CHAPTER III

PAPER I

NON-LINEAR ANALYSIS OF HEATED RHOMBIC PLATES

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ABSTRACT

This paper concerns a new approach to the investigation of non-linear behaviours of heated rhombic plates. A set of differential equations in oblique co-ordinates have been derived in this investigation. Numerical results showing central deflection parameters versus thermal load functions have been computed for different skew angles θ . For $\theta = 0^{o}$ the results obtained in the present study are in excellent agreement with the known results. It is believed that the results obtained for other different skew angles are completely new.

ANALYSIS

Let us consider a rhombic plate of skew angle 'θ' whose uniform thickness is 'h' and edge-length '2a'. The material of the plate is considered isotropic having mass density 'ρ', Young's Modulus 'E' and Poissn's Ratio 'ν'. The origin of the co-ordinates is located at the geometric centre of the plate

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(vide Fig.3 in Paper II, Chapter I). The deflections are considered to be of the same order of magnitude of the plate thickness, the edge-length being sufficiently large compared to the thickness.

Now the uncoupled set of differential equations in rectangular Cartesian co-ordinates, governing the thermal behaviours of elastic plates (vide Ref. 167) are given by

$$\nabla^4 W - \frac{12A}{h^2}\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) - \frac{6\lambda}{h^2}\Big[\nabla^2 W\Big\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\Big\}$$
$$+ 2\Big\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2\Big\} + 4\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial y}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right)\Big]$$
$$+ \frac{12\alpha_t \tau_0}{h^2}\sqrt{\lambda(1-\nu^2)}\ \nabla^2 W + (1+\nu)\alpha_t \nabla^2 \tau = \frac{q}{D} \quad . \qquad (1)$$

where

$$A = \frac{1}{2}\Big\{\left(\frac{\partial W}{\partial x}\right)^2 + \nu\left(\frac{\partial W}{\partial y}\right)^2\Big\} + \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y}$$
$$- (1+\nu)\alpha_t \tau_0 \quad . \qquad (2)$$

It is to be noted that in the derivation of the above equations (1) and (2) in rectangular Cartesian co-ordinates, the expression

$$(1-\nu^2)\Big[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial W}{\partial y}\right)^2\Big] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial W}{\partial x}\cdot\frac{\partial W}{\partial y}\right)^2 \cdot \frac{1}{2(1+\nu)} \quad .$$

in the total P.E. of the elastic plate (Ref.167) has been replaced by

$$\frac{\lambda}{4}\Big[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\Big]^2 .$$

As a consequence the partial differential equations governing the deflection of the plate have become uncoupled and the two decoupled differential equations (1) and (2) have been obtained.

In the present problem, the temperature is assumed to vary linearly with respect to the thickness direction z. We also note that (Ref.167)

$$T(x,y,z) = \tau_0(x,y) + z\,\tau(x,y)$$

in which

$$\tau_0 = \frac{1}{2}(T_1 + T_2) \ ; \quad \tau = \frac{1}{h}(T_1 - T_2) \ ;$$

where

$$T_1 = T(x,y,+\frac{h}{2}) \quad \text{and} \quad T_2 = T(x,y,-\frac{h}{2}).$$

Clearly τ_0 is the temperature in the middle plane and τ varies along the thickness of the plate and hence $\tau \neq \tau_0$.

The plan of the skew co-ordinates (x_1, y_1, θ) is shown in Fig.3 in the Paper II, Chapter I.

We now transform the equation (2) in oblique co-ordintes. For Simply-Supported plates the boundary conditions are

$$W = 0 \quad \text{at} \quad x_1 = \pm a \quad \text{and at} \quad y_1 = \pm a \ ;$$

$$\frac{\partial^2 W}{\partial x_1^2} = 0 \quad \text{at} \quad x_1 = \pm a \quad \text{and} \quad \frac{\partial^2 W}{\partial y_1^2} = 0 \quad \text{at} \quad y_1 = \pm a \ .$$

Then let us choose the deflection function for the Simply-Supported plate as

$$W = W_0 \cos\frac{\pi x_1}{2a} \cos\frac{\pi y_1}{2a} \tag{3}$$

which clearly satisfies the above-mentioned boundary conditions.

Now putting (3) in the transformed form of equation (2) in oblique co-ordinates and then integrating it over the entire surface of the plate, we obtain the value of A in the following form

$$A = \frac{\pi^2 W_0^2}{32a^2}(1+\nu+2\tan^2\theta) - (1+\nu)\alpha_t \tau_0 \,. \tag{4}$$

(As normal displacement W is our primary interest, the in-plane displacements u,v have been eliminated through integration by the choice of appropriate functions for such displacements). Again transforming the equation (1) in oblique co-ordinates, introducing (3) and (4) in the transformed equation and then applying the Galerkin's error minimising technique we get the following equation determining central deflection parameter W_o/h depending on thermal load function $q'a^4/Eh^4$

$$\Big[(1+2\tan^2\theta)\sec^2\theta - \frac{6S}{(1+\nu)\pi^2}\Big\{(1+\nu)(1+\nu+2\tan^2\theta) + 2\sqrt{\lambda(1-\nu^2)}\cdot\sec^2\theta\Big\}\Big]\left(\frac{W_o}{h}\right) + \frac{3}{8}\Big[(1+\nu+2\tan^2\theta)^2 + \frac{\lambda}{4}(8+49\tan^2\theta+29\tan^4\theta)\Big]\left(\frac{W_o}{h}\right)^3 = \frac{768(1-\nu^2)}{\pi^6}\left(\frac{q'a^4}{Eh^4}\right). \tag{5}$$

where

$$S = 2(a/h)^2(1+\nu)\alpha_t\tau_0$$

and

$$q' = q - D\alpha_t(1+\nu)\nabla^2\tau \,.$$

The equation (5) is applicable for the immovable edge conditions of the Simply-Supported skew plate. For the movable edge conditions we have A = 0, so that the equation (5) takes the form

$$\left[(1+2\tan^2\theta)\sec^2\theta - \frac{12S}{(1+\nu)\pi^2}\sqrt{\lambda(1-\nu^2)}\cdot\sec^2\theta\right]\left(\frac{W_0}{h}\right) + \frac{3\lambda}{32}\left(8+49\tan^2\theta+29\tan^4\theta\right)\left(\frac{W_0}{h}\right)^3 = \frac{768(1-\nu^2)}{\pi^6}\left(\frac{q'a^4}{Eh^4}\right). \qquad (6)$$

NUMERICAL RESULTS

Numerical results are presented here in tabular forms (Tables 1 and 2) for S = 0, 0.1, $\theta = 0^\circ, 15^\circ, 30^\circ$ and $q'a^4/Eh^4 = 2, 4, 8, 10$.

TABLE -1

$S = 0$, i.e. $\tau_o = 0$

$\frac{q'a^4}{Eh^4}$	W_o/h by Present Method						W_o/h by Berger's Method *		
	$\theta = 0^\circ$		$\theta = 15^\circ$		$\theta = 30^\circ$		(Ref.134) $\theta = 0^\circ$	$\theta = 15^\circ$	$\theta = 30^\circ$
	Movable Edge	Immovable Edge	Movable Edge	Immovable Edge	Movable Edge	Immovable Edge	Immovable Edge	Immovable Edge	Immovable Edge
2	1.30156	0.91435	1.08167	0.82069	0.6269	0.53604	0.9013	0.79972	0.53671
4	2.1909	1.3131	1.85443	1.20857	1.14734	0.84631	1.29017	1.16888	0.848
8	3.23354	1.78866	2.8581	1.67119	1.89675	1.22355	1.75406	1.60902	1.2266
10	3.73498	1.9613	3.2243	1.83866	2.17977	1.3597	1.92254	1.76847	1.36324

* Berger's method has been applied to the present problem by neglecting e_2, the second strain invariant, in the expression for total P.E. of the plate.

TABLE - 2

S = 0.1, i.e. $\tau_o \neq 0$

$\frac{q'a^4}{Eh^4}$	W_o/h by Present Method						W_o/h by Berger's Method *		
	$\theta = 0^o$		$\theta = 15^o$		$\theta = 30^o$		(Ref.134) $\theta = 0^o$	$\theta = 15^o$	$\theta = 30^o$
	Movable Edge	Immovable Edge	Movable Edge	Immovable Edge	Movable Edge	Immovable Edge	Immovable Edge	Immovable Edge	Immovable Edge
2	1.32786	0.94985	1.10168	0.83899	0.63597	0.55925	0.94058	0.83515	0.56109
4	2.22082	1.34324	1.87831	1.20992	1.1604	0.86901	1.32336	1.19954	0.87185
8	3.35106	1.81316	2.88067	1.65221	1.9111	1.24302	1.781	1.63412	1.24706
10	3.76118	1.98415	3.24585	1.81269	2.19385	1.37799	1.94764	1.79188	1.38247

* e_2 = 0 according to Berger's method.

OBSERVATIONS

From the numerical analysis of the undertaken problem the following observations are made :

(i) The nature of the central deflection of a skew plate under thermal loading is the same as that of the plate under mechanical loading, i.e. the central deflection increases continuously with the increase of loading for any edge conditions of the skew plate, whether movable or immovable.

(ii) Central deflection for movable edge conditions of the skew plate is always greater than that for immovable edge conditions of the plate, for the same loading in the two cases.

(iii) Irrespective of the edge conditions, the central deflection decreases with the increase of the skew angle.

(iv) The results for immovable edge conditions of the skew plate obtained by the present method, agree well with the results obtained by Berger's method. It is to be noted that Berger's method is a purely approximate method based on the neglect of e_2. But present study is based on Banerjee's hypothesis which suggests a modified strain-energy expression, and thus this model embraces less approximation (Ref.162) than that of Berger. Again Berger's method is meaningful only or immovable edge conditions of the plates.

and

(v) The deflections increase with τ_0.