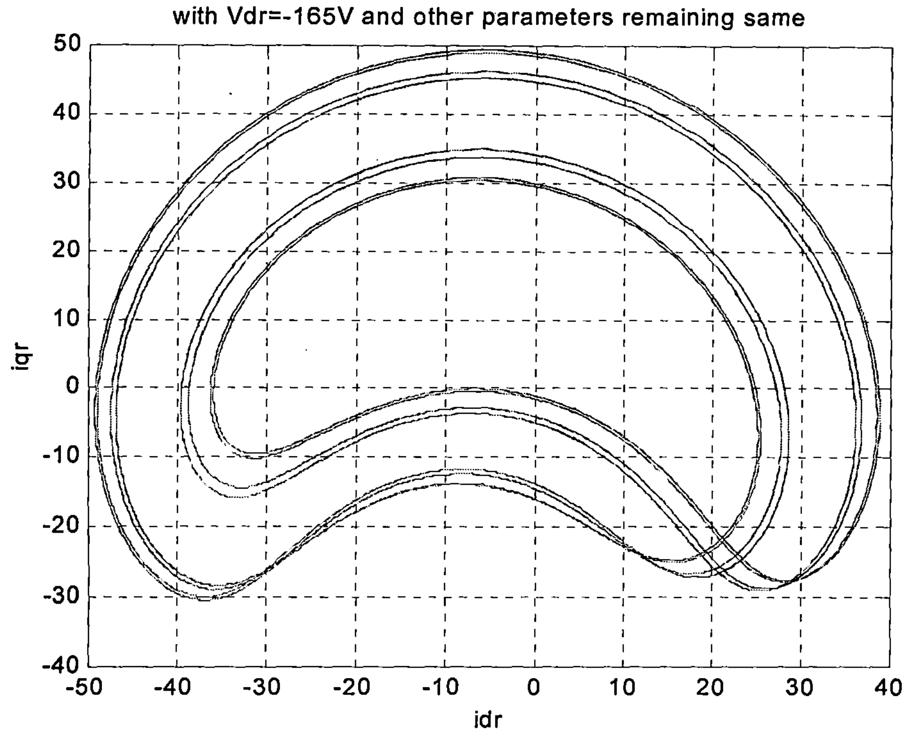


Fig.5.10:Phase Portrait with vdr=-165V

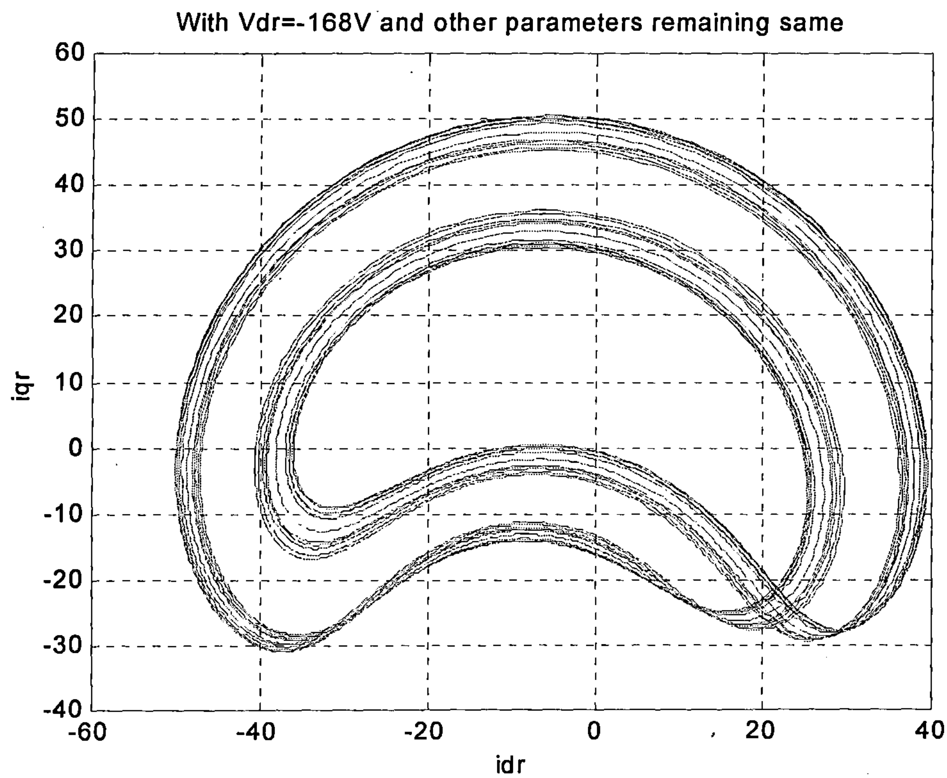


Fig5.11:Phase Portrait with vdr=-168V

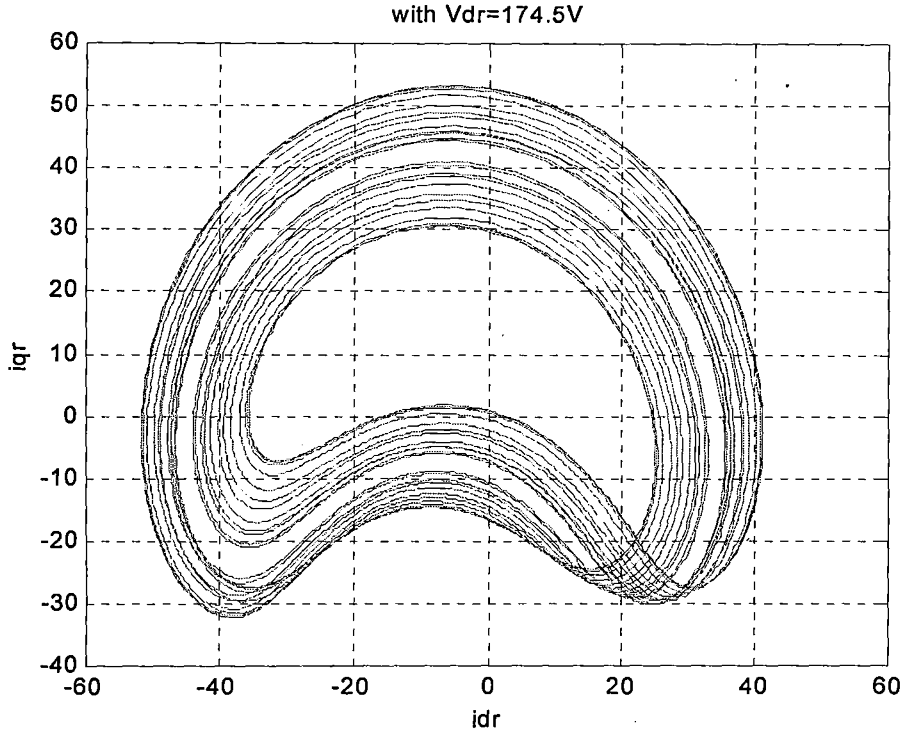


Fig.5.12:Phase Portrait with vdr=-174.5V

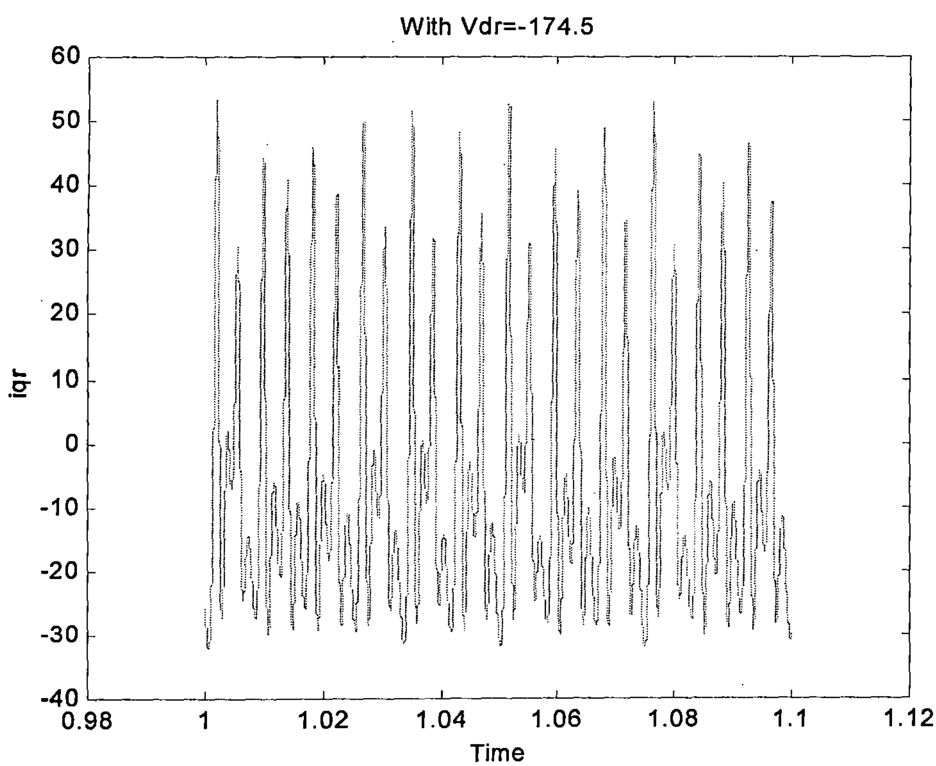


Fig. 5.13:Time domain plot with vdr=-174.5V

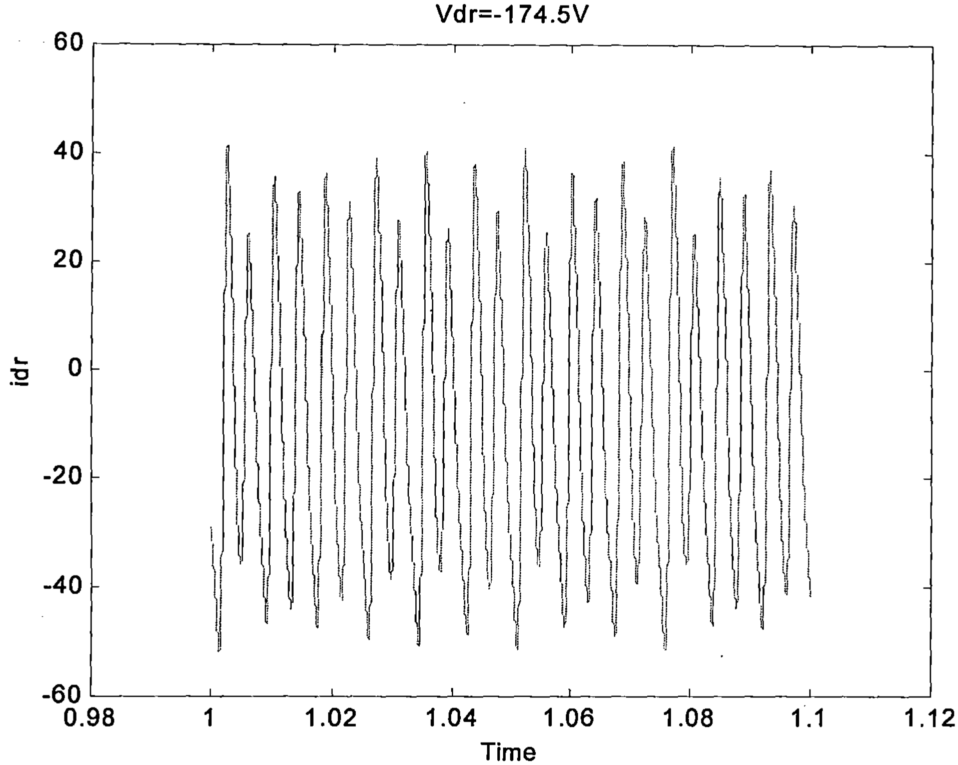


Fig. 5.14:Time domain plot with vdr=-174.5V

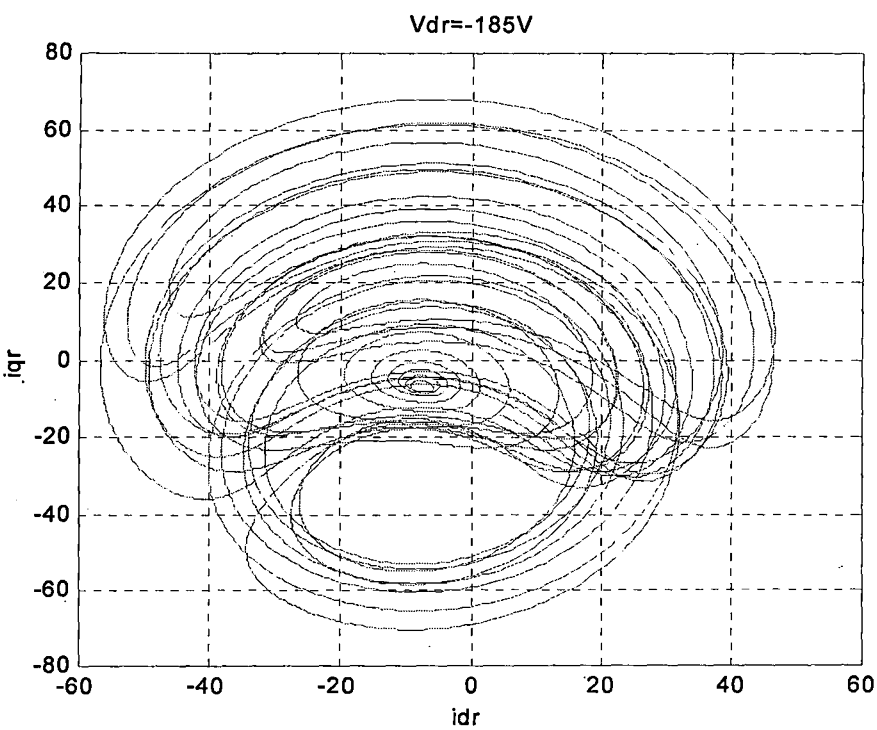


Fig.5.15:Phase Portrait with vdr=-185V

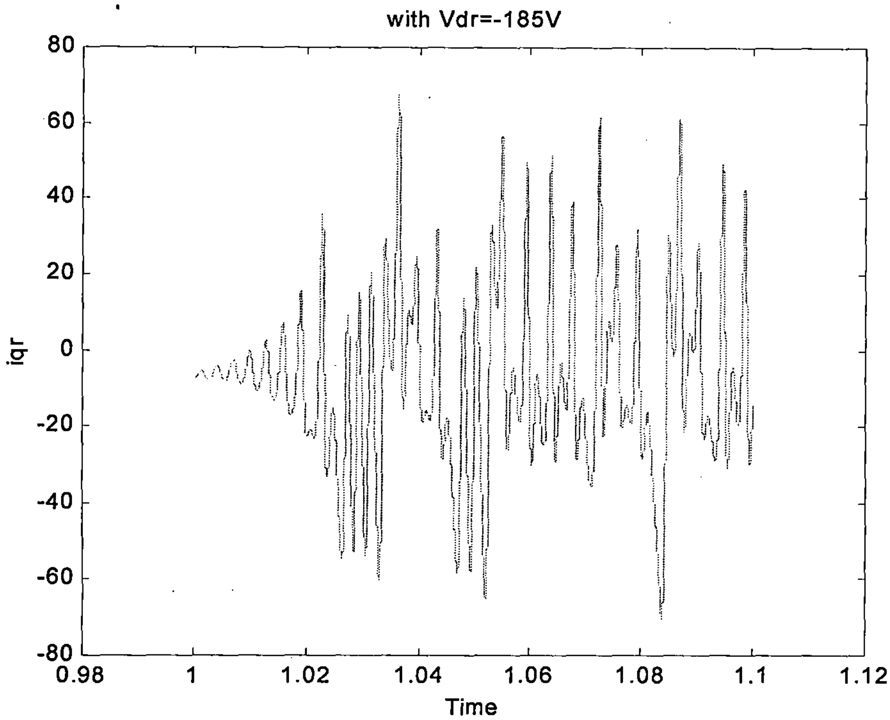


Fig. 5.16:Time domain plot with vdr=-185V

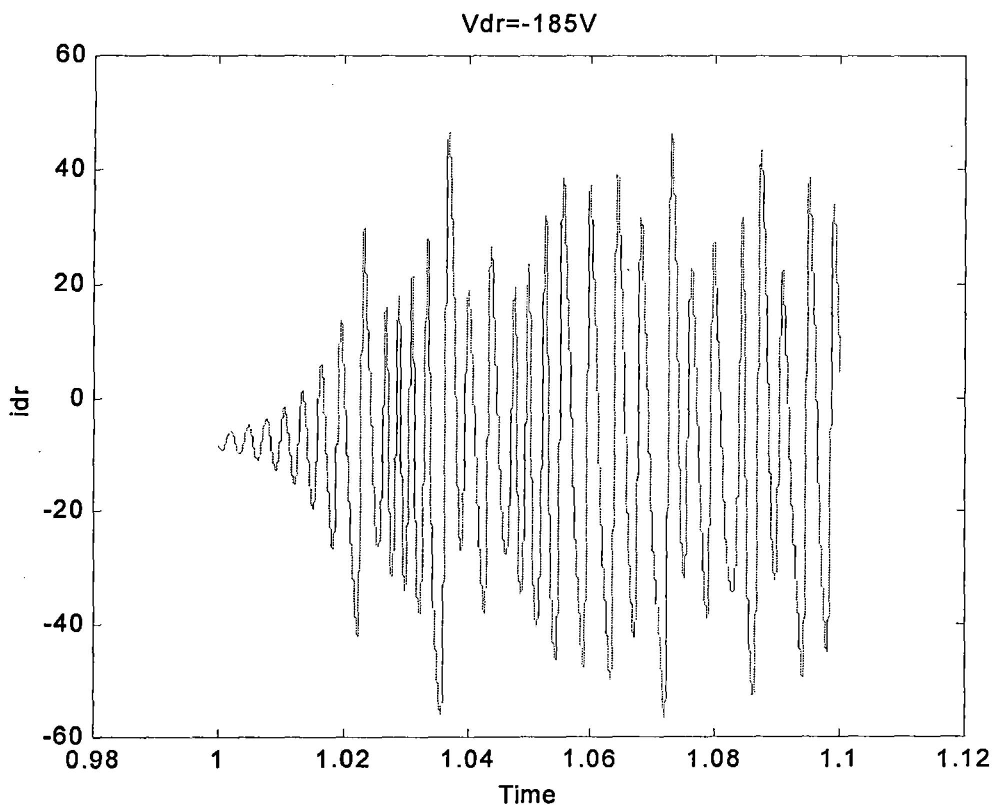


Fig. 5.17:Time domain plot with vdr=-185V

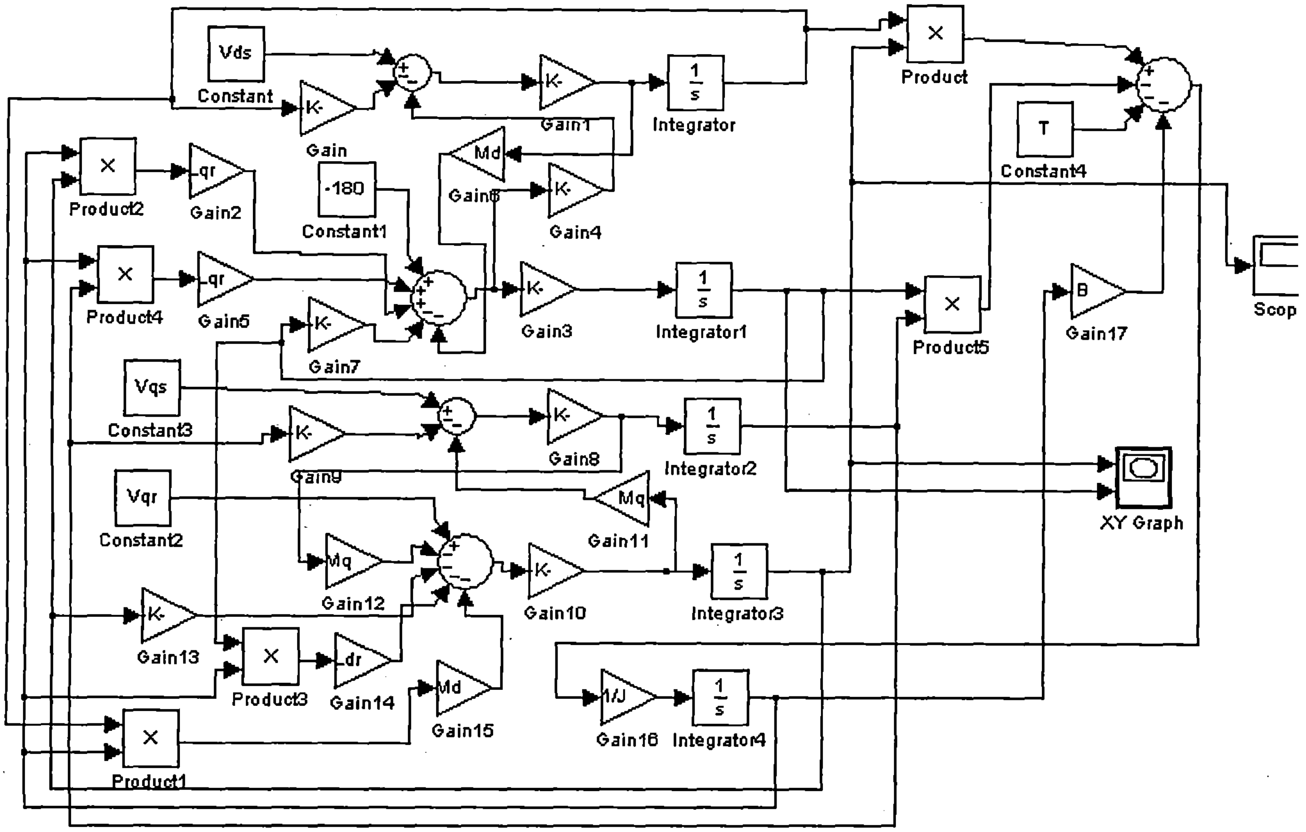


Fig.5.18: Simulink model for Simulation of Generalized Machine

Above results show that the generalized machine is a rich source of dynamics. With different sets of parameter values, it may show periodicity one, two four etc. and even it may become chaotic. The phase portraits also assure that the machine's dynamics is basically Lorenz like.

5.5 Nonlinear Phenomenon in the Approximate model of Generalized Machine:

The no. identified states of the systems of actual Generalized Electrical Machine is five, namely, $i_{ds}, i_{dr}, i_{qs}, i_{qr}, \omega_r$ and therefore, the GEM is basically a five dimensional system with 25 parameters. However, it is very difficult to deal with all of those 5 dimensions. Dimensions of the GEM may be reduced as per guidelines provided in [7],[40] basically by neglecting and eliminating the stator electric transients. Then, the direct and quadrature axis stator voltage equations are reduced to the following[41]:

$$v_{ds} = r_{ds} i_{ds} \qquad v_{qs} = r_{qs} i_{qs} \tag{5.17}$$

Hence reduced order equations for eqn.(2.15) is now having 3 dimensions and governing equations are as follows:

$$L_{dr} \frac{di_{dr}}{dt} = v_{dr} + M_q \omega_r i_{qs} - r_{dr} i_{dr} + \omega_r L_{qr} i_{qr}$$

$$L_{qr} \frac{di_{qr}}{dt} = v_{qr} - M_d \omega_r i_{ds} - \omega_r L_{dr} i_{dr} - r_{qr} i_{qr}$$

$$J \frac{d\omega_r}{dt} = i_{qr} M_d i_{ds} - M_q i_{dr} i_{qs} + (L_{dr} - L_{qr}) i_{qr} i_{dr} - B\omega_r + T_L \tag{5.18}$$

No of parameters of eqn.(2.36) may be reduced by suitable transformation as mentioned below:

$$x \equiv \alpha . i_{dr}$$
$$y \equiv \beta i_{qr}$$
$$z \equiv \gamma \omega_r$$
$$\tau \equiv \delta . t \tag{5.19}$$

The values of the co-efficients are suitably chosen as,

$$\alpha = \frac{M_d i_{ds} L_{dr}}{B.r_{qr}} \qquad \beta = \frac{M_d i_{ds} L_{qr}}{B.r_{qr}}$$

$$\gamma = \frac{L_{qr}}{r_{qr}} \qquad \delta = \frac{r_{qr}}{L_{qr}}$$

Thus the eqn.(2.36) are transformed into the equations:

$$\dot{x} = v'_{dr} - bx + dz + yz$$
$$\dot{y} = v'_{qr} - y + cz - xz$$
$$\dot{z} = a(y - z) - ex + fxy - T'_L \tag{5.20}$$

Here the parameter are as follows:

$$a = \frac{BL_{qr}}{Jr_{qr}} \qquad b = \frac{r_{dr} L_{qr}}{r_{qr} L_{dr}}$$

$$c = \frac{-M_d^2 i_{ds}^2}{Br_{qr}} \qquad d = \frac{M_d M_q i_{ds} i_{qs}}{Br_{qr}}$$

$$e = \frac{BM_q i_{qs} L_{qr}^2}{JM_d i_{ds} L_{dr} r_{qr}} \qquad f = \frac{(L_{dr} - L_{qr}) B^2 r_{qr}}{M_d^2 i_{ds}^2 L_{dr}}$$

$$v'_{dr} = \frac{v_{dr} M_d i_{ds} L_{qr}}{B r_{qr}^2} \qquad v'_{qr} = \frac{v_{qr} M_d i_{ds} L_{qr}}{B r_{qr}^2}$$

$$T'_L = \frac{T_L L_{qr}^2}{J r_{qr}^2} \tag{5.21}$$

Eqn presents the mathematical model of the GEM which can be used for the study of the dynamical behavior.

Nonlinear Phenomenon:

The mathematical model of the GEM represented by the eqn.(5.18) can be exploited for the study of its nonlinear phenomenon. To study the general nature of the dynamics and to simplify the analysis, as a special case, eqn.(5.18) is made autonomous. Hence we get the following:

$$\dot{x} = -bx + dz + yz$$

$$\dot{y} = -y + cz - xz$$

$$\dot{z} = a(y-z) - ex + fxy \tag{5.22}$$

The equilibrium point of eqn.(5.20) i.e.,(x_{eq}, y_{eq}, z_{eq})can be obtained by setting derivatives of it zero. By observation, origin(0,0,0) is a trivial solution of the eqn.(5.20). Hence, origin is an equilibrium point of the GEM. For other equilibrium points,

$$0 = -bx + dz + yz$$

$$0 = -y + cz - xz$$

$$0 = a(y-z) - ex + fxy$$

Eliminating x and y,

$$x_{eq} = \frac{cz^2 + dz}{b + z^2} \qquad y_{eq} = \frac{bcz - dz^2}{b + z^2}$$

$$az^4 + (ad + cdf + ce)z^3 + (2ab + d^2 f + de - abc - bc^2 f)z^2 + (abd + bce - bcdf)z + ab^2(1-c) = 0 \tag{5.23}$$

It is an quartic equation with four solutions. The nature of the roots depend on the coefficients. However, as the roots represent z_{eq} and related to the equilibrium points, only real roots are acceptable. So, except origin, the existence of other equilibrium points depend on the coefficients of eqn.(5.21). However, it is practically impossible to predict the nature of the roots by observation.

The quartic equation is of the form:

$$Az^4 + Bz^3 + Cz^2 + Dz + E = 0$$

its solution can be found by means of the following calculations:

$$F = -\frac{3B^2}{8A^2} + \frac{C}{A}$$

$$G = \frac{B^3}{8A^3} - \frac{BC}{2A^2} + \frac{D}{A}$$

$$H = \frac{-3B^4}{256A^4} + \frac{CB^2}{16A^3} - \frac{BD}{4A^2} + \frac{E}{A}$$

if G = 0, the quartic equation becomes a bi-quadric equation and then roots are given by

$$z = -\frac{B}{4A} \pm_s \sqrt{\frac{-F \pm_t \sqrt{F^2 - 4H}}{2}}$$

Otherwise,

$$P = -\frac{F^2}{12} - H$$

$$Q = -\frac{F^3}{108} + \frac{FH}{3} - \frac{G^2}{8}$$

$$R = \frac{Q}{2} \pm \sqrt{\frac{P^3}{27} + \frac{Q^2}{4}}$$

$$U = \sqrt[3]{R}$$

$$V = -\frac{5}{6}F - U + \begin{cases} U = 0 \rightarrow 0 \\ U \neq 0 \rightarrow \dfrac{P}{3U} \end{cases}$$

$$W = \sqrt{F + 2V}$$

$$z = -\frac{B}{4A} + \frac{\pm_s W \pm_t \sqrt{-\left(3G + 2V \pm_s \dfrac{2G}{W}\right)}}{2} \qquad (5.24)$$

The two $\pm_s$ must have the same sign, the $\pm_t$ is independent. To get all roots for eqn.(5.21), we are to find z for $\pm_s, \pm_t$ = +,+ and for +,− and for −,+ and for −,−.

For origin(0,0,0) and other existing equilibrium points, we can obtain the idea about the dynamical behavior and stability at their vicinity by determining the Jacobian at those equilibrium points(x_{eq}, y_{eq}, z_{eq}) as follows:

$$J = \begin{bmatrix} -b & z_{eq} & d + y_{eq} \\ -z_{eq} & -1 & c - x_{eq} \\ -e + fy_{eq} & a + fx_{eq} & -a \end{bmatrix} \tag{5.25}$$

For the eigen values:

$$\begin{vmatrix} -b-\lambda & z_{eq} & d + y_{eq} \\ -z_{eq} & -1-\lambda & c - x_{eq} \\ -e + fy_{eq} & a + fx_{eq} & -a-\lambda \end{vmatrix} = 0 \tag{5.26}$$

As it gives a cubic equation. It's roots and their nature are not readily available but those can be determined by Cardano's Formula. Therefore, the nonlinear phenomenon in Generalized Electrical Machine has been investigated numerically.

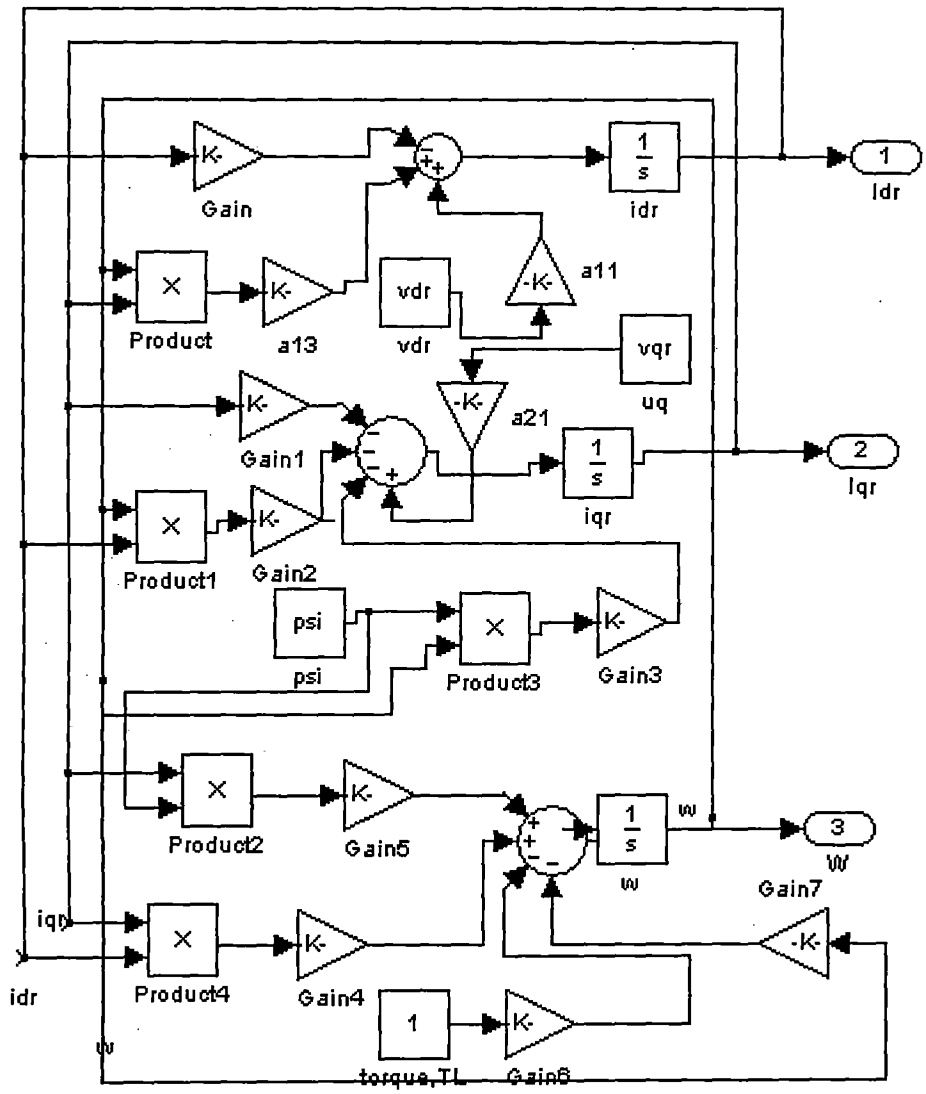


Fig.5.19: Simulink model for Simulation of Approximate Generalized Machine

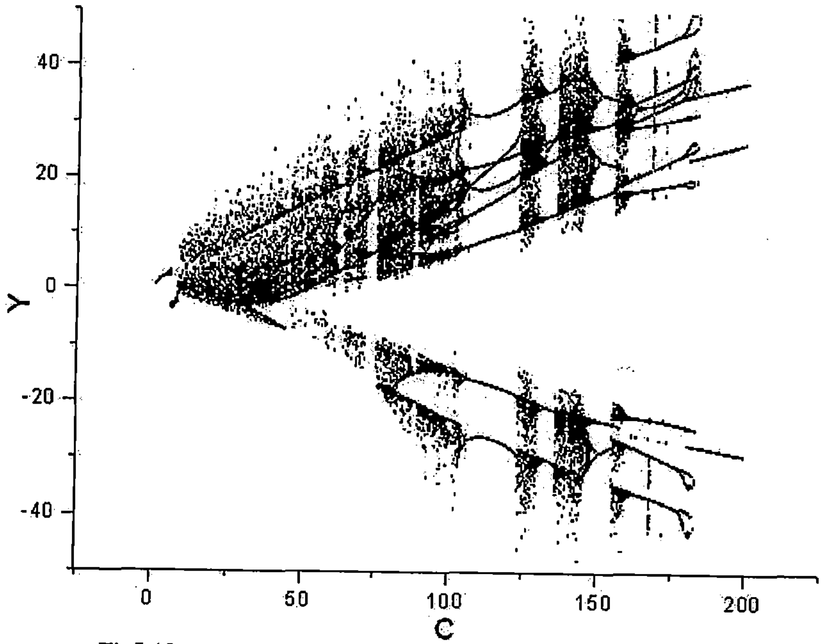


Fig5.19 Bifurcation Diagram for a=10,b=8/3,d=1,e=1,f=1

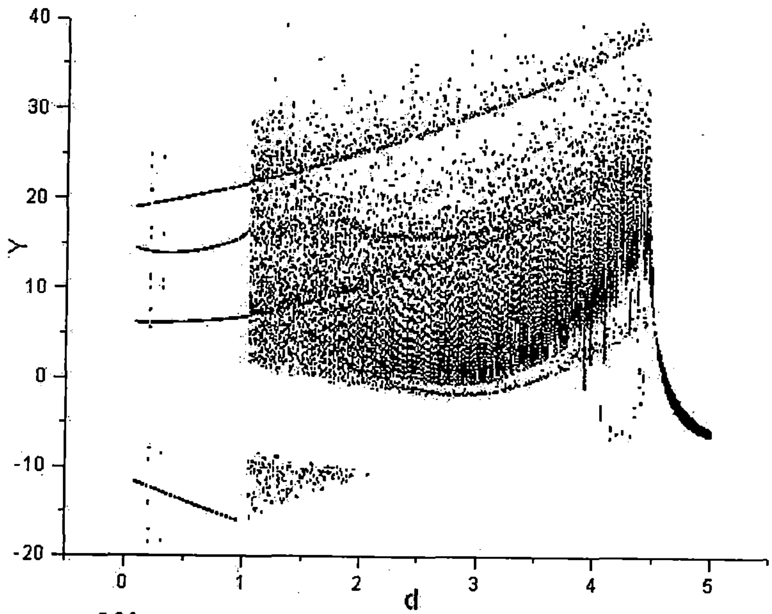


Fig. 5.20 : Bifurcation Diagram for a=10,b=8/3,c=75,e=1,f=1

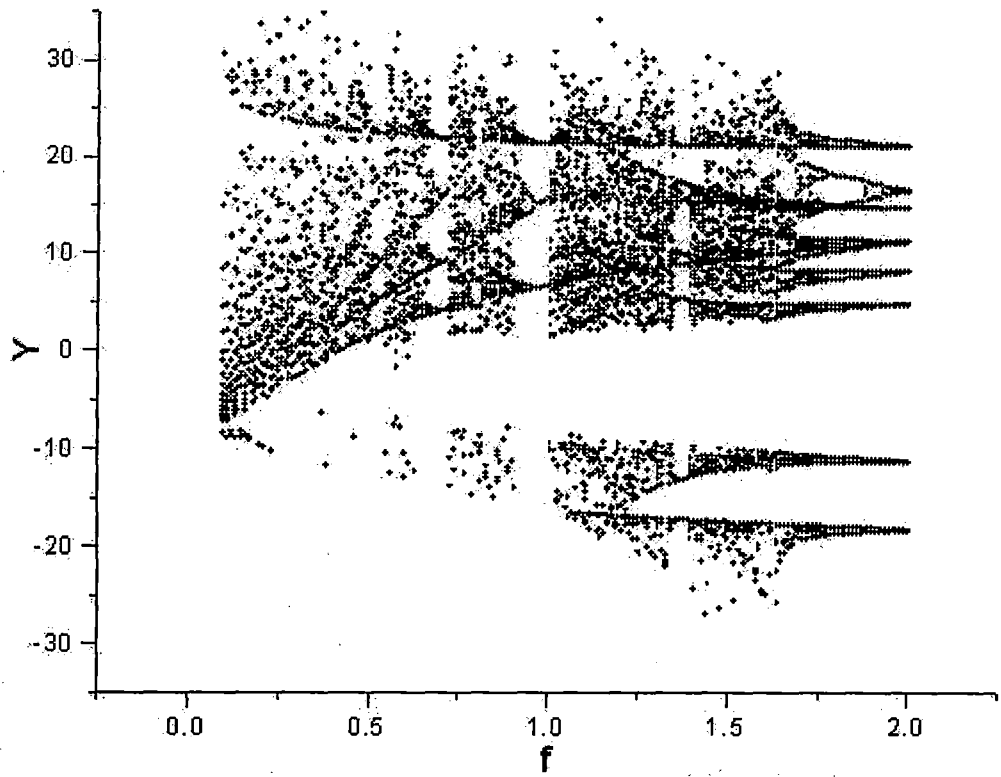


Fig5.21 : Bifurcation diagram for a=10,b=8/3,c=75,d=1,e=1

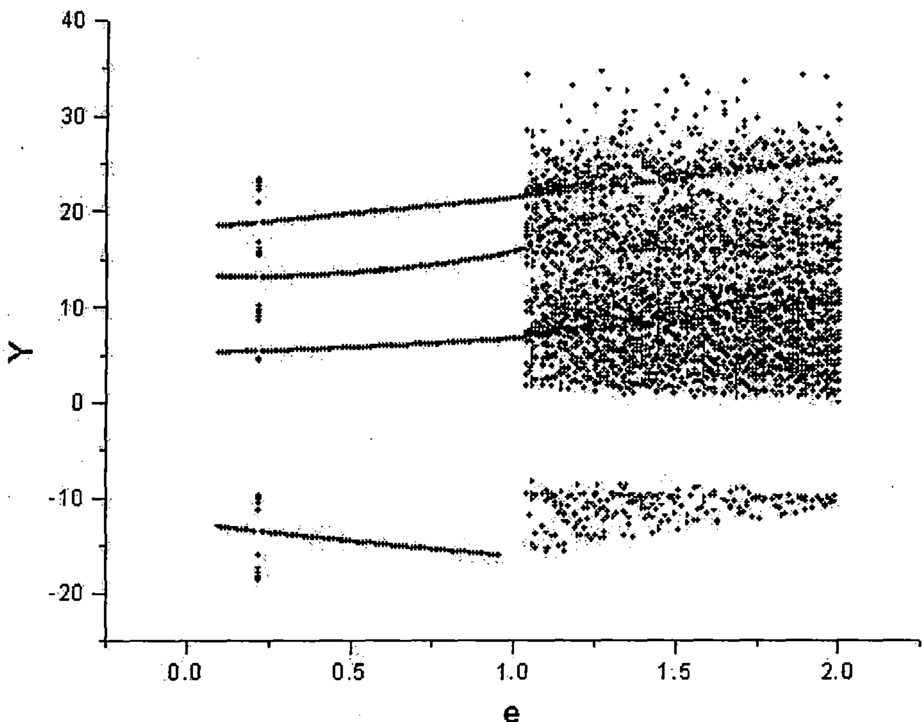


Fig5.22 Bifurcation Diagram for a=10,b=8/3,c=75,d=1,f=1

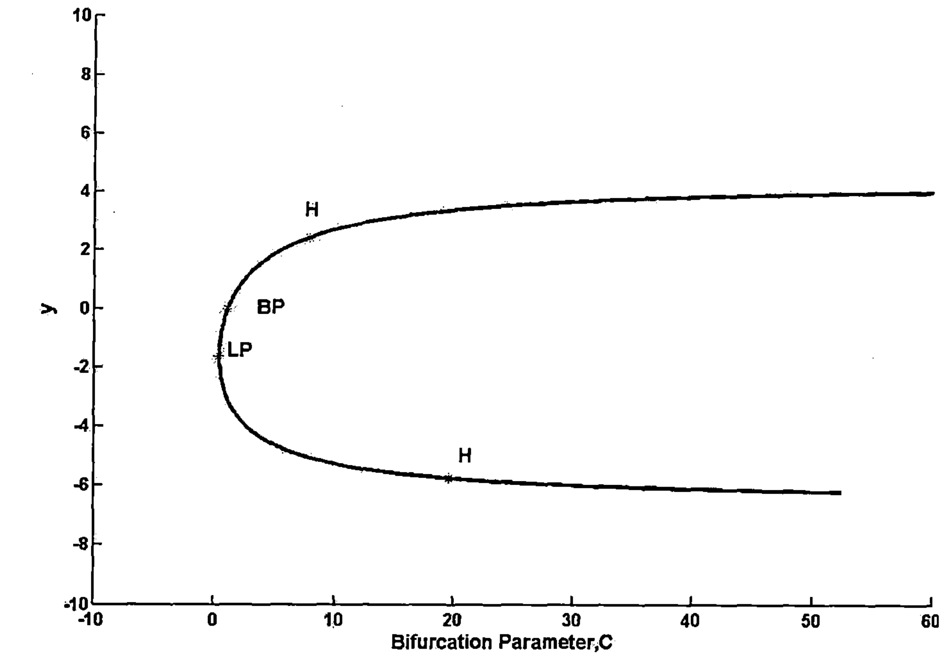


Fig5.23: Bifurcation Diagram with C as parameter

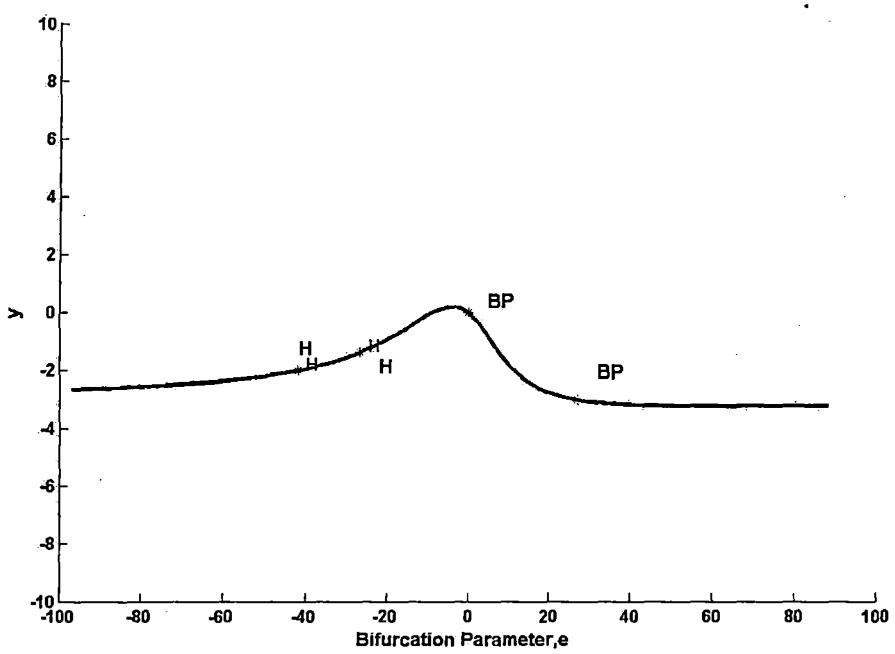


Fig.5.24: Bifurcation Diagram with e as parameter

Here, H=Hopf Bifurcation point, B=Branch Point, L=Limit point

5.6 Prediction of Chaos in conventional Electrical Machines:

Eqn.5.18 shows that the no. of terms in right hand side of autonomous Generalized Electrical Machine has 10 terms among which three terms are nonlinear. It exhibits marked nonlinear phenomenon. However, for conventional machines, no. of terms and nonlinear terms will be different. Their exact no. depend on the no. of existing winding in the said conventional machine and connection of the windings. So, here the question arises whether we can predict about the bifurcation and chaos of an existing conventional machine by the terms of the right hand side of the dynamic equations[90]. Some literature are available in this regard. The Poincare-Bendixson theorem states that chaos does not exist in a two-dimensional continuous-time autonomous system (or a second-order equation).Therefore, a necessary condition for a continuous-time autonomous system to be chaotic is to have at least three state variables with one nonlinear term in the right-hand side. In 1963, Lorenz found chaos in a simple system of three autonomous ordinary differential equations that has seven terms in the right-hand side with only two quadratic nonlinearities(xz and xy). In 1976, R¨ossler discovered a three-dimensional quadratic autonomous chaotic system, which also has seven terms on the right-hand side, but with only one quadratic nonlinearity (xz). Many investigators are interested to know whether there are three-dimensional autonomous chaotic systems with fewer than seven terms in the right-hand side including only one or two quadratic nonlinearities. In1979, Rossler found another simpler chaotic system which has only six terms with a single quadratic nonlinearity ($y2$). Recently, it has been proved that three-dimensional dissipative quadratic systems of ordinary differential equations, with a total of four terms in the right-hand side, cannot exhibit chaos. Very recently, this result was extended to three-dimensional conservative quadratic systems. Later, it was known that autonomous chaotic flow could be produced by a three-dimensional quadratic autonomous system having five terms on the right-hand side, with at least one quadratic nonlinearity, or having six terms with a single quadratic nonlinearity. Lately, chaotic flow in an algebraically simplest three-dimensional quadratic autonomous system was found by using jerky functions which has only five terms with a single quadratic nonlinearity ($y2$).

On the basis of above discussion, we can get some idea about the nonlinear phenomenon in the conventional electrical machines and drives. This is outlined below.

5.6.1 Synchronous Machine:

Conventional Synchronous Machine has four state variables and along with the dynamic equation, no. of state becomes five. So, the condition is similar to that of Generalized Machine. Therefore, system will be a Lorenz like system with higher dimension. Hence, it may be expected that similar nonlinear phenomenon may be occurred in conventional synchronous machine. If damper windings are present then no. of states will be increased by the no. of damper windings. However, nature will be same to that as observed in the case of conventional Synchronous machine[39].

In case of Permanent Magnet Synchronous Motor, no. of states are reduced due to the presence of permanent magnet. Total no. of state variable becomes three. Hence, it may show nonlinear phenomenon as discussed above. However, it has been noted that the system after proper normalization and after making it autonomous, it becomes similar to Lorenz equations. So, similar phenomenon is anticipated.

5.6.2 Induction Machine:

Induction machine may be squirrel cage type or slip ring type. Irrespective of type, it can be modeled using the approach of Generalized Machine which will comprise same no winding as found in Generalized Electrical Machine. Hence similar nonlinear behavior is expected in induction machine. In case single phase and double phase induction machines, no. of physical winding becomes fewer but no. of winding in the model using generalized approach is remaining earlier. Hence, similar nonlinear behavior is expected.

5.6.3 DC Machine:

DC Machines can be modeled using the Generalized approach with one winding on the stator and other winding on the rotor. The governing equation for stator winding is free from nonlinear term. So practically we are getting two state equations for DC machine. The generalized form of the DC Motor for different configuration will be different. however the most general of all is the compound motor. The generalized model of DC Compound Motor is described by the following:

$$v_{ds} = r_{ds} i_{ds} + L_{ds}\frac{di_{ds}}{dt} + M_d \frac{di_{se}}{dt}$$

$$v_{dse} = M_d \frac{di_{ds}}{dt} + r_{dse} i_{dse} + L_{dse} \frac{di_{dse}}{dt}$$

$$v_{qr} = M_d \omega i_{ds} + M_d \omega i_{dse} + r_{qr} i_{qr} + L_{qr} \frac{di_{qr}}{dt} \tag{5.27}$$

$$J\frac{d\omega}{dt} + B\omega = i_{qr} M_d i_{ds} + i_{qr} M_d i_{dse}$$

These equations are compared with the existing Lorenz like system's equation and considered the theory outlined in the section. It is found that the dynamics of Compound DC Motor will not contain any bifurcation and it will never become chaotic. Therefore, as per above discussion DC Machine cannot produce any nonlinear phenomenon. However, while used in drive due to external switching nonlinearity, nonlinear phenomenon might occur.

The model of Brushless DC motor is similar to that of permanent magnet synchronous motor . Therefore, similar nonlinear phenomenon is expected as it was predicted in case of permanent Magnet Synchronous Motor.

Finally, the system of equations of the conventional machines are basically Lorenz like. They may have three dimensions or dimensions may be higher. Therefore, the nonlinear phenomenon in Lorenz System should be well understood. Further, we should have some idea about the dynamics about the systems of Family of Lorenz like systems as the systems of conventional machines and drives may match one of them. If so, their dynamics can be predicted effortlessly. 26 such members of Lorenz like family are identified so far reported in Chapter 3. The dynamics of Lorenz system is also described in Appendix B as ready reference.

The common vocabularies and terminologies frequently used in the study of Nonlinear Dynamics are reported in Chapter 2 as a ready reference.

5.7 CONCLUSION:

Using the proposed method, the nonlinear phenomenon of Generalized Electrical Machine is studied. It has been revealed that the Generalized Machine is a Lorenz like system with higher dimension. For Generalized Machine, no. of state variables is 5. However, the system is a Lorenz like system. The phase portrait of the system confirms the same. The no. of parameters associated with the system is very large. The no. is 17 when exact system of equations are used. When the equations are normalized the parameters are 25. Therefore, variation of those large no. of parameters can produce a reach nonlinear phenomenon. However the route of bifurcation to be followed during this change in parameter will follow some typical path as pointed out and it was found for Lorenz like systems. While this process will be followed for some practical electrical machines the no. of parameters will be reduced and the dimension may be reduced and thereby the study will be simpler. The approximate model of generalized electrical machine is Lorenz like system with same no. of dimension as it was found in actual Lorenz system and similar dynamics as expected for Lorenz like system and as noticed for exact model of generalized machine is observed. The predicted nonlinear phenomenon on conventional electrical machine in the light of the nonlinear phenomenon in generalized machine may be studied further which may assure the applicability of the method proposed to study the nonlinear phenomenon of generalized machine and validate it.

Chapter **6**

NONLINEAR PHENOMENON IN ELECTRICAL MACHINES

6.1 General:

The study of bifurcation phenomenon for generalized machine as reported in the previous chapter can be used for the study of the same in other conventional machines and that can be validated. For its application, the equations of the dynamics of other machines have to be modified as per the existence of the coils. Similar process will be adopted to find out the route of the chaos. However, as mentioned earlier that the dynamic equation of the conventional machines as well as the generalized machine are similar to Lorenz like systems. Some of those systems are reported in the chapter 2 and their brief bifurcation phenomenon and the dynamics are outlined there so that those existing systems can be compared with the available equations of dynamics of the conventional machines[91].

6.2 Nonlinear Phenomenon in Permanent Magnet Synchronous Machines:

6.2.1 Mathematical Modeling:

The permanent magnet Synchronous machine is a machine containing two windings on the rotor. Along with the equation of motion of becomes 3. Hence dimension of the system is reduced and it is 3. No. of parameters also becomes fewer.

Detailed modeling of PM motor is required for proper simulation of the system. The d-q model has been developed [70]-[75]on rotor reference frame as shown in fig.6.1.At any time t, the rotating rotor d-axis makes and angle θr with the fixed stator phase axis and rotating stator mmf makes an angle α with the rotor d-axis. Stator mmf rotates at the same speed as that of the rotor.

The model of PMSM without damper winding has been developed on rotor reference frame using the following assumptions:

1) Saturation is neglected.

2) The induced EMF is sinusoidal.

3) Eddy currents and hysteresis losses are negligible.

4) There are no field current dynamics.

The dynamic d q modeling is used for the study of motor. It is done by converting the three phase voltages and currents to dqo variables by using Parks transformation.

Converting the phase voltages variables vabc to vdqo variables in rotor reference frame the following equations are obtained:

$$\begin{bmatrix} V_{ds} \\ V_{qs} \\ V_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \sin\theta_r & \sin(\theta_r - 120) & \sin(\theta_r + 120) \\ \cos\theta_r & \cos(\theta_r - 120) & \cos(\theta_r + 120) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} \tag{6.1}$$

It also can be converted to Vabc by using the transformation:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} \sin\theta_r & \cos\theta_r & 1 \\ \sin(\theta_r - 120) & \cos(\theta_r - 120) & 1 \\ \sin(\theta_r + 120) & \cos(\theta_r + 120) & 1 \end{bmatrix} \begin{bmatrix} V_{ds} \\ V_{qs} \\ V_0 \end{bmatrix} \tag{6.2}$$

Voltage equations are given by:

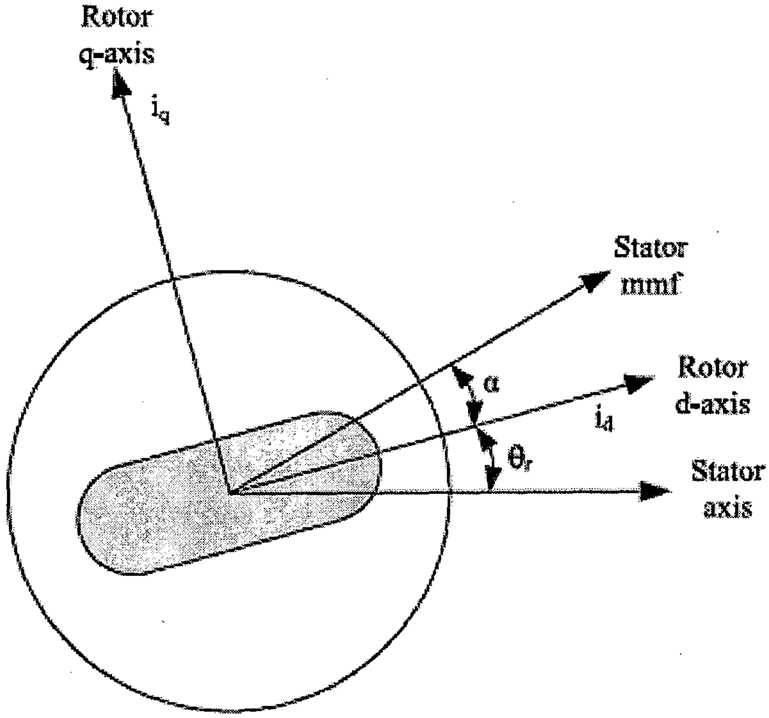


Fig. 6.1: d-q Phasors of permanent magnet Synchronous motor

Voltage equations are given by:

$$v_{ds} = L_{ds}\frac{di_{ds}}{dt} + R_1 i_{ds} - \omega L_{qs} i_{qs}$$
$$v_{qs} = L_{qs}\frac{di_{qs}}{dt} + R_1 i_{qs} + \omega L_{ds} i_{ds} + \omega\psi_r \tag{6.3}$$

And dynamic equation

$$J\frac{d\omega}{dt} = Te - T_L - B\omega \tag{6.4}$$

Where $T_e = n_p \psi_r i_{qs} + n_p (L_{ds} - L_{qs}) i_{ds} i_{qs}$

Here, V_{ds}, V_{qs} = Direct axis and Quadrature axis stator voltages

I_{ds}, I_{qs} = Direct axis and Quadrature axis stator currents

L_{ds}, L_{qs} = Direct axis and Quadrature axis stator inductances

ψ_r = permanent magnet flux

Np= no. of pole pair of the machine

J= moment of inertia of motor load combination

B= viscous damping

R1= stator winding resistances

6.2.2 Analytical Approach:

The analytical approach of studying nonlinear phenomenon in Generalized electrical machine discussed in chapter 5 is being adopted here for the study of the permanent magnet synchronous motor. It's mathematical model is already developed in previous section. As the rotor windings are replaced by a permanent magnet two state variables are eliminated and no. of state variables has been reduced to 3 [34],[35].

$[v] = \begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix}$, states are given by $\begin{bmatrix} i_{ds} \\ i_{qs} \\ \omega \end{bmatrix}$, then states equations derived from voltage equations and equation of the motion and given by

$$L_{ds}\frac{di_{ds}}{dt} = v_{ds} - R_1 i_{ds} + \omega L_{qs} i_{qs}$$

$$L_{qs}\frac{di_{qs}}{dt} = v_{qs} - R_1 i_{qs} - \omega L_{ds} i_{ds} - \omega\psi_r$$

$$J\frac{d\omega}{dt} = n_p \psi_r i_{qs} + n_p (L_{ds} - L_{qs}) i_{ds} i_{qs} - T_L - B\omega \qquad (6.5)$$

The equilibrium of the system will be obtained by setting the LHS of the equation s zero. This gives

$$I_{qs} = \frac{T_L}{n_p \psi_r} + \frac{B}{n_p \psi_r}\omega$$

$$I_{ds} = \frac{V_d}{R_1} + \frac{\omega L_{qs} T_L}{n_p \psi_r R_1} + \frac{\omega^2 L_{qs} B}{n_p \psi_r R_1} \qquad (6.6)$$

$$\varpi^3 + \frac{T_L}{B}\varpi^2 + \left(\frac{R_1^2}{L_{ds} L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds} L_{qs} B} + \frac{V_{ds} n_p \psi_r}{L_{ds} L_{qs} B} \right)\varpi + \left(\frac{R_1^2 T_L}{L_{ds} L_{qs} B} - \frac{V_{qs} n_p \psi_r R_1}{L_{ds} L_{qs} B} \right) = 0 \qquad (6.7)$$

with $a_2 = \frac{T_L}{B}$

$$a_1 = \left(\frac{R_1^2}{L_{ds} L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds} L_{qs} B} + \frac{V_{ds} n_p \psi_r}{L_{ds} L_{qs} B} \right)$$

$$a_0 = \left(\frac{R_1^2 T}{L_{ds} L_{qs} B} - \frac{V_{qr} n_p \psi_r R_1}{L_{ds} L_{qs} B} \right) \qquad (6.8)$$

The equilibrium point(I_{ds}, I_{qs}, ϖ) is described by the above equations. If the equilibrium is restricted in the real domain then D≤0 must hold.

Where $\quad D = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2$

$$p \equiv \frac{3a_1 - a_2^2}{3}$$

$$q \equiv \frac{9a_1 a_2 - 27a_0 - 2a_2^3}{27}$$

If D=0, ϖ has 3 real roots where two of them are equal and reminder is negative. The condition D=0 can be achieved without loosing the generality where T_L and v_{qs} can take deterministic values and v_{ds} can be adjusted accordingly. Other corresponding values of the state variables can be obtained from Equation 6.2

If D<0 , all roots are real and unequal. That condition also can be achieved by adjusting the parameters and other corresponding values of state variables for those equilibrium points can be calculated.

A general case may be worked out here. It is assumed that $V_{ds} = V_{qs} = T_L = 0$

Then

$$\varpi^3 + \left(\frac{R_1^2}{L_{ds}L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B} \right) \varpi = 0$$

It gives $\varpi_1 = 0$ There fore from Equation (6.2)

$$I_{ds1} = I_{qs1} = 0$$

That is, the origin is an equilibrium point. For other two nontrivial equilibrium point,

$$\varpi^2 + \left(\frac{R_1^2}{L_{ds}L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B} \right) = 0$$

$$\text{or } \varpi_{2,3} = \pm\sqrt{-\left(\frac{R_1^2}{L_{ds}L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B}\right)} \tag{6.9}$$

For the existence of those values,

$$\left(\frac{R_1^2}{L_{ds}L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B}\right) \le 0 \tag{6.10}$$

Corresponding values of the other state variables:

$$I_{ds2,3} = \varpi_{2,3}^2 = -\left(\frac{R_1^2}{L_{ds}L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B}\right)$$

$$I_{qs2,3} = \varpi_{2,3} = \sqrt{-\left(\frac{R_1^2}{L_{ds}L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B}\right)}$$

Like the Lorenz equation, the system will remain stable at the origin till $\left(\frac{R_1^2}{L_{ds}L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B}\right) \prec 0$

However, at $\left(\frac{R_1^2}{L_{ds}L_{qs}} + \frac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B}\right) = 0$, the origin will loose the stability in a pitchfork bifurcation at that point creating two nontrivial equilibria which are initially stable. To determine the stability of these nontrivial equilibria the Jacobian matrix has to be formed.

For nontrivial equilibria, Jacobian matrix is given by

$$J_{2,3} = \begin{bmatrix} \frac{-R_1}{L_{dr}} & \frac{\varpi_{2,3}L_{qr}}{L_{dr}} & \frac{L_{qr}I_{qr2,3}}{L_{dr}} \\ -\frac{\varpi_{2,3}L_{dr}}{L_{qr}} & \frac{-R_1}{L_{qr}} & \frac{-L_{dr}I_{dr2,3}-\Psi_r}{L_{qr}} \\ 0 & \frac{n_p\Psi_r}{J} & \frac{-B}{J} \end{bmatrix} \tag{6.11}$$

For eigen values,

$$\begin{vmatrix} \dfrac{-R_1}{L_{ds}}-\lambda & \dfrac{\varpi_{2,3}L_{qs}}{L_{ds}} & \dfrac{L_{qs}I_{qs2,3}}{L_{ds}} \\ -\dfrac{\varpi_{2,3}L_{ds}}{L_{qs}} & \dfrac{-R_1}{L_{qs}}-\lambda & \dfrac{-L_{ds}I_{ds2,3}-\psi_r}{L_{qs}} \\ 0 & \dfrac{n_p\psi_r}{J} & \dfrac{-B}{J}-\lambda \end{vmatrix} = 0$$

It gives

$$\lambda^3+\lambda^2\left(\frac{B}{J}+\frac{R_1}{L_{ds}}+\frac{R_1}{L_{qs}}\right)+\lambda\left(\frac{-\psi_r n_p R_1}{BL_{ds}L_{qs}J}+\frac{R_1 B}{L_{qs}J}+\frac{R_1 B}{L_{ds}J}\right)+\left(\frac{-n_p\psi_r L_{ds}I_{ds2,3}-n_p\psi_r^2}{L_{ds}L_{qs}J}+\frac{n_p\psi_r\varpi_{2,3}^2}{J}\right)$$

$$=0 \qquad (6.12)$$

Since two nontrival equilibria are symmetric, their stability must be same. Hopf Bifurcation[19] occurs when the corresponding Jacobian matrix has a pair of purely imaginary eigenvalues, with the remaining eigenvalues having nonzero real parts. For the bifurcation of two nontrival equilibria, i.e., values of the parameters for which either $\lambda=0$ or $\lambda=j\omega$ is the solution of the eigen value equation.

Setting $\lambda=0$, $\left(\dfrac{R_1^2}{L_{ds}L_{qs}}+\dfrac{\psi_r^2 n_p R_1}{L_{ds}L_{qs}B}\right)=0$ it gives pitchfork bifurcation which is already known.

For Hopf Bifurcation, putting $\lambda=j\omega$ in equation and equating real and imaginary parts:

$$\omega^2=\frac{\left(\dfrac{-n_p\psi_r L_{ds}I_{ds2,3}-n_p\psi_r^2}{L_{ds}L_{qs}J}+\dfrac{n_p\psi_r\varpi_{2,3}^2}{J}\right)}{\left(\dfrac{B}{J}+\dfrac{R_1}{L_{ds}}+\dfrac{R_1}{L_{qs}}\right)}=\left(\frac{-\psi_r n_p R_1}{BL_{ds}L_{qs}J}+\frac{R_1 B}{L_{qs}J}+\frac{R_1 B}{L_{ds}J}\right) \qquad (6.13)$$

Rearranging this relation the critical values of different parameters responsible for Hopf Bifurcation will be available. At that critical value corresponding to Hopf Bifurcation, the equilibria will be surrounded by the limit cycle. At a value of the parameter greater than the critical value, all three equilibria will become unstable.

The analytical results are similar to those obtained in case of Generalized machine as described in chapter 5. However, due to less no. of dimension analysis becomes easier to some extent and able to find the significant results like hopf bifurcation point analytically which was not possible for earlier case.

6.2.3 Numerical Approach:

The mathematical model of permanent magnet synchronous motor developed earlier is simulated to nonlinear dynamics of the machine[33].

Simulation is done for the following values of the parameters:

$np = 1$

$R1 = 0.9\Omega$

$Lds = 15mH,$

$Lqs = 15mH$

$J = 0.00005, Kgm^2$

$B = 0.017N/rad/s$

$\psi_r = 0.03$

$np = 1$

$T_L = 0$

Vds and Vqs are chosen as variable parameters. However, during experiment, they will not be readily available but phase voltages will be obtained.

Here the values of the parameters are chosen same as those are having for the motor under test to be described in the next section. This is done so that the results obtained from numerical analysis can be compared with the experimental results. However, during numerical analysis, the transformed dq0 state variables are used as it is done conventionally however, for experimental study only the real and actual state variables are available. Therefore, only rotor speed is considered for comparison while simulation results are being compared with the experimental results.

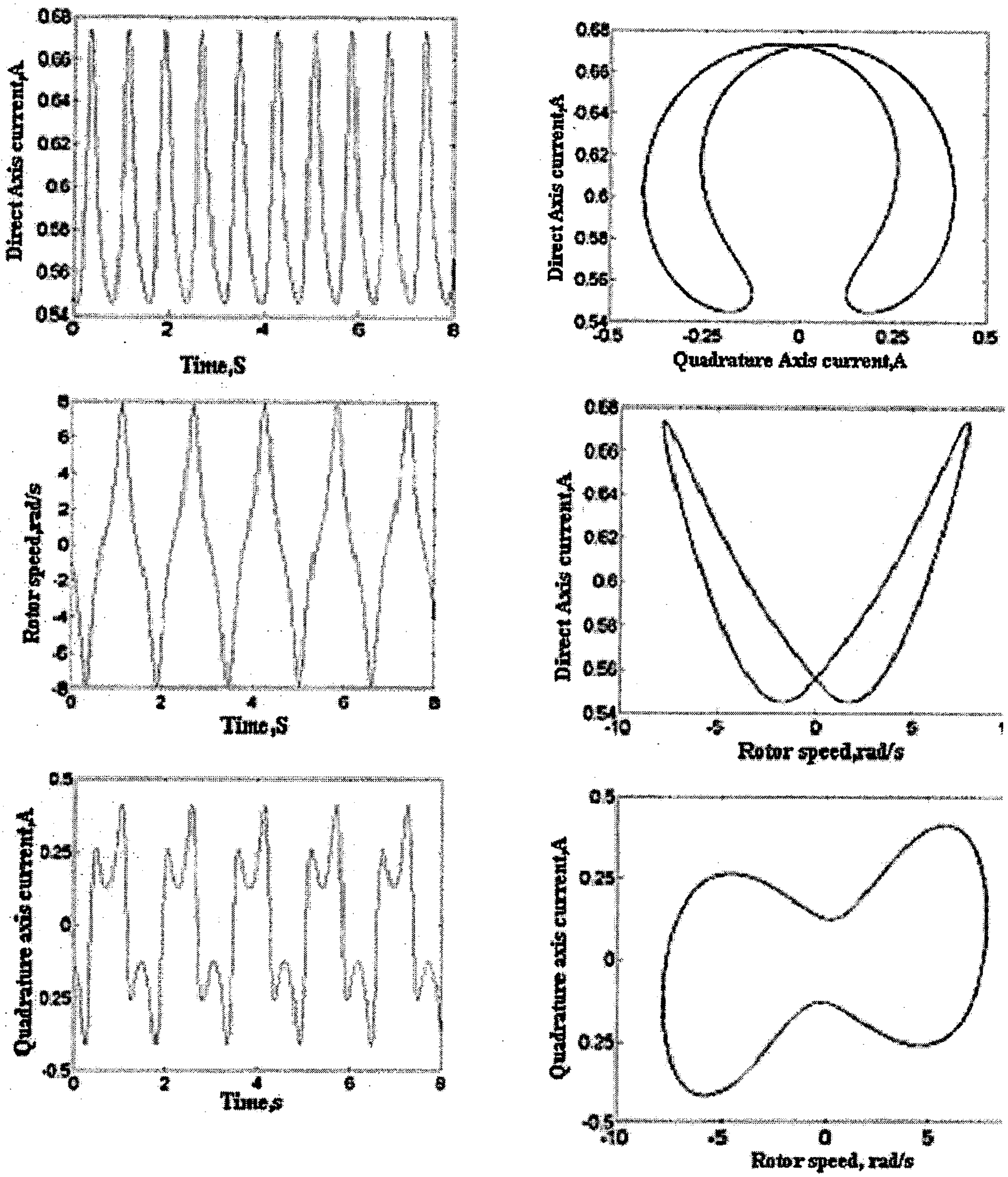


Fig6.2: Periodic waveforms and phase portraits of permanent magnet synchronous motor with the parameters as mentioned above and Vds=67V and Vqs=67V with TL=0

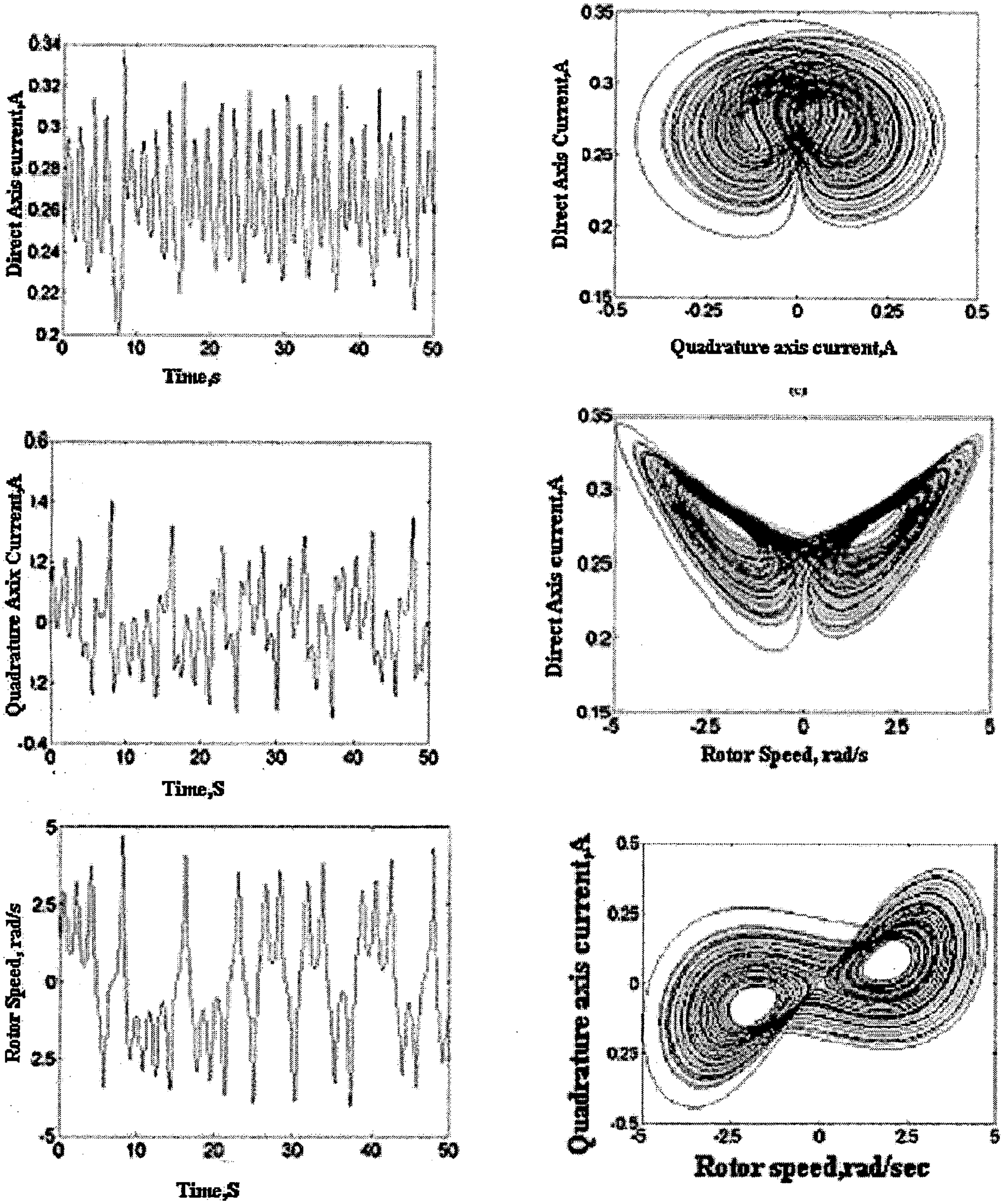


Fig6.3: Chaotic waveforms and phase portraits of permanent magnet synchronous motor with the parameters as mentioned above and Vds=115V and Vqs=115V with TL=0

6.2.3 Experimental Results:

Rating from the Nameplate:

C.Stall Torque=1.5Nm 2.4A

Max rpm = 6000

V =380Volt12 2A

IP =64/65 IC-400

Machine under test has the following parameter value:

$np = 1$

$R1 = 0.9\Omega$

$Lds = 15mH,$

$Lqs = 15mH$

$J = 0.00005, Kgm^2$

$B = 0.017N/rad/s$

$\psi_r = 0.03$

$T_L = 0$

Supply voltage are taken as variable parameters during the experiment. The dq variables, Vds and Vqs can be calculated using their measured value.

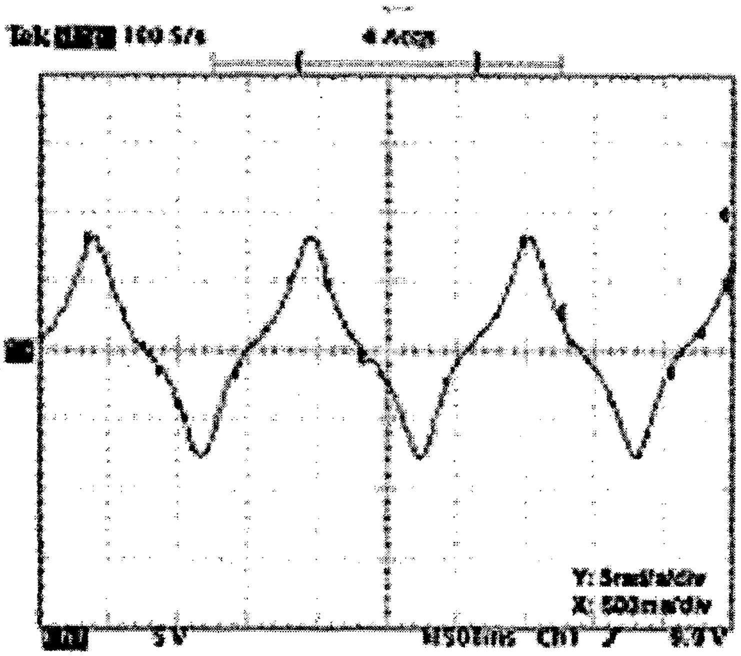


Fig6.4: Periodic waveforms and phase portraits of permanent magnet synchronous motor with the parameters as mentioned above and supply voltages are balanced with rms value of 67V
i.e., Vds= Vqs= 67V with TL=0

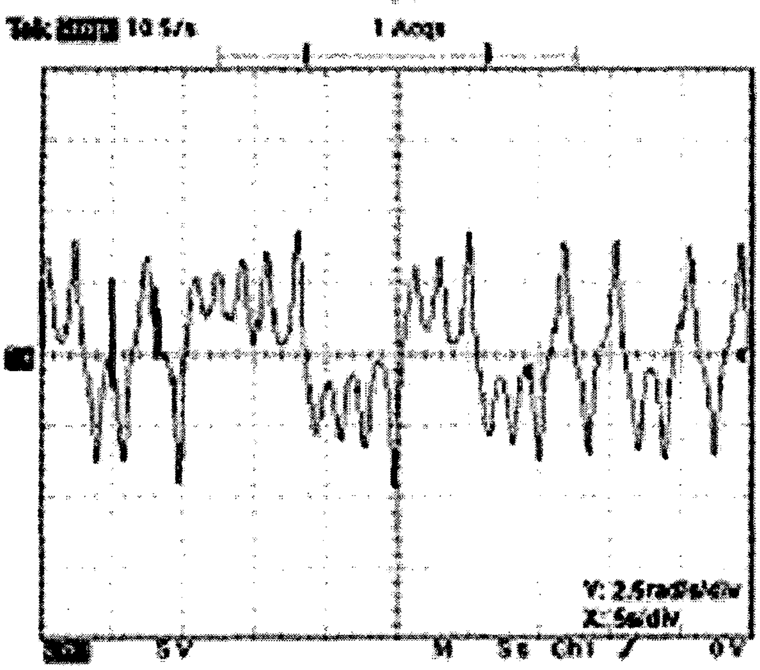


Fig6.5: chaotic waveform of permanent magnet synchronous motor with the parameters as mentioned above and supply voltages are balanced with rms value of 115V
i.e., Vds= Vqs= 115V with TL=0

As mentioned earlier, only directly measurable state variable i.e., rotor's speed as obtained from experiment and numerical study are readily available for comparison. Comparison clearly reveals that they are fairly same and hence it not only shows that the performance of the numerical analysis is satisfactory , it also validates that the approach adopted for the study of hypothetical generalized machine in chapter 5 is fairly general and can be adopted for other conventional electrical machines.

6.3 Nonlinear Phenomenon in Single Phase Induction Motor:

The Shaded pole induction motor is widely accepted for domestic appliances, especially cooling fans. It offers the definite advantages of simple structure, low cost as well as highly rugged and reliable. Its uniqueness is the use of the auxiliary winding, also called the shading winding, to produce a starting torque. As shown in Fig.6.6, a shaded-pole motor uses no starting switch. The stator poles are equipped with an additional winding in each corner, that is, the shade winding.

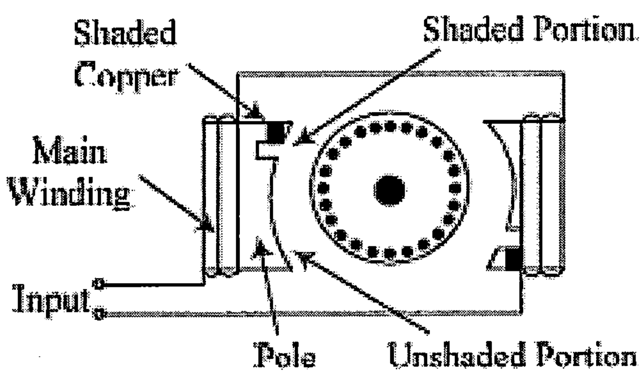


Fig.6.7: Shaded pole induction motor

These windings have no electrical connection for starting but use induced current to make a rotating magnetic field. The shaded pole structure of the SPIM enables the development of a rotating magnetic field by delaying the buildup of magnetic flux. A copper conductor isolates the shaded portion of the pole forming a complete turn around it. In the shaded portion, magnetic flux increases but is delayed by the current induced in the copper shield. Magnetic flux in the unshaded portion increases with the winding current forming a rotating field. The interaction of these two magnetic fields generates the starting torque for the motor. Normally, the effect of the shading winding is negligible when the motor reaches speed.

6.3.1 Mathematical Modeling:

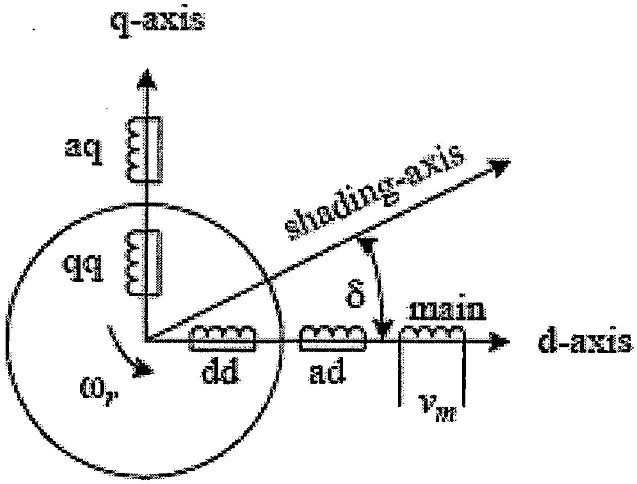


Fig.6.8: d-q model of Shaded pole induction motor

The d-q model of shaded pole induction motor[5],[6] is shown in fig.6.8.

When it is connected to supply,

$v_m = V\sin(2\pi ft + \theta)$

V= amplitude of the supply voltage

θ= initial phase angle

f= supply frequency

Then d-q model of the machine will be given by,

$$\begin{bmatrix} v_m \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} L_{mm} & L_{ma} & L_{md} & 0 \\ L_{ma} & L_{aa} & L_{ar}\cos\delta & -L_{ar}\sin\delta \\ L_{md} & L_{ar}\cos\delta & L_{dd} & 0 \\ 0 & -L_{ar}\sin\delta & 0 & L_{qq} \end{bmatrix} p \begin{bmatrix} i_{ms} \\ i_{as} \\ i_{dr} \\ i_{qr} \end{bmatrix} + \begin{bmatrix} R_m & 0 & 0 & 0 \\ 0 & R_a & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} i_{ms} \\ i_{as} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$+\omega \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & n_p L_{ar}\sin\delta & 0 & -n_p L_{qq} \\ n_p L_{md} & n_p L_{ar}\cos\delta & n_p L_{dd} & 0 \end{bmatrix} \begin{bmatrix} i_{ms} \\ i_{as} \\ i_{dr} \\ i_{qr} \end{bmatrix} \qquad (6.14)$$

i_{ms}= current in the main winding of stator

i_{as} =current in the shaded pole winding of stator

i_{dr} = current in the direct axis winding of rotor

i_{qr} = current in the quadrature axis winding of rotor

L_{mm} = self inductance of the main winding

L_{ma} =mutual inductance between main winding and shaded pole winding

L_{md} = mutual inductance between main winding and direct axis winding of rotor

L_{aa} = self inductance of the shaded pole winding

L_{ar} = mutual inductance between shaded pole winding and the rotor when the rotor is aligned with the shaded pole winding

L_{dd} = self inductance of the direct axis winding of rotor

L_{qq} = self inductance of the quadrature axis winding of rotor

R_m = resistance of the main winding of stator

R_a = resistance of the shaded pole winding of stator

R_r = resistance of the direct and quadrature axis winding of rotor

δ = angle between main winding and shaded pole winding

T_L =Load torque

B = viscous damping

n_p =no. of pole pair

J = moment of inertia

In matrix form,

$$[v] = \begin{bmatrix} v_m \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad [i] = \begin{bmatrix} i_{ms} \\ i_{as} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$[L] = \begin{bmatrix} L_{mm} & L_{ma} & L_{md} & 0 \\ L_{ma} & L_{aa} & L_{ar}\cos\delta & -L_{ar}\sin\delta \\ L_{md} & L_{ar}\cos\delta & L_{dd} & 0 \\ 0 & -L_{ar}\sin\delta & 0 & L_{qq} \end{bmatrix}$$

$$[G]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & n_p L_{ar}\sin\delta & 0 & -n_p L_{qq} \\ n_p L_{md} & n_p L_{ar}\cos\delta & n_p L_{dd} & 0 \end{bmatrix} \tag{6.15}$$

Therefore,

$$[v]=[L]p[i]+[R][i]+[G][i] \tag{6.16}$$

It is same to that used in chapter 5.

The equations can be rearranged in the form of state equations,

$$p[i]=[L]^{-1}([v]-[R][i]-\omega[G][i]) \tag{6.17}$$

Electromagnetic torque of the motor is given by,

$$T_e=n_p[L_{md}i_{ms}i_{qr}+L_{ar}i_{as}i_{dr}\sin\delta+L_{ar}i_{as}i_{qr}\cos\delta+(L_{dd}-L_{qq})i_{dr}i_{qr}] \tag{6.18}$$

Equation ofmotion is given by,

$$J\frac{d\omega}{dt}=T_e-B\omega-T_l$$

Thus total no. of state variables associated with the motor is 5. These are

$$[x]=\begin{bmatrix} i_{ms} \\ i_{as} \\ i_{dr} \\ i_{qr} \\ \omega \end{bmatrix}$$

The state equations can be expressed as,

$$p[x]=\left|[L]^{-1}([v]-[R][i]-\omega[G][i]);(T_e-B\omega-T_L)/J\right| \tag{6.19}$$

As no. of state variables is as good as that of Generalized electrical machines, it is very difficult to get fruitful result by adopting analytical approach. Therefore, numerical and experimental study are the only effective tools for the study of the dynamical behavior of the machine. As no. of state variable is more than 3, it is indicating that the machine under consideration is having a rich source of nonlinear phenomenon.

6.3.2 Numerical study:

Numerical study will be done to study the nonlinear phenomenon of the machine. Simulation of the system is done here with the parameters as they those are having to the motor under experimental study. This is done so that the simulation results can be compared with the experimental results.

However, all state variables used for simulations will not be available for measurement during experimental study. Then, the commonly available state variables will be compared. It does not affect the study because when a system becomes periodic or chaotic all state variables pass through similar states. Therefore, even comparison of only one state variable is sufficient.

Data used for simulation are given below:

$$
\begin{aligned}
&np=01\\
&Lmm=0.4H\\
&Ldd=0.4H\\
&Lqq=0.38H\\
&Lmd=0.35H\\
&Lmq=0.34H\\
&Lma=15mH\\
&Lar=17mH\\
&Rm=5.6\ ohm\\
&Ra=0.2\ ohm\\
&Rr=25\ ohm\\
&Laa=11mH\\
&\delta=28\ degree\\
&J=2.130\times10^{-5}Kg.m^2\\
&B=1.47\times10^{-4}Nms\\
&Tl=0
\end{aligned}
\tag{6.20}
$$

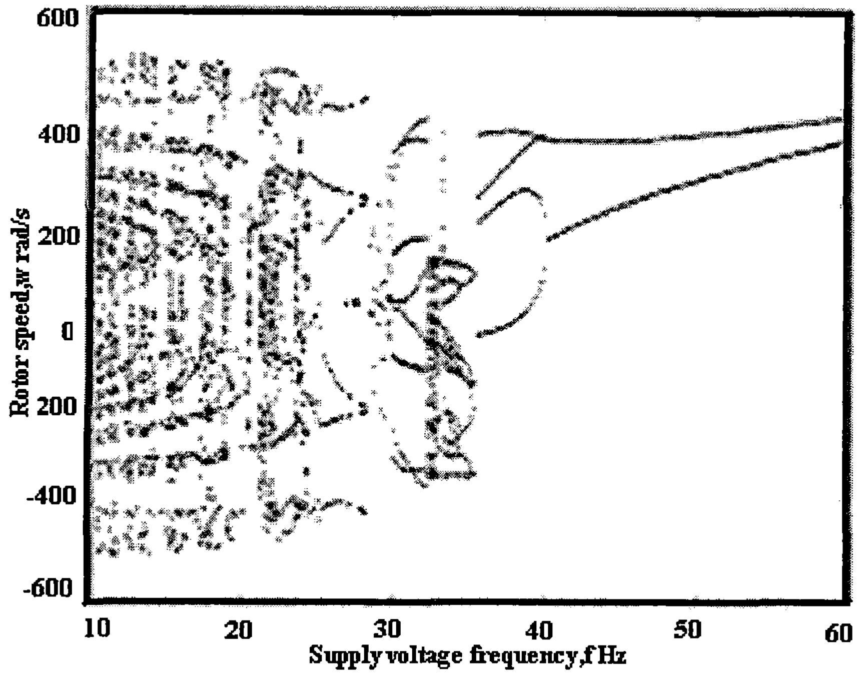


Fig: 6.9 Bifurcation diagram of single phase shaded pole induction motor with supply voltage=220V and supply frequency,f as Bifurcation parameter

From bifurcation diagram it is clear that frequency at about 60 Hz is almost periodicity becomes almost one. At frequency about3 8Hz, periodicity becomes four and at about 22Hz the motor becomes chaotic. The diagram also confirms that with certain sets of parameter values motor becomes chaotic. This was also expected as per the conclusion taken in chapter 5. Thus the simulation result validates the process described in chapter 5.

6.3.3 Experimental study:

The Name plate rating of the motor on which test is done, is given below:

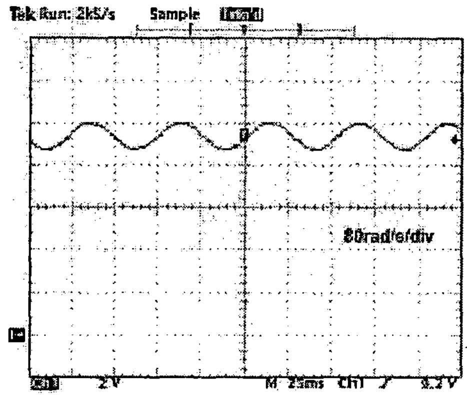
Hp =1.0 5.0A,

Volt =230V

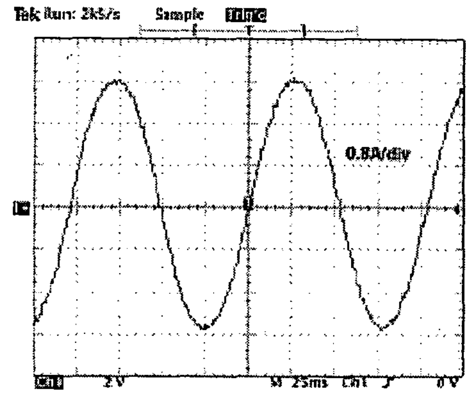
Hz =50

Shaded pole angle=28 degree

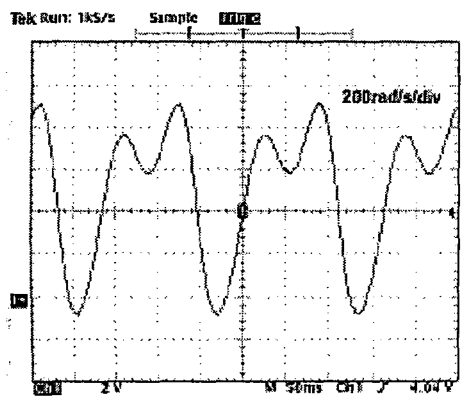
The values of the parameters for the machine used in the experiment are same to those used for simulation i.e., as given in(6.20). Actually those values are used for simulation so that the simulation results and experimental results can be compared. The state variables which can be observed directly during the experiments are rotors speed and the main winding currents on the stator. Those two state variables are observed from simulation and experimental results and compared.



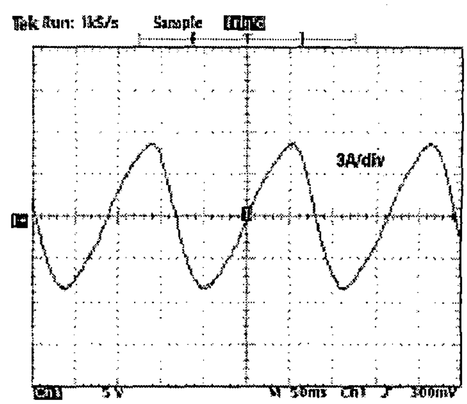
(a) rotor speed, ω with f=60 Hz



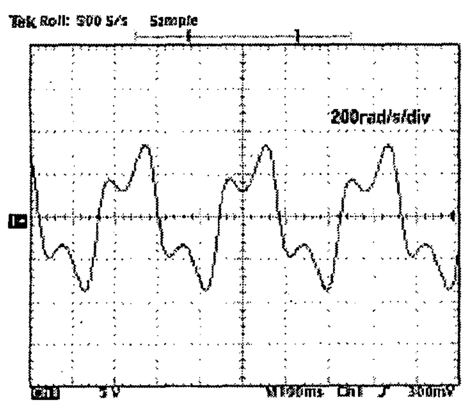
(b) stator main winding current, i_{ms} with f=60 Hz



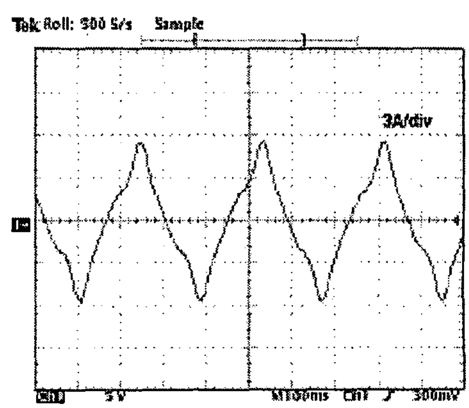
(c) rotor speed, ω with f=38 Hz



(d) stator main winding current, i_{ms} with f=38 Hz



(e) rotor speed, ω with f=22 Hz



(f) stator main winding current, i_{ms} with f=22 Hz

Fig: 6.10 periodic waveforms of single phase shaded pole induction motor with supply voltage=220V with different supply frequencies

Experimental results as found in Fig.6.10 clearly indicates the following thing:

- At frequency, f = 60Hz the motor operates with a periodicity equals to nearly one.
- At frequency, f= 38Hz the motor show period four operation
- At frequency, f=22Hz the operation of the motor is chaotic.

Hence, the experimental results reiterates the facts what was found during the simulation. It not only validates the simulation results but also reassures the effectiveness of the method described in the chapter 5 for generalized electrical machine.

6.4 Conclusion:

The process outlined in previous chapter is used to study the nonlinear phenomenon of some conventional machines. It is found that the method is effective for the machines it have been studied. The nonlinear phenomenon have been studied for two machines both by simulations and experiments. It is observed that similar results are obtained from simulation and experiments and the results are so as expected in chapter 5. This validates the proposed method of chapter 5. Also it has been found that for certain sets of parameters machines show that they may show the dynamics with one, two or more or even chaos. This not only confirms the effectiveness of the proposed method of chapter 5 to study the nonlinear phenomenon of generalized electrical machine but also approves that the method is general and can be adopted to study the nonlinear dynamical behavior of other conventional machines with minimum effort.

Chapter 7

CONCLUSION

The method of study of Nonlinear Phenomenon as outlined in the chapter 5 and same is used to study the same for conventional machines and found that it is very much effective. Following conclusions may be drawn:

- Generalized machine is a general form of representation of all conventional electrical machines. A method of studying nonlinear phenomenon of generalized machine is proposed and confirmed through a case study. As the generalized machine is a hypothetical machine with higher dimension, only numerical investigation can provide fruitful results, analytical approach cannot reach to the final result and experimental study is not possible.
- It also has been established that generalized machine is a rich source of nonlinear phenomenon with Lorenz like dynamics.
- In the light of nonlinear phenomenon of generalized machine, the nonlinear phenomenon of conventional machines are predicted.
- The proposed method of studying nonlinear behavior of generalized machine have been applied to conventional machines and their nonlinear dynamics have been studied successfully. It confirms the applicability of the proposed method. It also establishes that the proposed method can be applied to study the nonlinear dynamics of all conventional machines.
- Comparison of the results obtained from simulation and experiments assures that the proposed method accurately describes the nonlinear phenomenon of generalized and conventional machines and it also validates the proposed method.

FURTHER WORKS

The study of Nonlinear Phenomenon of Generalized Machine is effectively done and same is effectively applied on some conventional machines. Still some works are outlined here which can be carried out in future.

1. Though the effectiveness of the proposed method has been studied for some conventional machines, some machines are yet to be tried out. Those machines, especially the single phase and commutator machines may be modeled using generalized theory and effectiveness of proposed method as used for generalized machine may be explored on those models.
2. The nonlinearity arising due to saturation is not considered in the present work. Saturation may also be considered for Generalized machine and conventional machines and nonlinear phenomenon may then further be studied.
3. Conventional machines with inherent nonlinearity are now being used with switching mode power supply with switching nonlinearity in closed loop. Then their operation is becoming more complex. Those machines may be considered as the hybrid systems with both inherent and switching nonlinearity. The nonlinear phenomenon for those machines may further be studied[86],[11]-[16].

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