CHAPTER - III

THEORETICAL CONSIDERATIONS

3.1 GENERAL

The flow situations in the present study is similar to that of a plane turbulent wall jet in shallow tailwater, as will be apparent from Fig. 3.1. In the figure, B_0 and U_0 denote respectively the sluice gate opening (jet thickness) and the velocity of the jet at the efflux section which is assumed to be uniform for the entire depth at that section and to be constant with respect to time. A typical velocity profile is shown in Fig. 3.2 from which two distinct features of the velocity distribution are evident ; firstly, an inner layer close to the boundary which is affected by the boundary friction and secondly , an outer region which is affected by the jet diffusion and the free surface of the flow. The vertical distance from the boundary , where the velocity attains the maximum value, U at a section located at any distance, x from the sluice gate is designated as the boundary layer thickness, δ and the region of the velocity profile beyond that height forms the unbounded jet.

3.2 BASIC ASSUMPTIONS

In the analysis of the problem of scour due to such a jet, the following considerations have been taken into account :

(i) The flow is two dimensional ;

(ii) The velocities are high and the flow pattern does not depend on Reynold's number ;

(iii) The nature of velocity profile at a particular section does not change with respect to time ;

(iv) The basic variable parameters which can be controlled are L , B_0, H, q, D and d_g ; the dependent variable is U_0 (dependent on H) and the independent variables are x and t . The variables may be defined as follows,

L = length of rigid apron measured from the jet efflux section ; H = head of water causing the flow through the sluice ; D = depth of tailwater measured from undisturbed bed level ; q = discharge per metre width of the flume ; d_g = typical grain diameter of the bed materials in millimeter and t = any duration of time in minutes.

In view of the complexities of the present problem as will be evident from the following chapters, it is obvious that a complete theoretical solution of the problem is not possible. As such an experimental investigation of the problem has been carried out to obtain empirical relationships for describing the flow characteristics and the scour phenomena. The analysis and discussion of the experimental results have been presented in chapter - V . Herein, the investigator has endeavoured to analyse the ideas behind the development of various empirical expressions in terms of flow parameter and bed material characteristics.

3.3 EQUATION FOR BOUNDARY SHEAR

As one of the main objectives of the present investigation is to develop a transport function and to form thereto an expression for computing the maximum scour depth, in the first instance, it is necessary to obtain relationship for evaluating shear stress due to such flow situation. It is well known that the shear stress is related to the velocity distribution law within the boundary layer. Accordingly, the writer has attempted to derive a velocity distribution law within the boundary layer ; and thereafter an expression for critical shear stress τ_{oc} at equilibrium condition proceeding from the boundary layer equation.

The steady state boundary layer equations for a two-dimensional turbulent flow are given by,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \varepsilon\frac{\partial^2 u}{\partial y^2} \qquad \text{.......(3.1)}$$

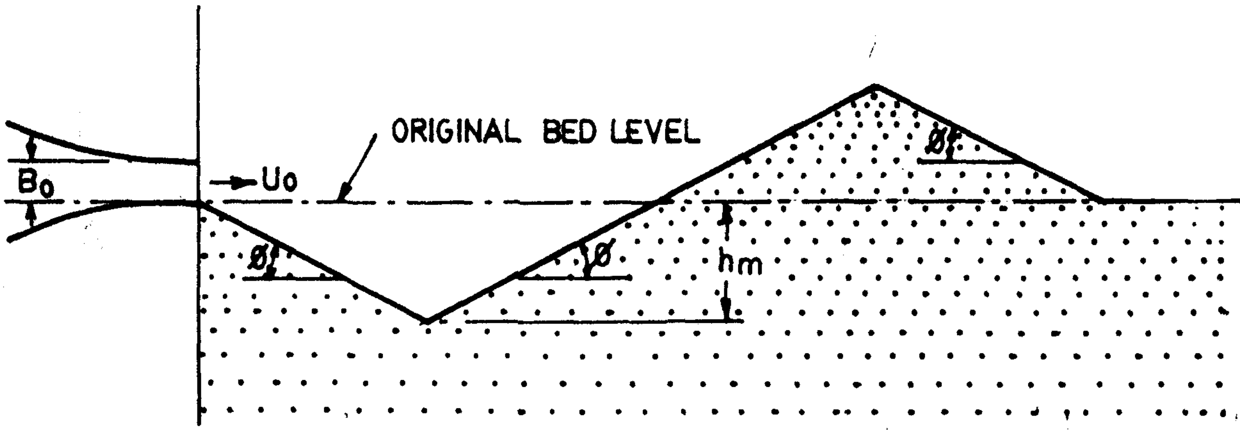

FIG. 2.1. A TYPICAL SCOUR PROFILE AFTER LAURSEN

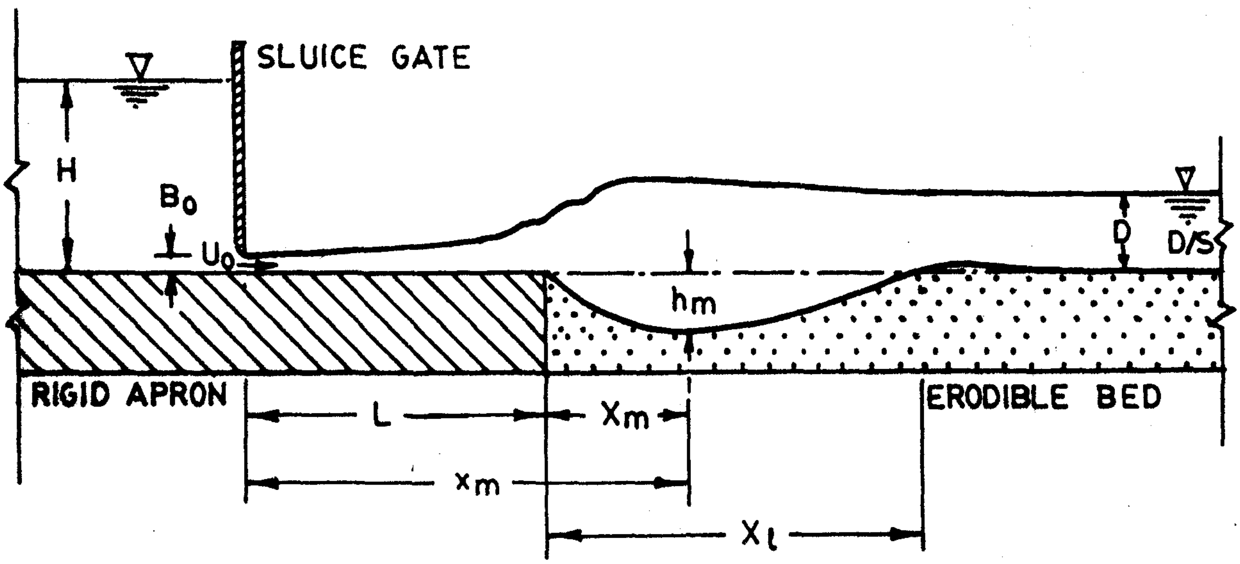

FIG. 3.1. SCHEMATIC DIAGRAM SHOWING THE DIFFUSED JET

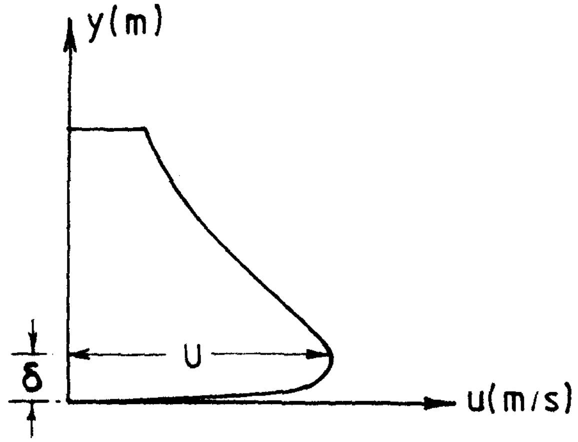

FIG.3.2. A TYPICAL VELOCITY PROFILE DUE TO A JET IN SHALLOW TAILWATER

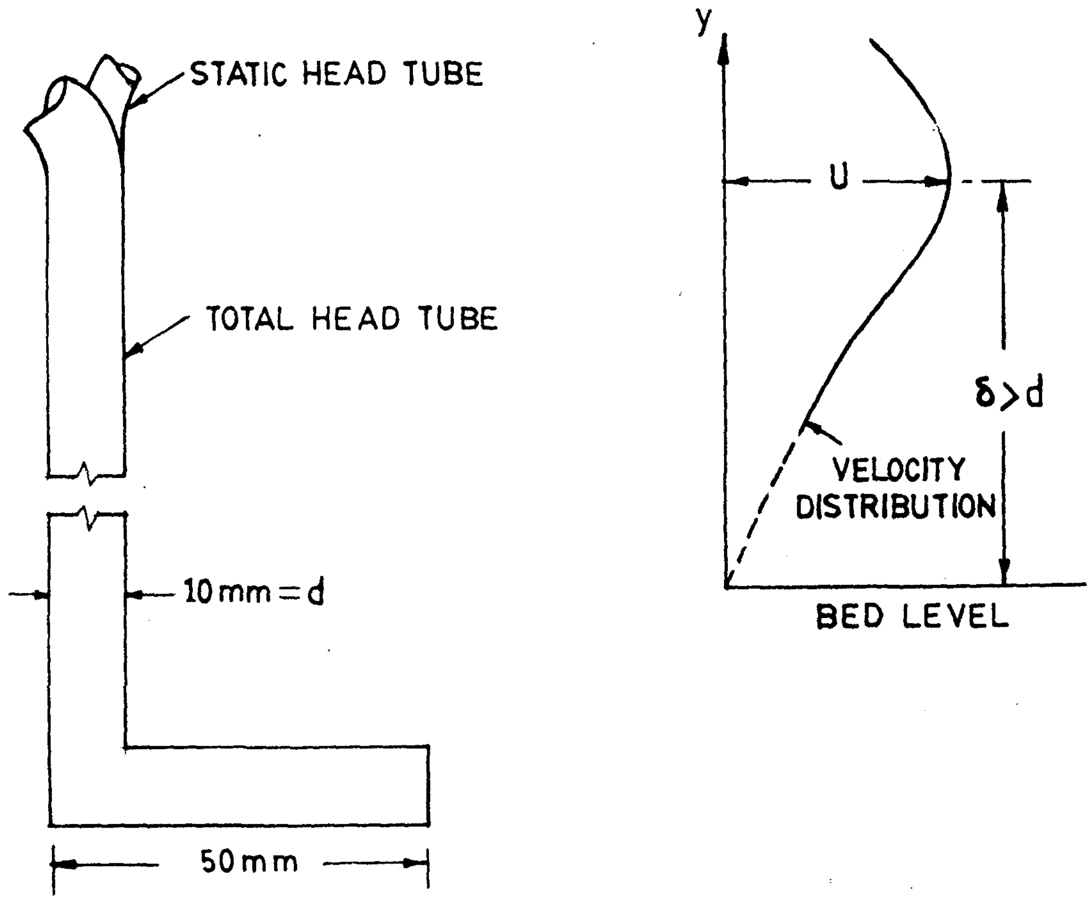

FIG. 3.3. A TYPICAL VELOCITY PROFILE AND A PRESTON TUBE FOR THE MEASUREMENT OF BOUNDARY SHEAR STRESS

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (3.2)$$

in which, u and v are the x and y component of velocity respectively, p is the intensity of pressure and ε is the kinematic eddy viscosity.

An exact solution of the above equations is very difficult. For application to engineering problems such as the present one , an approximate solution can be obtained by using the method based on integral theorem due to Vón Kármán. The basic concept of the method being that the solutions satisfy the differential equations only on the average. The steady state momentum integral equation derived from Eqs. 3.1 and 3.2 can be expressed as ,

$$\frac{d}{dx}\int_0^{\delta}u^2dy-U\frac{d}{dx}\int_0^{\delta}udy=-\frac{1}{\rho}\frac{dp}{dx}\delta-\frac{\tau_o}{\rho} \qquad (3.3)$$

substituting $-\frac{1}{\rho}\frac{dp}{dx}=U\frac{dU}{dx}$ and expressing in terms of the displacement thickness and the momentum thickness, Eq. 3.3 can be rewritten as follows,

$$\frac{d}{dx}\int_0^{\delta}u(U-u)dy+\frac{dU}{dx}\int_0^{\delta}(U-u)dy=\frac{\tau_o}{\rho} \qquad (3.4)$$

Expressing the Eq. 3.4 in terms of non-dimensional ratios of distance and velocity parameters viz. (y/δ) or η versus u/U or $f(\eta)$ we get,

$$\frac{d(U^2\delta)}{dx}\int_0^1\frac{u}{U}\left(1-\frac{u}{U}\right)d(y/\delta)+\frac{dU}{dx}\int_0^1U\delta\left(1-\frac{u}{U}\right)d(y/\delta)=\frac{\tau_o}{\rho} \qquad (3.5)$$

in which , u is the velocity in the flow direction at a particular location.
On simplification, Eq. 3.5 can be rewritten as,

$$\frac{d(U^2\delta)}{dx}I_1+U\delta\frac{dU}{dx}I_2=\frac{\tau_o}{\rho} \qquad (3.6)$$

in which, $I_1=\int_0^1[f(\eta)-f^2(\eta)]d\eta$ and $I_2=\int_0^1[1-f(\eta)]d\eta$

3.4 ANALYTICAL SOLUTION FOR THE EQUATION OF BOUNDARY SHEAR

To evaluate the shear stress by solving Eq. 3.6 one has to know the functional relationship governing various parameters such as diffusion characteristics of the jet, the growth of boundary layer thickness with distance and the velocity distribution law. These functional relationships have been derived from the experimental data in the form as described below.

For different experimental runs, velocity distribution profiles at various locations after the full development of scour hole have been shown in figures 5.2 – 5.5. From the figures it is apparent that the maximum velocity , U and the boundary layer thickness, δ vary from section to section. On the basis of the considerations as described earlier , the maximum velocity , U can be expressed as,

$$U=f_1(U_o\ ,\ B_o\ ,\ L\ ,\ d_g\ ,\ x) \qquad (3.7)$$

Following the procedure of Albertson et.al., at first , the diffusion characteristics of the jet has been studied through the plot of (U / U_o) versus (x / B_o) for various values of U_o, B_o, L and d_g. Then, through the study of the plotted curves the equation for estimating the maximum velocity U has been derived.

The growth of boundary layer thickness , δ with distance, x has been investigated through the non-dimensional plot of the experimental values of δ/D anainst x/L for various values of U_o, B_o, L and d_g. It has been observed that its growth is dependent on the distance x, L D and d_g which means , δ can be expressed as

$$\delta = f_2 (x , L , D , d_g) \qquad (3.8)$$

Coming to the velocity distribution law along the vertical , it can be observed from Figures 5.2 to 5.5 that on reaching the erodible bed, the velocity distribution suffers radical changes. Of particular interest to the investigator is the velocity profile at the location of maximum scour. Accordingly, the non-dimensional plot of the velocity distribution within the boundary layer has been made and it has been expressed as,

$$\frac{u}{U} = f(\eta) \qquad (3.9)$$

The expression for shear stress can now be obtained from Eq. 3.6 by substituting the values of $f(\eta)$, δ and U in terms of Eqs. 3.9, 3.8 and 3.7 respectively and then integrating within the boundary layer. Similarly, the expression for u can be obtained from Eq. 3.9 by substituting the values of U and δ in terms of Eqs. 3.7 and 3.8 respectively. Accordingly, the critical shear stress, τ_{oc} and u can be expressed as,

$$\tau_{oc} = f_3 (U_o, B_o, L, D, d_g, x) \qquad (3.10)$$

$$u = f_4 (U_o, B_o, L, D, d_g, x, y) \qquad (3.11)$$

3.5 TIME VARIATION OF BOUNDARY SHEAR STRESS

3.5.1 VELOCITY DISTRIBUTION LAW

The time variation of boundary shear stress, τ_t at the location of maximum scour has been evaluated from the dynamic pressure drop recorded by a Preston (1954) tube. The pressure drop Δp at various time intervals has been evaluated by integrating the velocity distribution law over the cross-sectional area of the Preston tube at that location . To evaluate τ_t in terms of Δp it is necessary to express u in terms of shear velocity, $u_* = \sqrt{\tau_o / \rho}$. This has been done in the following way.

From Eq. 3.9 u can be expressed as

$$u = U f(\eta) \qquad (3.12)$$

Since $f(\eta)$ is known, the integral functions I_1 and I_2 can also be evaluated. Thereafter , on substituting their values in Eq. 3.6 , U can be expresed as

$$U = \frac{\tau_o / \rho}{I_1 \, U \frac{d\delta}{dx} + C_1 \, \delta \frac{dU}{dx}} \qquad (3.13)$$

in which , $C_1 = (2I_1 + I_2)$.

Accordingly, from Eqs. 3.12, 3.13 , 3.7 and 3.8, it may be observed that u can as well be expressed in the following functional form,

$$u = f_5\,(\tau_o/\rho,\ U_o,\ B_o,\ L,\ D,\ d_g,\ x,\ y) \qquad \text{.......(3.14)}$$

3.5.2 PRESSURE-SHEAR RELATIONSHIP FOR PRESTON TUBE

The dynamic pressure drop, Δp measured by a Preston tube (Fig. 3.3) is related to the velocity distribution law in accordance with the following relationship,

$$\Delta p = \frac{1}{2A}\int_A \rho u^2 dA \qquad \text{.......(3.15)}$$

in which, A = Cross sectional area of the Preston tube = $\pi(r-t_p)^2$

r = external radius of Preston tube,

t_p = Wall thickness of Preston tube.

Now, $dA = 2\left[(r-t_p)^2 - (r-y)^2\right]^{0.5} dy$ and therefore, the expression for pressure drop becomes

$$\Delta p = \frac{\rho}{\pi(r-t_p)^2}\int_{t_p}^{(2r-t_p)}\left[u^2\left(t_p^2 - 2rt_p + 2ry - y^2\right)^{0.5}\right] dy \qquad \text{.......(3.16)}$$

Substituting the values of u in terms of Eq.3.14 into Eq. 3.16 and on evaluating the integral , τ_t can be found out and expressed as the following functional form,

$$\tau_t = f_6\,(\Delta p,\ \rho,\ U_o,\ B_o,\ D,\ L,\ d_g,\ t_p,\ r,\ x) \qquad \text{.......(3.17)}$$

The integral vide Eq. 3.16 has been evaluated by applying simpson's rule after substituting the numerical values of r and t_p. The empirical relationship vide Eq. (3.17) , on the other hand , has been formulated from experimental data.

3.6 FORMULATION OF LOCAL SCOUR AND SEDIMENT TRANSPORT FUNCTION

With the knowledge of shear distribution during the process of scour, the sediment transport function can be expressed in terms of the excess shear stress over the critical shear stress i. e., in terms of stage, $(u_*/u_{*o} - 1)$, u_{*o} being critical shear velocity which equals to $(\tau_{oc}/\rho)^{1/2}$. Converting the shear stress at any instant of time to the shear velocity, u_* , the transport stage at any instant of time can be evaluated , which, afterwards, has been used to correlate the weight rate of transport , i, with the fluid power of the jet causing scour, w.

The equation of continuity for sediment transport in two dimensional flow in a rectangular channel with the positive x-axis taken in the flow direction along the bed can be written as ,

$$\frac{\partial z}{\partial t} + \frac{1}{K}\frac{\partial i}{\partial x} = 0 \qquad \text{.......(3.18)}$$

in which, z = river-bed elevation measured from a datum plane.

K = bulk unit weight of the bed material.

In order to predict the scoured bed profile from the solution of the above equation, the relationships governing the flow characteristics and the sediment load are necessary. It has already been mentioned that the flow characteristics and the mechanics of sediment transport in the present flow situation is very complex and thereby precluding direct solution of the Eq. 3.18 . Accordingly, the writer proposes to develop empirical relationships to describe the scour hole geometry and sediment transport function based on the experimental results . In the following paragraphs the various parameters involved in the scour phenomena are described in the functional form which are subsequently correlated empirically.

With the reference to Fig. 3.1 , the most significant parameters to describe the scoured bed profiles are,

(i) X_m, the distance of the location of maximum scour from the end of rigid apron at any time ;

(ii) h_m , the maximum depth of scour at any time ;

(iii) X_ℓ , the length of the scour hole at any time.

obviously, all the three parameters at any instant of time, must be functions of scour volume , V_s.
The scour volume , on the other hand , will be a function of time, t corresponding to the given flow conditions which are governed by the parameters i. e., $B_o, U_o, L, D,$ and d_g. Hence, V_s can be expresed in the functional form as,

$$V_s = f_7 (U_o, B_o, L, D, d_g, t) \qquad (3.19)$$

Accordingly, the parameters X_m, h_m and X_ℓ can be expressed in the functional form as follows,

$$X_m = f_8 (U_o, B_o, L, D, d_g, t) \qquad (3.20)$$

$$h_m = f_9 (U_o, B_o, L, D, d_g, t) \qquad (3.21)$$

$$X_\ell = f_{10} (U_o, B_o, L, D, d_g, t) \qquad (3.22)$$

Since , the scour profiles are similar in nature (as will be discussed in chapter-V) with respect to time for the given bed material, the scour volume, V_s at any time, t can as well be expressed in terms of the ratio t/T, T being the equilibrium time of scour , as follows ,

$$V_s = f_{11} (U_o, B_o, L, d_g, t/T) \qquad (3.23)$$

The equilibrium time of scour, T can again be separately expressed as a function of U_o, L and d_g, i. e.,

$$T = f_{12} (U_o, L, d_g) \qquad (3.24)$$

From the functional relationship of V_s, the volume rate of sediment transport at any instant of time , q_s can be evaluated from the relationship,

$$q_s = \frac{dv_s}{dt} \qquad (3.25)$$

and the corresponding weight flow rate, i from the expression

$$i = (1-n)(S_s - 1)\gamma q_s \quad \text{.......(3.26)}$$

in which, γ = specific weight of water

n = porosity of bed material.

Following the procedure similar to Bagnold (1973), the transport function can be described by correlating the weight flow rate with the fluid power of the jet, $w = \gamma q H / B_o$ and the transport stage, $\left(\frac{u_*}{u_{*o}} - 1\right)$.