CHAPTER -I

A NOVEL COMPUTERIZED APPROACH FOR THE LARGE DEFLECTION OF THIN ELLIPTICAL PLATES UNDER MECHANICAL LOADING

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1.1 ABSTRACT

The non-linear behavior of elliptical plate with movable as well as unmovable edge conditions under mechanical loading has been investigated in the new light of computerized simulation technique using powerful shooting method as mathematical tool. This improved version eliminates the method of induction or assumption of the form of "W" i.e., the deflection parameter. The present investigation will definitely improve the solution methodology and make the solution accurate and general.

1.2 INTRODUCTION

Considerable interest has been shown in the past for solving the non-linear problems of thin plates of different shapes. It is evident that interest has been engenerated by the demands placed on the designers of sophisticated structures and space vessels of the present day technology industries. In this investigation the governing differential equation have been formulated by using the modified strain energy expression of Banerjee B. & Datta S.[15] in conjunction with the concept of concept of Constant Deflection Contour Lines to analyze the non-linear behavior of elliptical plates.

The present investigation deals with the problem in a new light of model building to achieve a new way of establishing the form of W' deflection in the following aspect.

1) The governing differential equation will be transformed to a suitable algebraic model for direct computerization of the model exploiting powerful shooting method as a mathematical tool.

2) The computerized generation of the form of deflection will give us a solid foundation for deflection. The data generated by the computerized simulation in conjunction with regression analysis will give us the form of deflection.
3) An almost exact and computerized mathematical model simulation along with the solutions have been developed. The results thus obtained will be compared with the well-known results available in the existing literature.

1.3. BASIC EQUATION

Let us consider x,y plane to be the middle plane if a thin elastic plate and directing the z- axis perpendicular to that plane, the intersections between the deflected surface z = w (x ,y) and plane z = constant, yield contours which after projection on to the z = 0 surface are the level curves called lines of equal deflection. Denote the family of such curves by u (x, y) = constant. If the boundary c of the plate is subjected to any combination of clamping and simple support , then clearly it will belong to the family of lines of Equal Deflection and without loss of generality one may consider that u = 0 on the boundary.

Following reference [15] and as carried in Majumdar and Jones [31] , one can have, [31]

$$\frac{d^3w}{du^3}\oint_{Cu} Rds + \frac{d^2w}{du^2}\oint_{Cu} Fds + \frac{dw}{du}\oint_{Cu} Gds -$$

$$\iint_{ru} D\left[\frac{q}{D} + \frac{12A}{h^2}\left(w_{xx} + \nu w_{yy}\right) + \frac{6\lambda}{h^2}\left\{\nabla^2 w\left|\left(w_x^2 + w_y^2\right) + 2\left(w_{xx}w_x^2 + 2w_x\right) + w_{yy}w_y^2\right|\right\}\right]dxdy = 0 \qquad \text{....(1}$$

where, h is the thickness of the plate with continuously distributed load q . All other symbols have their usual meaning as indicated in [1] and [3].

The flexural rigidity $D = \dfrac{Eh^3}{12\left(1-\nu^2\right)}$ -(1.2)

Where, E is Young's modulus and A is to be determined from

$$A=\frac{\partial u_1}{\partial x}+\nu\frac{\partial v}{\partial y}+\frac{1}{2}\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right] \qquad \ldots(1.3)$$

Integrating over the whole area of the plate, u_1 and v are the components of the displacement in x and y direction respectively. The value of λ is obtained from the condition of minimum potential energy.

1.4. FORMULATION OF THE PROBLEM UNDER INVESTIGATION.

Let us introduce the problem of non-linear behaviors of elliptic plate with movable as well as unmovable edge conditions under the influence of mechanical loading. From the consideration of symmetry, one may assume that the lines of equal deflections from a family of similar and similarly situated ellipses starting from the outer boundary as one of these lines. Therefore, the equation of lines of equal deflection may be conveniently taken to be of the form

$$u(x,y)=1-\frac{x^2}{a^2}-\frac{y^2}{b^2} \qquad (1.4)$$

Now, let us introduce a new variable f defined as follows :

$$f^2=1-u \qquad (1.5)$$

Following the calculations of the values of R,F,G and t_1 given in [31] and introducing the new variable 'f' into equation [1.1] , the following governing differential equation in terms of new variable f is obtained.

$$\frac{d^3w}{df^3}+\frac{1}{f}\frac{d^2w}{df^2}-\frac{1}{f^2}\frac{dw}{df}-4m^2\frac{dw}{df}-\frac{6\lambda}{h^2}\left(\frac{dw}{df}\right)^3-8q_1h=0 \qquad ---(1.6)$$

where, $$m^2=\frac{12Aa^2b^2(va^2+b^2)}{h^2(3a^4+2a^2b^2+3b^4)} \qquad \ldots.\ (1.7)$$

$$q_1=\frac{qa^4b^4}{2D(3a^4+2a^2b^2+3b^4)} \qquad \ldots.(1.8)$$

and $A = \frac{1}{2}(\frac{1}{a^2} + \frac{v}{b^2})\int_0^1 f(\frac{dw}{df})^2 df$ ...(1.9)

For movable edge , A=0, as the in-plane radial stress vanishes at the movable boundary.

1.5. COMPUTERIZED ANALYSIS

Almost without exception in engineering analysis situations where models are being constructed, the initial differential models set up to represent the behavior of a system will not be in any easily recognizable form for computation.

Consequently, it is common practice to repose the model [54]. So the describing differential model of equation [1.6] after reformation reduces to :

$$\frac{d^3\overline{w}}{df^3} + \frac{1}{f}\frac{d^2\overline{w}}{df^2} - \frac{1}{f^2}\frac{d\overline{w}}{df} - 4m^2\frac{d\overline{w}}{df} - \frac{6\lambda}{h^2}\left(\frac{d\overline{w}}{df}\right)^3 - 11.8424\overline{Q}f = 0 \text{ ------}(1.10)$$

where,

$\overline{w} = \frac{w}{h}$ = Dimensionless deflection parameter

$\overline{Q} = \frac{qb^4}{Eh^4}$ = Non dimensionless load function for elliptic plate where, a=2b

Since, in any numerical simulation efforts, the breakdown of any higher order differential equation into relatively simple set is highly significant and the fact that all slopes are simultaneously simulated has significant appeal when working engineering systems [53], we introduce the following transformation in equation (1.10).

$$\frac{d\bar{w}}{df} = x_1$$

$$\frac{d^2\bar{w}}{df^2} = x_2$$

$$\frac{d^3\bar{w}}{df^3} = \frac{dx_2}{df}$$

The equation (1.10) after transformation reduces to ,

$$\frac{dx_2}{df} = -\frac{1}{f}x_2 + \frac{1}{f^2}x_1 + 4m^2x_1 + 6\lambda x_1^3 + 11.8424\bar{Q}f \qquad \text{......(1.11)}$$

For the simulation of the model in the closed domain of 'f' i.e. $0 \le f \le 1$ we require the boundary conditions specified at both extremes . We adopt here the powerful "SHOOTING METHOD" to ascertain the boundary conditions. After introduction of the method taking $\gamma = 0.3$, a=2b.,$\lambda = 2\gamma^2$ for clamped edge and $\gamma = 0.3$, a=2b.,$\lambda = \gamma^2$ for simply supported edge, keeping three values in mind we developed software "GJBVP" following shooting method along with the Newton's method. The algorithm of the software is given below .

Step 1 The unspecified initial conditions of the system differentiated equations are guessed.

Step 2 A set of variational equations are developed which indicate the sensitivities of the dependent variables with respect to the general initial values.

Step 3 The system variational equations are integrated forward as a set simultaneous initial value differential equations.

Step 4 The guessed initial_ conditions are corrected using the variations (sensitivities) calculated in step 3.

Step 5 Step 2 to step 4 is repeated with corrected initial conditions., until the specified terminal values are achieved within a small convergence criterion of 0.0001.

The sequence of the software execution is as follows.

1. TITLE
2. MAIN PROGRAM
3. OUTPUT OPTIONS
4. SUBROUTINES
5. CONSTRUCTION OF JACOBIAN MATRIX
6. INVERSION OF JACOBIAN MATRIX
7. CALCULATION OF CORRECTION VECTOR
8. CHECK FOR CONVERGENCE
9. CORRECTION FOR INITIAL CONDITION
10. NON CONVERGENCE OPTIONS
11. CALL TO PRINTING AND PLOTTING SUBROUTINE
12. RETURN OPTIONS.

Subroutines:

1. Input Equations
2. Matrix Inversion
3. Input Integration parameter
4. Differential Equations.
5. Integration Methods
6. Print Table of results
7. Plotting options
8. Plotting .

The variational equations which modeled the describing differential equation formed as:

$$\frac{d^2w}{df} = y_1 \text{ ; } y_1 \text{ for } f = 0 \text{ known}$$

$$\frac{dw}{df} = y_2 ; y_2 \text{ for } f = 1 \text{ known}$$

$$w = y_3 ; y_3 \text{ for } f = 1 \text{ known}$$

4011. $\mathbf{G(1)} = 8q_1 f + 6\lambda y^3(2) + \frac{1}{f^2} y(2) - \frac{1}{f} y(1)$

4012 . G(2) = y(1)

4013. G(3) = y(2)

4014. $G(4) = 18\lambda y_2^2 + y(6) + \frac{1}{f^2} y(6) - \frac{1}{f} y(4)$

4015. $G(5) = 18\lambda y_2^2 + y(7) + \frac{1}{f^2} y(7) - \frac{1}{f} y(5)$

4016. $G(6) = y(4)$

4017. $G(7) = y(5)$

4018. $G(8) = y(6)$

4019. $G(9) = y(7)$

4020. End.

At the end of the execution of software and scanning we generate a data set to take into consideration of parabolic deflection variation for different case study. The details of data set as generated is given by Table 1 to 7.

For investigation of the dependence of W, the dependent variable with only independent variable 'f' is given in Table 8. We set the multiple equation and developed the software GJREG.BAS(attached floppy diskette).

Since we have fitted the generated data set into parabolic data set in the parabolic regression.

The software with this variational equations with different values of q generated the following sets of data taking the form of

$$\bar{w} = a + b_1 f^2 + b_2 f^4 \quad \text{-----------------}(1.12)$$

Table-1

f	$\frac{d^2\bar{w}}{df^2}$	$\frac{d\bar{w}}{df}$	$\bar{w}$	**Coefficient of Regression**
0.01	-0.5	-1.800021E–2	0.7024295	
0.109	-1.651243	-0.4009423	0.6962507	
0.208	-2.096696	-0.6018959	0.6471536	
0.307	-1.823285	-0.7987203	0.5776075	
0.406	-1.309729	-.0.9553849	0.4903630	$b_1 = -1.416029$
0.505	-0.6212565	-1.0523640	0.3904129	$b_2 = 0.7104033$
0.604	0.2401653	-1.0727260	0.2845160	$a = 0.7064713$
0.703	1.294738	-0.9984506	0.1811297	
0.802	2.565256	-0.8092235	0.061008E-2	
0.9009999	4.056680	-0.4831975	2.541494E-2	
0.9999999	5.745278	5.131662E-4	1.417194E-4	

Table-2

f	$\frac{d^2\overline{w}}{df^2}$	$\frac{d\overline{w}}{df}$	$\overline{w}$	**Coefficient of Regression**
0.01	-1.0	-2.013032E-2	0.7006085	
0.109	-1.874704	-0.3782283	0.6922241	
0.208	-2.16194	-0.5903706	0.6446538	
0.307	-1.854696	-0.7916392	0.5759994	
0.406	-1.328220	-.0.9506872	0.4893241	$b_1 = -1.40782$
0.505	-0.6332866	-1.0491440	0.3897637	$b_2 = 0.7046505$
0.604	0.2318089	-1.0704990	0.2841333	$a = 0.703793$
0.703	1.266562	-0.9969334	0.1809306	
0.802	2.560360	-0.8082485	9.053332E-2	
0.9009999	4.052521	-0.4826676	2.541206E-2	
0.9999999	5.741576	5.555915E-4	1.717489E-4	

Table-3

f	$\frac{d^2\overline{w}}{df^2}$	$\frac{d\overline{w}}{df}$	$\overline{w}$	**Coefficient of Regression**
0.01	-1.50	-2.226168E-2	0. 6987719	
0.109	-2.097536	-0.3555545	0.6881816	
0.208	-2.226981	-0.5788533	0.6421362	
0.307	-1.886001	-0.7845520	0.5788533	
0.406	-1.346636	-.0.9459747	0.4882715	$b_1 = -1.399605$
0.505	-0.6452526	-1.0459030	0.3890994	$b_2 = 0.6989126$
0.604	0.2235174	-1.0682440	0.2837381	$a = 0.7010974$
0.703	1.282456	-0.9953187	0.1807220	
0.802	2.555539	-0.8072319	9.045082E-2	
0.9009999	4.048443	-0.4820881	2.540799E-2	
0.9999999	5.737954	5.553863E-4	2.058744E-4	

Table-4

f	$\frac{d^2\overline{w}}{df^2}$	$\frac{d\overline{w}}{df}$	$\overline{w}$	**Coefficient of Regression**
0.01	-2.0	-2.439533E-2	0.6970258	
0.109	-2.32031	-0.3329479	0.684229	
0.208	-2.292273	-0.567419	0.6397014	
0.307	-1.917639	-0.7775773	0.5728208	
0.406	-1.365438	-0.9414104	0.4872792	$b_1 = -1.391564$
0.505	-0.6576587	-1.04285	0.3884789	$b_2 = 0.6931429$
0.604	0.2147199	-1.066225	0.2833656	$a = 0.6984925$
0.703	1.275772	-0.9941193	0.1805103	
0.802	2.55008	-0.806565	9.033358E-2	
0.9009999	4.043696	-0.4819234	2.533132E-2	
0.9999999	5.733672	5.744398E-4	1.23078E-4	

Table-5

F	$\frac{d^2\overline{w}}{df^2}$	$\frac{d\overline{w}}{df}$	$\overline{w}$	**Coefficient of Regression**
0.01	-2.50	-2.652577E-2	0.6951581	
0.109	-2.542963	-0.3102597	0.6801555	
0.208	-2.357268	-0.5558795	0.6371548	
0.307	-1.948937	-0.7744656	0.5711682	
0.406	-1.38386	-0.9366736	0.4862024	$b_1 = -1.383311$
0.505	-0.6696288	-1.039585	0.3877929	$b_2 = 0.6873887$
0.604	0.2064308	-1.063946	0.282951	$a = 0.6957661$
0.703	1.269677	-0.9925432	0.1802847	
0.802	2.545278	-0.8055225	9.023654E-2	
0.9009999	4.03964	-0.481316	2.531535E-2	
0.9999999	5.730074	8.045137E-4	1.4819821E-4	

Table-6

f	$\frac{d^2\overline{w}}{df^2}$	$\frac{d\overline{w}}{df}$	$\overline{w}$	**Coefficient of Regression**
0.01	-2.75	-2.758998E-2	0.6941994	
0.109	-2.654411	-0.2988899	0.6780941	
0.208	-2.389776	-0.5500882	0.6358591	
0.307	-1.964574	-0.7668882	0.5703216	
0.406	-1.393048	-0.9342819	0.4856459	$b_1 = -1.379153$
0.505	-0.6755815	-1.037926	0.3874342	$b_2 = 0.684507$
0.604	0.2023289	-1.062777	0.2827307	$a = 0.6943789$
0.703	1.266681	-0.9917209	0.180162	
0.802	2.542938	-0.8049612	9.018188E-2	
0.9009999	4.037677	-0.480966	2.0253055E-2	
0.9999999	5.72834	9.72271E-4	1.637805E-4	

Table-7

f	$\frac{d^2\overline{w}}{df^2}$	$\frac{d\overline{w}}{df}$	$\overline{w}$	**Coefficient of Regression**
0.01	-3.00	-2.865578E-2	0.693384	
0.109	-2.766237	-0.2875638	0.6761756	
0.208	-2.422687	-0.5443802	0.6347001	
0.307	-1.980634	-0.7634345	0.5696016	
0.406	-1.402689	-0.9320571	0.4852015	$b_1 = -1.375187$
0.505	-0.682037	-1.036482	0.3871688	$b_2 = 0.6185834$
0.604	0.1976553	-1.061873	0.28258	$a = 0.6931366$
0.703	1.263036	-0.9912259	0.1800796	
0.802	2.53988	-0.8047954	0.0901317	
0.9009999	4.034964	-0.4810846	2.525738E-2	
0.9999999	5.725864	5.974174E-4	9.104609E-5	

And so on.

For clamped edge conditions the boundary conditions and the parametric conditions are as follows :

CASE- a (Movable edge)

(i) $w = 0$ at f = 1 and (ii) $\frac{dw}{df} = 0$ at f = 1 for far end boundary conditions.

(iii) $\frac{d^2w}{df^2}$ = missing for both far end and near end

(iv) A = 0 i.e. $m^2 = 0$

CASE- b (Immovable edge)

The boundary and parametric conditions for immovale edges are as follows :

(i) $w = 0$ at f = 1

(ii) $\frac{dw}{df} = 0$ at f = 1 for far end boundary conditions

(iii) $\frac{d^2w}{df^2}$ = missing for both far end and near end

(iv) $A = 0.1848\frac{w_0^2h^2}{b^2}$ which in turn gives $m^2 = 0.3308\overline{w_0^2}$

For solving the describing differential equation using the parametric and boundary conditions as prescribed in (ii) , the major problem is the omission condition of the second derivative of the deflection , that is $\frac{d^2w}{df^2}$ at both the boundaries.

For sorting out this problem of missing condition of $\frac{d^2w}{df^2}$, we use powerful 'Non-linear shooting method' and found out the nature of variation of $\overline{w}$ with and f.

It is interesting to note that the variation of $\overline{w_0}$, is very very small with the change in initial value of $\frac{d^2w}{df^2}$, from which we found out the value of $\frac{d^2w}{df^2}$=-2.79678 at f = 0. [54]

From the obtained coefficient of regression (Tablel to Table 7) we get the form of 'w' as follows ;

$$w = 0.6990194\left(1 - 2.01f^2 + 0.9933f^4\right)$$

$$= 0.6990194(1 - f^2)^2$$

Where from we get the final form of 'w' as,

$$w = w_0(1 - f^2)^2 \qquad \ldots(1.13)$$

Hence executing the software GJBVP.BAS and GJREG.BAS and having the form of 'w' we get the following solution of Tables 8 to 10 for different load conditions as required for verification.

Table – 8

f	$\frac{d^2\bar{w}}{df^2}$	$\frac{d\bar{w}}{df}$	$\bar{w}$	**Total load** $(\bar{Q})$
0.01	-1.3648	-1.350885E-2	0.335783	
0.109	-1.303067	-0.1435055	0.3278335	
0.208	-1.168616	-0.2663939	0.3074342	
0.307	-0.9614929	-0.3724663	0.2756421	
0.406	-0.6783265	-0.4542704	0.2344875	$\bar{Q}=2$
0.505	-0.318446	-1.5042519	0.1867468	
0.604	0.120072	-1.5147338	0.1359488	
0.703	0.6403393	-0.477828	8.639419E-2	
0.802	1.245221	-0.3851534	4.318466E-2	
0.9009999	1.935289	-0.2284144	1.224927E-2	
0.9999999	2.707934	7.584095E-4	3.491594E-4	

Table – 9

f	$\frac{d^2\bar{w}}{df^2}$	$\frac{d\bar{w}}{df}$	$\bar{w}$	**Total load** $(\bar{Q})$
0.01	-2.729600	-2.639022E-2	0.6447204	
0.109	-2.546085	-0.2701384	0.6289828	
0.208	-2.246908	-0.5077319	0.5902259	
0.307	-1.840313	-0.7111180	0.5295606	
0.406	-1.302696	-0.8677954	0.4509645	$\bar{Q}=4$
0.505	-0.6294453	-0.9645983	0.35971	
0.604	0.1924598	-0.9875402	0.2624066	
0.703	1.184478	-0.9208842	1.671280E-2	
0.802	2.367220	-0.7467034	8.361485E-2	
0.9009999	3.785864	-0.4456758	2.346438E-2	
0.9999999	5.302291	8.5014141E-4	1.733732E-4	

Table – 10

f	$\frac{d^2\overline{w}}{df^2}$	$\frac{d\overline{w}}{df}$	$\overline{w}$	**Total load** ($\overline{Q}$)
0.01	-4.0946	-3.840044E-2		
0.109	-3.706624	-0.3733125	0.8931668	
0.208	-3.199834	-0.7142838	0.8388871	
0.307	-2.603377	-1.00278	0.7534025	
0.406	-1.849476	-1.224532	0.642531	$\overline{Q}=6$
0.505	-0.9254057	-1.3634	0.5136701	
0.604	0.2039742	-1.40103	0.3759046	
0.703	1.597926	-1.314297	0.2403533	
0.802	3.316539	-1.073881	0.12073	
0.9009999	5.378963	-0.6461711	3.389755E-2	
0.9999999	7.734737	8.625984E-4	2.769008E-2	

Hence , using the results from the Table 8 to 10

the final form of $\overline{w_0}$ with $\overline{Q}$ will be as:

$\overline{w_0} + 0.15\overline{w_0^3} = 1.26\overline{Q}$ for case (a) and

$\overline{w_0} + 0.1654\overline{w_0^3} = 0.3701\overline{Q}$ for case (b).

1.6 Numerical Results and conclusion

From the above equation we draw the graph as shown in the figure-1 Results are tabulated also in the following table.

Figure –1 presents a comparative study of the results obtained for dimensionless central deflection, $\frac{w_0}{h}$ versus dimensionless load function, $\frac{qb^4}{Eh^4}$,for immovable edges by [30],[45]. The present study gives us the results using ν=0.3,a=2b, $\lambda=\nu^2$ for simply supported edge.

Table –11 presents the results obtained for dimensionless central deflection $\frac{w_0}{h}$ versus dimensionless load function, $\frac{qb^4}{Eh^4}$, for movable edges by assuming ν=0.3,a=2b, $\lambda=\nu^2$ for simply supported edge. This form of $\overline{w_0}$ is in well agreement of the established well known results.

Table-11(Elliptic Plate with movable edges)

Dimensionless load function $[\frac{qb^4}{Eh^4}]$	Central Delection parameter $[\frac{w_0}{h}]$	
	Present Study	Known[49]
0.0	0.0	0.0
2.0	1.3485	1.3510
4.0	1.8331	1.8380
6.0	2.1604	2.1675
8.0	2.4163	2.4250
10.0	2.6311	2.6405
12.0	2.8117	2.8270
14.0	2.9702	2.9930
16.0	3.1330	3.1430

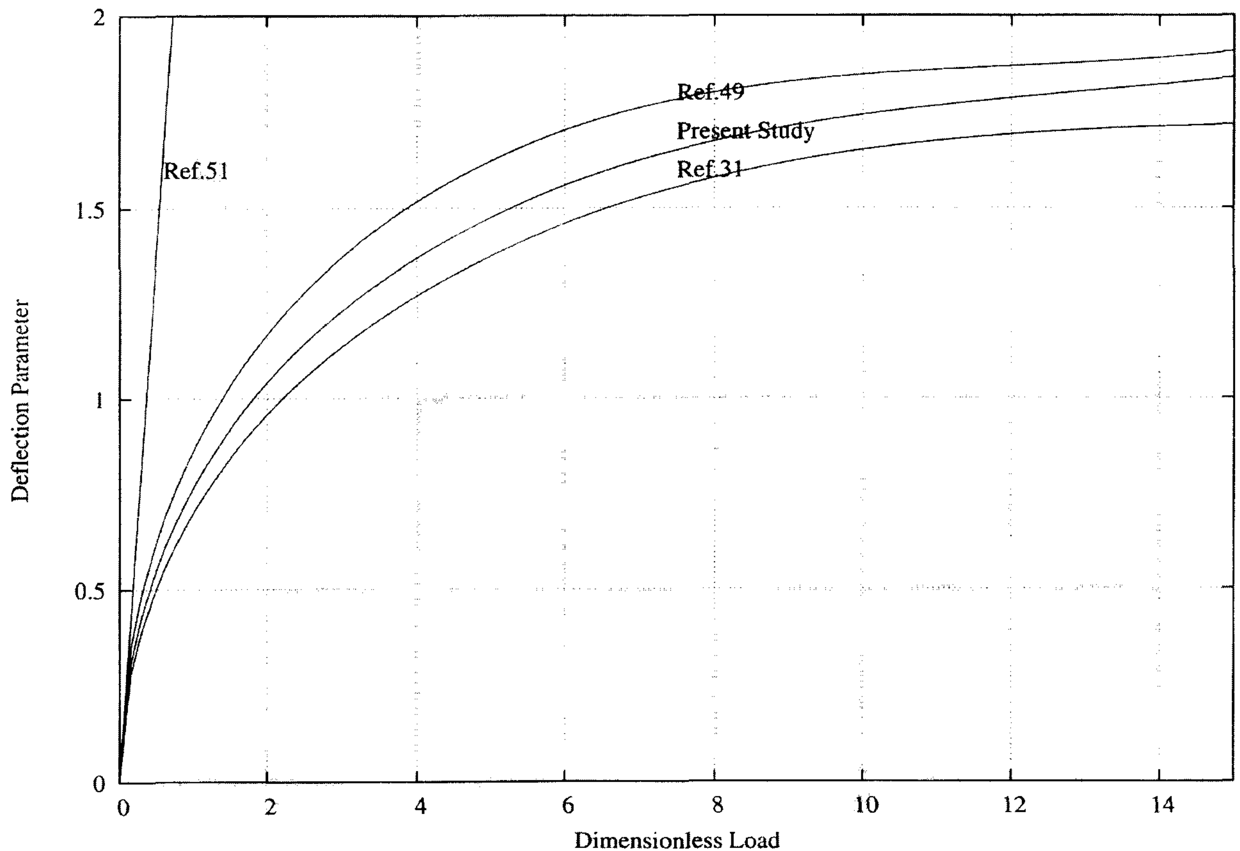
Hence we can write the following conclusions

1) single differential equation gives the behaviors of plates of any shape just changing ‘u’.

2) computation labor is minimum which is urgently needed to the present age

3) investigation is valid for movable as well as immovable edge conditions.

4) the program developed gives directly the form of the deflection form which is in exact agreement with the established one[49].



Deflection Parameter
Dimensionless Load
Ref.51
Ref.49
Present Study
Ref.31
0
0.5
1
1.5
2
0
2
4
6
8
10
12
14

Fig.1