PREFACE

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Structural members (commonly known as thin plates), whose one dimension is small in comparison with the other two dimensions, are frequently used in various machine parts and hence the scope of study of bending properties of such members is sufficiently broad. Within elastic limit, various plate problems have been dealt with by numerous eminent scientists and research workers. All these problems may be classified as

(i) Static Problems,

(ii) Dynamic Problems, and

(iii) Thermal Problems.

Any elastic behaviour (whether 'Static' or 'Dynamic' or 'Thermal') of plates is influenced by the following factors -

(i) Material properties defined by Young's Modulus 'E' and poisson's Ratio 'ν'. Both of 'E' and 'ν' may be constant or variable,

(ii) Geometry of the plate. Geometry of the plate may be simple such as Triangular, Rectangular, Circular, Elliptic etc. or may be complicated like cylindrical, Parabolic ,Polygonal or Skewed one,

(iii) Thickness of the plate 'h' which may be constant or varying as well,

(iv) Types of loading such as Transverse (or Lateral) Loading, In-plane Loading, Concentrated Loading, Edge Loading

Combined Loading etc.

and

(v) Nature of support i.e. edge conditions of the plate such as 'Simply-Supported' or 'Clamped' having 'Movable' or 'Immovable' states.

Whenever the deflections 'W' of a thin plate are small in comparison with its thickness 'h', a very approximate but satisfactory theory of bending of the plate by lateral loads can be developed by making the following assumptions-

(i) there is no deformation in the middle plane of the plate and hence this plane remains 'Neutral' during bending,

(ii) points of the plate lying initially on a normal-to-the-middle plane of the plate remain on the normal-to-the-middle surface of the plate after bending,

(iii) the stresses normal-to-the-middle plane of the plate, arising from the applied loading, can be disregarded.

These assumptions constitute the simplest and widely used "Classical Small Deflection Theory" or "Linear Theory", developed by Joseph Louis Lagrange in 1811. According to this theory all the stress components can be expressed by the deflection 'W' of the plate, which is a function of the two co-ordinates in the plane of the plate. This function has to satisfy a linear partial differential equation which together with the boundary conditions, completely defines 'W'. Thus the solution of this equation gives all the necessary information for calculating stresses at any point of the plate.

Following the above-mentioned linear theory, different works on the bending of thin plates have been carried out by numerous research workers and the Bibliography of these works has been nicely incorporated in the book, "Theory of Plates and Shells" by S. Timoshenko and S. Woinowski-Krieger (1962).

In most of the cases, bending of a plate is accompanied by strain in the middle plane, but, careful calculations show that the corresponding stresses in the middle plane are negligible if the deflections of the plate are small in comparison with its thickness. If the deflections are not small in comparison with the plate-thickness, these supplementary stresses in the middle plane of the plate must be taken into account in deriving the governing differential equations of the plate. The differential equations so obtained become non-linear in character and their solutions are much more complicated.

With the advent of modern technology and systems exposed to oppressive operation conditions, the linear hypothesis could no longer be retained. Whenever Forces, Deformations, Velocities, Temperatures and other factors become excessive, non-linear effects come into play and their influences can no longer be ignored. This situation occurs also in the particular field of applied mechanics involving plates and shallow shells. These elements, when used in modern structures, such as 'High-Speed Aeroplanes', 'Missiles' and 'Space-Vehicles', are often subject to large transverse deflections and reveal a clearly non-linear response.

Whenever the plate-thickness is considerably high, the prevalent theories for thin plates fail to explain the bending and vibration characteristics of plates. In actual technological and engineering design, plate – components become moderately thick, and considerable plate– thickness invites the study of complicated effects like 'Transverse Shear Deformation' and 'Rotatory Inertia' on large amplitude vibration. Naturally some "Thick-plate Theories" have been developed. These theories consider the problem of plates as a three-dimensional problem of elasticity. Increase in dimension of plates greatly complicates their elastic behaviours. The stress analysis becomes more involved in such problems, and till now, very few particular cases are completely solved.

METHODS OF ATTACK OF THE NON-LINEAR PROBLEMS OF THIN PLATES:

So far there has been a wide application of the three types of differential equations for the non-linear analyses of thin plates as well as thick plates. They are -

(i) Von-Ka'rma'n's Equations,

(ii) Berger's Equations, and

(iii) Banerjee's Equations.

(i) The first formulation of the theory of plates with a stretching of the middle plane and moderately large deflections was developed by Gehring (1860) and was improved upon by G. Kirchhoff (1883). The potential energy per unit area of the plates was written as a sum of a quadratic function of the quantities defining the Extension of the middle plane and a quadratic function of the quantities defining Bending. The equations of motion with finite displacements were deduced by the Principle of Virtual Work. The stress function satisfying the in-plane force equilibrium equations was introduced by A. Föppl (1907). The currently popular form of the two governing equations, in the rectangular cartesian co-ordinates, was given by T. Von-Ka'rma'n (1910). The equations of Von-Ka'rma'n are in the coupled form and involve Transverse Deflection and the Membrane Stress Function as unknown functions. These are difficult to solve. Several numerical methods have been employed by different investigators for solutions of Von-Ka'rma'n's equations.

Interesting works on Von-Karman's Equations

Among the authors who initially treated the non-linear analysis of plates, the works of S. Timoshenko (1937) and S. Way (1934, 1938) need special mention. Since then many authors investigated the different plate problems using Von-Ka'rma'n's Equations. Useful works in this field are due to S. Levy (1942), S. Levy and S. Greenman (1942), W.Z. Chien (1947), Chi-Teh Wang (1948), H.N. Chu and G. Herrmann (1955,1956), N.A. Weil and N.M. Newmark (1956), S.J. Medwadowski (1958), W.A. Nash (1959), W.A. Nash and I.D. Cooley (1959), N. Yamaki (1964), J.L. Nowinski (1962,1963,1964), A.M. Alwan (1964), J.L. Nowinski and I.A. Ismail (1965), G.Z. Harris and E.H. Mansfield (1967), J.B. Kennedy and Simon Ng. (1967), Robert Schmidt (1968), H.F. Bauer (1968), J.M. Whitney and A.W. Leissa(1969), M. Sathyamoorthy and K.A.V. Pandalai (1972) Richard Bolton (1972), J.L. Nowinski and H. Ohnabe (1973), K. Kanakaraju and C.R.V.Rao (1976), M.A. Sayed and R. Schmidt (1977), B.M. Karmakar (1978,1979), M. Sathyamoorthy (1978,1979), M. Sathyamoorthy and C.Y. Chia (1979, 1980), B. Banerjee and S. Dutta (1980), J.N. Reddy and W.C. Chao (1981), J.N. Reddy and C.L. Huang (1981), B. Banerjee (1982,1983,1984), S.K. Chaudhuri (1982, 1984), S. Das (1984), K. Kanakaraju and C.R.V. Rao (1986), P.C. Dumir(1988),G.L. Ostiguy and H. Nguyen (1988), Yen Kai-Yuan, Zheng Xiao-Jing and Zhou You-he (1989), M. Gorji (1989), H. Kobayashi and K. Sonoda,(1989), K.M. Liew and K.Y. Lam (1990), J.W. Zhang (1991), B. Singh and S. Chakravorty (1991), H. Kobayashi and K. Sonoda (1991), U.S. Gupta, R. Lal and S.K. Jain (1991), M. Ganapathi, T.K. Varadan and B.S. Sarma (1991), G.B. Chai (1991), and D.J. Gorman (1991),

All these workers solved the coupled form of Von-Kármán's equations by different numerical methods which are elegant but laborious.

(ii) H.M. Berger (1955) offered an approximate method for non-linear analyses of thin elastic plates by neglecting the second strain invariant in the expression for the total potential energy of the system. He investigated the large deflections of circular and rectangular plates both for clamped immovable and simply-supported immovable edges. Although no physical justification of his assumption was given in the paper, the results obtained in both the cases are in very good agreement with other known results. The speciality of Berger's equations is that the equations have been decoupled so that the solutions of the differential equations have been simplified and obtained in the closed form. Actually Berger's equations are linear in character. The essential non-linearity depends on the coupling parameter 'α'. These specialities have made Berger's equations very popular and many useful works on plates of immovable edges have been done following Berger's equations.

Useful works on Berger's Equations

The comprehensive works on Berger's equations which need special mention are due to T. Iwinski and J. L. Nowinski (1957), M.L.Williams (1958), W.A. Nash and J.R. Modeer (1959, 1960), S. Basuli (1961), J.E. Hassert and J.L. Nowinski (1962), T. Wah (1963), S.N. Sinha (1963), J.L. Nowinski and I.A. Ismail (1965), M.C. Pal (1967), B. Banerjee (1967,1968–1969), Cheng-Ih Wu and J.R. Vinson (1968,1969), M.C. Pal (1969–1970,1973), M. Sathyamoorthy and K.A.V. Pandalai (1970,1973, 1974), J. Mazumdar and R. Jones (1974), P. Biswas (1975), S. Dutta (1975,1976), M.M. Banerjee (1976), N. Kamiya (1976), B.M. Karmakar (1977), J. Mazumdar and R. Jones (1977), M. Sathyamoorthy (1977, 1978), N. Kamiya (1978), B. Banerjee and S. Dutta (1979), B. Banerjee (1982), S. Das (1984) and T. Das (1986).

It is to be noted that Berger's equations are meaningful only for immovable edge conditions of the plate structure. It leads to meaningless results for movable edge conditions. This has been pointed out by J.L. Nowinski and H. Ohnabe (1972), in an excellent research work.

(iii) In 1981, B. Banerjee and S. Dutta offered a modified strain energy expression to investigate non-linear problems on thin elastic plates. A set of decoupled differential equations are obtained under this modified strain energy expression. The authors have tested the accuracy of their equations by solving different non-linear plate problems. They have obtained sufficiently accurate results both for movable and immovable edge conditions. The equivalent hypothesis of Banerjee's equation is that the radial stretching of the plate is proportional to $(dW/dr)^2$. This is reasonable because under any type of loading and under any boundary condition the extra strain imposed by bending is represented by the term $(dW/dr)^2$. It is believed that Banerjee's equations are more welcome from the practical point of view because unlike Von-Kármán's equations, they are uncoupled and unlike Berger's equations they give reasonable results both for movable and immovable edge conditions of the plates.

Important works on Banerjee's Equations

So far, many important works have been carried out on Banerjee's equations. B. Banerjee and S. Dutta (1981), B. Banerjee (1984), G. Sinharay and B. Banerjee (1985,1986), R. Bhattacharjee and B. Banerjee (1988), S.K. Ghosh (1991), and,S. Dutta and B. Banerjee (1991), have utilised Banerjee's equations in different non-linear plate problems and have achieved satisfactory results in their respective cases.

From the survey of literature on non-linear elastic plate problems it is observed that most of the investigations, except a few, are confined to plates of simple geometry. In contrast to non-linear analyses of elastic plates of other geometries like triangular, rectangular, circular and elliptic, skew plates have not received as much attention. This may be due to their relatively difficult mathematical models.

For the rapid advancement of Science and technology today, elastic plates of different shapes and sizes are required to use in various instruments and appliances. Particularly skew plates (or oblique panels) find wide application in the Aircraft Industry and Spaceship Technology. Hence the study of the non-linear behaviours of skew plates is of paramount importance now-a-days. The most attractive work in this field is due to J.L. Nowinski (1964), who quite elegantly analysed the large amplitude oscillations of oblique panels with an initial curvature utlising Von-Kármán's field equations. The following particular cases are discussed in detail in this paper:

(i) Buckling of an oblique plate under uniaxial compressive load,

(ii) Free linear vibrations of a square plate,

(iii) Large deflections of a uniformly loaded square plate,

(iv) Snap-through phenomena of a curved oblique plate under uniform transverse load

and

(v) Free non-linear vibrations.

Also a numerical example concerning a rhombic plate is discussed in more detail. The author's observations are very important from the practical point of view.

Other interesting works on non-linear vibration problems of skew plates are due to M. Sathyamoorthy and K.A.V. Pandalai (1972,1973,1974). The authors have worked out nicely the investigations on non-linear flexural vibrations of simply-supported and clamped skew plates of isotropic, orthotropic as well as anisotropic materials. They have used Berger's uncoupled defferential equations in some cases and Von-Kármán's coupled differential equations in other cases. In each case they have got satisfactory results.

In analysing the large amplitude vibrations of clamped isotropic skew plates including Shear and Rotatory Inertia Effects M. Sathyamoorthy (1977,1978) has employed Berger's approximation as well as Von-Kármán-type equations to obtain satisfactory results for thick plate problems.

In 1980, using Von-Kármán's equations along with the aid of Hamilton's Principle, M. Sathyamoorthy and C.Y. Chia studied the large amplitude vibrations of anisotropic thick skew plates. They found excellent agreement between their results and those available from open literature.

In contrast to works on non-linear vibration problems of skew plates, the literature on non-linear deflection problems of skew plates is scanty. J.B. Kennedy and Simon Ng. (1965,1967) have analysed small and large deflection problems of clamped skewed plates under uniform pressure by the method based on the small parameter perturbation technique. The results are improved by successive approximations to the three displacement components of a point on the middle plane of the plate. Both numerical and graphical results are presented. Comparisons are made with existing results for skewed plates with small deflections as well as with results for rectangular plates with small and large deflection behaviours. Good agreement has been shown in these cases. The authors have also obtained some experimental results for clamped skewed plates governed by the small deflection theory. But they could not compare their large deflection results for clamped skew plates (immovable edges only) directly neither with any existing theoretical results nor with any experimental results. However their observations are noteworthy from practical point of view.

R. S. Srinivasan and S.V. Ramachandran (1975) have analysed quite elegantly the large deflections of skew plates of variable thickness by using the Newton-Raphson Procedure. This is probably the first attempt of its kind.

It is interesting to note that most of the above investigations are carried out on skew plates of clamped edges only and the cases of simply-supported edge conditions have not received proper attention. It is noteworthy that an interesting work on a simply-supported skew plate could be located where D.J. Gorman (1991) quite elegantly carried out accurate analytical type solution for the free vibrations of simply-supported parallelogram plates.

Again, as far as it is known, no work has been apparently devoted to the investigations of non-linear behaviours of heated elastic skew plates having various applications in modern design, especially in Air-craft Industry, Spaceship Technology and Astronautical Engineering. Also no attempt on the non-linear behaviours of skewed sandwich plates has been reported in the literature as yet.

It is seen that in solving various elastic problems, different methods have been employed. Based on the geometry of the elastic structure and the nature of the problem concerning it (e.g. static, dynamic or thermally induced) one method has got beauty and advantage over the other. Most of the earlier workers have utilised either Von-Kármán's classical equations or Berger's differential equations in their investigations on skew plates. But as already mentioned Von-Kármán's classical equations are in the coupled form and the transformations of these coupled equations in oblique co-ordinates involve much mathematical complexity. So this entails difficulty in solution as well. On the other hand

Berger's differential equations, although decoupled, are questionable mainly because it fails miseravly for movable edge conditions.

The object of modern research work is to find solutions of natural problems in such a way that the method employed and the level of presentation are lucid, computational labour is minimum and the results predicted are sufficiently accurate for the practical purpose.

The purpose of the present thesis is to present a simplified approach to the non-linear analyses of isotropic elastic rhombic plates (skew plates having aspect ratio 1) under different types of loadings and edge conditions. Banerjee's well-known hypothesis has been utilised in these investigations. A set of uncoupled differential equations has been obtained in oblique co-ordinates and solved by the Galerkin's Error Minimising Technique. Accuracy of the method has been tested in some static cases by comparison with experimental results and with other known results from the open literature. It has been observed that the results of the present study are in very good agreement with those obtained in open literature. The main advantages of the present study are that

(i) unlike the Von-Kármán's differential equations, the present differential equations are in the uncoupled form and thus easy to solve,

(ii) unlike Berger's uncoupled differential equations, the present uncoupled differential equations give results both for immovable as well as movable edge conditions with sufficient amount of accuracy,

and

(iii) from the same cubic equation, results both for immovable as well as movable edge conditions can be obtained with minimum computational labour, offering an additional advantage.

The present thesis has been divided into five chapters. In the first chapter (containing two papers), the non-linear static behaviours of thin isotropic elastic plates of uniform thickness, have been studied. The first paper of this chapter deals with large deflections of rhombic plates with simply-supported edge conditions. Non-linear static behaviours of rhombic plates have been analysed following Banerjee's hypothesis. Calculations have been done for different skew angles. To test the accuracy of the theoretical results thus obtained, experiments are carried out on copper-made and steel-made plates. The theoretical results are found to be in excellent agreement with those obtained from the experimental data.

The second paper in the first chapter,is devoted to the investigations of the non-linear static behaviours of clamped thin rhombic plates under uniform lateral pressure. Numerical results for different skew angles are presented. Here also accuracy of Banerjee's hypothesis has been tested by comparison with the experimental results and with other theoretical results.

The second chapter of the thesis contains only one paper on large deflection analyses of rhombic plates of vabiable thickness. In this paper both the non-linear static and dynamic behaviours of simply-supported rhombic plates of linearly varying thickness have been presented following Banerjee's hypothesis. Various numerical results showing the combined effects of skew angle and thickness variation parameter on the deflections and vibrations of the rhombic plates are given. Proper comparison has also been done.

The third chapter of the thesis contains one paper devoted to non-linear analysis of heated rhombic plates. This paper seems to predict some new information on thermal behaviours of rhombic plates. For 0^{o}-skew angle, the results obtained in the present study have been compared with the other results found in open literature. It is believed that the results obtained for other skew angles are completely new.

The fourth chapter also contains one paper on non-linear static and dynamic behaviours of the freely supported skewed sandwich plates. Banerjee's hypothesis has been employed in these investigations. Numerical results have been obtained for different skew angles. For 0^{o}- skew angle, results obtained are compared with other known results. Here also the results for other skew angles are believed to be new.

The last chapter of the thesis concerns with the non-linear analysis of moderately thick plates. This chapter also contains one paper. In this paper, the large amplitude free flexural vibrations of clamped as well as simply-supported isotropic elastic rhombic plates are investigated showing the effects of Shear Deformation and Rotatory Inertia, Various numerical results have been presented in tabular forms and in few cases they are compared with existing results.

From the numerical results obtained in the present study for different rhombic plates with different edge conditions and under Static, Dynamic and Thermal loadings, it is observed that the present study yields sufficiently accurate results from the practical point of view. Also for uncoupled form of differential equations Banerjee's equations transformed in oblique co-ordinates are simple and yield results both for immovable as well as movable edge conditions with minimum computational labour. Thus the present project seems to be more acceptable for practical purposes.

NOTATIONS

(x,y,z)	Rectangular Cartesian Co-ordinates,
(x_1,y_1)	Oblique Co-ordinates on the xy-plane,
θ	Skew Angle,
W	Deflection normal-to-the-middle-plane of the plate,
W_o	Maximum Central Deflection,
u,v	Inplane Displacements,
h	Thickness of the plate ,
p	Thickness-variation Parameter,
Q	Load Function,
q	Intensity of a Continuously Distributed Load,
E	Modulus of Elasticity in Tension and Compression,
G	Modulus of Elasticity in Shear,
ν	Poisson's Ratio,
D	Flexural Rigidity of the Plate $= Eh^3/12(1-\nu^2)$,
C_p	Speed of Wave Propagation along the Surface of the Plate,
α_t	Co-efficient of Linear Expansion,
ρ	Density of the Plate Material,
t	Time Parameter,
T	Linear Period of Vibration of the Plate,
T^*	Non-linear Period of Vibration of the Plate,
ω	Vibration Frequency of the Plate,
A_{oo}	Initial Amplitude of Vibration of the Plate,
$\bar{\beta}$	Non-dimensional Amplitude,
$A,\bar{\alpha},I_1^m$	Coupling Parameters,
τ_o	Temperature in the Middle Plane of the Plate,
∇^2	Laplacian Operator,

$\beta = w_o/h$ Central Deflection Parameter,

$\lambda = \nu^2$ for Simply-Supported Edge Condition of the elastic plate,

$\lambda = 2\nu^2$ for Clamped Edge Condition of the elastic plate.