CHAPTER I

NON-LINEAR STATIC ANALYSES OF THIN RHOMBIC PLATES

PAPER I

LARGE DEFLECTIONS OF RHOMBIC PLATES- A NEW APPROACH*

ABSTRACT

In this paper non-linear static behaviour of Simply-Supported rhombic plates has been analysed following Banerjee's hypothesis. A set of uncoupled differential equations is obtained in oblique co-ordinates and solved by applying the Galerkin techinque. The case of a Simply-Supported rhombic plate is discussed in detail. Calculations have been carried out for different skew angles. To test the accuracy of the theoretical results so obtained, experiments are carried out on copper and steel rhombic plates. The theoretical results are found to be in excellent agreement with those obtained from the experimental data.

ANALYSIS

Let us consider a rhombic plate of an elastic, isotropic material, having uniform thickness 'h'. Let the size

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of each side of the skew plate be 'a' which is sufficiently large compared to 'h'. The origin of the rectangular Cartesian co-ordinate (x,y) is located at one of the corners of the skew plate (vide Fig.1). The plate is considered to be Simply-Supported along its edges and loaded uniformly all over.

Following Banerjee's hypothesis[162], the differential equations, referred to the system of rectangular Cartesian co-ordinates governing the deflections of the plate are

$$\nabla^4 W - \frac{12A}{h^2}\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) - \frac{6\lambda}{h^2}\left\{\nabla^2 W\left[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right]\right.$$
$$+2\left[\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2\right]$$
$$\left.+4\left(\frac{\partial^2 W}{\partial x \partial y}\right)\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial y}\right)\right\} = \frac{q}{D}. \qquad (1)$$

where

$$A = \frac{1}{2}\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \nu\left(\frac{\partial W}{\partial y}\right)^2\right\} + \frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} \qquad (2)$$

is a constant depending on the surface and edge conditions of the plate, and ∇^2 is the Laplacian operator.

For a skew plate, let us adopt a system of oblique co-ordinates (x_1, y_1, θ) as shown in Fig.1, θ being the skew angle. Clearly, $x = x_1 \cos\theta$, and $y = y_1 + x_1 \sin\theta$ (3)

are the co-ordinate transformation equations. Hence the partial differential operators become

$$\frac{\partial}{\partial x} \equiv \sec\theta\left(\frac{\partial}{\partial x_1} - \sin\theta\frac{\partial}{\partial y_1}\right); \quad \frac{\partial}{\partial y} \equiv \frac{\partial}{\partial y_1}; \quad \frac{\partial^2}{\partial y^2} \equiv \frac{\partial^2}{\partial y_1^2};$$

$$\frac{\partial^2}{\partial x^2} \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\frac{\partial^2}{\partial x_1 \partial y_1}\right) + \tan^2\theta\frac{\partial^2}{\partial y_1^2};$$

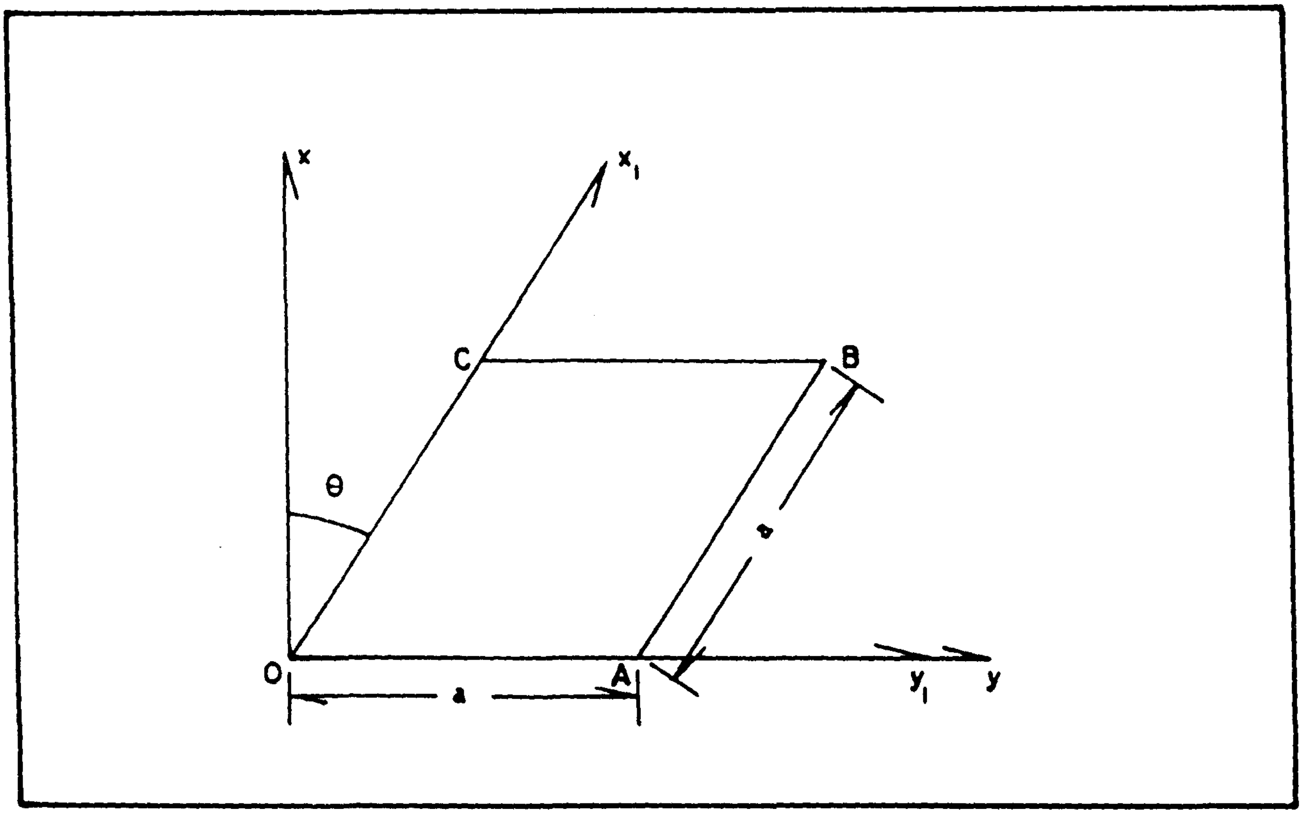

FIG. 1. PLAN FORM OF SKEW PLATE.

$$\frac{\partial^2}{\partial x \partial y} \equiv \operatorname{Sec}\theta\left(\frac{\partial^2}{\partial x_1 \partial y_1} - \operatorname{Sin}\theta \frac{\partial^2}{\partial y_1^2}\right) ;$$

$$\nabla^2 \equiv \operatorname{Sec}^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\operatorname{Sin}\theta \frac{\partial^2}{\partial x_1 \partial y_1} + \frac{\partial^2}{\partial y_1^2}\right)$$

and

$$\nabla^4 \equiv \operatorname{Sec}^4\theta\left[\frac{\partial^4}{\partial x_1^4} - 4\operatorname{Sin}\theta\left(\frac{\partial^4}{\partial x_1^3 \partial y_1} + \frac{\partial^4}{\partial x_1 \partial y_1^3}\right) + 2\left(1 + 2\operatorname{Sin}^2\theta\right)\frac{\partial^4}{\partial x_1^2 \partial y_1^2} + \frac{\partial^4}{\partial y_1^4}\right]. \quad (4)$$

Using these operators, transforming the differential equations (1) and (2) in oblique co-ordinates, we arrive at the following set of transformed differential equations :

$$\begin{aligned} &\operatorname{Sec}^4\theta\left[\frac{\partial^4 W}{\partial x_1^4} - 4\operatorname{Sin}\theta\left(\frac{\partial^4 W}{\partial x_1^3 \partial y_1} + \frac{\partial^4 W}{\partial x_1 \partial y_1^3}\right) + 2\left(1+2\operatorname{Sin}^2\theta\right)\frac{\partial^4 W}{\partial x_1^2 \partial y_1^2} \right.\\ &\left. + \frac{\partial^4 W}{\partial y_1^4}\right] - \frac{12A}{h^2}\left[\operatorname{Sec}^2\theta\left(\frac{\partial^2 W}{\partial x_1^2} - 2\operatorname{Sin}\theta\frac{\partial^2 W}{\partial x_1 \partial y_1}\right) + \tan^2\theta\frac{\partial^2 W}{\partial y_1^2} \right.\\ &\left. + \nu\frac{\partial^2 W}{\partial y_1^2}\right] - \frac{6\lambda}{h^2}\left\{\operatorname{Sec}^4\theta\left(\frac{\partial^2 W}{\partial x_1^2} - 2\operatorname{Sin}\theta\frac{\partial^2 W}{\partial x_1 \partial y_1} + \frac{\partial^2 W}{\partial y_1^2}\right)\left[\left(\frac{\partial W}{\partial x_1}\right)^2 \right.\right.\\ &\left. + \left(\frac{\partial W}{\partial y_1}\right)^2 - 2\operatorname{Sin}\theta\left(\frac{\partial W}{\partial x_1}\right)\left(\frac{\partial W}{\partial y_1}\right)\right] + 2\left[\operatorname{Sec}^4\theta\left(\frac{\partial^2 W}{\partial x_1^2} - 2\operatorname{Sin}\theta\frac{\partial^2 W}{\partial x_1 \partial y_1}\right.\right.\\ &\left.\left. + \operatorname{Sin}^2\theta\frac{\partial^2 W}{\partial y_1^2}\right)\left(\frac{\partial W}{\partial x_1} - \operatorname{Sin}\theta\frac{\partial W}{\partial y_1}\right)^2 + \frac{\partial^2 W}{\partial y_1^2}\left(\frac{\partial W}{\partial y_1}\right)^2\right]\\ &\left. + 4\operatorname{Sec}^2\theta\left(\frac{\partial^2 W}{\partial x_1 \partial y_1} - \operatorname{Sin}\theta\frac{\partial^2 W}{\partial y_1^2}\right)\left(\frac{\partial W}{\partial x_1} - \operatorname{Sin}\theta\frac{\partial W}{\partial y_1}\right)\left(\frac{\partial W}{\partial y_1}\right)\right\}\\ &= \frac{q}{D}. \end{aligned} \quad (5)$$

and

$$\begin{aligned} A = \frac{1}{2}&\left\{\operatorname{Sec}^2\theta\left[\left(\frac{\partial W}{\partial x_1}\right)^2 - 2\operatorname{Sin}\theta\left(\frac{\partial W}{\partial x_1}\right)\left(\frac{\partial W}{\partial y_1}\right) + \operatorname{Sin}^2\theta\left(\frac{\partial W}{\partial y_1}\right)^2\right]\right.\\ &\left. + \nu\left(\frac{\partial W}{\partial y_1}\right)^2\right\} + \operatorname{Sec}\theta\left(\frac{\partial u}{\partial x_1} - \operatorname{Sin}\theta\frac{\partial u}{\partial y_1}\right) + \nu\frac{\partial v}{\partial y_1}. \end{aligned} \quad (6)$$

Now to solve the problem, let us assume

$$W = W_0 \operatorname{Sin}\frac{\pi x_1}{a} \operatorname{Sin}\frac{\pi y_1}{a} \quad (7)$$

W_0 being the maximum central deflection. Clearly W satisfies the following Simply-Supported edge conditions

$$W = 0 \text{ at } x_1 = \pm a \text{ and at } y_1 = \pm a \; ;$$

$$\frac{\partial^2 W}{\partial x_1^2} = 0 \text{ at } x_1 = \pm a \text{ and } \frac{\partial^2 W}{\partial y_1^2} = 0 \text{ at } y_1 = \pm a .$$

For the value of A, let us integrate equation (6) over the whole area of the plate. Then we have

$$\int_0^a\int_0^a A\cos\theta\, dx_1 dy_1 = \frac{1}{2}\int_0^a\int_0^a \Big\{ \sec^2\theta \Big[\Big(\frac{\partial W}{\partial x_1}\Big)^2 + \sin^2\theta \Big(\frac{\partial W}{\partial y_1}\Big)^2 - 2\sin\theta \Big(\frac{\partial W}{\partial x_1}\Big)\Big(\frac{\partial W}{\partial y_1}\Big)\Big] + \nu \Big(\frac{\partial W}{\partial y_1}\Big)^2 \Big\} \cos\theta \, dx_1 dy_1 .$$

After integration, we get

$$A = \frac{\pi^2 W_0^2}{8a^2}\left(1 + \nu + 2\tan^2\theta\right) \tag{8}$$

For movable edge conditions the value of A will be zero. Here, it is to be noted that, since the normal displacements are our primary interest, the in-plane displacements in equation (2) have been eliminated through integration by choosing suitable expressions for them, compatible with their boundary conditions.

Now, applying Galerkin's method of approximation to the transformed differential equation (5) and keeping in mind the value of A from equation (8), we get the following cubic equation determining β ($= W_0 / h$).

$$\left(1+\sin^2\theta\right)\beta + \frac{3}{8}\Big\{\Big[(1+\nu) + (1-\nu)\sin^2\theta\Big]^2 + \nu^2\left(5+\sin^2\theta\right)\Big\}\beta^3 = \frac{4}{\pi^6}\left(\frac{q a^4}{D h}\right)\cos^4\theta \tag{9a}$$

For the sake of comparison we adopt, now, the well-known equation of Berger[36], where e_2 has been neglected. The corresponding cubic equation determining the central deflection parameter (for immovable edges only) takes the following form (after applying Galerkin's technique) :

$$(1+\sin^2\theta)\beta + 1.5\,\beta^3 = \frac{4}{\pi^6}\left(\frac{qa^4}{Dh}\right)\cos^4\theta \quad . \tag{9b}$$

NUMERICAL CALCULATIONS

For a steel plate we have $E = 2 \times 10^{12}$ dyne/cm^2. and $\nu = 0.3$, for which equation (9a) becomes

$$(1+\sin^2\theta)\beta + \frac{3}{8}\left[(1.3+0.7\sin^2\theta)^2 + 0.09(5+\sin^2\theta)\right]\beta^3$$
$$= 22.66\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \quad . \tag{10a}$$

whereas for a copper plate we have $E = 1.25 \times 10^{12}$ dyne/cm^2. and $\nu = 0.333$, so that equation (9a) becomes

$$(1+\sin^2\theta)\beta + \frac{3}{8}\left[(1.333+0.667\sin^2\theta)^2 + 0.11(5+\sin^2\theta)\right]\beta^3$$
$$= 35.46\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \quad . \tag{10b}$$

Also, for a steel plate equation (9b) becomes

$$(1+\sin^2\theta)\beta + 1.5\,\beta^3$$
$$= 22.66\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \quad . \tag{10c}$$

and for a copper plate it becomes

$$(1+\sin^2\theta)\beta + 1.5\beta^3$$
$$= 35.46 \times 10^{-15}\left(\frac{q a^4}{h^4}\right)\cos^4\theta \quad . \qquad (10d)$$

METHOD OF EXPERIMENT

A sketch of the apparatus used for the experimental purpose is shown in Fig.2. Two skew boxes with upper side open are constructed, each of whose four side walls are made of steel. Each vertical wall of one box is 16 cm. and of the other is 14 cm. The upper side of each wall is made sharp (knife edge), care being taken to see that all the knife edges lie on the same horizontal plane. The walls of the box with sides 16 cm. long are welded in such a manner that the two opposite angles are each 75° and the other two opposite angles are each 105°. Two opposite angles of the second box with sides 14 cm. long are each 60° and the other two opposite angles are each 120°. Two holes are drilled on two opposite sides of each box and fitted with short metal pipes, one of which acts as an air inlet and the other as an air outlet.

For the experiment with the first skew box, the centre of the box is first found and then a plumb-line is set as an indicator along the vertical line on which the centre of the box lies. For the free movable boundary conditions one Test plate (which is approximately mirror surfaced) is symme-

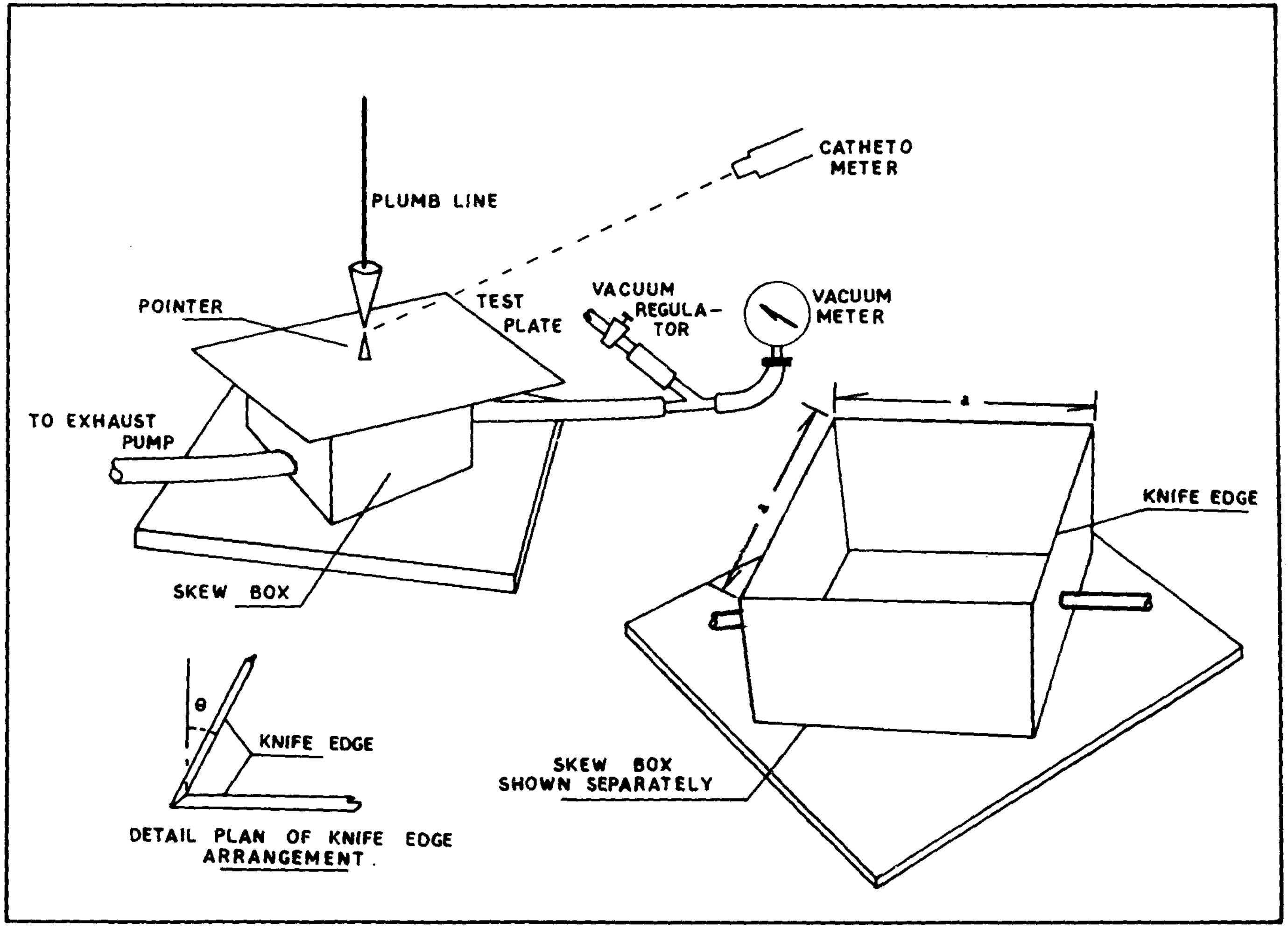

FIG. 2. THE EXPERIMENTAL ARRANGEMENT.

trically placed on the knife edges of the box and a pointer is fixed on the upper surface of the Test plate with some adhesive along the plumb-line. The outlet pipe is then joined to an exhaust pump by rubber tubing and the inlet pipe is joined to a standard vacuum meter and an air pressure regulator (as shown in the sketch). Along the contact line beneath the Test plate some thick grease is used to make the box perfectly airtight, (Grease does not apply any appreciable tension on the plate). When the exhaust pump operates, the box becomes evacuated, thereby causing the depression of the Test plate by the excess outside air pressure, which is uniform all over the effective skew part of the Test plate. The central deflection of the Test plate is easily measured with the help of a precision cathetometer set at a distance of approximately 1.5m. from the pointer.

To make the free boundaries of a skew plate immovable, four pieces of steel collars are taken whose lengths are equal to the length of outer boundary line of the skew plate. The collars are kept outside the box in contact with the lower surface of the plate and with the side walls of the box and then the collars are tightly clamped with the Test plate using nuts and bolts in sufficient number well outside the boundary of skew section.

Tables 1 and 2 present a comparative view of the various theoretical and experimental values of the central deflection parameter β ($= W_o / h$) for different values of the load function Q ($= qa^4/Dh$), for the case of steel plate and copper plate respectively.

Comparison tables (showing theoretical vs experimental results as well as showing changes for immovable edge conditions from movable edge conditions).

TABLE 1

(For Steel Plate)

a = 16cm., when $\theta = 15^{\circ}$; a = 14cm., when $\theta = 30^{\circ}$; h = 0.1343cm.

$\beta = w_o/h$, when $\theta = 15^{\circ}$

$Q = \frac{qa^4}{Dh}$	Movable Edges			Immovable Edges				
	From Banerjee hypothesis	From Experiment	Percentage of error	From Berger Method	From Banerjee hypothesis	From Experiment	Percentage of error: From Berger Method	Percentage of error: From Banerjee hypothesis
111.72	0.3716	0.3872	4.0 %	0.3285	0.3454	0.36485	9.96 %	5.33 %
223.44	0.7038	0.7372	4.5 %	0.5378	0.5914	0.6329	15 %	6.56 %
335.16	0.98592	1.0201	3.3 %	0.6843	0.7703	0.8414	18.67 %	8.45 %
446.88	1.22484	1.2882	4.9 %	0.7982	0.9107	0.9903	19.40 %	8 %
558.6	1.4303	1.5115	5.4 %	0.8924	1.027	1.1244	20.60 %	8.66 %

Contd.........

TABLE 1 (Continued)

$Q = \frac{qa^4}{Dh}$	Movable Edges			Immovable Edges				
	From Banerjee hypothesis	From Experiment	Percentage of error	From Berger Method	From Banerjee hypothesis	From Experiment	Percentage of error: From Berger Method	Percentage of error: From Banerjee hypothesis
			$\beta = W_o/h$, when $\theta = 30^o$					
65.5	0.12208	0.134	8.90 %	0.1202	0.1209	0.12658	5.00 %	4.49 %
131	0.2427	0.25316	4.13 %	0.2301	0.2346	0.25316	9.11 %	7.33 %
196.5	0.3604	0.37975	5.10 %	0.3254	0.3369	0.36485	10.80 %	7.66 %
262	0.4742	0.4989	5.00 %	0.40782	0.4276	0.46165	11.66 %	7.40 %
327.5	0.5835	0.61802	5.59 %	0.4795	0.5081	0.55845	14.20 %	9.00 %

Average percentage of error by utilising Banerjee's hypothesis is only around 6% for skew angles $\theta = 15^o$, and $\theta = 30^o$ whereas by utilising Berger method it is around 17% for $\theta = 15^o$ and 10% for $\theta = 30^o$

TABLE 2

(For Copper Plate).

$a = 16$cm., when $\theta = 15^{\circ}$; $a = 14$cm., when $\theta = 30^{\circ}$; $h = 0.0789$cm.

$\beta = W_o/h$, when $\theta = 15^{\circ}$

$Q = \frac{qa^4}{Dh}$	Movable Edges			Immovable Edges				
	From Banerjee hypothesis	From Experiment	Percentage of error	From Berger Method	From Banerjee hypothesis	From Experiment	Percentage of error: From Berger Method	Percentage of error: From Banerjee hypothesis
1467.53	2.3727	2.4208	2 %	1.36820	1.57802	1.673	18.22 %	5.68 %
2935.06	3.2506	3.308	1.70 %	1.79580	2.08753	2.23067	19.50 %	6.40 %
4402.59	3.8437	3.9924	3.72 %	2.0891	2.43578	2.6109	19.98 %	6.70 %
5870.12	4.307	4.4867	4 %	2.32003	2.7095	2.90241	20.07 %	6.65 %
7337.65	4.6935	4.90494	4.30 %	2.5138	2.93893	3.1559	20.35 %	6.90 %

Contd ....

TABLE 2 (Continued)

$\beta = w_o/h$, when $\theta = 30^\circ$

$Q = \frac{qa^4}{Dh}$	Movable Edges			Immovable Edges				
	From Banerjee hypothesis	From Experiment	Percentage of error	From Berger Method	From Banerjee hypothesis	From Experiment	Percentage of error	
							From Berger Method	From Banerjee hypothesis
860.2	1.2602	1.2801	1.55 %	0.8553	0.9283	0.9886	13.50 %	6.10 %
1720.4	1.9429	2.0279	4.20 %	1.1901	1.3095	1.40684	15.50 %	6.92 %
2580.6	2.4064	2.5095	4.10 %	1.4156	1.5657	1.6857	16.00 %	7.12 %
3440.8	2.765	2.9404	6.00 %	1.5913	1.7649	1.90114	16.30 %	7.17 %
4301	3.0616	3.2319	5.30 %	1.7376	1.9307	2.09125	16.91 %	7.70 %

Average percentage of error by utilising Banerjee's hypothesis is only around 5 % for skew angles $\theta = 15^\circ$ and $\theta = 30^\circ$ whereas by utilising Berger method it is around 20 % for $\theta = 15^\circ$ and around 15 % for $\theta = 30^\circ$.

N.B. - Errors are calculated considering experimental results as standard (sacrificing instrumental and personal errors).

OBSERVATIONS

It is observed from the two tables that the results of the present study are in excellent agreement with those obtained from the experimental analysis. It is well-known that Berger's method fails[114] miserably under movable edge conditions. The results for simply-supported immovable edges, obtained by Berger's method (as shown in the Tables 1 and 2) show that this method is not even acceptable from the practical point of view. It is worth noting that Berger's method always gives less deflections for a given load. The errors of Berger's method (as shown in Tables 1 and 2) are certainly questionable from the view point of safety design.

It is observed that deflections for movable edges are always greater than those for immovable edges. This is quite expected from the practical point of view, because movable edge conditions give stress-free boundary and, hence, there are large energy changes in the boundary.

Here the results for skew angles $\theta = 15^{\circ}$ and 30° only have been considered, because, for greater values of the skew angles the effect of non-linearity does not play important role in design, and the study of linear analysis serves the practical purpose.

PAPER II

NON-LINEAR BEHAVIOURS OF CLAMPED RHOMBIC PLATES- A NEW APPROACH

ABSTRACT

In this paper non-linear static behaviours of clamped (along all edges) thin rhombic plates under uniform normal pressure have been analysed following Banerjee's hypothesis. Numerical results for different skew angles are presented. Comparisons (both numerically and graphically) are made with available existing results for skewed plates. The effects of skew angle on large deflections are carefully investigated. To test the accuracy of the theoretical results, so obtained, experiments have also been carried out on copper-made and steel-made plates. Both the cases of movable and immovable edge conditions have been delt with. It is observed that the present theoretical results are to the close proximity of the results obtained from the experimental analysis.

ANALYSIS

Let us consider a rhombic plate as shown in Fig.3. It is of an isotropic, elastic material, whose uniform thickness

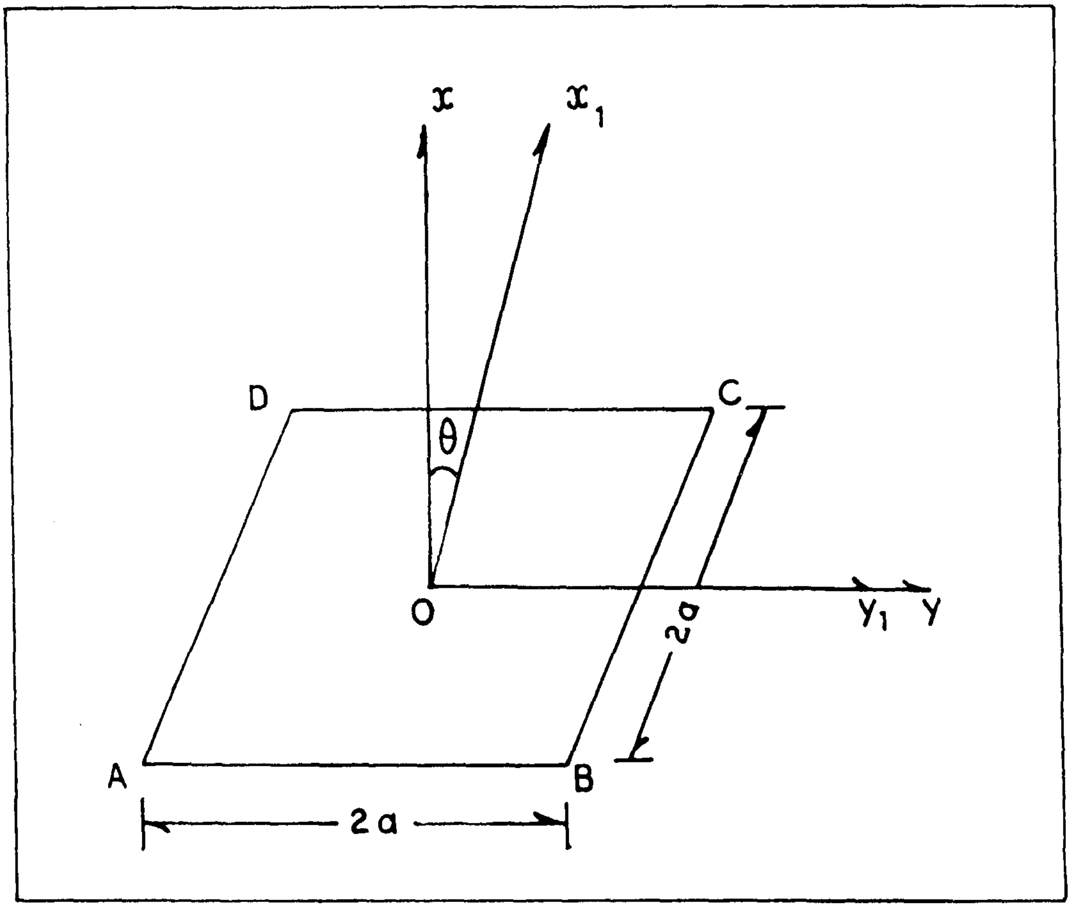

FIG.3: PLAN FORM OF SKEW PLATE AND CO-ORDINATE SYSTEM

is 'h'. Also let the sides of the rhombic plate be each of '2a', sufficiently great compared to 'h'. The origin of the rectangular cartesian co-ordinates (x,y) is located at the geometric centre of the skewed plate. The plate is considered to be clamped along its four edges and loaded uniformly all over.

Following Banerjee's hypothesis, the differential equations, referred to the system of rectangular cartesian co-ordinates are transformed in oblique co-ordinates as in paper I.

Now to solve the differential equations (5) and (6) in paper I, we assume

$$W = W_o \cos^2 \frac{\pi x_1}{2a} \cos^2 \frac{\pi y_1}{2a} \quad . \tag{11}$$

W_o being the maximum central deflection. Clearly the deflection function W satisfies all the boundary conditions for a skew plate clamped along its four edges, viz.

$$W = o \text{ at } x_1 = \pm a \text{ and at } y_1 = \pm a.$$

$$\frac{\partial W}{\partial x_1} = o \text{ at } x_1 = \pm a \text{ and } \frac{\partial W}{\partial Y_1} = o \text{ at } y_1 = \pm a.$$

To determine the value of A, we are to integrate equation (6), as usual, over the whole area of the skew plate. Thus we have

$$\int_{-a}^{+a}\int_{-a}^{+a} A\cos\theta \, dx_1 dy_1 = \frac{1}{2}\int_{-a}^{+a}\int_{-a}^{+a}\Big[\sec^2\theta\Big\{\Big(\frac{\partial W}{\partial x_1}\Big)^2 + \sin^2\theta\Big(\frac{\partial W}{\partial y_1}\Big)^2 - 2\sin\theta\Big(\frac{\partial W}{\partial x_1}\Big)\Big(\frac{\partial W}{\partial y_1}\Big)\Big\} + \nu\Big(\frac{\partial W}{\partial y_1}\Big)^2\Big]\cos\theta \, dx_1 dy_1 .$$

After integration we get

$$A = \frac{3\pi^2 W_0^2}{128a^2}\left(1 + \nu + 2\tan^2\theta\right) .$$

(12)

Now applying Galerkin's method of approximation to the transformed differential equation (5) and keeping in mind the value of A from (12) we get the following cubic equation determining the deflection parameter $\beta = W_o/h$

$$\left(2 + \sin^2\theta\right)\beta + \left\{\frac{27}{128}\left(1 + \nu + 2\tan^2\theta\right)^2 \cos^4\theta + \frac{30\nu^2}{128}\left(13 + 5\sin^2\theta\right)\right\}\beta^3 = \frac{4}{\pi^4}\cos^4\theta\, Q .$$

(13)

where $Q = qa^4/Dh$ is the load function.

NUMERICAL CALCULATIONS

For a copper plate we have, $E = 1.25 \times 10^{12}$ dyne /cm^2. and $\nu = 0.333$, so that for such a plate, the equation (13) becomes

$$\left(2 + \sin^2\theta\right)\beta + \left\{\frac{27}{128}\left(1.333 + 2\tan^2\theta\right)^2 \cos^4\theta + \frac{3.327}{128}\left(13 + 5\sin^2\theta\right)\right\}\beta^3 = 0.041\cos^4\theta\, Q .$$

(14)

whereas for a steel plate we have $E = 2 \times 10^{12}$ dyne/cm^2. and $\nu = 0.3$, so that for such a plate, the equation (13) becomes

$$(2+\sin^2\theta)\beta + \left\{\frac{27}{128}(1.3 + 2\tan^2\theta)^2 \cos^4\theta + \frac{2\cdot 7}{128}(13 + 5\sin^2\theta)\right\}\beta^3 = 0.041\cos^4\theta\, Q \,. \quad (15)$$

METHOD OF EXPERIMENT

The sketch of the apparatus used for the experimental purpose is shown in Fig.4. Three skew boxes with upper side open are constructed each of whose four side-walls are made of steel of 6mm. thickness. Each vertical wall of two boxes is 16cm. long and of the other is 14cm. long. The upper side of each wall is made sharp (knife edge), care being taken so that all the knife-edges lie on the same horizontal plane. The walls of the boxes with sides 16cm.long are arc-welded in such a manner that the two opposite angles are each 75°and other two opposite angles are each 105°. The two opposite angles of the 3rd box with sides 14cm. long are each 60°and the other two opposite angles are each 120°. Two holes are drilled on two opposite side-walls of each box and then fitted with short metal pipes, one of which acts as an air inlet and the other as an air outlet.

For the experiment with one skew box, the centre of the box is first found and then a plumb-line is set as indicator along the same vertical line on which the centre of the box lies. For the clamped movable boundary conditions the experimental plate (which is approximately mirror surfaced) is

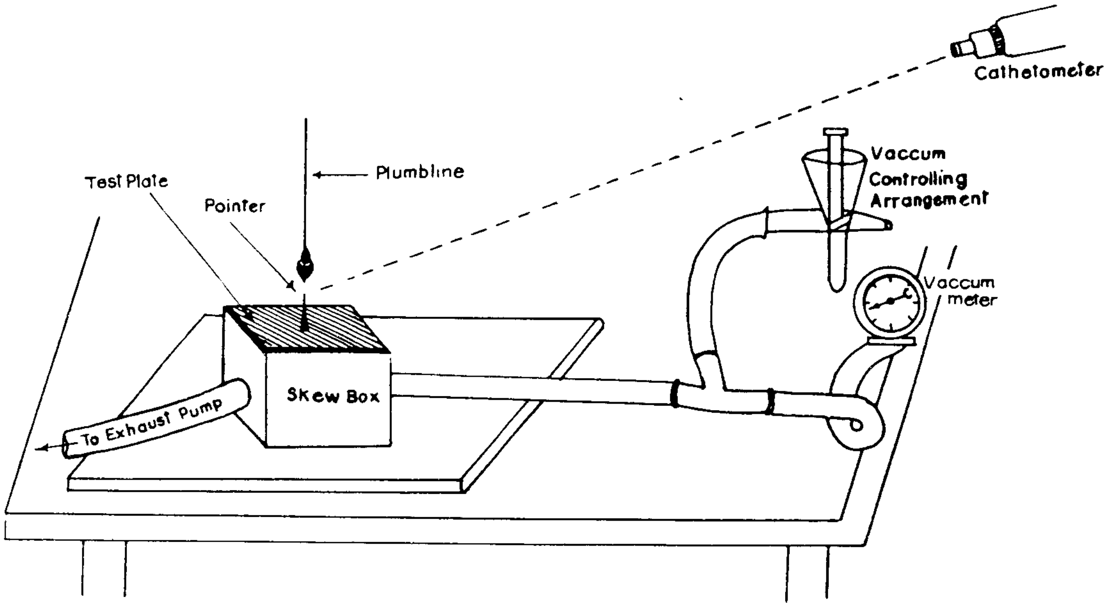

FIG-4. THE EXPERIMENTAL ARRANGEMENT

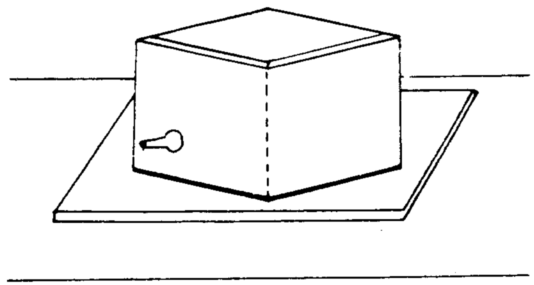

FIG-4a THE SKEW BOX SHOWN SEPARATELY

symmetrically placed on the knife-edged side of the box and a pointer (made of very thin plastic rod) is fixed on the upper surface of the experimental plate with some adhesive, along the plumb-line. The contact lines of the plate with the knife-edges are uniformly arc-welded from outside. The outlet pipe of the box is connected to a vacuum pump and the inlet pipe is connected to a standard vacuum meter and an air inlet regulator (using a T-section as shown in the sketch). When the vacuum pump operates, the box is evacuated, thereby causing the depression of the experimental plate by the excess outside air pressure, which is uniform all over the effective skew part of the plate and the pressure is noted from the pressure gauge. The central deflection of the plate is easily measured with the help of a precesion cathetometer set at a distance of approximately 1.5 metres from the pointer.

To make the boundaries of the skew plate 'clamped immovable', the elongated portions of the plate beyond the knife-edge of the box are cut and then the whole boundary of the experimental plate is are-welded with the supporting knife edges covering the full thickness (h) of the experimental plate at its boundary. The experimental procedure is now, as usual.

The following tables and graphs show a comparative study of the central deflection parameter β Vs. load function Q obtained by the theoretical and experimental methods. For movable edge conditions A = O,as usual.

TABLE 1

Showing comparison of results obtained from different theories and experiment for copper-made skew-plate ($\nu = 0.333$, $E = 1.25 \times 10^{12}$ dyne/cm^2, $\theta = 15^o$, a = 8cm., and h = 0.0789cm.)

Value of Q	Movable Edges				
	Uniform pressure applied in the Expt.(inch of Hg)	Value of W_0 from the Expt. (cm)	Value of β by the Expt.	Value of β by Banerjee's hypothesis	Percentage of error w.r. to Expt.
20	-	-	-	-	-
50	-	-	-	-	-
61.08	2"	0.073	0.92522	0.92284	0.257 %
100	-	-	-	-	-
122.16	4"	0.121	1.5336	1.5202	0.874 %
150	-	-	-	-	-
183.24	6"	0.158	2.00253	1.93987	3.13 %
200	-	-	-	-	-
244.32	8"	0.182	2.30672	2.26643	1.75 %
250	-	-	-	-	-
300	-	-	-	-	-
305.4	10"	0.205	2.59823	2.5366	2.372 %

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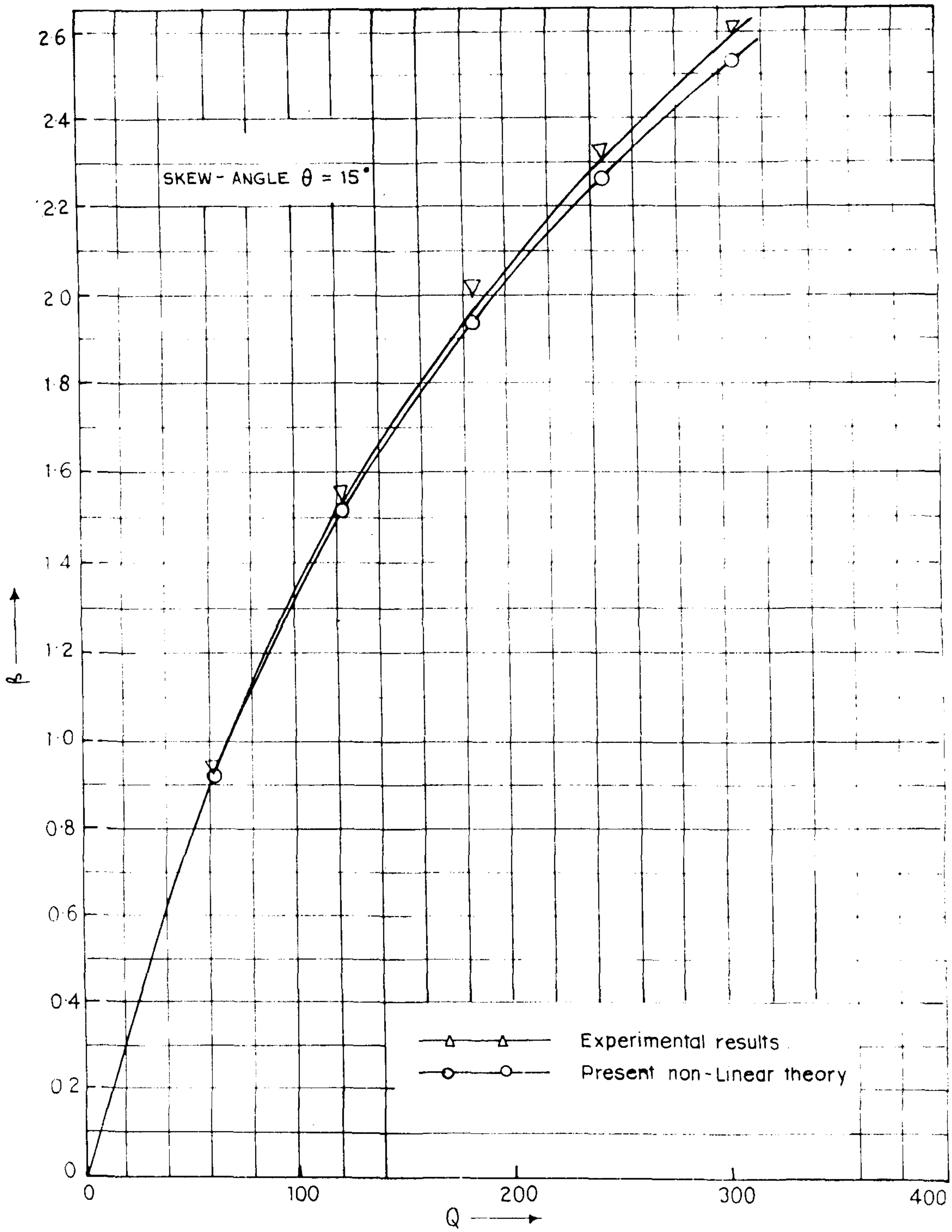

FIG. 5: GRAPHS SHOWING VARIATION OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC COPPER PLATE ($\nu = 0{\cdot}333$) WHOSE EDGES ARE KEPT MOVABLE

TABLE 1 (Continued)

Value of Ω	Immovable Edges						
	Uniform pressure applied in the Expt. (inch of Hg)	Value of W_o from the Expt. (cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Value of β from Kennedy and Simon's curve (Ref.85)	Percentage of error by Banerjee's hypothesis w.r. to Expt.	Percentage of error by Kennedy and Simon's theory w.r. to Expt.
20	-	-	-	0.3321	0.28	-	-
50	-	-	-	0.7254	0.62	-	-
61.08	2"	0.068	0.86185	0.84025	0.71	2.5 %	17.62 %
100	-	-	-	1.161	0.975	-	-
122.16	4"	0.105	1.3308	1.3054	1.11	1.91 %	16.6 %
150	-	-	-	1.4616	1.25	-	-
183.24	6"	0.131	1.66033	1.622	1.4	2.43 %	15.68 %
200	-	-	-	1.6946	1.475	-	-
244.32	8"	0.151	1.91382	1.86695	1.63	2.45 %	14.83 %
250	-	-	-	1.8873	1.65	-	-
300	-	-	-	2.053	1.8	-	-
305.4	10"	0.168	2.12928	2.0696	1.815	2.8 %	14.76 %

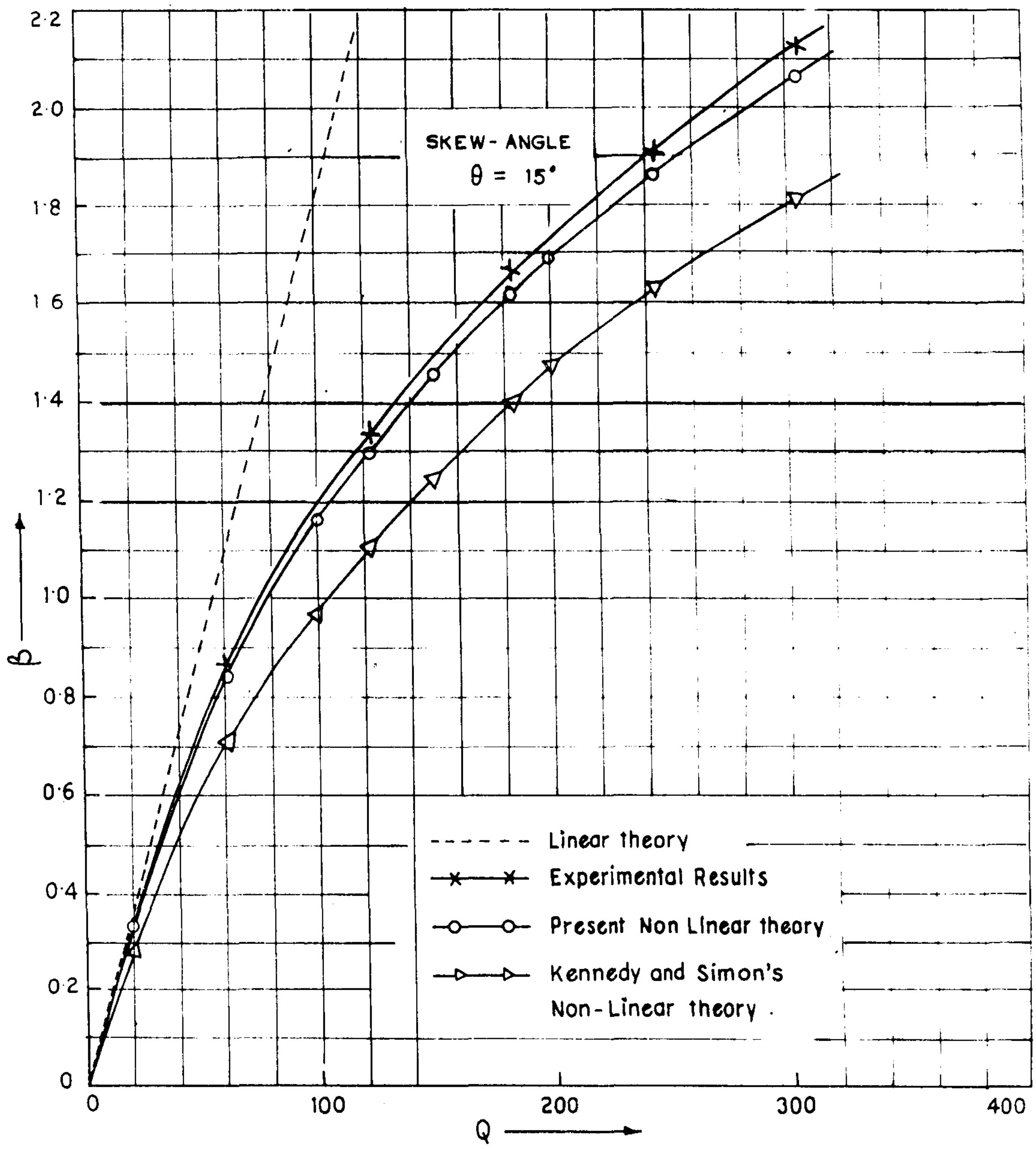

FIG. 6: GRAPHS SHOWING DEVIATIONS OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC COPPER-PLATE WHOSE EDGES ARE KEPT IMMOVABLE (ϑ = 0·333).

TABLE 2

Showing comparison of results obtained from different theories and experiment for copper-made skew plate ($\nu = 0.333$, $E = 1.25 \times 10^{12}$ dyne/cm^2., $\theta = 30^\circ$, a = 7cm., and h = 0.0789cm.)

Value of Q	Movable Edges				
	Uniform pressure applied in the Expt.(inch of Hg)	Value of W_0 from the Expt. (cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Percentage of error w.r.to Expt.
20	—	—	—	—	—
50	—	—	—	—	—
53.7	3"	0.042	0.53232	0.52642	1.11 %
100	—	—	—	—	—
107.4	6"	0.078	0.98859	0.9567	3.22 %
150	—	—	—	—	—
161.1	9"	0.107	1.35615	1.29435	4.55 %
200	—	—	—	—	—
214.8	12"	0.129	1.63498	1.5676	4.12 %
250	—	—	—	—	—
268.5	15"	0.148	1.8758	1.797	4.2 %
300	—	—	—	—	—

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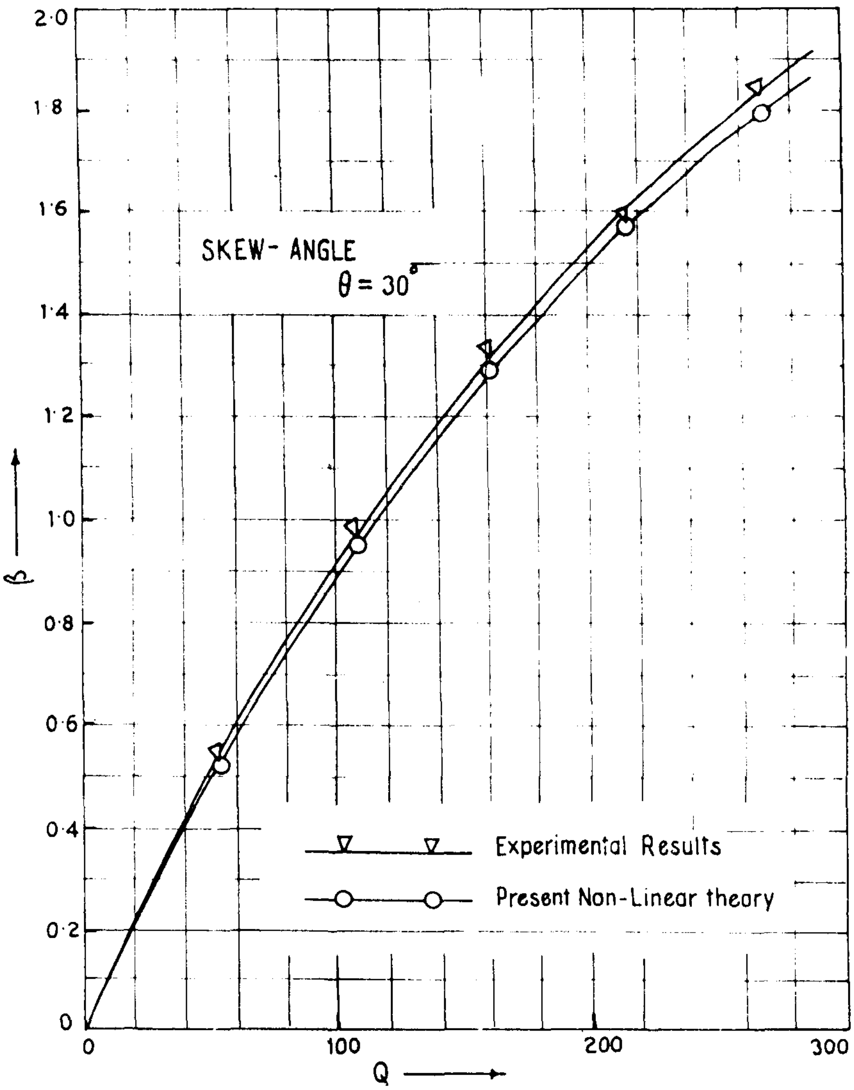

FIG. 7: GRAPHS SHOWING VARIATION OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC COPPER-PLATE ($\nu = 0{\cdot}333$) WHOSE EDGES ARE KEPT MOVABLE.

TABLE 2 (Continued)

Value of Q	Immovable Edges						
	Uniform pressure applied in the Expt. (inch of Hg)	Value of W_o from the Expt. (cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Value of β from Kennedy and Simon's curve (Ref.85)	Percentage of error by Banerjee's hypothesis w.r. to Expt.	Percentage of error by Kennedy and Simon's theory w.r. to Expt.
20	-	-	-	0.2019	0.22	-	-
50	-	-	-	0.47282	0.45	-	-
53.7	3"	0.04	0.50697	0.5028	0.475	0.822 %	6.3 %
100	-	-	-	0.8117	0.78	-	-
107.4	6"	0.07	0.8872	0.8611	0.83	2.94 %	6.45 %
150	-	-	-	1.0733	1.03	-	-
161.1	9"	0.09	1.14068	1.12156	1.08	1.676 %	5.32 %
200	-	-	-	1.274	1.225	-	-
214.8	12"	0.106	1.34347	1.3261	1.275	1.29 %	5.1 %
250	-	-	-	1.4404	1.39	-	-
268.5	15"	0.12	1.5209	1.49573	1.445	1.65 %	5 %
300	-	-	-	1.5837	1.525	-	-

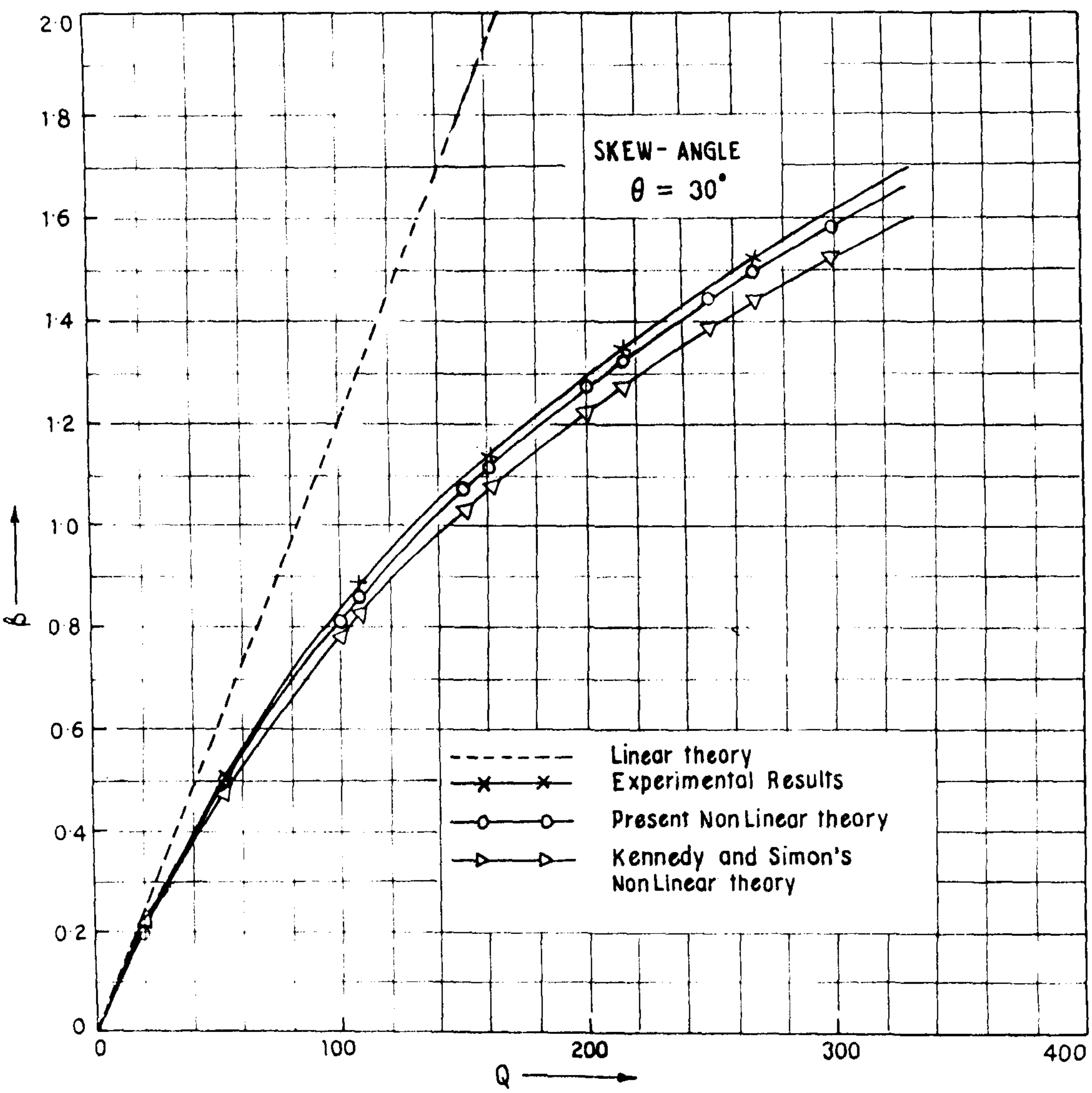

FIG. 8 : GRAPHS SHOWING DEVIATIONS OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC COPPER-PLATE WHOSE EDGES ARE KEPT IMMOVABLE ($\nu = 0{\cdot}333$) .

TABLE 3

Showing comparison of results obtained from the present non-linear theory and experiment for steel-made skew plate ($\nu = 0.3$, $E = 2 \times 10^{12}$ dyne/cm^2. , $\theta = 15^\circ$, a = 8cm. and h = 0.1343cm.)

Value of Q	Movable Edges				
	Uniform normal pressure applied in the Expt. (inch of Hg)	Value of W_o from the Expt. (cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Percentage of error w.r. to the Expt.
9.31	4"	0.023	0.17126	0.16019	6.46 %
18.62	8"	0.044	0.32762	0.3172	3.18 %
27.93	12"	0.065	0.48399	0.46827	3.248 %
37.24	16"	0.084	0.625465	0.61182	2.182 %
46.55	20"	0.104	0.7744	0.747	3.54 %

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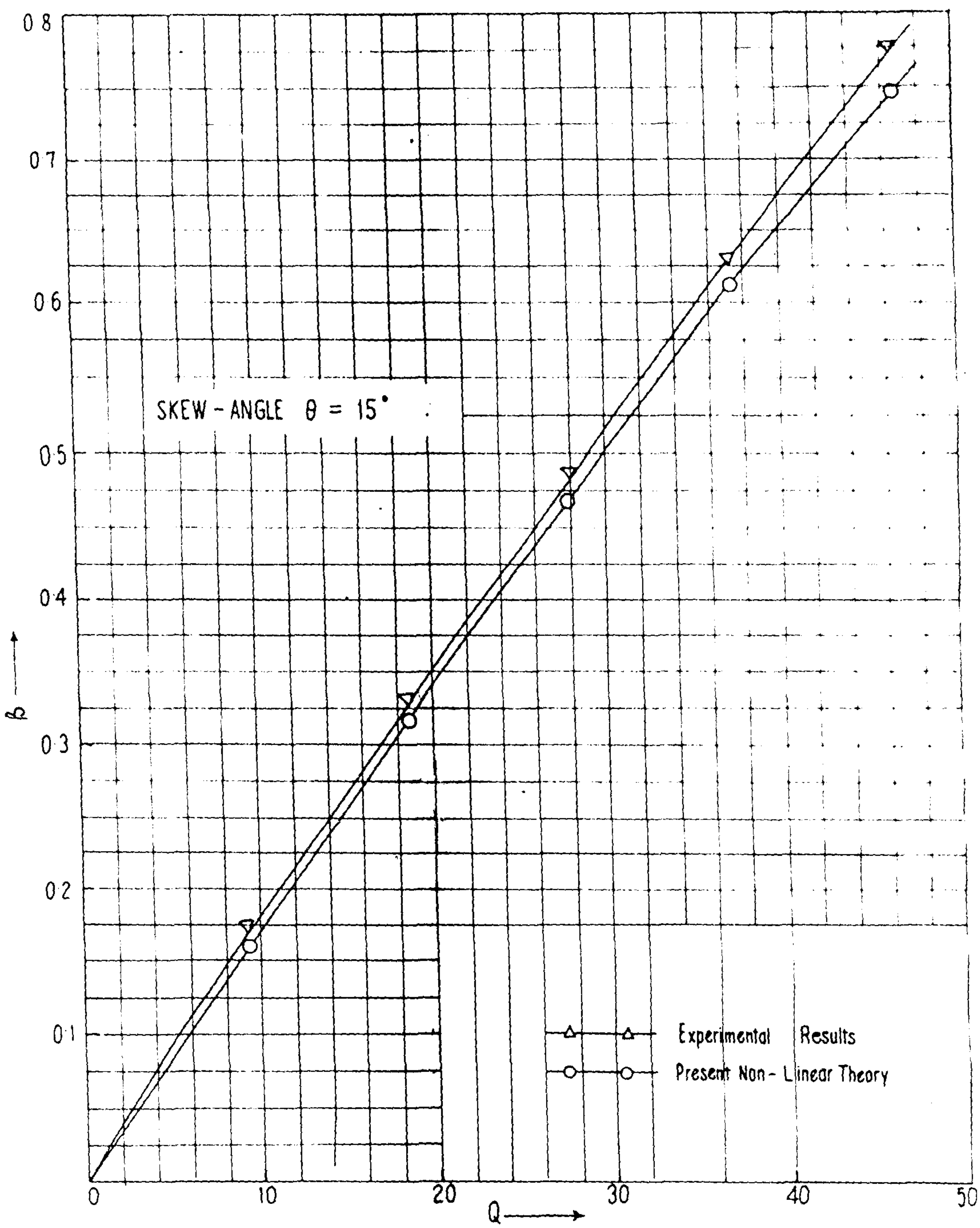

FIG-9: GRAPHS SHOWING DIVIATIONS OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC STEEL PLATE WHOSE EDGES ARE KEPT MOVABLE, (ν = 0·3).

TABLE 3 (Continued)

Value of Q	Immovable Edges				
	Uniform normal pressure applied in the Expt. (inch of Hg)	Value of W_0 from the Expt.(cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Percentage of error w.r.to the Expt.
9.31	4"	0.022	0.16381	0.15944	2.67 %
18.62	8"	0.043	0.32018	0.31176	2.63 %
27.93	12"	0.063	0.4691	0.45248	3.543 %
37.24	16"	0.08	0.59568	0.58024	2.592 %
46.55	20"	0.096	0.71482	0.69562	2.686 %

N.B :- All the errors are calculated considering our experimental results as standard, sacrificing instrumental (if any) and personal errors

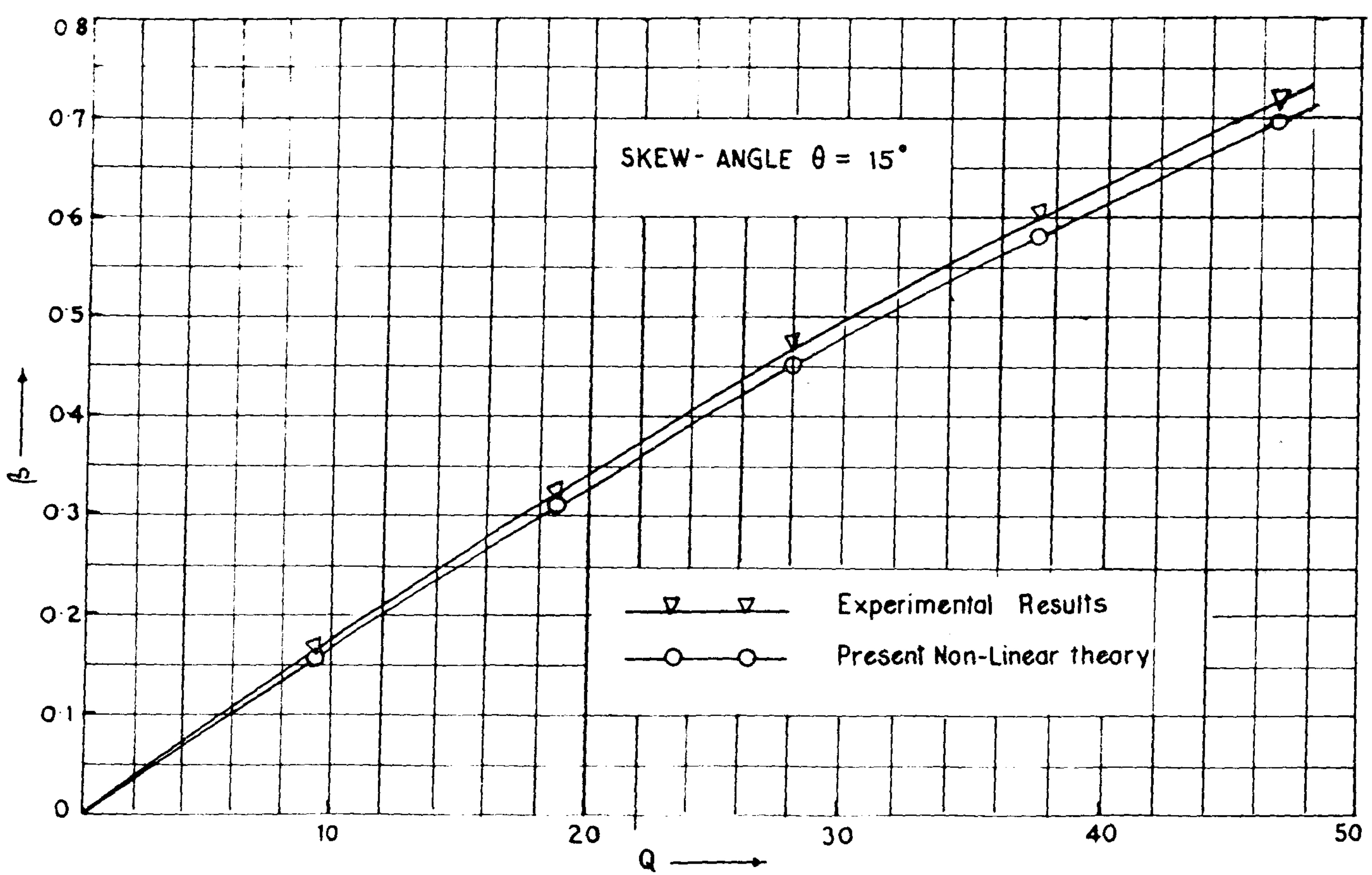

FIG. 10: GRAPHS SHOWING VARIATION OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC STEET-PLATE WHOSE EDGES ARE KEPT IMMOVABLE ($\nu = 0{\cdot}3$)

TABLE 4

Showing the variation of results obtained by the present theory due to change of edge conditions for steel-made skew plate ($\nu = 0.3$, $E = 2 \times 10^{12}$ dyne/cm^2. $\theta = 30^{\circ}$)

Value of Q	Movable edges	Immovable Edges	Difference of β due to change of Edge conditions
	Value of β from Banerjee's hypothsis.	Value of β from Banerjee's hypothesis	
20	0.2039	0.20221	0.00169
60	0.5879	0.5568	0.0311
100	0.9207	0.8312	0.0895
160	1.3334	1.14	0.1934
200	1.5515	1.3031	0.2484
260	1.83692	1.5078	0.32912
300	2.0024	1.6254	0.377

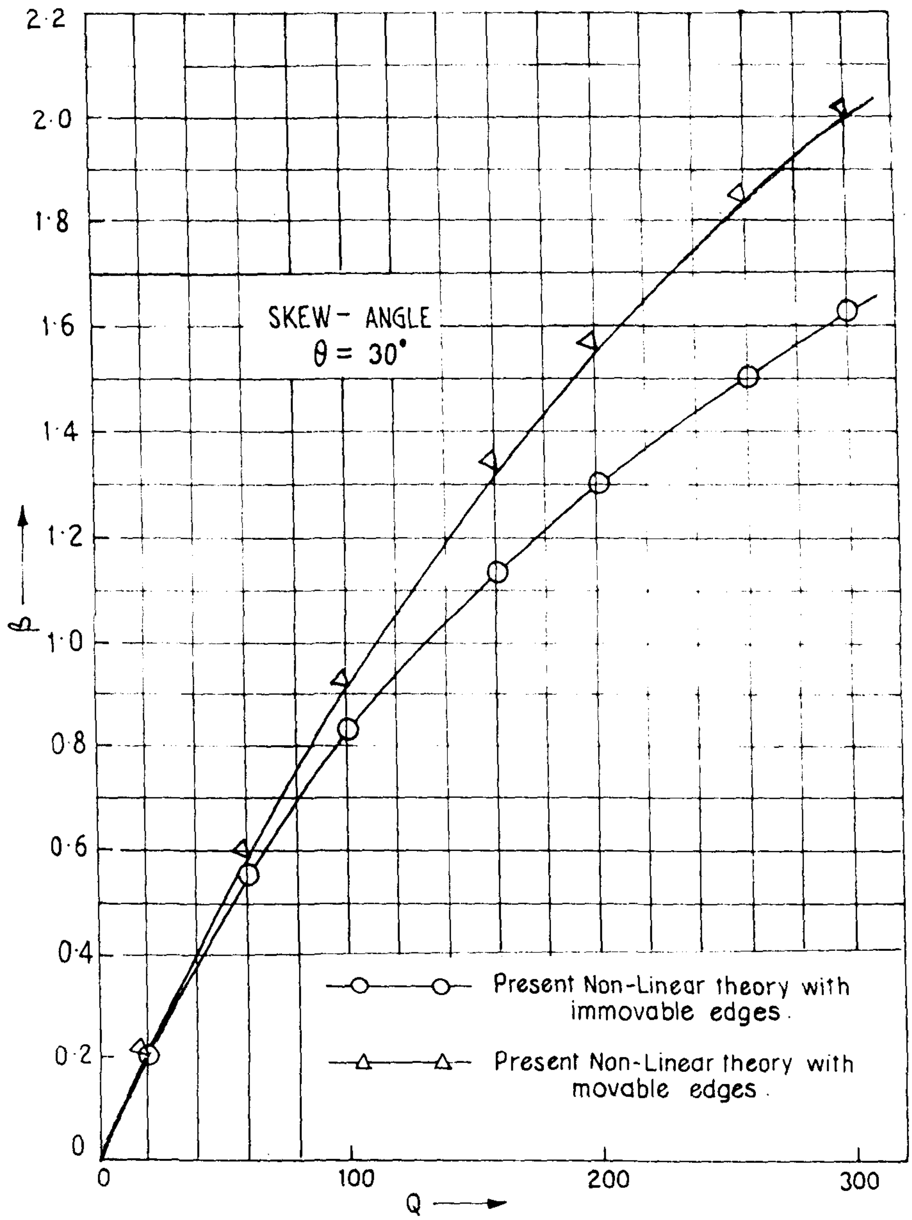

FIG. 11: GRAPHS SHOWING VARIATIONS OF CENTRAL DEFLECTIONS OF A STEEL-PLATE ($\nu = 0{\cdot}3$) DUE TO CHANGE OF EDGE CONDITIONS.

OBSERVATIONS

From the numerical tables and graphs we find that the present non-linear theory gives better results than kennedy and Simon's non-linear approach [85]. Because almost in all the cases our percentage of error with respect to the experimental results remain within 5 percent, whereas, the errors in the results according to Kennedy and Simon's theory vary from the lowest 5 percent to the highest 17.62 percent with respect to the experimental results. It appears that deviations in the results of Kennedy and Simon's theory are due to application of perturbation technique which embraces more approximation. Moreover the perturbation technique requires much computational labour. It is interesting to note that Kennedy and Simon did not compare their large deflection results for skewed plates with other results available in open literatures or with any experimental results. They only compared their results for a rectangular/ square plate where $\theta = 0^{\circ}$.

It is observed, from the present results, that the maximum central deflection of a rhombic plate decreases with the increase in skew angle. This may be due to the increased rigidity of the obtuse corners of the plate with the increase in skew angles. Thus we may conclude that the effect of the non-linear terms on the deflection diminishes with the increase in skew angle and hence the large deflection curves tend to become increasingly linear for large skew angles.

It is also observed that, the deflections for movable edges are always greater than those for the immovable edges (Ref. table 4). This is quite expected from the practical point of view, because movable edge conditions give stress-free boundary and hence there is large energy changes in the boundary.

NOTE :

Like simply-supported skew plate here also results for skew angles $\theta = 15^{o}$ and $\theta = 30^{o}$ have only been considered, because, for greater values of the skew angle the influence of non-linearity does not play much of significant role in design and the study of linear analysis serves the practical purpose.