CHAPTER - II

REVIEW OF LITERATURE

2.1 GENERAL

To the best of the writers knowledge, the flow characteristics and local scour caused by a horizontal water jet issuing from a sluice with shallow tailwater and flowing over a rigid apron, then on to an erodible bed have not yet been investigated. However, a large number of studies concerning general flow characteristics and local scour caused by water jets have been carried out by many investigators. A brief review of the relevant works is presented below.

2.2 SCOUR DUE TO VERTICAL JET

Rouse (1939) conducted systematic experiments upon the rate of scour of a bed composed of cohesionless sediment by a vertical jet of clear water. In that study he noted the variation of scour profile with time and the results have been plotted in terms of (h_m/s) and $(\log\ wt/s)$ for different (U_0/w) values. He then observed it to be a straight line which means that the time development of scour hole follows a logarithmic law, where

- s = distance of erodible bed from jet efflux section,
- h_m = maximum scour depth at any instant of time,
- w = geometric mean fall velocity of the sediment,
- U_o = efflux velocity of jet,
- t = duration of time.

The study also indicated that,

(i) the depth of scour in uniform material is dependent solely upon the size and velocity of the jet, the fall velocity of the sediment, and the duration of scouring action;

(ii) the relative rate of scour produced by a given jet at a given stage depends only upon the ratio of jet velocity to fall velocity, the rate being near to zero as the ratio approaches unity,

(iii) selective sorting of graded material occurs, so that with a wide variation in size of bed material the bottom of the hole gradually becomes paved with a progressively coarser material, thus decreasing the effective fall velocity of the sediment and reducing the rate of scour;

(iv) if the sediment is carried in by the jet exactly at the rate at which a clear jet would scour, the scour hole will be stabilised; that means an increase in the rate of sediment inflow would cause a reversal in the scour process (i.e., deposition) and hance gradual filling of the hole until equilibrium is again established.

He recommended that these facts are directly applicable either to model experiments for prototype structures or to the design and maintenance of the structures themselves.

Dodiah, Albertson and Thomas (1953) reported the experiments on scour due to vertical solid and hollow jets and scour below free overfall. The erodible bed was composed of narrow size range gravel covered by a pool of water having various depths.

In analysing the problem of scour by vertical solid and hollow jets the authors assumed the general relationship expressing the phenomenon to be dependent on the following variables,

$$\phi_1 = f(b,\ U_o,\ a,\ t,\ \rho,\ \rho_s,\ w,\ \sigma_w,\ h_m) = 0 \qquad (2.1)$$

where

- b = depth of pool above the original bed level,
- a = area of the jet
- ρ = density of water,
- ρ_s = density of the sediment,
- σ_w = standard deviation of the fall velocity.

The above expression can as well be expressed as a function of the following dimensionless parameters

$$\phi_2 = f(b/\sqrt{a},\ U_o/w,\ wt/\sqrt{a},\ \rho_s/\rho,\ \sigma_w,\ h_m/\sqrt{a}) = 0 \quad \text{.....(2.2)}$$

on further simplification, considering density ratio and standard deviation to be constant the equation becomes

$$\frac{h_m}{b} = \phi_3(b/\sqrt{a},\quad U_o/w,\quad wt/b) = 0 \quad \text{.....(2.3)}$$

Based on experimental data the functional relationship of the above equation has been formulated and these are, for solid jet

$$\frac{h_m}{b} = \frac{0.023\sqrt{a}}{b}\log\left[\frac{wt}{b}\right]^{\left(\frac{U_o}{w}-1\right)} - 0.022\frac{b}{\sqrt{a}} + 0.4 \quad \text{.....(2.4)}$$

and for hollow jet

$$\frac{h_m}{b} = \frac{0.023\sqrt{a}}{b}\log\left[\frac{wt}{b}\right]^{\left(\frac{U_o}{w}-1\right)} - 0.032\frac{b}{\sqrt{a}} + 0.5 \quad \text{.....(2.5)}$$

Consideration of these equations shows that the scour from a solid jet is nearly the same as that with a hollow jet, the minor difference being due to the slight variation of the coefficients. The conclusions of these findings are as follows:

(i) Scour is directly proportional to the geometric progression of time. In other words, the state of equilibrium of the process of scour connot be expected either at any depth or after any period of time.

(ii) The magnitude of scour decreases with a decrease in the ratio of jet velocity to the fall velocity, approaching zero as this ratio approaches unity.

(iii) Scour increases with an increase in the depth of water over the erodible bed until the depth reaches a critical value. Any further increase in depth will diminish the resulting scour.

(iv) For a given area and velocity of jet the scour resulting from a hollow jet as compared with a solid jet appears to indicate identical trend.

Coming to the study of scour at the base of a free overfall Dodiah et al (1953) assumed the following functional relationship governing scour depth by dimensional analysis,

$$\frac{h_m}{b} = \phi_4(H/b,\quad q/Hw,\quad qt/H^2,\quad \sigma_w) \quad \text{.....(2.6)}$$

Where, H = height of fall from the bed level upstream to the bed level downstream,
q = discharge per unit width of the crest.

In order to determine the relationship between the variables expressed in Eq. 2.6, experimental arrangement was made for a sudden drop in elevation from one horizontal to a lower horizontal one. The depth of flow over the edge of the crest was controlled only by the discharge per unit length of the crest and the tailwater depth was varied. On the basis of the experimental results h_m versus t curves were plotted for various values of H, q and b and by the process of curve fitting following empirical equations were developed for the narrow and wider size-range materials respectively.

$$\frac{h_m}{b} = [0.29 + 0.07\ \log\ qt/H^2](q/Hw)^{\frac{1}{2}}[H/b]^{3(q/Hw)^{\frac{1}{3}}} \quad \text{.....(2.7)}$$

$$\frac{h_m}{b} = [0.49 + 0.04\ \log\ qt/H^2](q/Hw)^{\frac{2}{3}}[H/b]^{2(q/Hw)^{\frac{1}{6}}} \quad \text{.....(2.8)}$$

The salient features of this investigation are as follows :

(i) The depth of scour continues to increase with a geometric progression of time.

(ii) An increase in discharge causes a greater increase in depth of scour than is caused by a corresponding increase in drop height or change in depth of tailwater.

(iii) A critical depth of tailwater is reached at which either an increase or a decrease in tailwater causes a decrease in scour depth.

(iv) A 50% decrease in deviation of size distribution resulted in a 50% increase in depth of scour when $(qt / H^2) = 3 \times 10^5$

(v) The experimental data compare well with the equation of Schoklitsch (1937) for the smaller depths of scour. For the greater depths of scour, however, the equation of Schoklitsch predicts a depth of scour only half of that which actually occured.

Altinbilek and Okyay (1973) carried out experimental study of localised scour on a horizontal sand bed under vertical jet, based on the application of continuity principle to scour hole and adopting the control volume approach. The approach is to formulate the rate of change of the scour hole volume and the volume rate of transport separately and then to integrate the differential equation in order to obtain scour depth as a function of time. Model tests were then carried out to determine the development and the geometry of the scour hole and also to obtain a functional relationship of the equilibrium depth. Further, dimensionless scour depth versus time charts were prepared and the measured scour depth was compared with the predicted ones for a number of model tests.

Westrich and Kobus (1973) investigated the erosion of a uniform sand bed by continuous and pulsating jets. Their investigation has indicated that the momentum flux of the jet and the distance between nozzle and sediment bed determines the rate of scour. Under fully turbulent conditions, the relative dimensions of the scour hole depend only upon the time parameter and the ratio between the fictitious axial jet velocity at the bed and fall velocity of the sediment particles. Two distinct values of ratio exist at which the eroding action of the jet becomes particularly effective. The study further indicates that at the same mean volume flux, the erosion rate can be more than doubled by pulsations of the jet velocity. For the relative distance between nozzle and bed investigated, the continuous jet has the least erosion capacity as compared to pulsating jets.

Uyumaz (1988) investigated the scour phenomena on noncohesive soils below a vertical gate considering simultaneous flow over and under the gate. In his experiments two different noncohesive soils were used as the bed material. On the basis of experimental data he developed a graphical relation for the dimensionless scour hole geometry by plotting Y / L_h against X / L_h, in which, Y is the depth of scour below initial bed level, X is the horizontal distance from gate and L_h is the distance of peak of dune of the scour hole from the gate. The geometry of the scour hole was expressed as a function of q_o / q_u, in which, q_o =discharge per unit length over the gate and q_u =discharge per unit length under the gate. Similar type of dimensionless scour geometry for flow only under or only over the gate was also developed by Erkek and Uyumaz (1976), and by Uyumaz (1985). In order to develop an expression for the maximum scour depth, Uyumaz (1988) plotted the values of maximum scour depth against the discharge per unit width. He observed that in the case of discharge over the gate the depth of scour was greater than that in the case of discharge under the gat and that in the case of simultaneous flow over and under the gate, the scour was smaller compared to the two former cases. Ultimately, the following expression for maximum scour depth was developed,

$$d_o + h_1 = w \frac{h^{0.5}\, q^{0.6}}{d_{90}^{0.4}} \qquad (2.9)$$

in which,
- d_o = scour depth in metre,
- h_1 = downstream water depth in metre,
- h = head of water in metre,
- q = discharge per unit length in $m^3/s/m$,
- d_{90} = grain diameter corresponding to 90% finer
- w = a coefficient depending on the ratio q_o / q_u

2.3 SCOUR DUE TO HORIZONTAL JET

Laursen (1952) carried out experiments to study the scour of a flat sand bed due to two dimensional horizontal jet of water. He noted that the development of the scour hole depth, h with elapsed time, t and the scour hole geometry remains constant with time. Considering the scour hole area as an equilateral triangle with the initial bed level as the base and the inclination ϕ of the two equal sides being equal to the angle of repose of the bed material in submerged condition, the maximum depth of scour hole, h_m at any instant of time, t, was found as the depth of the triangle (Fig.2.1). The experiments were carried out with three different sizes of quartz sand using a constant value of jet velocity, U_0 for each run.

The study revealed very interesting information on the nature of scour and these are summarised as follows:

(i) The rate of scour is equal to the difference between the capacity for transport out of the scoured area and the rate of supply to the area.

(ii) The rate of scouring decreases as the flow section is enlarged by erosion.

(iii) A limiting value of scour for given initial condition exists which is reached theoretically at infinitely long time.

Mathematically, the above particulars were expressed in the following form,

$$f(B) = \phi_5[\psi(B), \ \psi(h), \ t] \qquad (2.10)$$

in which,

$f(B)$ = mathematical expression of boundary,

$\psi(B)$ = sediment transport rate out of the scour zone as a function of boundary shape and position,

$\psi(h)$ = rate of supply of sediment to the scoured zone.

Tarapore (1956) carried out an experimental investigation to study the scour of sand bed by a horizontal jet. He used velocity and the size of the jet as the parametrs to describe the flow pattern which, for high velocity and submerged jet condition, was considered to be independent of Reynold's and Froude number influence. To describe the characteristics of the sediment, he used the parameter "Critical tractive force".

With the above assumptions, Laursen's equation was rewritten in the following form,

$$f_1(B) = \phi_6\left[\frac{V}{\sqrt{\tau_{oc}/\rho}}, \frac{Vt}{B_o}\right] \qquad (2.11)$$

in which, $f_1(B)$ = dimensionless function of bed configuration

V = a typical velocity

τ_{oc} = Critical shear stress for the sediment composing the bed.

This study indicate that greater the $(V/\sqrt{\tau_{oc}/\rho})$ ratio, the greater is the rate of scour and finally a limiting extent of scour is reached. Due to the enlargement of scoured section, the Velocity along the boundary decreases to a point at which the boundary shear stress equals the critical shear stress of the sediment composing the bed when no further sediment is transported.

To demonstrate that a state of equilibrium in the evolution of scour can be reached after sufficiently long time, the author carried out experimental run lasting upto 23 days and based on this investigation he developed an empirical relation connecting maximum scour depth with time and expressed in the following form,

$$h_m = m + \log t^n \qquad (2.12)$$

in which , m and n are coefficients.

Rajaratnam and Berry (1977) studied the erosion by circular turbulent wall jets issuing into a stagnant ambient fluid. They conducted experiments on erosion of loose beds of sand and polystyrene by submerged jets of air and water. In the case of submerged jet of air the erodible bed was of sand as well as polystyrene and in case of submerged jet of water the bed was of sand only. They found that the maximum depth of erosion in terms of jet diameter at the nozzle is mainly a function of the densimetric Froude number and this functional relation is essentially the same for air-sand, air-polysyrene and water-sand system. They also studied the evolution of scour profiles and found that the end state is reached in an asymptotic manner. They developed the graphical relations for the scour profiles and the maximum depth of scour in non-dimensional form.

Rajaratnam (1980) investigated the erosion of a loose non-cohesive bed caused by submerged circular turbulent wall jets issuing into a cross flow. In that study he used only a single jet diameter and used three specimen of sands of different median sizes and one polystrene as the bed materials. The experiments were conducted on air-sand, air-polystyrene and water-sand systems by using air jet as well as water jet. He studied the evolution of scour hole both in the direction of axis of scour hole and in the direction of width of the scour hole and observed that in each direction, the scour profiles were similar in nature. It was also observed that at the ultimate stage, the scour profiles were in asymptotic state. He developed the graphical relation for asymptotic scour profile in non-dimensional form and expressed the maximum depth of scour as

$$\frac{\epsilon_{m\infty}}{d} = (F_o,\ \alpha) \quad (2.13)$$

in which, $\epsilon_{m\infty}$ = asymptotic value of maximum depth of erosion below original bed level,

d = diameter of the jet at the nozzle,

F_o = densimetric Froude number

α = ratio of jet to free stream Velocity of cross flow.

Apart from this he developed the graphical relation for the maximum axial length of scour and the maximum height of the ridge above original bed level and observed that the values of these parameters depend on the diameter of the jet at the nozzle and the densimetric Froude number.

Rajatanam (1981) investigated the erosion of sand beds caused by plane turbulent wall jets of water when the tailwater depth was much larger than the thickness of the jet (deep jet). He determined the maximum asymptotic depth of erosion, $\epsilon_{m\infty}$, and the distance of maximum scour point from the nozzle (or gate producing the jet), $x_{m\infty}$. The characteristics of the ridge that forms at the downstream end of scour hole was also studied and was observed that the values of $\epsilon_{m\infty}$ and $x_{m\infty}$, and the characteristics of the ridge were functions mainly of the densimetric Froude number,

$$F_o = \frac{U_o}{(gD\Delta\rho)^{1/2}}$$

in which, U_o = Velocity of the jet

g = acceleration due to gravity

D = mean size of sand

$\Delta\rho$ = the difference between the mass density of the sand, ρ_s and that of the fluid, ρ

Hassan and Narayanan (1985) investigated the flow characteristics and the similarity of scour profiles downstream from a rigid apron due to a water jet issuing from a sluice. The wall jet developed on the rigid apron was allowed to encounter a bed of cohesionless sand which was fully submerged by tailwater. They measured the depths of scour at different times and locations. The mean velocity profiles were measured along the rigid apron and in the scour hole. On the basis of experimental data they developed a graphical relation for mean Velocity distribution along rigid apron. An empirical expression for the decay of maximum mean velocity along rigid apron was developed in the following form,

$$\frac{U_m}{U_1} = 3.83\left(\frac{x_L}{y_1}\right)^{-0.5} \quad (2.14)$$

in which, U_m = maximum mean Velocity,

U_1 = average velocity at the vena Contracta,

x_L = streamwise distance from sluice

y_1 = thickness of jet at vena Contracta.

From the velocity profiles in the scour hole, they developed the following empirical relation for the mean velocity in the outer layer,

$$\frac{U}{U_m} = e^{-0.693(y'/\delta_1)^{1.5}} \quad (2.15)$$

in which, U = mean velocity

y' = height from the bed level at which the mean velocity equals to U_m

δ_1 = height of the level at which the mean velocity equals to 0.5 U_m from the level at which the mean velocity is U_m.

They observed that the self-similarity of mean velocity profiles did not exist in the inner layer. However, they developed the following empirical relations for the decay of maximum mean velocity,

$$\frac{U_m}{U_{mA}} = e^{-0.138\left(\frac{x}{\delta_{1A}}\right)^{1.055}} \quad , \text{ for } 0 < \frac{x}{\delta_{1A}} < 4.28 \qquad \text{(2.16)}$$

$$\frac{U_m}{U_{mA}} = 0.61 \left(\frac{x}{\delta_{1A}}\right)^{-0.1} , \text{ for } \frac{x}{\delta_{1A}} \geq 4.28 \qquad \text{(2.17)}$$

in which, U_{mA} = maximum mean velocity at the end of rigid apron.

x = streamwise distance from the end of rigid apron

δ_{1A} = length scale for outer layer at the end of rigid apron.

To predict the maximum scour depth with time they proposed the following relation,

$$\frac{1}{(n+1)} \cdot \frac{d[h^{n+1}]}{dt} = \beta_1 \left[\frac{U_m^2}{g(s-1)}\right]^n U_m \left[\frac{U_m^2}{g(s-1)D}\right] \qquad \text{(2.18)}$$

in which, h = maximum scour depth,

β_1 = a dimensionless coefficient,

n = a constant

s = relative density of sediment,

D = sediment size,

t = time.

The values of β_1 and n were determined from the experimental results concerning the depth of scour versus time.

Ali and Lim (1986) studied the local scour caused by submerged wall jets. They investigated the scour produced by two-dimensional and three-dimensional horizontal turbulent jets submerged by tailwater. Two separate sets of experimental arrangements were made for the study; in one set-up the water jet was produced by a pipe with rectangular cross-section and in the other set-up the jet was formed by a sluice gate. The measurements were taken for the maximum scour depth, d_m, centre line scoured bed profiles and time evolution of these scour characteristics. They developed the expressions for the volume of scour, V_s and the maximum depth of scour, d_m at any time, t in the following forms,

$$\frac{V_s}{R^3} = A_1 \left(\frac{U_o t}{R}\right)^{B_1} \qquad \text{(2.19)}$$

$$\frac{d_m}{R} = A_2 \left(\frac{U_o t}{R}\right)^{B_2} \qquad \text{(2.20)}$$

in which, R = ratio of area of jet to its perimeter,

U_o = mean Velocity of jet

t = time required to scour corresponding volume of sediment

A_1, A_2, B_1 and B_2 are coefficients whose values depend on the ratio B / b_o or H / d_o ; on an average,

$$A_1 = 1.52(\frac{b_o}{B})^{0.83}, \text{ and } B_1 = 0.01\frac{B}{b_o} + 0.59$$

in which, B = inlet width of a two-dimensional channel,

b_o = width of rectangular jet,

d_o = sluice gate opening or diameter of jet,

H = tailwater depth.

For the average values of H/d_o,

$$\frac{d_m}{R} = 0.9\left(\frac{U_o t}{R}\right)^{0.33} \quad \text{.....(2.21)}$$

However, V_s and d_m were correlated as,

$$\frac{V_s}{R^3} = 187.72\left(\frac{d_m}{R}\right)^{2.28} \quad \text{.....(2.22)}$$

for two-dimensional jets , and

$$\frac{V_s}{R^3} = 49.36\left(\frac{d_m}{R}\right)^{1.89} \quad \text{.....(2.23)}$$

for three-dimensional jets.

The authors expressed the limiting depth of scour, $d_{m\infty}$ by the following relationship,

$$\frac{d_{m\infty}}{L} = 2.3\left(\frac{U_o}{w}\right)^{1/2}\left(\frac{d_{50}}{L}\right)^{3/8}F_o^{\,3/4} - 1.19 \quad \text{.....(2.24)}$$

The length sclae L was defined as,

(a) L = d_o for a deeply submerged two-dimensional jet issuing from a well rounded nozzle ($d_o = b$) and with $H/d_o > 16.0$

(b) L = b for a deeply submerged two-dimensional jet issuing from a sharp sluice gate (b is the depth of water at the vena contracta)

(c) L = R for two-dimensional and three-dimensional jets with a minimum (or shallow) tailwater depth $(H = d_o)$

The authors expressed the geometry of scour profile by the relation,

$$\frac{x_m}{R} = a_1\left(\frac{U_o t}{R}\right)^{b_1} \quad \text{.....(2.25)}$$

the variations in a_1 and b_1 being given by,

$$a_1 = 1.55\left(B/b_o\right)^{0.74} \text{ and } b_1 = 0.33\left(b_o/B\right)^{0.59}$$

They observed the similarity of velocity distributions in the scour hole and developed the following expression,

$$\frac{u}{u_m} = \exp\left[-0.693\left(\frac{y-\delta_1}{y_m-\delta_1}\right)^2\right] \quad \text{.....(2.26)}$$

in which, u = horizontal component of mean velocity,

u_m = maximum velocity at a section,

y = normal distance from the boundary,

y_m = normal distance from the axis to the point where $u = 0.5u_m$,

δ_1 = surface layer thickness of velocity profile.

The authors also computed the boundary shear stresses, τ_b along the scoured bed profiles at t = 15 mins and t = 90 mins, using the relations proposed by Melvika and Raudkivi (1977) and by Ludwieg and Tillmann (1970).

Chatterjee, Ghosh and Chatterjee (1994) investigated the local scour and sediment transport due to a submerged horizontal jet issuing from a sluice and flowing over a rigid apron, then on to an erodible bed. Experiments were performed on two erodible beds ; one consisting of sand ($d_{50} = 0.76$mm) and the other of gravel ($d_{50} = 4.3$mm). In the case of sand bed, the ratio D / B_o varies from 5.86 to 14.65 and in the case of gravel bed D / B_o varies from 6.04 to 15.10, D and B_o being the mean flow depth in the undisturbed zone and the sluice gate opening, respectively. For each type of bed material, experimental runs were undertaken for differnt gate openings and for various discharges. Two identical runs were performed in each test. In the first run, measurements were taken of the scour profiles at various instants of time, as well as at the equilibrium stage, the time required to reach the equilibrium stage, the velocity distribution at equilibrium stage at several locations along the central plane of the jet, the downstream flow depth in the undisturbed zone and the discharge. In the second run (identical to first one), the dynamic pressure in a Preston tube at the location of maximum scour at various instants of time during the development of scour hole were measured. The equilibrium stage was assumed to be reached when no grain movement was observed at the location of maximum scour. The authors analyzed the experimental data and developed various empirical relationships governing the scouring process.

The time required to reach equilibrium stage, T, applicable to both sand and gravel beds was obtained as

$$T = e^{4.67 d_g^{0.172} U_o^{-0.667 d_g^{-0.014}}} \quad \text{.........(2.27)}$$

in which, e = naperian base,

$d_g = d_{50}$ = grain diameter corresponding to 50% finer,

U_o = efflux velocity of jet.

The volume of scour , V_s , at any time, t, was expressed as

$$V_s = 0.474 U_o^2 B_o (t/T)^{0.456} \quad \text{.........(2.28)}$$

$$V_s = 0.374 U_o^2 B_o (t/T)^{0.343} \quad \text{.........(2.29)}$$

for the sand and the gravel beds respectively. Replacing T in terms of Eq. 2.27 and incorporating the effect of grain diameter, V_s was finally expressed as

$$V_s = 0.456 U_o^2 B_o d_g^{-0.136} \left(e^{-2.031 d_g^{0.008} . U_o^{-0.667 d_g^{-0.014}}}\right) t^{0.435 d_g^{-0.164}} \quad \text{.........(2.30)}$$

The distance of maximum scour point, X_M and the distance of the peak of dune, X_D from the end of rigid apron were expressed as

$$X_M = 0.6 V_s^{0.374} \quad \text{.........(2.31)}$$

$$X_D = 2.684 V_s^{0.45} \quad \text{.........(2.32)}$$

Replacing V_s in terms of Eq 2.30, the following expressions for X_M and X_D were obtained,

$$X_M = 0.447\ U_o^{0.748}\ B_o^{0.374}\ d_g^{-0.051} \left(e^{-0.759 d_g^{0.008} U_o^{-0.667 d_g^{-0.014}}}\right) t^{0.163 d_g^{-0.164}} \quad \text{.........(2.33)}$$

$$X_D = 1.885\ U_o^{0.9}\ B_o^{0.45}\ d_g^{-0.061} \left(e^{-0.914 d_g^{0.008} U_o^{-0.667 d_g^{-0.014}}}\right) t^{0.196 d_g^{-0.164}} \quad \text{.........(2.34)}$$

To study the similarity of scour profiles, the authors plotted h / X_D against X / X_D for the sand and gravel beds separately ; h being the depth of scour at a distance X from the end of the rigid apron. It was observed that the scour profiles were similar in nature and independent of time but dependent on the grain size characteristics of the bed.

To develop an expression for the maximum scour depth, h_m at any time, t , the authors first correlated h_m with V_s as,

$$h_m = 0.379 V_s^{0.496} \qquad \text{.......... (2.35)}$$

$$h_m = 0.513 V_s^{0.549} \qquad \text{.......... (2.36)}$$

for the sand and gravel beds, respectively. Substituting the values of V_s in terms of Eq. 2.30 and then incorporating the effect of grain diameter, h_m was finally expressed in terms of t as,

$$h_m = 0.267 d_g^{0.233} \left[U_o^2 \, B_o \, d_g^{-0.136} \left(e^{-2.031 d_g^{0.008} U_o^{-0.667 d_g^{-0.014}}} \right) \right]^{0.504 d_g^{0.058}} . t^{0.219 d_g^{-0.106}} \qquad \text{......... (2.37)}$$

Similarity criterion for the scour pattern was developed by correlating h_m with jet characteristics and fall velocity of bed materials. They observed that the development of scour depth was faster at the initial stage compared to the later stages. The expressions for h_m at the initial stage were given by,

$$\frac{h_m}{L} = 0.00385 \left[\log \left\{ \left(\frac{W_m t}{L} \right) \left(\frac{W_m L}{U_o B_o} \right)^5 \right\}^{(U_o/W_m)-1} - \frac{L}{B_o} \right] + 0.0184 \qquad \text{.......... (2.38)}$$

for sand beds and

$$\frac{h_m}{L} = 0.04 \left[\log \left\{ \left(\frac{W_m t}{L} \right) \left(\frac{W_m L}{U_o B_o} \right)^5 \right\}^{(U_o/W_m)-1} - \frac{L}{B_o} \right] + 0.338 \qquad \text{.......... (2.39)}$$

for gravel beds. Those at the later stages are given by,

$$\frac{h_m}{L} = 0.00205 \left[\log \left\{ \left(\frac{W_m t}{L} \right) \left(\frac{W_m L}{U_o B_o} \right)^5 \right\}^{(U_o/W_m)-1} - \frac{L}{B_o} \right] + 0.046 \qquad \text{.......... (2.40)}$$

for sand beds and

$$\frac{h_m}{L} = 0.00685 \left[\log \left\{ \left(\frac{W_m t}{L} \right) \left(\frac{W_m L}{U_o B_o} \right)^5 \right\}^{(U_o/W_m)-1} - \frac{L}{B_o} \right] + 0.128 \qquad \text{.......... (2.41)}$$

for gravel beds ; L and W_m being the length of rigid apron measured from jet efflux section and fall velocity corresponding to d_{50} size of bed materials, respectively.

The maximum scour depth at equilibrium, H_m was expressed in terms of the Froude number based on the efflux thickness of the jet. The authors observed that H_m was independent of grain size of bed materials. The expression of H_m applicable to both sand and gravel beds was given as

$$\frac{H_m}{B_o} = 0.775 \left[\frac{U_o}{(g B_o)^{1/2}} \right] \qquad \text{.......... (2.42)}$$

The expression for volume rate of transport, q_s was obtained by differentiating V_s with respect to t , i.e., $q_s = \frac{dV_s}{dt}$. Using Eq. 2.30, the expression for q_s was obtained for both sand and gravel beds as,

$$q_s = 0.198\ U_o^2\ B_o d_g^{-0.3}\left(e^{-2.031 d_g^{0.008} . U_o^{-0.667 d_g^{-0.014}}}\right) . t^{0.435 d_g^{-0.164} - 1} \quad \text{.......................................(2.43)}$$

The authors investigated the similarity criterion for sediment transport and developed a generalised transport equation correlating the weight rate of sediment transport with the fluid power of jet, the transport stage $[(u_{*t} / u_{*o}) - 1]$, and a nondimensional parameter characterising the diffusion of the jet. Depending on the value of the transport stage, two relationships described the transport laws as follows,

$$i/w = 1.61 \times 10^{-5} \left[\left\{\left(\frac{u_{*t}}{u_{*o}}\right) - 1\right\}\left(\frac{x_m}{d_g}\right)^{1/2}\right]^{1.5} \quad \text{..(2.44)}$$

$$i/w = 1.05 \times 10^{-5} \left[\left\{\left(\frac{u_{*t}}{u_{*o}}\right) - 1\right\}\left(\frac{x_m}{d_g}\right)^{1/2}\right]^{2.0} \quad \text{..(2.45)}$$

in which,

i	=	weight rate of sediment transported per meter width per minute,
w	=	fluid power of jet $= (\rho q \Delta h / B_o)$,
u_{*o}	=	Critical shear velocity $= (\tau_{oc}/\rho)^{\frac{1}{2}}$,
u_{*t}	=	shear velocity at any time $= (\tau_{oc}/\rho)^{\frac{1}{2}}$,
x_m	=	the distance of maximum scour point from the sluice,
ρ	=	mass density of water,
q	=	discharge per meter width,
Δh	=	head causing water to flow
τ_{oc}	=	Critical shear stress for the sediment composing the bed,
τ_t	=	Boundary shear stress at time, t, during the scour process.

Eq. 2.44 is valid for the range $20 < [(u_{*t} / u_{*o}) - 1](\frac{x_m}{d_g})^{\frac{1}{2}} \leq 60$ and Eq. 2.45 is valid for the range

$$70 \leq [(u_{*t} / u_{*o}) - 1](\frac{x_m}{d_g})^{\frac{1}{2}} \leq 230$$

2.4 SCOUR DUE TO INCLINED JET

An analytical study of the mechanics of scour due to a three dimensional jet was carried out by Iwagaki et. al. (1958). The salient feature of their investigation was the existence of three different regions of scour development. These being corresponding to maximum jet diflection which disappears when the angle of impingement, θ on the tailwater reaches approximately 61°, the minimum jet deflection when θ is less than 61° and lastly the final equilibrium.

Francis and Ghosh (1974) studied the local scour of a horizontal bed of granular material due to a 45° inclined jet drowned in a relatively large depth of water with a view to formulating suitable scaling laws for sand bed modelling. The authors extended the work of Carstens (1966) partly by showing that $(N_s^2 - N_{sc}^2)$ might be expressed in the form of transport stage, $(\frac{u_*}{u_{*o}} - 1)$ and partly by showing that q_s might be more effectively and generally expressed in terms of fluid power of jet which has a strong resemblance to the stream power concept proposed by Bagnold (1973) for striaght uniform channels of erodible bed material, where

q_s = volume rate of scour per unit width,

u_* = shear velocity $=\sqrt{\tau_o / \rho}$

u_{*o} = critical shear velocity $= \sqrt{\tau_{oc} / \rho}$

N_s = sediment number $= \frac{V}{[(S_s-1)gd_g]^{0.5}}$

S_s = specific gravity of the particles

d_g = Typical grain diameter of surface particles

N_{sc} = Lowest value of the sediment number, for which the scour will occur.

The authors proposed that the modelling parameters were transport stage, geometric parameters and relative roughness. Experimentally, a bed of granular material was eroded away and the weight rate of transport was determined for the 25% of the equilibrium volume. The results were correlated with the fluid power of the jet and the transport stage. Since the shear stress had not been measured, the stage had been expressed in terms of typical flow velocity near the bed. Accordingly,

$$\left(\frac{u_* - u_{*o}}{u_{*o}}\right) = \frac{K}{0.2W_m}(V - V_o) \quad \text{.......(2.46)}$$

in which, W_m = fall velocity corresponding to d_{50} size,

V_0 = typical velocity just to cause sediment to move,

K = constant of proportionality that connects u_* with V as well as u_{*o} with V_o for a particular geometry.

The weight flow rate of sediment, i, and the fluid power per unit area of sand bed, w had been expressed as,

$$i = \rho(S_s - 1)gq_s \qquad \text{and} \qquad w = \rho V^3$$

Studies carried out by Albertson et.al (1950) indicates that for the case of a jet diffusing out in a larger volume of flow, the centre line velocity, U at a distance x can be expressed as

$$U = 2.28U_o(B_o / x)^{1/2} \quad \text{.......(2.47)}$$

Using the centre line velocity U as the typical velocity V, w can be expressed as $w = \rho(2.28)^3 U_o^3(B_o / x)^{\frac{3}{2}}$ and therefore,

$$\frac{i}{w} = \frac{g(S_s - 1)}{11.85}\left(\frac{q_s}{q}\right)\left(\frac{x}{U_o^2}\right)\left(\frac{x}{B_o}\right)^{0.5} \quad \text{.......(2.48)}$$

Using the experimental data, the parameters i/w and the 'stage' were computed from Eqs. 2.46 and 2.48 and the results were plotted in a log diagram to show that

$$\frac{i}{w} \alpha \left(\frac{u_*}{u_{*o}}\right)^{3/2} \times f(B_o / d_g)$$

Secondly, considering fluid power w_1 issuing per unit area of the nozzle, the ratio, i / w_1 was expressed ultimately as

$$\frac{i}{w_1} = (S_s - 1)\left(\frac{q_s}{q}\right)\left(\frac{B_o}{\Delta h}\right) \quad \text{.......(2.49)}$$

in which, Δh = head causing the flow through the gate.

i / w_1 also was found to be expressible as

$$\frac{i}{w_1}\alpha\left(\frac{u_*}{u_{*o}}-1\right)^{3/2} \text{ with } (x/d_g) \text{ as a further variable.}$$

Hence, for high stages and for a given geometry, (q_s/q) should be proportional to $(\text{stage})^{\frac{3}{2}}$. Accordingly, if the geometry of both grain and flow is preserved and if $[(S_s-1)q_s/q]$ is modelled to the same scale as the prototype, W_m must be selected so that

$$\left(\frac{\sqrt{\Delta h}}{W_m}\right)^{\frac{3}{2}}_{\text{mod el}} = \left(\frac{\sqrt{\Delta h}}{W_m}\right)^{\frac{3}{2}}_{\text{proyotype}}$$

The authors pointed out that the relative roughness terms (B_o/d_g) or (x/d_g) are surprisingly important. Accurate geometric scaling of grain diameter d_g therefore seems to be very important in experiments concerned with the rate of erosion ; and this contrasts with the lack of influence of d_g on model experiments where only the equilibrium depth of erosion is under investigation.

Mason (1988) studied the effects of air entrainment on plunge pool scour. The jet of water was caused to impinge on the erodible gravel bed at an angle of 45°. The experimental set up was calibrated in such a way that the unit flow q, head drop H, and the air/water ratio β could be individually varied in a controlled manner to establish their effects on scour depth. Air was added artificially and the jet was introduced below tailwater level to eliminate unknown amounts of natural boundary entrainment at the water surface. The experiments were conducted on the jets (without or with entrained air) to produce two dimensional scour patterns and the maximum depth of scour was noted in each test run. Analysing the test data of that study and of the study of Mason and Arumugam (1985), the author developed the following expresion for the maximum scour depth, D in the case of the jets without entrained air,

$$D = 3.27\frac{q^{0.60}\ H^{0.05}\ h^{0.15}}{g^{0.30}\ d^{0.10}} \qquad \text{.......(2.50)}$$

in which, h = tailwater depth above unscoured bed level,

d = mean size of particles of bed materials,

For the jets with entrained air, the resulting expression was

$$D = 3.39\frac{q^{0.60}\ (1+\beta)^{0.30}\ h^{0.16}}{g^{0.30}\ d^{0.06}} \qquad \text{.......(2.51)}$$

in which, β was the ratio of entrained air to water. The author obtained the value of β using relation proposed by Ervine (1976), which was as follows,

$$\beta 0.13\left(1-\frac{V_e}{V}\right)\left(\frac{H}{t}\right)^{0.446} \qquad \text{.......(2.52)}$$

in which, V = impact velocity of jet

V_e = the minimum jet velocity required to entrain air.

t = jet thickness at impact.

According to Ervine (1976), $V_e = 1.10\ \text{m/s}$. The author noted that the depth of scour decreased when air was entrained, compared to the scour developed by the water jet acting alone and he concluded that the expressions suggested were aplicable to both model and prototype.

Blaisdell and Anderson (1991) studied the scour due to an inclined jet of water issuing from a pipe outlet and impinging on cohesionless bed. Their study was aimed at the design of pipe plunge pool Energy Dissipator. The experiments were performed using different bed material sizes, discharges, height of the efflux section from tailwater level and tailwater depth. To describe the scour pattern the authors developed the expressions for the horizontal distance of maximum scour point from the pipe exit, X_m ; the ultimate maximum scour depth from tailwater level, $(Z_{m,u})$; and time dependent scour depth. From the experimental observations it was noted that (X_m / X_j) varied with the discharge (Q), over a range from 1.07 for $Q/(gD^5)^{1/2} = 0.5$ to 0.54 for $Q/(gD^5)^{1/2} = 5$; X_j being computed distance from pipe exit to the bottom of scour hole and D is the diameter of pipe. (X_m / X_j) versus $Q/(gD^5)^{1/2}$ were plotted to obtain three different curves for different limits i.e., for upper limit, for lower limit and mean curve respectively as follow,

$$\frac{X_m}{X_j} = 2.09\ e^{-0.26\ Q/(gD^5)^{1/2}} \quad \text{..........(2.53)}$$

$$\frac{X_m}{X_j} = 0.80\ e^{-0.086\ Q/(gD^5)^{1/2}} \quad \text{..........(2.54)}$$

and

$$\frac{X_m}{X_j} = 1.15\ e^{-0.15\ Q/(gD^5)^{1/2}} \quad \text{..........(2.55)}$$

The authors pointed out that these three curves intersect at $X_m / X_j = 0.5$ and $Q/(gD^5)^{1/2} = 5.5$ and they should not be extrapolated beyond this range.

Following realtionships for the ultimate maximum scour depth were derived for the two ranges of pipe height, i. e., for $Z_p / D \leq 1.0$,

$$\frac{Z_{m,u}}{D} = -7.5\left[1 - e^{-0.6(F_d - 2)}\right] \quad \text{..........(2.56)}$$

and for $Z_p / D > 1.0$,

$$\frac{Z_{m,u}}{D} = -10.5\left[1 - e^{-0.35(F_d - 2)}\right] \quad \text{..........(2.57)}$$

in which, Z_p = height of pipe above (+) or below (-) tailwater level,

F_d = densimetric Froude number = $\dfrac{V_p}{[g d_{50}(\rho_s - \rho)\rho]}$

V_p = jet plunge velocity.

To determine the maximum scour depth at any time, the authors proposed the following relations,

$$\frac{Z_m}{D} = -\text{Anti}\log(x + y_o - \sqrt{x^2 + a^2}) \quad \text{..........(2.58)}$$

in which, y_o = $\log(\frac{-Z_{m,u}}{D})$

x = $\log(\frac{t\ V_p}{D_p})$

t = time in seconds

D_p = diameter of jet at the plunge point

a = 1.75 (constant)

$(\frac{Z_{m,u}}{D})$ being computed from Eqs. 2.56 and 2.57

Procedure and criteria for designing plunge pool energy dissipators was also described in this study.

2.5 SCOUR DUE TO HORIZONTAL JET IN SHALLOW TAILWATER

Rajaratnam and Macdougall (1983) did the pioneering investigation on erosion by plane turbulent wall jets in shallow tailwater. The experiments were performed with two types of sands with the mean size, D (50% finer than this size) equal to 2.38 mm and 1.0 mm, maintaining tailwater depth at a value approximately equal to the jet thickness, b_0 (minimum tailwater). In all the experimental runs, the asymptotic eroded bed profile was measured with a point gauge, only after the scour profile appeared to be invarient with time. Analysing the experimental data the authors plotted two curves, one showing the relationship between $\varepsilon_{m\infty} / b_0$ and F_0 and the other between $X_{m\infty} / b_0$ and F_0; where $\varepsilon_{m\infty}$ = maximum depth of asymptotic erosion, $X_{m\infty}$ = distance of point of maximum scour depth from the jet efflux section, F_0 = densimetric Froude number = $U_0 / (gD\Delta\rho)^{1/2}$,

in which, U_0 = velocity of jet at the efflux section,

D = mean size of sand

$\Delta\rho$ = difference between the mass density of sand and that of the fluid.

The test results were compared with those obtained by Rajaratnam(1981) and it was observed that the value $\varepsilon_{m\infty}$ in the case of shallow tailwater is less than that in the case of large tailwater depth (i.e., deep jet). On the other hand, the value of $X_{m\infty}$, in the case of shallow tailwater is greater than that in the case of deep tailwater indicating that the maximum scour section is located further downstream from the sluice gate. To study the similarity of asymptotic eroded bed profile, the authors plotted $[\varepsilon / \varepsilon_m]_\infty$ against $[X / X_m]_\infty$ and obtained a mean curve for the bed profile where ε is the scour depth at a distance X from the jet efflux section. The mean scoured bed profile mostly resembled the sine curve. An interesting observation of this study was that there was no ridge formation at the downstream end of the scour hole, in contrast with the case of deeply submerged jet. All materilas removed from the scour hole was transported by the flow. While perfroming experiments, the authors observed that during the development of asymptotic eroded bed profile, the flow in the vicinity of the gate (jet efflux section) fluctuated continuously from a surface jet to hydraulic jump and thereafter to submerged hydraulic jump and back to surface jet. the intensity as well as the frequency of such flow sequences reduced considerably as the asympotic state was approached.

Johnston (1990) investigated the scour hole development due to a plane jet of water entering into shallow tailwater. Through dimensional considerations and physical reasoning, the appropriate functional groups were identified and experimental data were used to evaluate the functional relationships. He identified that for the flow of a palne jet emerging through a slot into a channel with erodible bed, any feature of the flow field, N_x can be expressed as

$$N_x = f\left(F_d, \frac{P}{d_o}, \frac{D}{d_o}, \frac{d_{50}}{d_o}\right) \qquad (2.59)$$

in which, P = vertical distance from jet centre line to bed,

d_o = slot width,

D = vertical distance from jet centre line to the free surface,

d_{50} = mean grain diameter of sediment,

F_d = densimetric Froude number = $U_o / \left(\frac{\rho_s - \rho}{\rho} g d_{50}\right)^{1/2}$

where, U_0 = source velocity, ρ = mass density of fluid and ρ_s = mass density of sediment.

The author described three scourhole regime states, bed jet (deeply submerged jet), surface jet (shallow jet) and bed-surface jet (deep to shallow jet). Experiments were performed with two sand beds consisting of sand of uniform size (d_{50}= 0.7 mm and 1.8 mm) at four submergence depths (D / d_0= 0.52 , 2.33, 3.48 and 4.03. He observed that at the highest tailwater depth ($D/d_0 = 4.03$) the jet mainstream clearly attached itself to the erodible bed and very quickly developed a scourhole with a ridge positioned just downstream of the hole, i.e., a "Bed Jet" scourhole regime was formed as described by Rajaratnam (1981) and Hassan and Narayanan (1985). At the lowest tailwater depth ($D/d_0 = 0.52$) a different pattern developed, where, the hole was longer and shallower in the stream-wise direction, than the bed jet case; also, the ridge was less pronounced and tended to move downstream. In this case the jet was attached to the free surface, and not to the erodible bed. Such a scour hole development is described as a "surface jet" regime and has similar characteristics to the scourhole described by Rajaratnam and Macdougall (1983). During the tests with two intermediate depths (D / d_0= 2.33, 3.48) the jet was initially attached to the bed and a bed jet scourhole regime was developing ; however, at a particular instant the development suddenly changed and the scour hole filled in as the downstream mound slumped back into the scour hole. Just as this pattern began to stabilize, there was a further sudden change and the bed jet regime was re-established. The position of the jet was observed to flick between the boundaries in accordance with the changing of the scour hole developments. Such a scour hole development was identified as a "Bed-surface Jet" regime.

On closer scruitiny of the experimental data the author observed that apart from submergence parameter (D/d_0) the slot offset parameter (P/d_0) plays an important role in flicking the jet between boundaries and forming different scourhole regimes. Analysing the test data, the author prepared a number of non-dimensional plots where the scourhole regimes were classified into groups of submergence depths (D / d_0)and shown in terms of densimetric Froude number F_d and slot offset (P / d_0). These non-dimensional plots reflect that with very shallow tailwater depths(D / d_0 being 0.25 to 1.5), the scour holes are nearly all of the surface jet regime whereas, with the larger depths($D / d_0 > 6.5$) , the scourholes are all of the bed jet regime. At the depths between these extremes a pattern of scourhole regimes is apparent, but not completely obvious. In order to clarify the deliveation of these regimes the author proposed a separate length scale, L_d ,considering strong effects of momentum and buoyancy in the two-dimensional jet situations. He considered $L_d = d_0 F_d^{1.33}$ and plotted ($P / d_0 F_d^{1.33}$) against F_d for different groups of ($D / d_0 F_d^{1.33}$). From these plots important observations were as follows,

(a) the flicking of bed-surface regime is most likely to occur when (D / L_d) is approximately equal to (P / L_d),

(b) the reduction in size of the surface only zones and the increase in size of bed only zones occur as the tailwater depth increases,

(c) a similar trend is followed by the two sub-sets of the bed-surface jet regimes.

The author analysed the test data of Rajaratam (1981), Rajaratnam and Macdougall (1983), Ali and Lim (1986), Lim (1985) and Iwagaki et al.(1965) and observed that all results consistently fall into the same regimes as specified by the author. The time was expressed in non-dimensional form as $T = U_0 t / d_0$ and the maximum depths of scour holes for different regimes at corresponding times were assessed. It was observed that with both the bed and the surface regimes an increase in time results in an increase in scour hole depth, which is more pronounced in the bed case. This trend was not apparent in the bed-surface regime where scour hole depth remains relatively constant. The scouring rates in the cases of surface regime and bed-surface regime were 40% and 70% respectively of that of corresponding bed regime. On the other hand, with the bed regime , an increase in submergence produces an increase in scour hole depth whereas with the surface regime, the depths remian relatively constant and such a trend is less prominent with the bed-surface regime. However, the main conclusions drawn by the author were as follows ,

(a) a submergence parameter (D / L_d), as bed offset parameter (P / L_d) and densimetric Froude number have the dominant influences on the flow pattern ;

(b) for fairly small tailwater depth ($D / L_d < 0.15$), the moderate bed offset (P / L_d being from zero to 0.55) and small densimetric Froude numbers ($F_d < 5$) a surface jet scour hole regime is likely to occur, where the jet is attached to the free surface;

(c) for large depth values ($D/L_d > 0.6$) with large bed offset ($P/L_d \geq 0.7$) a bed jet regime is observed where the jet is attached to the bed ;

(d) with moderate depths (D/L_d being form 0.15 to 0.5) and moderate bed offsets (P/L_d being from zero to 0.55), a bed-surface jet regime occurs where the jet flicks between boundaries. The percentage of time that the jet is attached to the bed is directly related to the tailwater depth.

2.6 GENERALIZED SCOUR STUDIES

Based on the experimental data of localised scour resulting from different types of flow obstructions, Carstens (1966) formulated sediment transport functions of localised scour which when intergrated give the scour depth functions.

The author considered the rate of transport to be dependent on the ratio of the disturbing to the resisting force. Considering the forces on a typical particle resting on the surface of the bed, the ratio was expressed as,

$$\frac{\Sigma F_M}{\Sigma F_R} = \frac{K_1\sqrt{C_L^2+C_D^2}}{K_2}\left[\frac{V^2}{(S_s-1)gd_g}\right] \qquad \text{.....(2.60)}$$

in which, F_M = the dusturbing force on the particle,

F_R = effective weight of the particle,

K_1 & K_2 = dimensionless particle shape factors,

C_D & C_L = coefficients of drag and lift respectively on the particle.

Since, K_1, K_2, C_D and C_L are all essentially related to the sediment particle characteristics, the Eq. 2.60 can as well be expressed as

$$\frac{\Sigma F_M}{\Sigma F_R} = \left[f(\text{sediment}-\text{grain}-\text{geometry})\right].N_s^2 \qquad \text{.....(2.61)}$$

in which N_s = sediment number after Carstens = $V/[(S_s-1)gd_g]^{1/2}$

Since the local rate of scour varies over the surface of the scour hole and it drastically decreases as the depth of scour, h, increases and since the sediment transport rate is a functiona of the force ratio, Carstens(1966) expressed the Eq. 2.61 in a dimensionless form, i.e.,

$$\left[\frac{q_s}{VBd_g}\right] = f\left[\left(N_s^2 - N_{sc}^2\right), \frac{h}{L}, \text{ disturbance geometry, sediment}-\text{grain}-\text{geometry}\right] \qquad \text{......(2.62)}$$

in which N_{sc} = lowest value of sediment number for which scour will occur.

B = width of the scour hole,

L = Pertinent dimension of the obstruction.

The validity of the above mentioned relationship was checked by analysing the experimental data of localised scour due to (i) a defined scour hole (ii) scour associated with dunes, (iii) two-dimensional jet scour, (iv) scour around a vertical cylinder, and (v) scour around a cylinder lying on the bed. However, a typical analysis covering the experimental data of Laursen (1952) due to a two dimensinal jet is given below.

With reference to Fig. 2.1 it can be easily seen that the rate of sediment transport, q_s out of the scour hole is equal to the rate of change of scour-hole volume V_s, i. e.,

$$q_s = \frac{dV_s}{dt} \qquad \text{.....(2.63)}$$

For Laursen's experiment (Fig.2.1),

$$\frac{q_s}{B}=\left[\frac{4}{\tan\phi}\ h_m\left(\frac{dh_m}{dt}\right)\right] \qquad (2.64)$$

Using the experimental results of h_m as a function of t, the sediment transport rate, q_s out of the scour hole was calculated and the values were plotted in the form of Eq. 2.62 and thereby yeilding the function,

$$\left(\frac{q_s}{U_o B d_g}\right)=1.9\left(\bar{10}^3\right)\left(N_s^2-4\right)^{5/2}\left(\frac{h_m}{B_o}\right) \qquad (2.65)$$

In the Eq. 2.65, the reference velocity, V was taken to be the velocity of the jet issuing from the nozzle, U_0. the pertinent dimension, L was taken as the thickness of the jet B_0. The value of N_{sc} for d_g of 0.24 mm, 0.69 mm and 1.6 mm was chosen as two.

Substituting Eq. 2.65 into Eq. 2.64

$$\frac{\left(\frac{h_m}{B_o}\right)^5}{\tan\phi}\cdot\frac{d\left(\frac{h_m}{B_o}\right)}{d\left(\frac{U_o t}{B_o}\right)}=4.75\left(\bar{10}^4\right)\left(N_s^2-4\right)^{5/2}\left(\frac{d_g}{B_o}\right) \qquad (2.66)$$

On integration, Eq. 2.66 yeilds

$$\left(\frac{h_m}{B_o}\right)^6=2.85\left(\bar{10}^3\right)\left(N_s^2-4\right)^{5/2}\tan\phi\left(\frac{d_g}{B_o}\right)\left(\frac{U_o t}{B_o}\right)+c \qquad (2.67)$$

Which is the scour depth versus time function. The constant of integration c in Eq. 2.67 can be determined by initial conditions and by the time required for the scour hole to develop in the form as shown in Fig.2.1. The initial condition that $h_m = 0$ when $t = 0$ was used to determine that c =0

Carstens (1966) compared the Eq. 2.67 with the Laursen's (1952) data of scour depth versus time and it was found that the equation was a good empirical relation applicable to such type of scour situation.

2.7 STUDIES ON FLOW CHARACTERISTICS OF WALL JET

The above mentioned reviews are concerned with the studies of local scour due to various flow situations. In the following paragraphs the studies made to investigate the flow Characteristics of turbulent wall jets are summarised, since theoretically the present study is analogous to the solution of a turbulent wall jet problem over a partly rigid and partly erodible bed.

The earliest known work on the plane turbulent wall jet was done by Forthmann (1936). In his experimental work the author observed the self preserving nature of the wall jet, and that the boundary-layer thickness varied linearly with distance and the maximum velocity varied inversely as the square root of the distance. Further, from the data he determined that the velocity in the inner layer varied as the one-seventh power of the distance from the wall.

The theoretical solution of the problem of wall jet whether radial or plane, laminar or turbulent, was first investigated by Glauert (1956) proceeding from continuity and momemtum equations. The author developed appropriate boundary layer equations and tried for a similarity solution in which the form of velocity distribution across the jet does not vary along its length. Two similarity exponents, giving the variation of maximum velocity and jet width with respect to time, was determined and from boundary layer equations a single relation between them was obtained. For analysing the turbulent wall jet the author replaced the molecular viscosity by an effective eddy viscosity assuming it to be constant across the breadth of the jet. The velocity profile thus derived was found to be identical with the corresponding laminar flow which did not satisfy the flow conditions of the turbulent wall jet near the boundary. Hence the author modified his prior solution by introducing an eddy viscosity as required to satisfy the law due to Blasius for flow in a pipe. Ultimately, he presented a solution leading to definite prediction of velocity distribution of Reynold's number requiring a single constant to be determined experimentally.

The detailed investigation of turbulent plane wall jet was done by Schwarz and Cosart (1961). Starting from momentum and continuity equations the authors presented a theoretical solutions to predict the growth of boundary layer, decay of maximum velocity, development of shear stress and eddy viscosity assuming that the viscous stresses were negligibly small. They had shown that the boundary layer thickness, δ varied linearly with distance, x ; the variation of maximum velocity being of the order of x^{α}, where α , the exponent, had been empirically determined to be -0.555. To correlate all the velocity data over the entire range of experimental conditions, the authors assumed a single velocity scale U and a single length scale, δ . The empirical expressions were developed to correlate the length and velocity scales with the velocity and width of the issuing jet. To express the velocity distribution law in the inner layer the velocity data obtained from a wind tunnel experiment were plotted and it was observed that the velocity profile varied as the $1/(14 \pm 1)$ th power of the distance from the wall, with nearly 90% of the inner layer following this relationship, However from the velocity distribution law, the Reynold's shear stress, eddy viscosity and the shear stress at the wall were evaluated. Finally, in view of the existence of two flow layers in a wall jet, one being influenced by the other, the authors arrived at the following conclusions,

(i) "The law of the wall" is not applicable to the turbulent wall jet in the form that was obtained by experimentation in the turbulent layers, and turbulent pipe and channel flows.

(ii) The power-law representation commonly used for the turbulent boundaries to describe the mean velocity is appreciably modified in the wall jet resulting in the modification of skin-friction coefficient.

(iii) In the outer layer of the wall jet and near the position of maximum velocity, there is a considerable difference in the velocity profile from that of a free-jet or a mixing layer.

(iv) The square of the hyperbolic secant will not repesent the velocity distribution since the value of the eddy viscosity is not constant across the flow and approaches infinity at the maximum.

Rajaratnam (1965) experimentally investigated the problem of submerged hydraulic jump considering it to be the case of a plane turbulent wall jet under adverse pressure gradient over which a backward flow was provided. Using the experimental results the author presented an analysis of the forward flow in the submerged jump as a plane wall jet. The backward flow was evaluated using the results of Liu (1949) and Henry (1950). These two were jioned together to predict the characteristics of the submerged jump.

The author selected the velocity scale as the maximum velocity, U and the length scale as the distance from the wall , δ_1 where the velocity , u equals to U/2. δ_1 and U were expressed in terms of velocity and width of issuing jet, the distance from the slot, x , and some coefficients. It was shown that δ_1 increases linearly with x and the coefficients in the expression depend mainly on the Froude number of issuing jet. The reference velocity U also decreases linearly with x and the coefficients in the expression depend mainly on submergence factor. The velocity distribution in the wall jet was found to be similar and follows the Schwarz-Cosart (1961) curve closely for y/δ_1 approximate equal to one and falls below it for higher values of y/δ_1. The surface velocity , u_s, was expressed in terms of the velocity of the issuing jet, distance from the slot and the length of the surface roller. Using the results of Liu (1949) and Henry (1950), the velocity distribution and depth of backward flow was predicted. Combining these two analysis, the author predicted the surface profile, velocity distribution, pressure plus momentum, energy and drag coefficient at any section in a given submerged jump.

Ghosh and Chatterjee (1976), and Chatterjee and Ghosh (1980) investigated the flow characteristics of a two dimensinal horizontal jet (issued from sluice) meeting a large depth of water (deep jet) and flowing over a rigid apron, then on to an erodible bed. The aim of this study was to evaluate the hydraulic parameters involved in the phenomena of local scour and sediment transport caused by the jet of water as described above. Experiments were performed with one sand bed ($d_{50} = 0.76$ mm) and one gravel bed ($d_{50} = 4.3$ mm) for a constant length of rigid apron, L equals to 0.66m; and for varying gate opening (jet thickness), B_0 and discharge per unit width , q. The flow depth, however, in the undisturbed zone, D was kept more or less constant and was maintained at 0.293m and 0.302 m for the sand bed and the gravel bed respectively. In each test run the measurements were taken for velocity distribution at the equilibrium stage at several locations along the central plane of the jet, time required to reach the equilibrium state, depth of maximum scour at equilibrium state, the scour profiles at different instants of time during the development of scour hole and the dynamic pressure drop in a preston tube at the location of maximum sour at various instants of time during the development of scour hole.

The author studied the mechanism of diffusion of jet below the sluice using the procedure adopted by Albertson et.al.(1950) in the case of study of duffusion of "Free Jet" and it was observed that the reduction of maximumvelocity, U can be expressed as a function of the efflux velocity, U_0, jet thickness, B_0 ,length of rigid apron, L , grain size, d_g and the distance from sluice opening, x. The laws governing the diffusion process are different for the rigid apron and erodible bed. The empirical relationships governing the diffusion process over the rigid apron and the erodible bed were derived separately in the non-dimensional form as follows,

$$\frac{U}{U_o} = C_1(B_o/L)^{n_1} - C_2(B_o/L)^{n_2}(\frac{x}{B_o}) \quad \text{.....(2.68)}$$

$$\frac{U}{U_o} = C_3(L/B_0)^{n_3}(\frac{x}{B_o})^{n_4} \quad \text{.....(2.69)}$$

Eqs. 2.68 and 2.69 are for the rigid apron and for the erodible bed, respectively. The values of the coeffiecients, C_1, C_2, C_3 and the exponents n_1, n_2, n_3 and n_4 for sand bed were different from those of gravel bed which confirms the effect of grain diameter of bed material on the diffusion process.

The authors developed the expressions for the growth of boundary layer on both the rigid apron and the erodible bed. The numerical values of the boundary layer thickness, δ (as obtained from velocity distribution diagram) were plotted against the corresponding distance from sluice gate, x on log-diagrams for sand and gravel bed separately. From this plot it was observed that along the rigid apron, the growth of boundary layer is slow as compared to the region covering the erodible bed and there is a transition in between. Accordingly, δ had been correlated with x for three different regions, i.e., rigid apron, the transition zone and the erodible bed respectively as follows,

$$\delta = C_4 x^{n_5}$$

$$\delta = C_5 x^3 + C_6 x^2 + C_7 x + C_8$$

and $$\delta = C_9 x^{n_6}$$

These equations were also expressed in the non-dimensional form as follows,

$$\frac{\delta}{D_m} = C_{10}(^x/L)^{n_5} \quad \text{.....(2.70)}$$

$$\frac{\delta}{D_m} = C_{11}(x/L)^3 + C_{12}(x/L)^2 + C_{13}(x/L) + C_{14}, \quad \text{.....(2.71)}$$

$$\frac{\delta}{D_m} = C_{15}(^x/L)^{n_6} \quad \text{.....(2.72)}$$

D_m and L being the mean value of downstream flow depth and length of rigid apron respectively. However, the values of coefficients and the expponents in these equations for sand bed were different from those of gravel bed, indicating the effect of grain diameter of bed materials on boundary layer growth.

The authors determined the velocity distribution law along the rigid apron and at the location of maximum scour using the measured velocity profiles. In case of the rigid apron a graphical relation was developed by plotting y/δ against u/U , u being the velocity at a height y from the rigid apron. On superimposition of the velocity distribution due to classical wall jet, some deviation was observed for this flow situation in the region beyond the boundary layer and such a considerable disagreement was expected in view of the introduction of an erodible bed. To derive the velocity distribution law at the localtion of maximum scour, the non-dimensional plots u/U versus $y/\delta = \eta$ were made separately for the sand and the gravel bed. The velocity distribution law within the boundary layer was expressed in the polynomial form as follows,

for sand bed,

$$\frac{u}{U} = -0.795\eta^3 + 1.02\eta^2 + 0.773\eta \quad \text{.....(2.73)}$$

and for gravel bed,

$$\frac{u}{U} = -1.96\eta^3 + 2.7\eta^2 + 0.26\eta \quad \text{.....(2.74)}$$

To use the velocity distribution law in the estimation of the time variation of boundary shear stress using dynamic pressure drop recorded by a Preston tube, the velocity distribution at the zone close to the bed was expressed in the form of power law for the sand and the gravel bed respectively as follows,

$$\frac{u}{U} - 1.188(\eta)^{1.19} \quad \text{....for } 0 \le \eta \le 0.41 \tag{2.75}$$

$$\frac{u}{U} = -1.227(\eta)^{1.273} \quad \text{...for } 0 \le \eta \le 0.36 \tag{2.76}$$

The expression for critical shear stress, τ_{oc} acting at the location of maximum scour (at the equilibrium state of scouring process) was derived starting from Vón Kármán's integral equation and using the relations developed for U, δ and η, which were obtained for the sand and the gravel bed respectively as follows,

$$\frac{\tau_{oc}}{\rho} = -0.2827\ U_0^2\ D_m\ L^{1.38}\ B_0^{0.38}\ x^{-2.76} \tag{2.77}$$

$$\frac{\tau_{oc}}{\rho} = -0.132\ U_0^2\ D_m\ L^{-1.45}\ B_0^{1.09}\ x^{-0.64} \tag{2.78}$$

The negative sign indicates that the boundary shear stress acts in the direction opposite to that of the flow.

The time variation of boundary shear stress, τ_t at the location of maximum scour was correlated with the dynamic pressure drop, Δp recorded by a Preston (1954) tube at various instants of time during the development of scour hole. The expression for τ_t was obtained using Vón Kármán's integral equation and the expressions developed for U , u, δ , Δp for a Preston tube. The expressions for τ_t for the sand bed and the gravel bed were obtained respectively as follows,

$$\tau_t = 105\ U_0\ D_m^{2.19}\ L^{-4.68}\ B_0^{0.19}\ x^{3.49}(\Delta p)^{0.5} \tag{2.79}$$

$$\tau_t = 8.15\ U_0\ D_m^{2.273}\ L^{-7.375}\ B_0^{0.545}\ x^{5.83}(\Delta p)^{0.5} \tag{2.80}$$

Calculating the values of τ_t from Eqs. 2.79 and 2.80 and those of τ_{oc} from Eqs. 2.77, and 2.78, a non-demensional plot of τ_t / τ_{oc} versus t/T was made separately for the sand and the gravel bed. The fitted equations were obtained as follows,

for the sand bed,

$$\frac{\tau_t}{\tau_{oc}} = 8.93\ (t/T)^{-0.38} \quad \text{.............for } 0 < t/T \le 0.1 \tag{2.81}$$

$$\frac{\tau_t}{\tau_{oc}} = e^{2.79(1-t/T)^{0.198}} \quad \text{...........for } 0.2 \le t/T \le 1.0 \tag{2.82}$$

and for gravel bed,

$$\frac{\tau_t}{\tau_{oc}} = 11.53(t/T)^{-0.38} \quad \text{.......for } 0 < t/T \le 0.1 \tag{2.83}$$

$$\frac{\tau_t}{\tau_{oc}} = e^{3.2(1-t/T)^{0.155}} \quad \text{...........for } 0.2 \le t/T \le 1.0 \tag{2.84}$$

Form the Eqs. 2.81, 2.82, 2.83 and 2.84 it was apparent that the effect of grain size characteristics of the bed material on the development of boundary shear was increasingly felt as the equilibrium stage was reached.

2.8 DIFFUSION CHARACTERISTICS OF FREE JET

Albertson et.al (1950) carried out the classical investigation of the diffusion of a free jet into the surrounding fluid. They observed that for plane two dimensional jet such as that issuing out of a slot the zone of flow establishment is reached at a distance of approximately five times the slot opening, the zone of flow establishment being defined as the region where the lateral eddies do not penetrate right upto the central plane of the jet. Thereafter, the decay of the maximum velocity occurs inversely to the square root of the distance along the jet axis in accordance with following relationship,

$$\frac{U}{U_0} = \left[\frac{1}{\sqrt{\pi}\, C_1} \cdot \frac{B_0}{X}\right]^{1/2} \qquad (2.85)$$

C_1 being a coefficient of the diffusion process.

Through the assumtions of hydrostatic pressure distribution across the jet, dynmic similarity of the diffusion process, and normal probability variation of the velocity, analytical expressions were developed for the volume flux ratio and energy flux ratio in the zone of flow establishment as well as in the zone of established flow.

For the zone of flow establishment,

$$\frac{Q}{Q_0} = 1 + \sqrt{\pi}(\sqrt{2} - 1)C_1\left(\frac{X}{B_0}\right) \qquad (2.86)$$

$$\frac{E}{E_0} = 1 + \sqrt{\pi}\left(\sqrt{2/3} - 1\right)C_1\left(\frac{X}{B_0}\right) \qquad (2.87)$$

For the zone of established flow,

$$\frac{Q}{Q_0} = \left[2\sqrt{\pi}\, C_1 \left(\frac{X}{B_0}\right)\right]^{1/2} \qquad (2.88)$$

$$\frac{E}{E_0} = \left[\frac{2}{3\sqrt{\pi} C_1}\left(\frac{B_0}{X}\right)\right]^{1/2} \qquad (2.89)$$

in which,

Q_0 = efflux volume flux,

Q = volume flux past successive normal sections,

E_0 = energy flux at the efflux section,

E = energy flux at successive sections.

The numerical value of the coefficient C_1 was determined experimentally and the magnetude was found to be 0.109.

2.9 CONCLUDING REMARKS

The above survey of literature proveides a good picture of the progress so far made in the study of flow characteristics and local erosion due to different types of jets. The subject being so wide and of varied interest to the profession, there exists no generalised relationship applicable to all types of flow situations, and it is necessary to ensure pertinent theoretical considerations for the analysis of every individual problem and the analytical results are to be substantiated by developing relevant relations based on experimental data. Accordingly, in the next chapter, the theoretical considerations are made taking into account the effect of various variables inveloved in the present study and in the subsequent chapters, the analytical epressions developed for the present flow situation have been substantiated through the derivation of various empirical relations based on the experimental data.