NON-LINEAR ANALYSES OF RHOMBIC PLATES— A SIMPLIFIED APPROACH

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PREFACE

-: PREFACE :-

Structural members (commonly known as thin plates), whose one dimension is small in comparison with the other two dimensions, are frequently used in various machine parts and hence the scope of study of bending properties of such members is sufficiently broad. Within elastic limit, various plate problems have been dealt with by numerous eminent scientists and research workers. All these problems may be classified as

(i) Static Problems,

(ii) Dynamic Problems, and

(iii) Thermal Problems.

Any elastic behaviour (whether 'Static' or 'Dynamic' or 'Thermal') of plates is influenced by the following factors -

(i) Material properties defined by Young's Modulus 'E' and poisson's Ratio 'ν'. Both of 'E' and 'ν' may be constant or variable,

(ii) Geometry of the plate. Geometry of the plate may be simple such as Triangular, Rectangular, Circular, Elliptic etc. or may be complicated like cylindrical, Parabolic ,Polygonal or Skewed one,

(iii) Thickness of the plate 'h' which may be constant or varying as well,

(iv) Types of loading such as Transverse (or Lateral) Loading, In-plane Loading, Concentrated Loading, Edge Loading

Combined Loading etc.

and

(v) Nature of support i.e. edge conditions of the plate such as 'Simply-Supported' or 'Clamped' having 'Movable' or 'Immovable' states.

Whenever the deflections 'W' of a thin plate are small in comparison with its thickness 'h', a very approximate but satisfactory theory of bending of the plate by lateral loads can be developed by making the following assumptions-

(i) there is no deformation in the middle plane of the plate and hence this plane remains 'Neutral' during bending,

(ii) points of the plate lying initially on a normal-to-the-middle plane of the plate remain on the normal-to-the-middle surface of the plate after bending,

(iii) the stresses normal-to-the-middle plane of the plate, arising from the applied loading, can be disregarded.

These assumptions constitute the simplest and widely used "Classical Small Deflection Theory" or "Linear Theory", developed by Joseph Louis Lagrange in 1811. According to this theory all the stress components can be expressed by the deflection 'W' of the plate, which is a function of the two co-ordinates in the plane of the plate. This function has to satisfy a linear partial differential equation which together with the boundary conditions, completely defines 'W'. Thus the solution of this equation gives all the necessary information for calculating stresses at any point of the plate.

Following the above-mentioned linear theory, different works on the bending of thin plates have been carried out by numerous research workers and the Bibliography of these works has been nicely incorporated in the book, "Theory of Plates and Shells" by S. Timoshenko and S. Woinowski-Krieger (1962).

In most of the cases, bending of a plate is accompanied by strain in the middle plane, but, careful calculations show that the corresponding stresses in the middle plane are negligible if the deflections of the plate are small in comparison with its thickness. If the deflections are not small in comparison with the plate-thickness, these supplementary stresses in the middle plane of the plate must be taken into account in deriving the governing differential equations of the plate. The differential equations so obtained become non-linear in character and their solutions are much more complicated.

With the advent of modern technology and systems exposed to oppressive operation conditions, the linear hypothesis could no longer be retained. Whenever Forces, Deformations, Velocities, Temperatures and other factors become excessive, non-linear effects come into play and their influences can no longer be ignored. This situation occurs also in the particular field of applied mechanics involving plates and shallow shells. These elements, when used in modern structures, such as 'High-Speed Aeroplanes', 'Missiles' and 'Space-Vehicles', are often subject to large transverse deflections and reveal a clearly non-linear response.

Whenever the plate-thickness is considerably high, the prevalent theories for thin plates fail to explain the bending and vibration characteristics of plates. In actual technological and engineering design, plate – components become moderately thick, and considerable plate– thickness invites the study of complicated effects like 'Transverse Shear Deformation' and 'Rotatory Inertia' on large amplitude vibration. Naturally some "Thick-plate Theories" have been developed. These theories consider the problem of plates as a three-dimensional problem of elasticity. Increase in dimension of plates greatly complicates their elastic behaviours. The stress analysis becomes more involved in such problems, and till now, very few particular cases are completely solved.

METHODS OF ATTACK OF THE NON-LINEAR PROBLEMS OF THIN PLATES:

So far there has been a wide application of the three types of differential equations for the non-linear analyses of thin plates as well as thick plates. They are -

(i) Von-Ka'rma'n's Equations,

(ii) Berger's Equations, and

(iii) Banerjee's Equations.

(i) The first formulation of the theory of plates with a stretching of the middle plane and moderately large deflections was developed by Gehring (1860) and was improved upon by G. Kirchhoff (1883). The potential energy per unit area of the plates was written as a sum of a quadratic function of the quantities defining the Extension of the middle plane and a quadratic function of the quantities defining Bending. The equations of motion with finite displacements were deduced by the Principle of Virtual Work. The stress function satisfying the in-plane force equilibrium equations was introduced by A. Föppl (1907). The currently popular form of the two governing equations, in the rectangular cartesian co-ordinates, was given by T. Von-Kármán (1910). The equations of Von-Kármán are in the coupled form and involve Transverse Deflection and the Membrane Stress Function as unknown functions. These are difficult to solve. Several numerical methods have been employed by different investigators for solutions of Von-Kármán's equations.

Interesting works on Von-Karman's Equations

Among the authors who initially treated the non-linear analysis of plates, the works of S. Timoshenko (1937) and S. Way (1934, 1938) need special mention. Since then many authors investigated the different plate problems using Von-Kármán's Equations. Useful works in this field are due to S. Levy (1942), S. Levy and S. Greenman (1942), W.Z. Chien (1947), Chi-Teh Wang (1948), H.N. Chu and G. Herrmann (1955,1956), N.A. Weil and N.M. Newmark (1956), S.J. Medwadowski (1958), W.A. Nash (1959), W.A. Nash and I.D. Cooley (1959), N. Yamaki (1964), J.L. Nowinski (1962,1963,1964), A.M. Alwan (1964), J.L. Nowinski and I.A. Ismail (1965), G.Z. Harris and E.H. Mansfield (1967), J.B. Kennedy and Simon Ng. (1967), Robert Schmidt (1968), H.F. Bauer (1968), J.M. Whitney and A.W. Leissa(1969), M. Sathyamoorthy and K.A.V. Pandalai (1972) Richard Bolton (1972), J.L. Nowinski and H. Ohnabe (1973), K. Kanakaraju and C.R.V.Rao (1976), M.A. Sayed and R. Schmidt (1977), B.M. Karmakar (1978,1979), M. Sathyamoorthy (1978,1979), M. Sathyamoorthy and C.Y. Chia (1979, 1980), B. Banerjee and S. Dutta (1980), J.N. Reddy and W.C. Chao (1981), J.N. Reddy and C.L. Huang (1981), B. Banerjee (1982,1983,1984), S.K. Chaudhuri (1982, 1984), S. Das (1984), K. Kanakaraju and C.R.V. Rao (1986), P.C. Dumir(1988),G.L. Ostiguy and H. Nguyen (1988), Yen Kai-Yuan, Zheng Xiao-Jing and Zhou You-he (1989), M. Gorji (1989), H. Kobayashi and K. Sonoda,(1989), K.M. Liew and K.Y. Lam (1990), J.W. Zhang (1991), B. Singh and S. Chakravorty (1991), H. Kobayashi and K. Sonoda (1991), U.S. Gupta, R. Lal and S.K. Jain (1991), M. Ganapathi, T.K. Varadan and B.S. Sarma (1991), G.B. Chai (1991), and D.J. Gorman (1991),

All these workers solved the coupled form of Von-Kármán's equations by different numerical methods which are elegant but laborious.

(ii) H.M. Berger (1955) offered an approximate method for non-linear analyses of thin elastic plates by neglecting the second strain invariant in the expression for the total potential energy of the system. He investigated the large deflections of circular and rectangular plates both for clamped immovable and simply-supported immovable edges. Although no physical justification of his assumption was given in the paper, the results obtained in both the cases are in very good agreement with other known results. The speciality of Berger's equations is that the equations have been decoupled so that the solutions of the differential equations have been simplified and obtained in the closed form. Actually Berger's equations are linear in character. The essential non-linearity depends on the coupling parameter 'α'. These specialities have made Berger's equations very popular and many useful works on plates of immovable edges have been done following Berger's equations.

Useful works on Berger's Equations

The comprehensive works on Berger's equations which need special mention are due to T. Iwinski and J. L. Nowinski (1957), M.L.Williams (1958), W.A. Nash and J.R. Modeer (1959, 1960), S. Basuli (1961), J.E. Hassert and J.L. Nowinski (1962), T. Wah (1963), S.N. Sinha (1963), J.L. Nowinski and I.A. Ismail (1965), M.C. Pal (1967), B. Banerjee (1967,1968–1969), Cheng-Ih Wu and J.R. Vinson (1968,1969), M.C. Pal (1969–1970,1973), M. Sathyamoorthy and K.A.V. Pandalai (1970,1973, 1974), J. Mazumdar and R. Jones (1974), P. Biswas (1975), S. Dutta (1975,1976), M.M. Banerjee (1976), N. Kamiya (1976), B.M. Karmakar (1977), J. Mazumdar and R. Jones (1977), M. Sathyamoorthy (1977, 1978), N. Kamiya (1978), B. Banerjee and S. Dutta (1979), B. Banerjee (1982), S. Das (1984) and T. Das (1986).

It is to be noted that Berger's equations are meaningful only for immovable edge conditions of the plate structure. It leads to meaningless results for movable edge conditions. This has been pointed out by J.L. Nowinski and H. Ohnabe (1972), in an excellent research work.

(iii) In 1981, B. Banerjee and S. Dutta offered a modified strain energy expression to investigate non-linear problems on thin elastic plates. A set of decoupled differential equations are obtained under this modified strain energy expression. The authors have tested the accuracy of their equations by solving different non-linear plate problems. They have obtained sufficiently accurate results both for movable and immovable edge conditions. The equivalent hypothesis of Banerjee's equation is that the radial stretching of the plate is proportional to $(dW/dr)^2$. This is reasonable because under any type of loading and under any boundary condition the extra strain imposed by bending is represented by the term $(dW/dr)^2$. It is believed that Banerjee's equations are more welcome from the practical point of view because unlike Von-Kármán's equations, they are uncoupled and unlike Berger's equations they give reasonable results both for movable and immovable edge conditions of the plates.

Important works on Banerjee's Equations

So far, many important works have been carried out on Banerjee's equations. B. Banerjee and S. Dutta (1981), B. Banerjee (1984), G. Sinharay and B. Banerjee (1985,1986), R. Bhattacharjee and B. Banerjee (1988), S.K. Ghosh (1991), and, S. Dutta and B. Banerjee (1991), have utilised Banerjee's equations in different non-linear plate problems and have achieved satisfactory results in their respective cases.

From the survey of literature on non-linear elastic plate problems it is observed that most of the investigations, except a few, are confined to plates of simple geometry. In contrast to non-linear analyses of elastic plates of other geometries like triangular, rectangular, circular and elliptic, skew plates have not received as much attention. This may be due to their relatively difficult mathematical models.

For the rapid advancement of Science and technology today, elastic plates of different shapes and sizes are required to use in various instruments and appliances. Particularly skew plates (or oblique panels) find wide application in the Aircraft Industry and Spaceship Technology. Hence the study of the non-linear behaviours of skew plates is of paramount importance now-a-days. The most attractive work in this field is due to J.L. Nowinski (1964), who quite elegantly analysed the large amplitude oscillations of oblique panels with an initial curvature utlising Von-Kármán's field equations. The following particular cases are discussed in detail in this paper:

(i) Buckling of an oblique plate under uniaxial compressive load,

(ii) Free linear vibrations of a square plate,

(iii) Large deflections of a uniformly loaded square plate,

(iv) Snap-through phenomena of a curved oblique plate under uniform transverse load

and

(v) Free non-linear vibrations.

Also a numerical example concerning a rhombic plate is discussed in more detail. The author's observations are very important from the practical point of view.

Other interesting works on non-linear vibration problems of skew plates are due to M. Sathyamoorthy and K.A.V. Pandalai (1972,1973,1974). The authors have worked out nicely the investigations on non-linear flexural vibrations of simply-supported and clamped skew plates of isotropic, orthotropic as well as anisotropic materials. They have used Berger's uncoupled defferential equations in some cases and Von-Kármán's coupled differential equations in other cases. In each case they have got satisfactory results.

In analysing the large amplitude vibrations of clamped isotropic skew plates including Shear and Rotatory Inertia Effects M. Sathyamoorthy (1977,1978) has employed Berger's approximation as well as Von-Kármán-type equations to obtain satisfactory results for thick plate problems.

In 1980, using Von-Kármán's equations along with the aid of Hamilton's Principle, M. Sathyamoorthy and C.Y. Chia studied the large amplitude vibrations of anisotropic thick skew plates. They found excellent agreement between their results and those available from open literature.

In contrast to works on non-linear vibration problems of skew plates, the literature on non-linear deflection problems of skew plates is scanty. J.B. Kennedy and Simon Ng. (1965,1967) have analysed small and large deflection problems of clamped skewed plates under uniform pressure by the method based on the small parameter perturbation technique. The results are improved by successive approximations to the three displacement components of a point on the middle plane of the plate. Both numerical and graphical results are presented. Comparisons are made with existing results for skewed plates with small deflections as well as with results for rectangular plates with small and large deflection behaviours. Good agreement has been shown in these cases. The authors have also obtained some experimental results for clamped skewed plates governed by the small deflection theory. But they could not compare their large deflection results for clamped skew plates (immovable edges only) directly neither with any existing theoretical results nor with any experimental results. However their observations are noteworthy from practical point of view.

R. S. Srinivasan and S.V. Ramachandran (1975) have analysed quite elegantly the large deflections of skew plates of variable thickness by using the Newton-Raphson Procedure. This is probably the first attempt of its kind.

It is interesting to note that most of the above investigations are carried out on skew plates of clamped edges only and the cases of simply-supported edge conditions have not received proper attention. It is noteworthy that an interesting work on a simply-supported skew plate could be located where D.J. Gorman (1991) quite elegantly carried out accurate analytical type solution for the free vibrations of simply-supported parallelogram plates.

Again, as far as it is known, no work has been apparently devoted to the investigations of non-linear behaviours of heated elastic skew plates having various applications in modern design, especially in Air-craft Industry, Spaceship Technology and Astronautical Engineering. Also no attempt on the non-linear behaviours of skewed sandwich plates has been reported in the literature as yet.

It is seen that in solving various elastic problems, different methods have been employed. Based on the geometry of the elastic structure and the nature of the problem concerning it (e.g. static, dynamic or thermally induced) one method has got beauty and advantage over the other. Most of the earlier workers have utilised either Von-Kármán's classical equations or Berger's differential equations in their investigations on skew plates. But as already mentioned Von-Kármán's classical equations are in the coupled form and the transformations of these coupled equations in oblique co-ordinates involve much mathematical complexity. So this entails difficulty in solution as well. On the other hand

Berger's differential equations, although decoupled, are questionable mainly because it fails miseravly for movable edge conditions.

The object of modern research work is to find solutions of natural problems in such a way that the method employed and the level of presentation are lucid, computational labour is minimum and the results predicted are sufficiently accurate for the practical purpose.

The purpose of the present thesis is to present a simplified approach to the non-linear analyses of isotropic elastic rhombic plates (skew plates having aspect ratio 1) under different types of loadings and edge conditions. Banerjee's well-known hypothesis has been utilised in these investigations. A set of uncoupled differential equations has been obtained in oblique co-ordinates and solved by the Galerkin's Error Minimising Technique. Accuracy of the method has been tested in some static cases by comparison with experimental results and with other known results from the open literature. It has been observed that the results of the present study are in very good agreement with those obtained in open literature. The main advantages of the present study are that

(i) unlike the Von-Kármán's differential equations, the present differential equations are in the uncoupled form and thus easy to solve,

(ii) unlike Berger's uncoupled differential equations, the present uncoupled differential equations give results both for immovable as well as movable edge conditions with sufficient amount of accuracy,

and

(iii) from the same cubic equation, results both for immovable as well as movable edge conditions can be obtained with minimum computational labour, offering an additional advantage.

The present thesis has been divided into five chapters. In the first chapter (containing two papers), the non-linear static behaviours of thin isotropic elastic plates of uniform thickness, have been studied. The first paper of this chapter deals with large deflections of rhombic plates with simply-supported edge conditions. Non-linear static behaviours of rhombic plates have been analysed following Banerjee's hypothesis. Calculations have been done for different skew angles. To test the accuracy of the theoretical results thus obtained, experiments are carried out on copper-made and steel-made plates. The theoretical results are found to be in excellent agreement with those obtained from the experimental data.

The second paper in the first chapter,is devoted to the investigations of the non-linear static behaviours of clamped thin rhombic plates under uniform lateral pressure. Numerical results for different skew angles are presented. Here also accuracy of Banerjee's hypothesis has been tested by comparison with the experimental results and with other theoretical results.

The second chapter of the thesis contains only one paper on large deflection analyses of rhombic plates of vabiable thickness. In this paper both the non-linear static and dynamic behaviours of simply-supported rhombic plates of linearly varying thickness have been presented following Banerjee's hypothesis. Various numerical results showing the combined effects of skew angle and thickness variation parameter on the deflections and vibrations of the rhombic plates are given. Proper comparison has also been done.

The third chapter of the thesis contains one paper devoted to non-linear analysis of heated rhombic plates. This paper seems to predict some new information on thermal behaviours of rhombic plates. For 0^{o}-skew angle, the results obtained in the present study have been compared with the other results found in open literature. It is believed that the results obtained for other skew angles are completely new.

The fourth chapter also contains one paper on non-linear static and dynamic behaviours of the freely supported skewed sandwich plates. Banerjee's hypothesis has been employed in these investigations. Numerical results have been obtained for different skew angles. For 0^{o}- skew angle, results obtained are compared with other known results. Here also the results for other skew angles are believed to be new.

The last chapter of the thesis concerns with the non-linear analysis of moderately thick plates. This chapter also contains one paper. In this paper, the large amplitude free flexural vibrations of clamped as well as simply-supported isotropic elastic rhombic plates are investigated showing the effects of Shear Deformation and Rotatory Inertia, Various numerical results have been presented in tabular forms and in few cases they are compared with existing results.

From the numerical results obtained in the present study for different rhombic plates with different edge conditions and under Static, Dynamic and Thermal loadings, it is observed that the present study yields sufficiently accurate results from the practical point of view. Also for uncoupled form of differential equations Banerjee's equations transformed in oblique co-ordinates are simple and yield results both for immovable as well as movable edge conditions with minimum computational labour. Thus the present project seems to be more acceptable for practical purposes.

NOTATIONS

(x,y,z)	Rectangular Cartesian Co-ordinates,
(x_1,y_1)	Oblique Co-ordinates on the xy-plane,
θ	Skew Angle,
W	Deflection normal-to-the-middle-plane of the plate,
W_o	Maximum Central Deflection,
u,v	Inplane Displacements,
h	Thickness of the plate ,
p	Thickness-variation Parameter,
Q	Load Function,
q	Intensity of a Continuously Distributed Load,
E	Modulus of Elasticity in Tension and Compression,
G	Modulus of Elasticity in Shear,
ν	Poisson's Ratio,
D	Flexural Rigidity of the Plate $= Eh^3/12(1-\nu^2)$,
C_p	Speed of Wave Propagation along the Surface of the Plate,
α_t	Co-efficient of Linear Expansion,
ρ	Density of the Plate Material,
t	Time Parameter,
T	Linear Period of Vibration of the Plate,
T^*	Non-linear Period of Vibration of the Plate,
ω	Vibration Frequency of the Plate,
A_{oo}	Initial Amplitude of Vibration of the Plate,
$\bar{\beta}$	Non-dimensional Amplitude,
$A,\bar{\alpha},I_1^m$	Coupling Parameters,
τ_o	Temperature in the Middle Plane of the Plate,
∇^2	Laplacian Operator,

$\beta = w_o/h$ Central Deflection Parameter,

$\lambda = \nu^2$ for Simply-Supported Edge Condition of the elastic plate,

$\lambda = 2\nu^2$ for Clamped Edge Condition of the elastic plate.

CONTENTS

P.T.O.

CHAPTER I

NON-LINEAR STATIC ANALYSES OF THIN RHOMBIC PLATES

PAPER I

LARGE DEFLECTIONS OF RHOMBIC PLATES-
A NEW APPROACH*

ABSTRACT

In this paper non-linear static behaviour of Simply-Supported rhombic plates has been analysed following Banerjee's hypothesis. A set of uncoupled differential equations is obtained in oblique co-ordinates and solved by applying the Galerkin techinque. The case of a Simply-Supported rhombic plate is discussed in detail. Calculations have been carried out for different skew angles. To test the accuracy of the theoretical results so obtained, experiments are carried out on copper and steel rhombic plates. The theoretical results are found to be in excellent agreement with those obtained from the experimental data.

ANALYSIS

Let us consider a rhombic plate of an elastic, isotropic material, having uniform thickness 'h'. Let the size

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of each side of the skew plate be 'a' which is sufficiently large compared to 'h'. The origin of the rectangular Cartesian co-ordinate (x,y) is located at one of the corners of the skew plate (vide Fig.1). The plate is considered to be Simply-Supported along its edges and loaded uniformly all over.

Following Banerjee's hypothesis[162], the differential equations, referred to the system of rectangular Cartesian co-ordinates governing the deflections of the plate are

$$\nabla^4 W - \frac{12A}{h^2}\left(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\right) - \frac{6\lambda}{h^2}\left\{\nabla^2 W\left[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right]\right.$$
$$+2\left[\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2\right]$$
$$\left.+4\left(\frac{\partial^2 W}{\partial x\partial y}\right)\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial y}\right)\right\} = \frac{q}{D}. \quad (1)$$

where

$$A = \frac{1}{2}\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \nu\left(\frac{\partial W}{\partial y}\right)^2\right\} + \frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} \quad (2)$$

is a constant depending on the surface and edge conditions of the plate, and ∇^2 is the Laplacian operator.

For a skew plate, let us adopt a system of oblique co-ordinates (x_1, y_1, θ) as shown in Fig.1, θ being the skew angle. Clearly, $x = x_1 \cos\theta$, and $y = y_1 + x_1 \sin\theta$ (3)

are the co-ordinate transformation equations. Hence the partial differential operators become

$$\frac{\partial}{\partial x} \equiv \sec\theta\left(\frac{\partial}{\partial x_1} - \sin\theta\frac{\partial}{\partial y_1}\right); \quad \frac{\partial}{\partial y} \equiv \frac{\partial}{\partial y_1}; \quad \frac{\partial^2}{\partial y^2} \equiv \frac{\partial^2}{\partial y_1^2};$$

$$\frac{\partial^2}{\partial x^2} \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\frac{\partial^2}{\partial x_1\partial y_1}\right) + \tan^2\theta\frac{\partial^2}{\partial y_1^2};$$

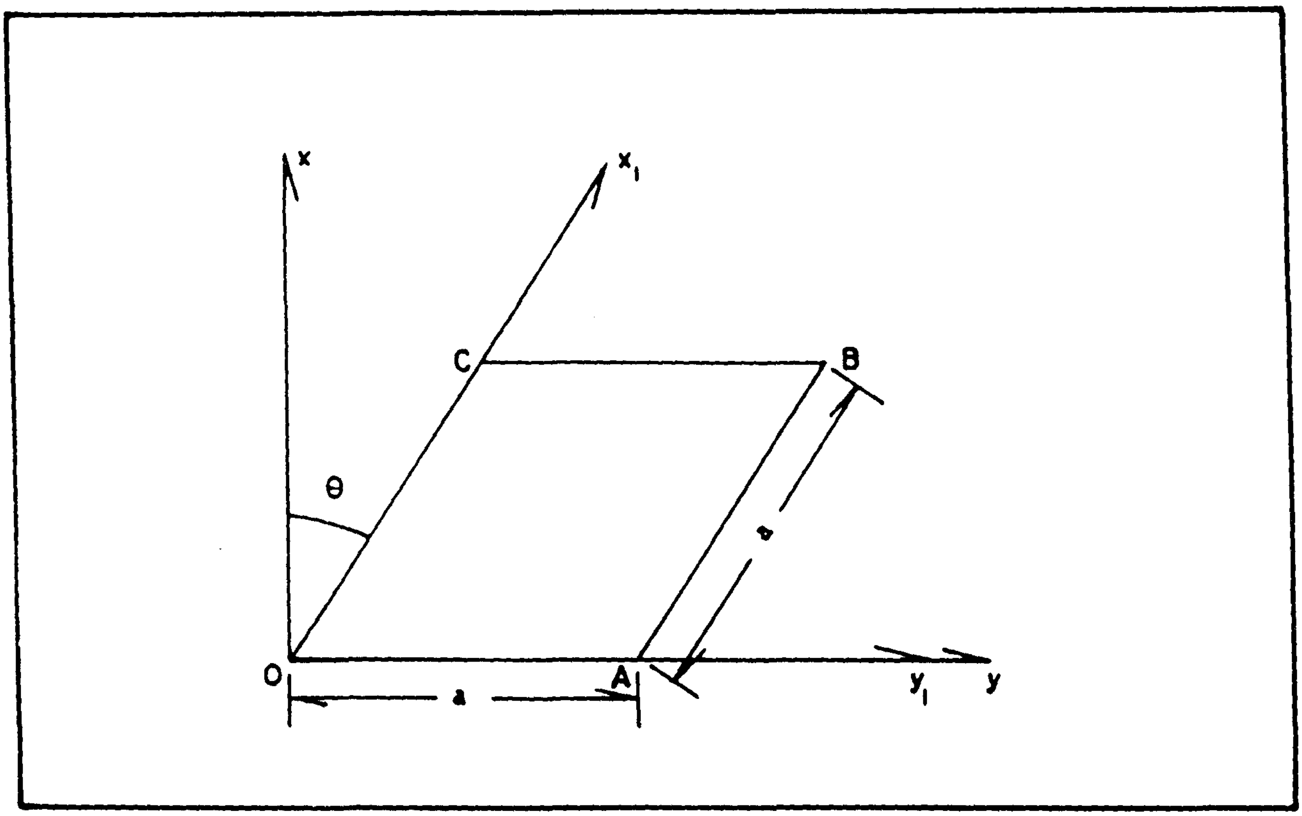

FIG. 1. PLAN FORM OF SKEW PLATE.

$$\frac{\partial^2}{\partial x \partial y} \equiv \sec\theta\left(\frac{\partial^2}{\partial x_1 \partial y_1} - \sin\theta \frac{\partial^2}{\partial y_1^2}\right) \; ;$$

$$\nabla^2 \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta \frac{\partial^2}{\partial x_1 \partial y_1} + \frac{\partial^2}{\partial y_1^2}\right)$$

and
$$\nabla^4 \equiv \sec^4\theta\left[\frac{\partial^4}{\partial x_1^4} - 4\sin\theta\left(\frac{\partial^4}{\partial x_1^3 \partial y_1} + \frac{\partial^4}{\partial x_1 \partial y_1^3}\right) + 2\left(1+2\sin^2\theta\right)\frac{\partial^4}{\partial x_1^2 \partial y_1^2} + \frac{\partial^4}{\partial y_1^4}\right] . \qquad (4)$$

Using these operators, transforming the differential equations (1) and (2) in oblique co-ordinates, we arrive at the following set of transformed differential equations :

$$\sec^4\theta\left[\frac{\partial^4 W}{\partial x_1^4} - 4\sin\theta\left(\frac{\partial^4 W}{\partial x_1^3 \partial y_1} + \frac{\partial^4 W}{\partial x_1 \partial y_1^3}\right) + 2\left(1+2\sin^2\theta\right)\frac{\partial^4 W}{\partial x_1^2 \partial y_1^2} + \frac{\partial^4 W}{\partial y_1^4}\right] - \frac{12A}{h^2}\left[\sec^2\theta\left(\frac{\partial^2 W}{\partial x_1^2} - 2\sin\theta\frac{\partial^2 W}{\partial x_1 \partial y_1}\right) + \tan^2\theta\frac{\partial^2 W}{\partial y_1^2} + \nu\frac{\partial^2 W}{\partial y_1^2}\right] - \frac{6\lambda}{h^2}\left\{\sec^4\theta\left(\frac{\partial^2 W}{\partial x_1^2} - 2\sin\theta\frac{\partial^2 W}{\partial x_1 \partial y_1} + \frac{\partial^2 W}{\partial y_1^2}\right)\left[\left(\frac{\partial W}{\partial x_1}\right)^2 + \left(\frac{\partial W}{\partial y_1}\right)^2 - 2\sin\theta\left(\frac{\partial W}{\partial x_1}\right)\left(\frac{\partial W}{\partial y_1}\right)\right] + 2\left[\sec^4\theta\left(\frac{\partial^2 W}{\partial x_1^2} - 2\sin\theta\frac{\partial^2 W}{\partial x_1 \partial y_1} + \sin^2\theta\frac{\partial^2 W}{\partial y_1^2}\right)\left(\frac{\partial W}{\partial x_1} - \sin\theta\frac{\partial W}{\partial y_1}\right)^2 + \frac{\partial^2 W}{\partial y_1^2}\left(\frac{\partial W}{\partial y_1}\right)^2\right] + 4\sec^2\theta\left(\frac{\partial^2 W}{\partial x_1 \partial y_1} - \sin\theta\frac{\partial^2 W}{\partial y_1^2}\right)\left(\frac{\partial W}{\partial x_1} - \sin\theta\frac{\partial W}{\partial y_1}\right)\left(\frac{\partial W}{\partial y_1}\right)\right\} = \frac{q}{D} . \qquad (5)$$

and
$$A = \frac{1}{2}\left\{\sec^2\theta\left[\left(\frac{\partial W}{\partial x_1}\right)^2 - 2\sin\theta\left(\frac{\partial W}{\partial x_1}\right)\left(\frac{\partial W}{\partial y_1}\right) + \sin^2\theta\left(\frac{\partial W}{\partial y_1}\right)^2\right] + \nu\left(\frac{\partial W}{\partial y_1}\right)^2\right\} + \sec\theta\left(\frac{\partial u}{\partial x_1} - \sin\theta\frac{\partial u}{\partial y_1}\right) + \nu\frac{\partial v}{\partial y_1} . \qquad (6)$$

Now to solve the problem, let us assume

$$W = W_o \sin\frac{\pi x_1}{a}\sin\frac{\pi y_1}{a} \qquad (7)$$

W_o being the maximum central deflection. Clearly W satisfies the following Simply-Supported edge conditions

$$W = 0 \text{ at } x_1 = \pm a \text{ and at } y_1 = \pm a \ ;$$

$$\frac{\partial^2 W}{\partial x_1^2} = 0 \text{ at } x_1 = \pm a \text{ and } \frac{\partial^2 W}{\partial y_1^2} = 0 \text{ at } y_1 = \pm a .$$

For the value of A, let us integrate equation (6) over the whole area of the plate. Then we have

$$\int_0^a \int_0^a A \cos\theta \, dx_1 dy_1 = \frac{1}{2} \int_0^a \int_0^a \Big\{ \sec^2\theta \Big[\Big(\frac{\partial W}{\partial x_1}\Big)^2 + \sin^2\theta \Big(\frac{\partial W}{\partial y_1}\Big)^2 - 2 \sin\theta \Big(\frac{\partial W}{\partial x_1}\Big)\Big(\frac{\partial W}{\partial y_1}\Big) \Big] + \nu \Big(\frac{\partial W}{\partial y_1}\Big)^2 \Big\} \cos\theta \, dx_1 dy_1 .$$

After integration, we get

$$A = \frac{\pi^2 W_0^2}{8a^2}\left(1 + \nu + 2\tan^2\theta\right) \tag{8}$$

For movable edge conditions the value of A will be zero . Here, it is to be noted that, since the normal displacements are our primary interest, the in-plane displacements in equation (2) have been eliminated through integration by choosing suitable expressions for them, compatible with their boundary conditions.

Now, applying Galerkin's method of approximation to the transformed differential equation (5) and keeping in mind the value of A from equation (8), we get the following cubic equation determining β ($= W_0 / h$).

$$\left(1 + \sin^2\theta\right)\beta + \frac{3}{8}\Big\{ \left[(1+\nu) + (1-\nu)\sin^2\theta \right]^2 + \nu^2\left(5 + \sin^2\theta\right) \Big\} \beta^3 = \frac{4}{\pi^6}\left(\frac{q a^4}{Dh}\right)\cos^4\theta \ . \tag{9a}$$

For the sake of comparison we adopt, now,the well-known equation of Berger[36], where e_2 has been neglected. The corresponding cubic equation determining the central deflection parameter (for immovable edges only) takes the following form (after applying Galerkin's technique) :

$$(1+\sin^2\theta)\beta + 1.5\,\beta^3 = \frac{4}{\pi^6}\left(\frac{qa^4}{Dh}\right)\cos^4\theta \quad . \tag{9b}$$

NUMERICAL CALCULATIONS

For a steel plate we have $E = 2 \times 10^{12}$ dyne/cm^2. and $\nu = 0.3$, for which equation (9a) becomes

$$(1+\sin^2\theta)\beta + \frac{3}{8}\left[(1.3+0.7\sin^2\theta)^2 + 0.09(5+\sin^2\theta)\right]\beta^3 = 22.66 \times 10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \quad . \tag{10a}$$

whereas for a copper plate we have $E = 1.25 \times 10^{12}$ dyne/cm^2. and $\nu = 0.333$, so that equation (9a) becomes

$$(1+\sin^2\theta)\beta + \frac{3}{8}\left[(1.333+0.667\sin^2\theta)^2 + 0.11(5+\sin^2\theta)\right]\beta^3 = 35.46 \times 10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \quad . \tag{10b}$$

Also, for a steel plate equation (9b) becomes

$$(1+\sin^2\theta)\beta + 1.5\,\beta^3 = 22.66 \times 10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \quad . \tag{10c}$$

and for a copper plate it becomes

$$(1+\sin^2\theta)\beta + 1.5\beta^3$$
$$= 35.46\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \quad \text{.} \qquad (10d)$$

METHOD OF EXPERIMENT

A sketch of the apparatus used for the experimental purpose is shown in Fig.2. Two skew boxes with upper side open are constructed, each of whose four side walls are made of steel. Each vertical wall of one box is 16 cm. and of the other is 14 cm. The upper side of each wall is made sharp (knife edge), care being taken to see that all the knife edges lie on the same horizontal plane. The walls of the box with sides 16 cm. long are welded in such a manner that the two opposite angles are each 75^{o} and the other two opposite angles are each 105^{o}. Two opposite angles of the second box with sides 14 cm. long are each 60^{o} and the other two opposite angles are each 120^{o}. Two holes are drilled on two opposite sides of each box and fitted with short metal pipes, one of which acts as an air inlet and the other as an air outlet.

For the experiment with the first skew box, the centre of the box is first found and then a plumb-line is set as an indicator along the vertical line on which the centre of the box lies. For the free movable boundary conditions one Test plate (which is approximately mirror surfaced) is symme-

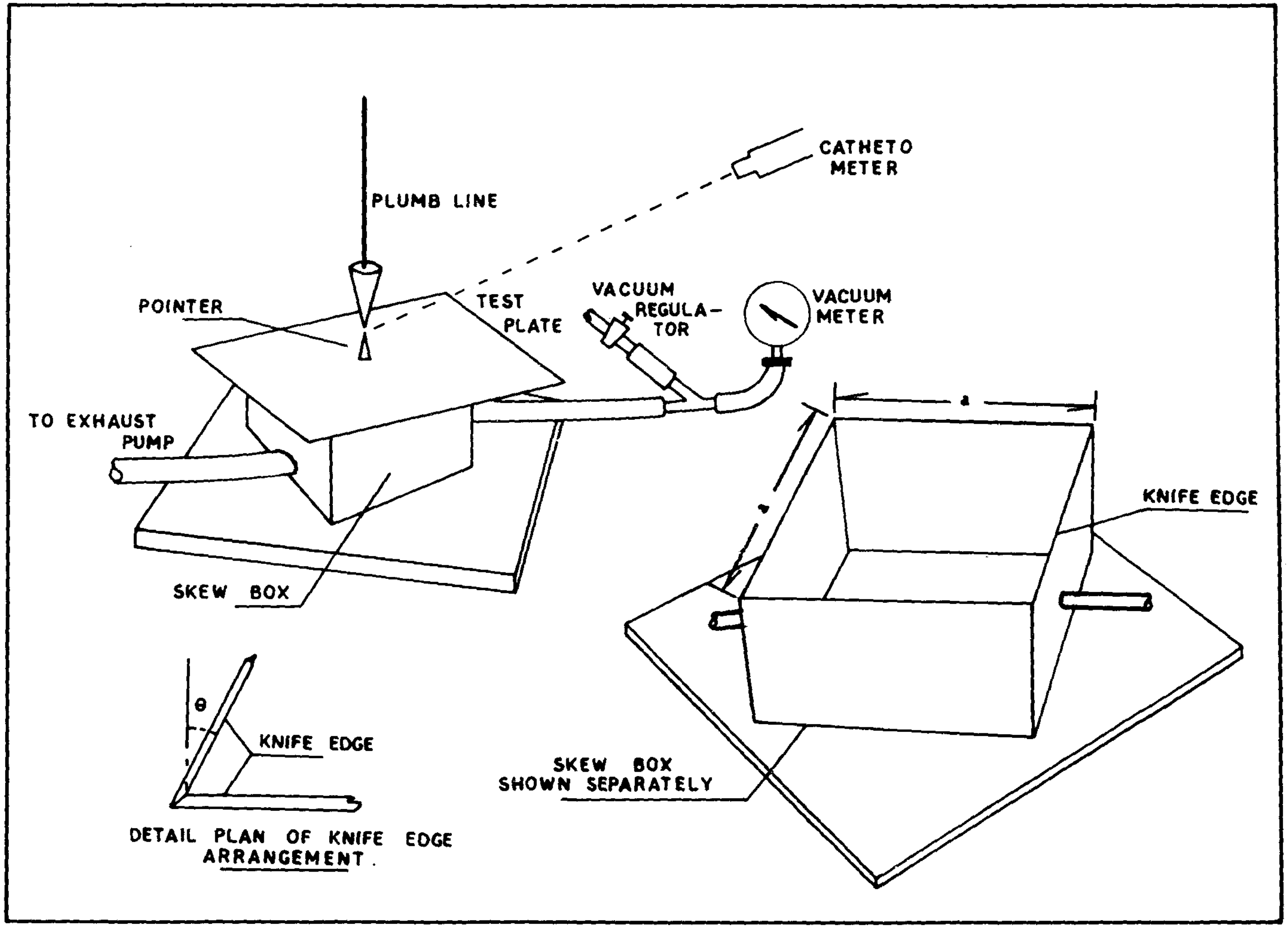

FIG. 2. THE EXPERIMENTAL ARRANGEMENT.

trically placed on the knife edges of the box and a pointer is fixed on the upper surface of the Test plate with some adhesive along the plumb-line. The outlet pipe is then joined to an exhaust pump by rubber tubing and the inlet pipe is joined to a standard vacuum meter and an air pressure regulator (as shown in the sketch). Along the contact line beneath the Test plate some thick grease is used to make the box perfectly airtight, (Grease does not apply any appreciable tension on the plate). When the exhaust pump operates, the box becomes evacuated, thereby causing the depression of the Test plate by the excess outside air pressure, which is uniform all over the effective skew part of the Test plate. The central deflection of the Test plate is easily measured with the help of a precision cathetometer set at a distance of approximately 1.5m. from the pointer.

To make the free boundaries of a skew plate immovable, four pieces of steel collars are taken whose lengths are equal to the length of outer boundary line of the skew plate. The collars are kept outside the box in contact with the lower surface of the plate and with the side walls of the box and then the collars are tightly clamped with the Test plate using nuts and bolts in sufficient number well outside the boundary of skew section.

Tables 1 and 2 present a comparative view of the various theoretical and experimental values of the central deflection parameter β ($= W_o$ / h) for different values of the load function Q ($= qa^4/Dh$), for the case of steel plate and copper plate respectively.

Comparison tables (showing theoretical vs experimental results as well as showing changes for immovable edge conditions from movable edge conditions).

TABLE 1

(For Steel Plate)

a = 16cm., when $\theta = 15^{\circ}$; a = 14cm., when $\theta = 30^{\circ}$; h = 0.1343cm.

$\beta = W_o/h$, when $\theta = 15^{\circ}$

$Q = \frac{qa^4}{Dh}$	Movable Edges			Immovable Edges				
	From Banerjee hypothesis	From Experiment	Percentage of error	From Berger Method	From Banerjee hypothesis	From Experiment	Percentage of error	
							From Berger Method	From Banerjee hypothesis
111.72	0.3716	0.3872	4.0 %	0.3285	0.3454	0.36485	9.96 %	5.33 %
223.44	0.7038	0.7372	4.5 %	0.5378	0.5914	0.6329	15 %	6.56 %
335.16	0.98592	1.0201	3.3 %	0.6843	0.7703	0.8414	18.67 %	8.45 %
446.88	1.22484	1.2882	4.9 %	0.7982	0.9107	0.9903	19.40 %	8 %
558.6	1.4303	1.5115	5.4 %	0.8924	1.027	1.1244	20.60 %	8.66 %

Contd.........

TABLE 1 (Continued)

$Q = \frac{qa^4}{Dh}$	Movable Edges			Immovable Edges				
	$\beta = W_o/h$, when $\theta = 30^o$							
	From Banerjee hypothesis	From Experiment	Percentage of error	From Berger Method	From Banerjee hypothesis	From Experiment	Percentage of error: From Berger Method	Percentage of error: From Banerjee hypothesis
65.5	0.12208	0.134	8.90 %	0.1202	0.1209	0.12658	5.00 %	4.49 %
131	0.2427	0.25316	4.13 %	0.2301	0.2346	0.25316	9.11 %	7.33 %
196.5	0.3604	0.37975	5.10 %	0.3254	0.3369	0.36485	10.80 %	7.66 %
262	0.4742	0.4989	5.00 %	0.40782	0.4276	0.46165	11.66 %	7.40 %
327.5	0.5835	0.61802	5.59 %	0.4795	0.5081	0.55845	14.20 %	9.00 %

Average percentage of error by utilising Banerjee's hypothesis is only around 6% for skew angles $\theta = 15^o$, and $\theta = 30^o$ whereas by utilising Berger method it is around 17% for $\theta = 15^o$ and 10% for $\theta = 30^o$

TABLE 2

(For Copper Plate).

$a = 16$cm., when $\theta = 15^{\circ}$; $a = 14$cm., when $\theta = 30^{\circ}$; $h = 0.0789$cm.

$\beta = w_o/h$, when $\theta = 15^{\circ}$

$Q = \frac{qa^4}{Dh}$	Movable Edges			Immovable Edges				
	From Banerjee hypothesis	From Experiment	Percentage of error	From Berger Method	From Banerjee hypothesis	From Experiment	Percentage of error: From Berger Method	Percentage of error: From Banerjee hypothesis
1467.53	2.3727	2.4208	2 %	1.36820	1.57802	1.673	18.22 %	5.68 %
2935.06	3.2506	3.308	1.70 %	1.79580	2.08753	2.23067	19.50 %	6.40 %
4402.59	3.8437	3.9924	3.72 %	2.0891	2.43578	2.6109	19.98 %	6.70 %
5870.12	4.307	4.4867	4 %	2.32003	2.7095	2.90241	20.07 %	6.65 %
7337.65	4.6935	4.90494	4.30 %	2.5138	2.93893	3.1559	20.35 %	6.90 %

Contd ....

TABLE 2 (Continued)

$\beta = w_o/h$, when $\theta = 30^\circ$

$Q = \frac{qa^4}{Dh}$	Movable Edges: From Banerjee hypothesis	Movable Edges: From Experiment	Movable Edges: Percentage of error	Immovable Edges: From Berger Method	Immovable Edges: From Banerjee hypothesis	Immovable Edges: From Experiment	Immovable Edges: Percentage of error From Berger Method	Immovable Edges: Percentage of error From Banerjee hypothesis
860.2	1.2602	1.2801	1.55 %	0.8553	0.9283	0.9886	13.50 %	6.10 %
1720.4	1.9429	2.0279	4.20 %	1.1901	1.3095	1.40684	15.50 %	6.92 %
2580.6	2.4064	2.5095	4.10 %	1.4156	1.5657	1.6857	16.00 %	7.12 %
3440.8	2.765	2.9404	6.00 %	1.5913	1.7649	1.90114	16.30 %	7.17 %
4301	3.0616	3.2319	5.30 %	1.7376	1.9307	2.09125	16.91 %	7.70 %

Average percentage of error by utilising Banerjee's hypothesis is only around 5 % for skew angles $\theta = 15^\circ$ and $\theta = 30^\circ$ whereas by utilising Berger method it is around 20 % for $\theta = 15^\circ$ and around 15 % for $\theta = 30^\circ$.

N.B. - Errors are calculated considering experimental results as standard (sacrificing instrumental and personal errors).

OBSERVATIONS

It is observed from the two tables that the results of the present study are in excellent agreement with those obtained from the experimental analysis. It is well-known that Berger's method fails[114] miserably under movable edge conditions. The results for simply-supported immovable edges, obtained by Berger's method (as shown in the Tables 1 and 2) show that this method is not even acceptable from the practical point of view. It is worth noting that Berger's method always gives less deflections for a given load. The errors of Berger's method (as shown in Tables 1 and 2) are certainly questionable from the view point of safety design.

It is observed that deflections for movable edges are always greater than those for immovable edges. This is quite expected from the practical point of view, because movable edge conditions give stress-free boundary and, hence, there are large energy changes in the boundary.

Here the results for skew angles $\theta = 15^{\circ}$ and 30° only have been considered, because, for greater values of the skew angles the effect of non-linearity does not play important role in design, and the study of linear analysis serves the practical purpose.

PAPER II

NON-LINEAR BEHAVIOURS OF CLAMPED RHOMBIC PLATES- A NEW APPROACH

ABSTRACT

In this paper non-linear static behaviours of clamped (along all edges) thin rhombic plates under uniform normal pressure have been analysed following Banerjee's hypothesis. Numerical results for different skew angles are presented. Comparisons (both numerically and graphically) are made with available existing results for skewed plates. The effects of skew angle on large deflections are carefully investigated. To test the accuracy of the theoretical results, so obtained, experiments have also been carried out on copper-made and steel-made plates. Both the cases of movable and immovable edge conditions have been delt with. It is observed that the present theoretical results are to the close proximity of the results obtained from the experimental analysis.

ANALYSIS

Let us consider a rhombic plate as shown in Fig.3. It is of an isotropic, elastic material, whose uniform thickness

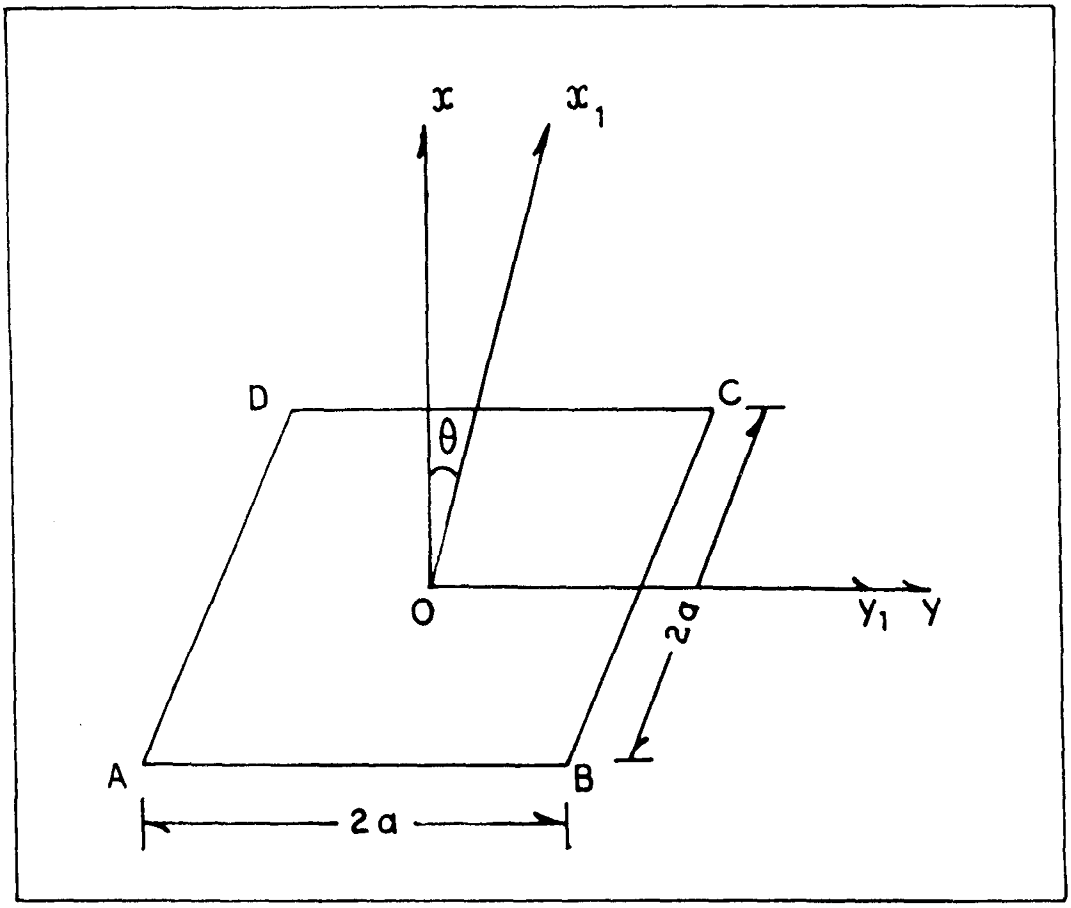

FIG.3: PLAN FORM OF SKEW PLATE AND CO-ORDINATE SYSTEM

is 'h' . Also let the sides of the rhombic plate be each of '2a', sufficiently great compared to 'h'. The origin of the rectangular cartesian co-ordinates (x,y) is located at the geometric centre of the skewed plate. The plate is considered to be clamped along its four edges and loaded uniformly all over.

Following Banerjee's hypothesis, the differential equations, referred to the system of rectangular cartesian co-ordinates are transformed in oblique co-ordinates as in paper I.

Now to solve the differential equations (5) and (6) in paper I, we assume

$$W = W_o \cos^2 \frac{\pi x_1}{2a} \cos^2 \frac{\pi y_1}{2a} \quad . \tag{11}$$

W_o being the maximum central deflection. Clearly the deflection function W satisfies all the boundary conditions for a skew plate clamped along its four edges, viz.

$$W = o \text{ at } x_1 = \pm a \text{ and at } y_1 = \pm a.$$

$$\frac{\partial W}{\partial x_1} = o \text{ at } x_1 = \pm a \text{ and } \frac{\partial W}{\partial Y_1} = o \text{ at } y_1 = \pm a.$$

To determine the value of A, we are to integrate equation (6), as usual, over the whole area of the skew plate. Thus we have

$$\int_{-a}^{+a}\int_{-a}^{+a} A\cos\theta\, dx_1 dy_1 = \frac{1}{2}\int_{-a}^{+a}\int_{-a}^{+a}\Big[\operatorname{Sec}^2\theta\Big\{\Big(\frac{\partial W}{\partial x_1}\Big)^2 + \operatorname{Sin}^2\theta\Big(\frac{\partial W}{\partial y_1}\Big)^2 - 2\operatorname{Sin}\theta\Big(\frac{\partial W}{\partial x_1}\Big)\Big(\frac{\partial W}{\partial y_1}\Big)\Big\} + \nu\Big(\frac{\partial W}{\partial y_1}\Big)^2\Big]\cos\theta\, dx_1 dy_1 \, .$$

After integration we get

$$A = \frac{3\pi^2 W_o^2}{128 a^2}\left(1 + \nu + 2\tan^2\theta\right) .$$

(12)

Now applying Galerkin's method of approximation to the transformed differential equation (5) and keeping in mind the value of A from (12) we get the following cubic equation determining the deflection parameter $\beta = W_o/h$

$$\left(2 + \sin^2\theta\right)\beta + \left\{\frac{27}{128}\left(1 + \nu + 2\tan^2\theta\right)^2 \cos^4\theta + \frac{30\nu^2}{128}\left(13 + 5\sin^2\theta\right)\right\}\beta^3 = \frac{4}{\pi^4}\cos^4\theta\, Q .$$

(13)

where $Q = qa^4/Dh$ is the load function.

NUMERICAL CALCULATIONS

For a copper plate we have, $E = 1.25 \times 10^{12}$ dyne $/cm^2$. and $\nu = 0.333$, so that for such a plate, the equation (13) becomes

$$\left(2 + \sin^2\theta\right)\beta + \left\{\frac{27}{128}\left(1.333 + 2\tan^2\theta\right)^2 \cos^4\theta + \frac{3.327}{128}\left(13 + 5\sin^2\theta\right)\right\}\beta^3 = 0.041\cos^4\theta\, Q .$$

(14)

whereas for a steel plate we have $E = 2 \times 10^{12}$ dyne/cm^2. and $\nu = 0.3$, so that for such a plate, the equation (13) becomes

$$(2+\sin^2\theta)\beta + \left\{\frac{27}{128}(1.3 + 2\tan^2\theta)^2 \cos^4\theta + \frac{2.7}{128}(13 + 5\sin^2\theta)\right\}\beta^3 = 0.041 \cos^4\theta \, Q \; . \qquad (15)$$

METHOD OF EXPERIMENT

The sketch of the apparatus used for the experimental purpose is shown in Fig.4. Three skew boxes with upper side open are constructed each of whose four side-walls are made of steel of 6mm. thickness. Each vertical wall of two boxes is 16cm. long and of the other is 14cm. long. The upper side of each wall is made sharp (knife edge), care being taken so that all the knife-edges lie on the same horizontal plane. The walls of the boxes with sides 16cm.long are arc-welded in such a manner that the two opposite angles are each $75^{\circ}$and other two opposite angles are each 105°. The two opposite angles of the 3rd box with sides 14cm. long are each $60^{\circ}$and the other two opposite angles are each 120°. Two holes are drilled on two opposite side-walls of each box and then fitted with short metal pipes, one of which acts as an air inlet and the other as an air outlet.

For the experiment with one skew box, the centre of the box is first found and then a plumb-line is set as indicator along the same vertical line on which the centre of the box lies. For the clamped movable boundary conditions the experimental plate (which is approximately mirror surfaced) is

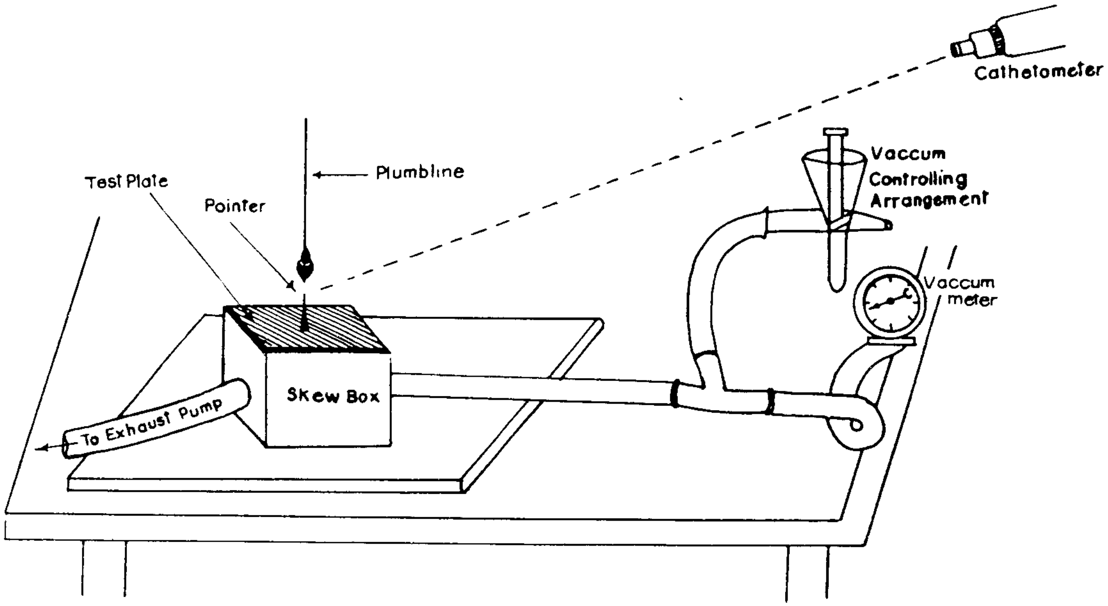

FIG-4. THE EXPERIMENTAL ARRANGEMENT

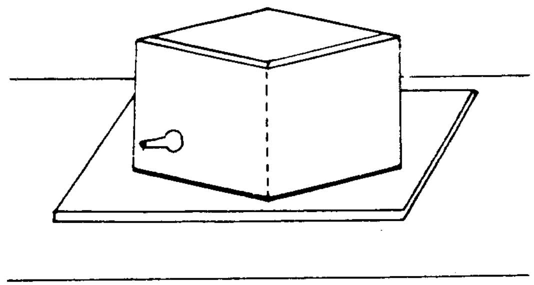

FIG-4a THE SKEW BOX SHOWN SEPARATELY

symmetrically placed on the knife-edged side of the box and a pointer (made of very thin plastic rod) is fixed on the upper surface of the experimental plate with some adhesive, along the plumb-line. The contact lines of the plate with the knife-edges are uniformly arc-welded from outside. The outlet pipe of the box is connected to a vacuum pump and the inlet pipe is connected to a standard vacuum meter and an air inlet regulator (using a T-section as shown in the sketch). When the vacuum pump operates, the box is evacuated, thereby causing the depression of the experimental plate by the excess outside air pressure, which is uniform all over the effective skew part of the plate and the pressure is noted from the pressure gauge. The central deflection of the plate is easily measured with the help of a precesion cathetometer set at a distance of approximately 1.5 metres from the pointer.

To make the boundaries of the skew plate 'clamped immovable', the elongated portions of the plate beyond the knife-edge of the box are cut and then the whole boundary of the experimental plate is are-welded with the supporting knife edges covering the full thickness (h) of the experimental plate at its boundary. The experimental procedure is now, as usual.

The following tables and graphs show a comparative study of the central deflection parameter β Vs. load function Q obtained by the theoretical and experimental methods. For movable edge conditions A = 0, as usual.

TABLE 1

Showing comparison of results obtained from different theories and experiment for copper-made skew-plate ($\nu = 0.333$, $E = 1.25 \times 10^{12}$ dyne/cm^2, $\theta = 15^o$, a = 8cm., and h = 0.0789cm.)

Value of Q	Movable Edges				
	Uniform pressure applied in the Expt.(inch of Hg)	Value of W_0 from the Expt. (cm)	Value of β by the Expt.	Value of β by Banerjee's hypothesis	Percentage of error w.r. to Expt.
20	-	-	-	-	-
50	-	-	-	-	-
61.08	2"	0.073	0.92522	0.92284	0.257 %
100	-	-	-	-	-
122.16	4"	0.121	1.5336	1.5202	0.874 %
150	-	-	-	-	-
183.24	6"	0.158	2.00253	1.93987	3.13 %
200	-	-	-	-	-
244.32	8"	0.182	2.30672	2.26643	1.75 %
250	-	-	-	-	-
300	-	-	-	-	-
305.4	10"	0.205	2.59823	2.5366	2.372 %

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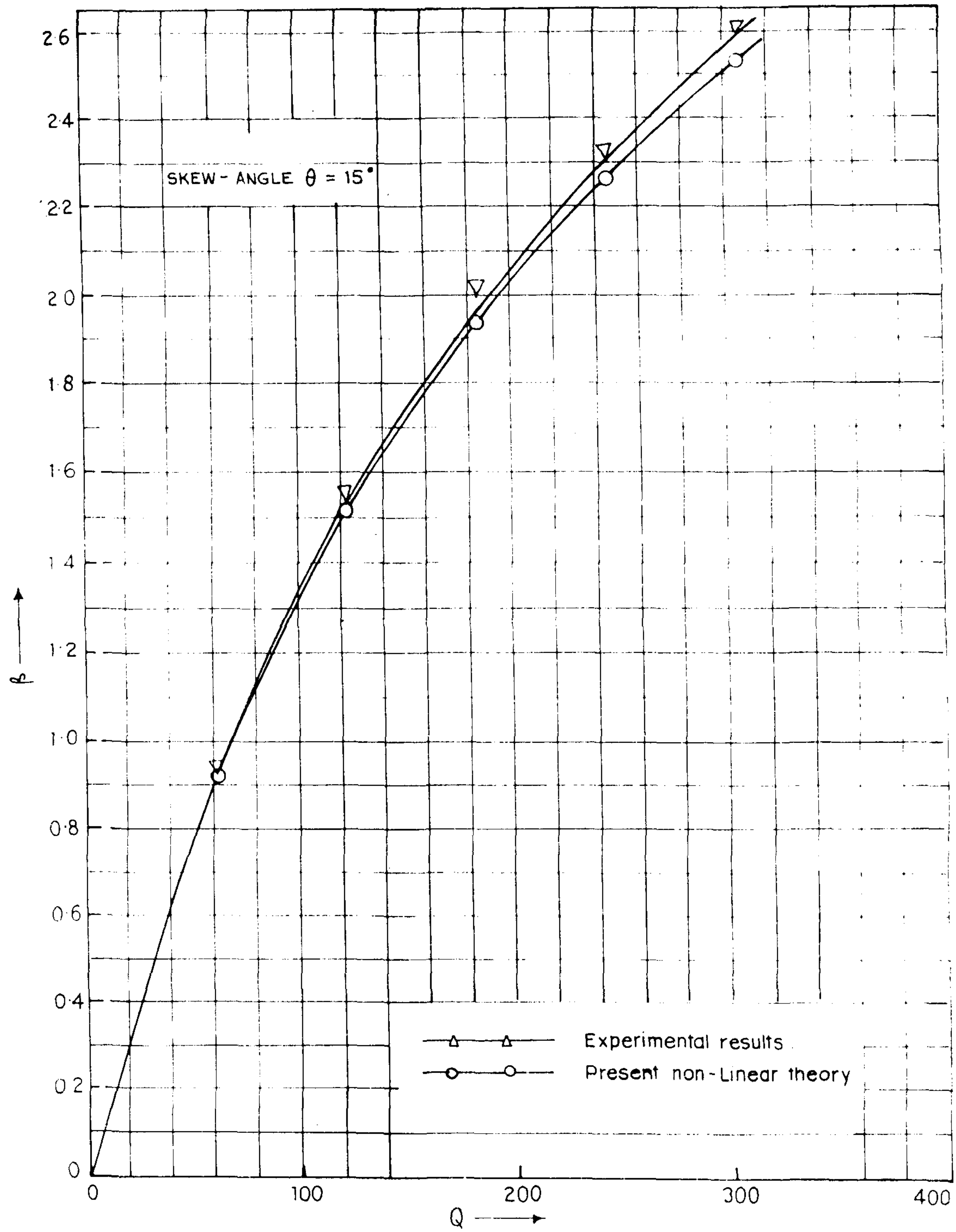

FIG. 5: GRAPHS SHOWING VARIATION OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC COPPER PLATE ($\nu = 0{\cdot}333$) WHOSE EDGES ARE KEPT MOVABLE

TABLE 1 (Continued)

Value of Ω	Immovable Edges						
	Uniform pressure applied in the Expt. (inch of Hg)	Value of W_o from the Expt. (cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Value of β from Kennedy and Simon's curve (Ref.85)	Percentage of error by Banerjee's hypothesis w.r. to Expt.	Percentage of error by Kennedy and Simon's theory w.r. to Expt.
20	-	-	-	0.3321	0.28	-	-
50	-	-	-	0.7254	0.62	-	-
61.08	2"	0.068	0.86185	0.84025	0.71	2.5 %	17.62 %
100	-	-	-	1.161	0.975	-	-
122.16	4"	0.105	1.3308	1.3054	1.11	1.91 %	16.6 %
150	-	-	-	1.4616	1.25	-	-
183.24	6"	0.131	1.66033	1.622	1.4	2.43 %	15.68 %
200	-	-	-	1.6946	1.475	-	-
244.32	8"	0.151	1.91382	1.86695	1.63	2.45 %	14.83 %
250	-	-	-	1.8873	1.65	-	-
300	-	-	-	2.053	1.8	-	-
305.4	10"	0.168	2.12928	2.0696	1.815	2.8 %	14.76 %

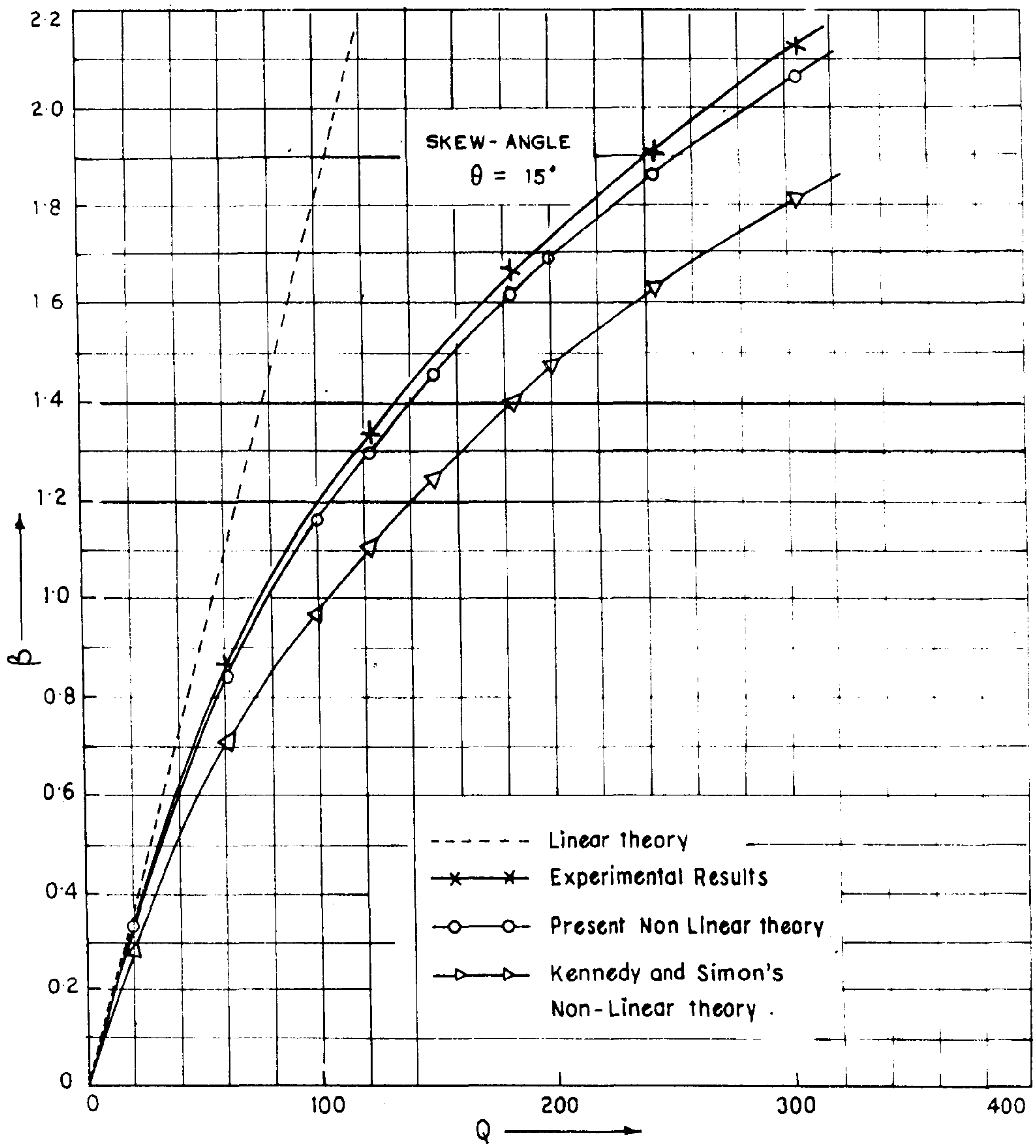

FIG. 6: GRAPHS SHOWING DEVIATIONS OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC COPPER-PLATE WHOSE EDGES ARE KEPT IMMOVABLE (ϑ = 0·333).

TABLE 2

Showing comparison of results obtained from different theories and experiment for copper-made skew plate ($\nu = 0.333$, $E = 1.25 \times 10^{12}$ dyne/cm^2., $\theta = 30^\circ$, a = 7cm., and h = 0.0789cm.)

Value of Q	Movable Edges				
	Uniform pressure applied in the Expt.(inch of Hg)	Value of W_0 from the Expt. (cm.)	Value of β ,by the Expt.	Value of β from Banerjee's hypothesis	Percentage of error w.r.to Expt.
20	—	—	—	—	—
50	—	—	—	—	—
53.7	3"	0.042	0.53232	0.52642	1.11 %
100	—	—	—	—	—
107.4	6"	0.078	0.98859	0.9567	3.22 %
150	—	—	—	—	—
161.1	9"	0.107	1.35615	1.29435	4.55 %
200	—	—	—	—	—
214.8	12"	0.129	1.63498	1.5676	4.12 %
250	—	—	—	—	—
268.5	15"	0.148	1.8758	1.797	4.2 %
300	—	—	—	—	—

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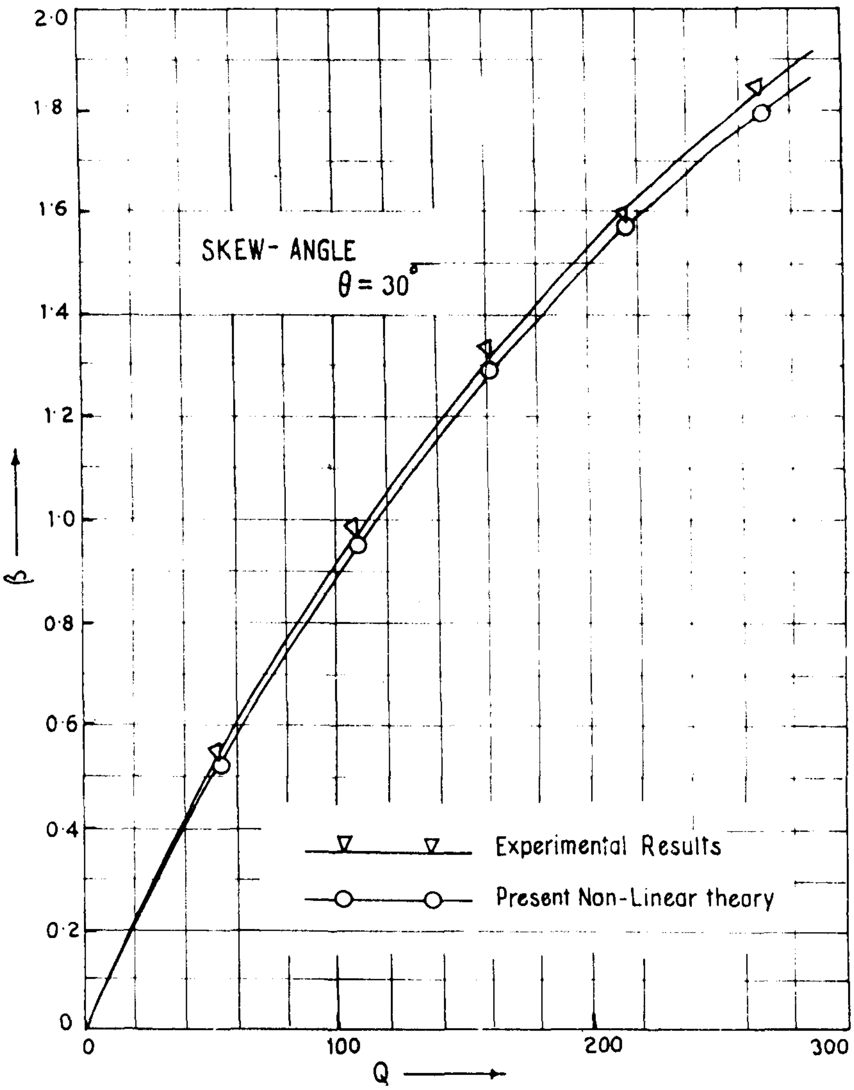

FIG. 7: GRAPHS SHOWING VARIATION OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC COPPER-PLATE ($\nu = 0{\cdot}333$) WHOSE EDGES ARE KEPT MOVABLE.

TABLE 2 (Continued)

Value of Q	Immovable Edges						
	Uniform pressure applied in the Expt. (inch of Hg)	Value of W_o from the Expt. (cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Value of β from Kennedy and Simon's curve (Ref.85)	Percentage of error by Banerjee's hypothesis w.r. to Expt.	Percentage of error by Kennedy and Simon's theory w.r. to Expt.
20	-	-	-	0.2019	0.22	-	-
50	-	-	-	0.47282	0.45	-	-
53.7	3"	0.04	0.50697	0.5028	0.475	0.822 %	6.3 %
100	-	-	-	0.8117	0.78	-	-
107.4	6"	0.07	0.8872	0.8611	0.83	2.94 %	6.45 %
150	-	-	-	1.0733	1.03	-	-
161.1	9"	0.09	1.14068	1.12156	1.08	1.676 %	5.32 %
200	-	-	-	1.274	1.225	-	-
214.8	12"	0.106	1.34347	1.3261	1.275	1.29 %	5.1 %
250	-	-	-	1.4404	1.39	-	-
268.5	15"	0.12	1.5209	1.49573	1.445	1.65 %	5 %
300	-	-	-	1.5837	1.525	-	-

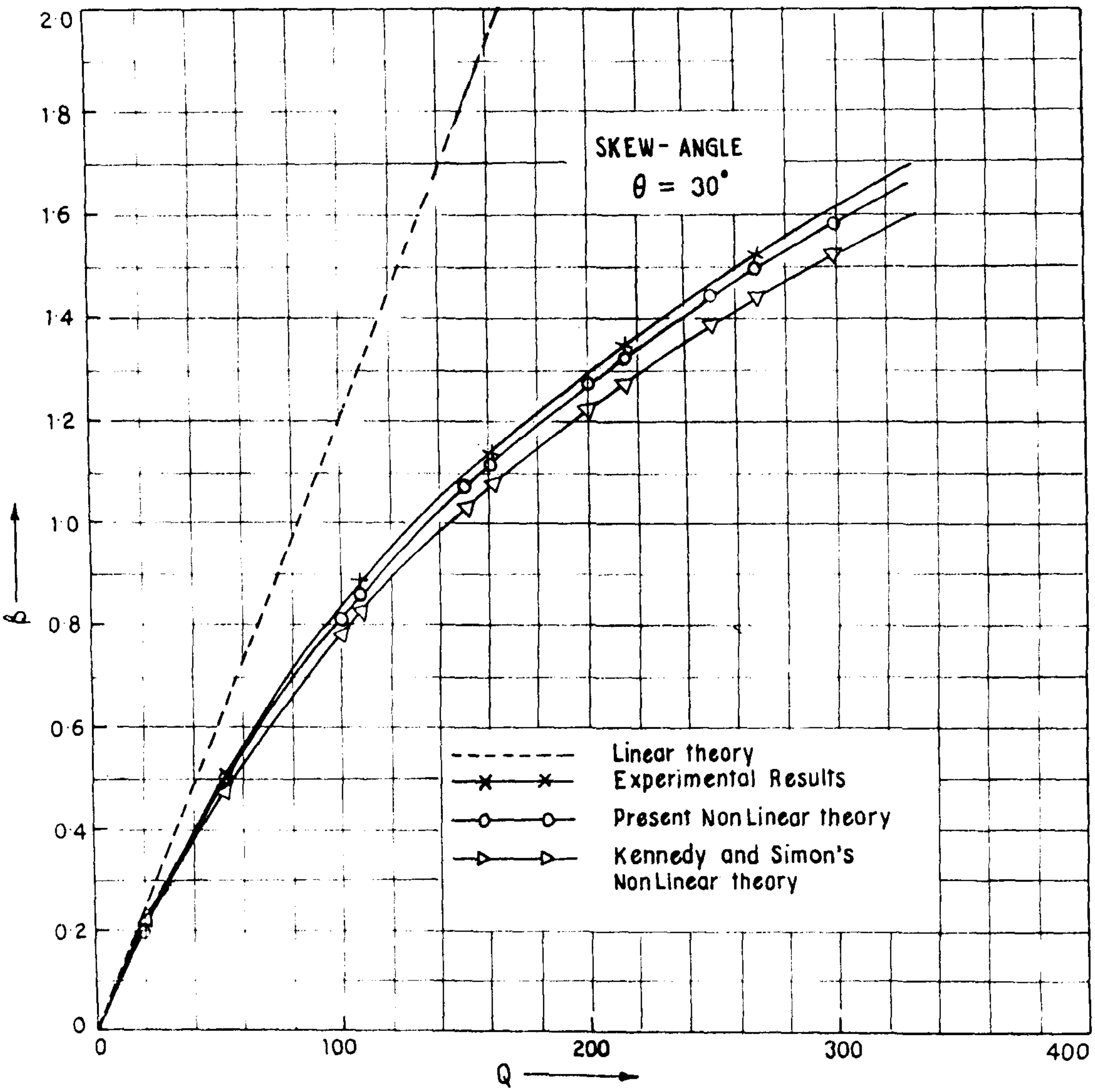

FIG. 8 : GRAPHS SHOWING DEVIATIONS OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC COPPER-PLATE WHOSE EDGES ARE KEPT IMMOVABLE ($\nu = 0.333$) .

TABLE 3

Showing comparison of results obtained from the present non-linear theory and experiment for steel-made skew plate ($\nu = 0.3$, $E = 2 \times 10^{12}$ dyne/cm^2. , $\theta = 15^{\circ}$, a = 8cm. and h = 0.1343cm.)

Value of Q	Movable Edges				
	Uniform normal pressure applied in the Expt. (inch of Hg)	Value of W_o from the Expt.(cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Percentage of error w.r. to the Expt.
9.31	4"	0.023	0.17126	0.16019	6.46 %
18.62	8"	0.044	0.32762	0.3172	3.18 %
27.93	12"	0.065	0.48399	0.46827	3.248 %
37.24	16"	0.084	0.625465	0.61182	2.182 %
46.55	20"	0.104	0.7744	0.747	3.54 %

Contd ....

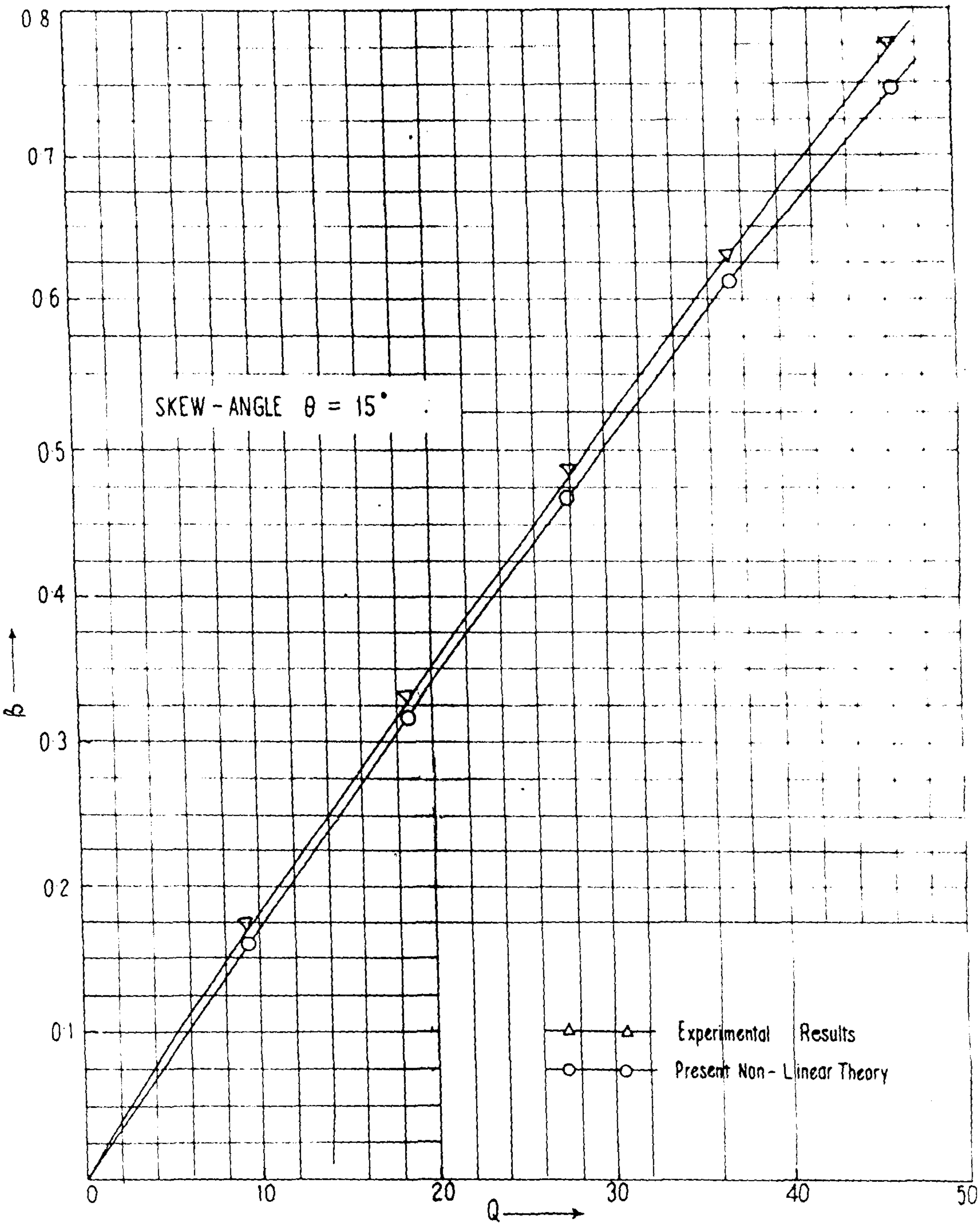

FIG-9: GRAPHS SHOWING DIVIATIONS OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC STEEL PLATE WHOSE EDGES ARE KEPT MOVABLE, (ν = 0·3).

TABLE 3 (Continued)

Value of Q	Immovable Edges				
	Uniform normal pressure applied in the Expt. (inch of Hg)	Value of W_0 from the Expt.(cm.)	Value of β by the Expt.	Value of β from Banerjee's hypothesis	Percentage of error w.r.to the Expt.
9.31	4"	0.022	0.16381	0.15944	2.67 %
18.62	8"	0.043	0.32018	0.31176	2.63 %
27.93	12"	0.063	0.4691	0.45248	3.543 %
37.24	16"	0.08	0.59568	0.58024	2.592 %
46.55	20"	0.096	0.71482	0.69562	2.686 %

N.B :- All the errors are calculated considering our experimental results as standard, sacrificing instrumental (if any) and personal errors

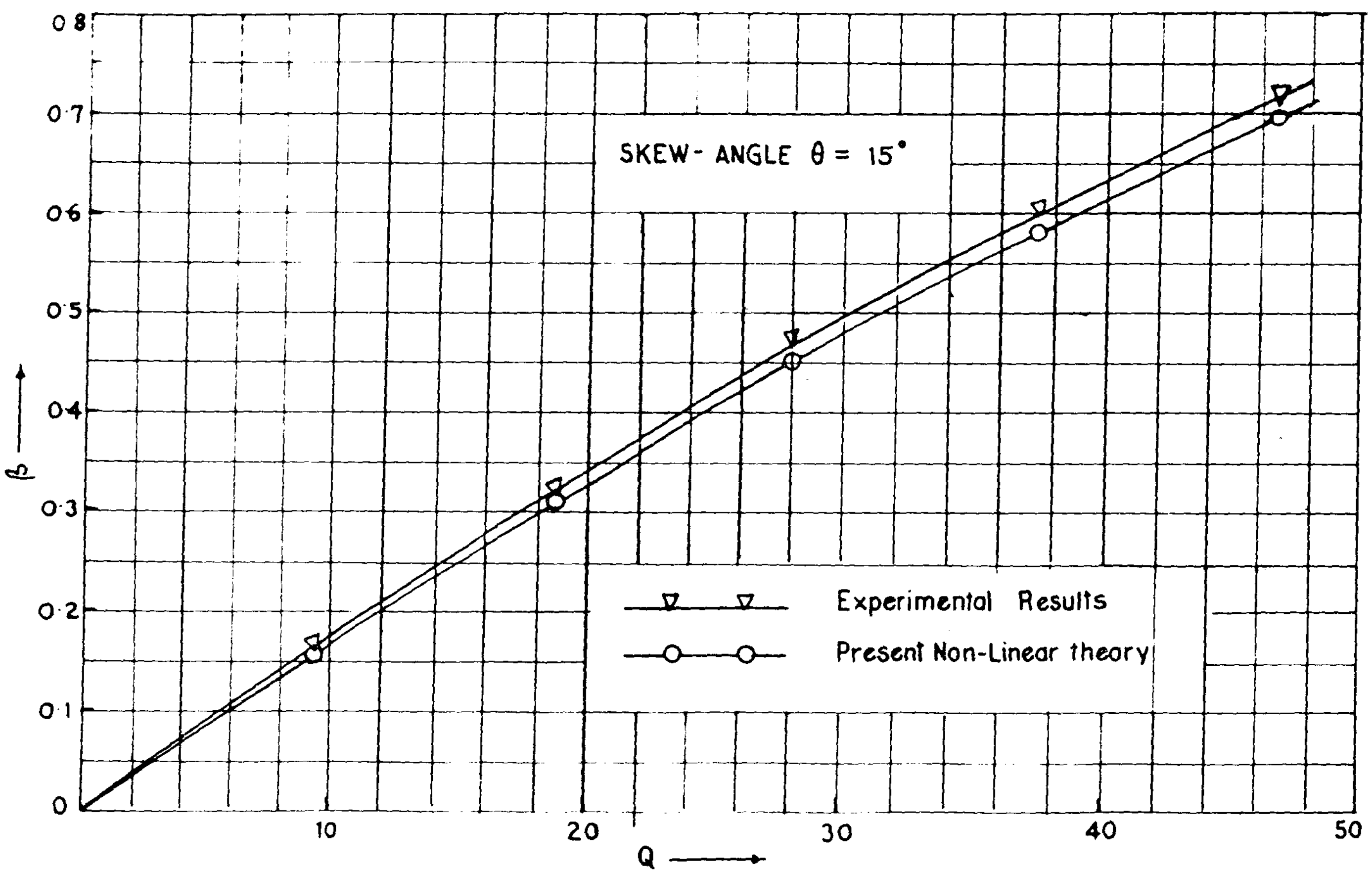

FIG. 10: GRAPHS SHOWING VARIATION OF CENTRAL DEFLECTIONS OF A CLAMPED RHOMBIC STEET-PLATE WHOSE EDGES ARE KEPT IMMOVABLE ($\nu = 0 \cdot 3$)

TABLE 4

Showing the variation of results obtained by the present theory due to change of edge conditions for steel-made skew plate (ν = 0.3, E = 2 X 10^{12} dyne/cm^2. θ = 30°)

Value of Ω	Movable edges	Immovable Edges	Difference of β due to change of Edge conditions
	Value of β from Banerjee's hypothsis.	Value of β from Banerjee's hypothesis	
20	0.2039	0.20221	0.00169
60	0.5879	0.5568	0.0311
100	0.9207	0.8312	0.0895
160	1.3334	1.14	0.1934
200	1.5515	1.3031	0.2484
260	1.83692	1.5078	0.32912
300	2.0024	1.6254	0.377

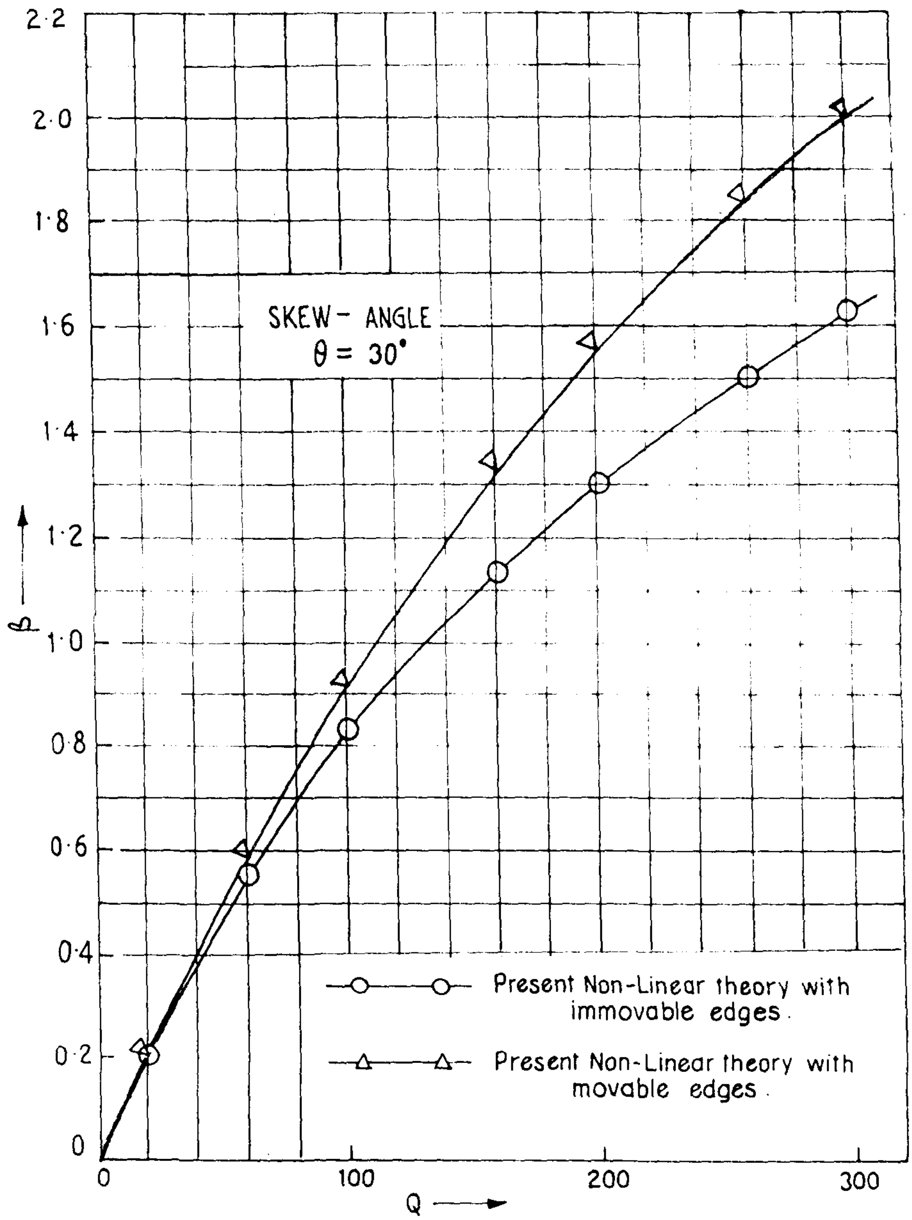

FIG. 11: GRAPHS SHOWING VARIATIONS OF CENTRAL DEFLECTIONS OF A STEEL-PLATE ($\nu = 0{\cdot}3$) DUE TO CHANGE OF EDGE CONDITIONS.

OBSERVATIONS

From the numerical tables and graphs we find that the present non-linear theory gives better results than kennedy and Simon's non-linear approach [85] . Because almost in all the cases our percentage of error with respect to the experimental results remain within 5 percent,whereas,the errors in the results according to Kennedy and Simon's theory vary from the lowest 5 percent to the highest 17.62 percent with respect to the experimental results. It appears that deviations in the results of Kennedy and Simon's theory are due to application of perturbation technique which embraces more approximation. Moreover the perturbation technique requires much computational labour. It is interesting to note that Kennedy and Simon did not compare their large deflection results for skewed plates with other results available in open literatures or with any experimental results. They only compared their results for a rectangular/ square plate where $\theta = 0^{\circ}$.

It is observed, from the present results, that the maximum central deflection of a rhombic plate decreases with the increase in skew angle. This may be due to the increased rigidity of the obtuse corners of the plate with the increase in skew angles. Thus we may conclude that the effect of the non-linear terms on the deflection diminishes with the increase in skew angle and hence the large deflection curves tend to become increasingly linear for large skew angles.

It is also observed that, the deflections for movable edges are always greater than those for the immovable edges (Ref. table 4). This is quite expected from the practical point of view, because movable edge conditions give stress-free boundary and hence there is large energy changes in the boundary.

NOTE :

Like simply-supported skew plate here also results for skew angles $\theta = 15^{\circ}$ and $\theta = 30^{\circ}$ have only been considered, because, for greater values of the skew angle the influence of non-linearity does not play much of significant role in design and the study of linear analysis serves the practical purpose.

CHAPTER II

LARGE DEFLECTION ANALYSES OF SKEW PLATES OF VERIABLE THICKNESS

PAPER I

NON-LINEAR ANALYSIS OF RHOMBIC PLATES OF VARIABLE THICKNESS

ABSTRACT

This paper deals with the non-linear static and dynamic behaviours of a simply-supported rhombic plate (skew plate of aspect ratio 1) of linearly varying thickness. Banerjee's hypothesis has been followed to form a set of decoupled differential equations and then the Galerkin's procedure has been utilised to solve the equations. Various numerical results for a rhombic plate of isotropic material, under both static and dynamic loadings have been computed and compared with the other results known from literature. It is seen that the present approach offers sufficiently accurate results for both movable and immovable edge conditions.

GOVERNING EQUATIONS

Let us consider a rhombic plate of elastic isotropic material having thickness varying linearly, the central thickness being 'h_o' and thickness-variation parameter being 'p'. Let the size of each side of the plate be '2a' which is sufficiently large compared to 'h_o'. The plate is considered to be simply-supported along its edges and its faces respond to the bending and membrane actions.

We now posit a rectangular cartesian co-ordinate system (x,y,z,) at the centre of the plate, (x,y) being in the middle plane and z the thickness direction positive downwards. Also let us set an oblique co-ordinate system (x_1, y_1, θ) at the same origin, (x_1, y_1) being parallel to the sides of the plate, and θ the skew angle of the plate (Vide Fig.3 in paper II Chapter I). Obviously

$$x = x_1 \cos\theta \quad \text{and} \quad y = y_1 + x_1 \sin\theta$$

are the co-ordinate transformation equations.

Now following Banerjee's hypothesis, the differential equations in rectangular cartesian co-ordinate system governing the deflections and vibrations of plates of linearly varying thickness will be

$$\begin{aligned} & h^3\nabla^4 W + 6h^2\left(\frac{dh}{dx}\right)\nabla^2\left(\frac{\partial W}{\partial x}\right) + 6h\left(\frac{dh}{dx}\right)^2\left(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\right) \\ & -A_1\left(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\right) - 6\lambda h\Big[\Big\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\Big\}\Big\{\nabla^2 W \\ & + \frac{1}{h}\left(\frac{dh}{dx}\right)\left(\frac{\partial W}{\partial x}\right)\Big\} + 2\Big\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2 \\ & + 2\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial y}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right)\Big\}\Big] = \frac{q}{16L} \quad . \end{aligned} \tag{1}$$

for non-linear static deflections under uniform loading, where A_1 is a constant given by

$$\frac{A_1}{12h} = \frac{1}{2}\Big\{\left(\frac{\partial W}{\partial x}\right)^2 + \nu\left(\frac{\partial W}{\partial y}\right)^2\Big\} + \frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} \quad . \tag{2}$$

and

$$h^3 \nabla^4 W + 6h^2\left(\frac{dh}{dx}\right)\nabla^2\left(\frac{\partial W}{\partial x}\right) + 6h\left(\frac{dh}{dx}\right)^2\left(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\right)$$

$$-A_2\left(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\right) - 6\lambda h\left[\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right\}\left\{\nabla^2 W\right.\right.$$

$$\left. + \frac{1}{h}\left(\frac{dh}{dx}\right)\left(\frac{\partial W}{\partial x}\right)\right\} + 2\left\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2\right.$$

$$\left.\left. + 2\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial y}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right)\right\}\right] + \frac{\rho h}{L}\frac{\partial^2 W}{\partial t^2} = 0 \quad . \qquad (3)$$

for non-linear free elastic vibrations, where A_2 is a time - dependent constant given by

$$\frac{A_2}{12h} = \frac{1}{2}\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \nu\left(\frac{\partial W}{\partial y}\right)^2\right\} + \frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} \quad . \qquad (4)$$

In both the equations (1) and (3)

$$L = \frac{E}{12(1-\nu^2)} \quad , \quad D = \frac{Eh_o^3}{12(1-\nu^2)} \quad .$$

The thickness variation is expressed by

$$h = h_o(1 + px/a) \, ,$$

where $p < 1$.

ANALYSIS

(A) Non-linear static behaviours of skew plates of variable thickness -

We consider, here, the bending of simply-supported rhombic plate of variable thickness with constrained in-plane displacements at the boundaries. We now transform equation (1) and (2) in oblique co-ordinates by the transformation operators as given in Paper I, Chapter I.

We choose , as usual, the deflection function W in the following form for simply-supported edge conditions

$$W = W_o \cos\frac{\pi X_1}{2a} \cos\frac{\pi Y_1}{2a} \tag{5}$$

Now integrating the transformed equation (2) over the entire area of the plate we get

$$A_1 = 6h_o p\cos\theta\left\{\frac{\pi^2 W_o^2}{8a^2}\left(1+\nu+2\tan^2\theta\right)\right\}\Big/\log_e\frac{(1+p\cos\theta)}{(1-p\cos\theta)} . \tag{6}$$

Again introducing (5) and (6) in the transformed form of equation (1) and then applying the Galerkin's procedure, we arrive at the following cubic equation determining the non-dimensional central deflection $\beta = W_o/h_o$ of the simply-supported rhombic plate of variable thickness

$$\left[(1+2\tan^2\theta)\left\{\sec^2\theta+\left(1-\frac{6}{\pi^2}\right)p^2\right\}+\frac{6p^2}{\pi^2}\left(1-\nu+2\tan^2\theta\right)\right]\beta$$
$$+\frac{3}{8}\left[\lambda\left(5+11\tan^2\theta+6\tan^4\theta\right)\right.$$
$$\left.+2p\cos\theta\left(1+\nu+2\tan^2\theta\right)^2\Big/\log_e\frac{(1+p\cos\theta)}{(1-p\cos\theta)}\right]=\frac{4}{\pi^6}Q \ . \tag{7}$$

where $Q = qa^4/Dh_o$ is the load function parameter, q being the load per unit area of the plate .

NUMERICAL RESULTS

Table 1 shows different numerical results of the central deflections of a rhombic plate of variable thickness having $\nu = 0.3$; load parameters are taken the same as in Paper I, Chapter I. It is to be noted that the results for thickness variation parameter p = 0 (i.e. for a plate of constant thickness), agree exactly with those in Paper I, Chapter I, both for movable and immovable edge conditions, which have been experimentally verified by the author. The results for other values of p are new. (Note - For movable edge conditions $A_1 = 0$).

TABLE 1

Static Deflections
of Rhombic Plates.

		W_o/h_o					
Skew Angle θ	Load Parameter qa^4/Dh_o	Immovable Edge			Movable Edge		
		p = 0.1	p = 0.2	p = 0.3	p = 0.1	p = 0.2	p = 0.3
15^o	111.72	0.3434	0.3374	0.3277	0.3675	0.3595	0.3472
	335.16	0.7682	0.7618	0.7513	0.9775	0.8935	0.9370
	558.60	1.0255	1.0204	1.0121	1.4210	1.4040	1.3765
30^o	111.72	0.2009	0.1974	0.1916	0.2061	0.2022	0.1958
	335.16	0.5094	0.5083	0.4991	0.5840	0.5824	0.5663
	558.60	0.7301	0.7239	0.7153	0.9250	0.9120	0.8900

(B) Non-linear dynamic behaviours of skew plates of variable thickness -

Let us now consider free vibrations of variable thickness rhombic plates. Neglecting in-plane inertia, transforming equation (4) in oblique co-ordinates, choosing

$$W = W_o F(t) \cos \frac{\pi X_1}{2a} \cos \frac{\pi Y_1}{2a} \tag{8}$$

for fundamental mode of vibration and then integrating the transformed equation over the whole domain of the plate we get

$$A_2 = 6h_o p \cos\theta \frac{\pi^2 W_o^2 F^2}{8a^2}(1+\nu+2\tan^2\theta) \Big/ \log_e \frac{(1+p\cos\theta)}{(1-p\cos\theta)} . \tag{9}$$

Here W_o is the initial amplitude of vibration and F(t) is some unspecified function of time. It is to be noted that, we are interested in the normal displacement W only and so the in-plane displacements u and v are eliminated here also by considering suitable expressions for them compatible with the boundary conditions of the plate.

Now transforming equation (3) in oblique co-ordinates, inserting (8) and (9) in the transformed equation and then applying the Galerkin's procedure we get the following differential equation for time function

$$\frac{d^2F}{d\tau^2} + \Big[(1+2\tan^2\theta)\frac{\pi^4}{4}\Big\{\sec^2\theta + \Big(1-\frac{6}{\pi^2}\Big)p^2\Big\} + \frac{3}{2}\pi^2 p^2 (1-\nu +2\tan^2\theta)\Big]F + \frac{3\pi^4}{32}\Big[\lambda(5+11\tan^2\theta+6\tan^4\theta) +2(1+\nu+2\tan^2\theta)^2 \cdot p\cos\theta \Big/ \log_e \frac{(1+p\cos\theta)}{(1-p\cos\theta)}\Big]\beta^2 F^3 = 0 . \tag{10}$$

where $\beta = W_o/h_o$, the non-dimensional amplitude and

$$\tau = (Lh_o^2/\rho a^4)^{\frac{1}{2}} t$$

some time function.

The equation (10) is in the form $\ddot{F} + AF + BF^3 = 0$, the familiar Duffing's Equation. With the initial conditions $F(0) = 1$ and $\dot{F}(0) = 0$, the solution of equation (10) is the well-known elliptic integral $F(t) = C_n(\omega^*, t, k)$. Then the ratio of the non-linear frequency ω^* to the linear frequency ω is given by

$$\frac{\omega^*}{\omega} = \sqrt{1 + B/A}$$

where

$$A = \Big[(1+2\tan^2\theta)\frac{\pi^4}{4}\Big\{\sec^2\theta + \big(1-\frac{6}{\pi^2}\big)p^2 + \frac{3}{2}\pi^2 p^2 (1-\nu+2\tan^2\theta)\Big] .$$

and

$$B = \frac{3\pi^4}{32}\Big[\lambda(5+11\tan^2\theta+6\tan^4\theta) + 2(1+\nu+2\tan^2\theta)^2 \cdot p\cos\theta \Big/ \log_e\frac{(1+p\cos\theta)}{(1-p\cos\theta)} .$$

NUMERICAL RESULTS

Numerical results of the ratio ω^*/ω are shown in Tables 2 and 3. Table 2 shows the results for a square plate ($\theta = 0^\circ$) compared with those obtainable from ref.174 , after converting the shell equations into plate equations. It is seen that the results agree perfectly. Table 3 shows the results for rhombic plates with skew angles $\theta = 15^\circ$, 22.5° and 30° and thickness variation parameters p = 0,0.1, 0.2 and 0.3. These results are new to the author's sincere belief. Here the results for skew angles higher than 30° are not considered, because for greater values of θ, the effect of non-liearity does not play important role in design. (Note - For movable edge conditions $A_2 = 0$).

TABLE 2

Showing Dynamic Characteristics of Square Plates $\theta = 0^{\circ}$

ω^*/ω

Edge condition	W_o/h_o	p = 0		p = 0.1		p = 0.2		p = 0.3	
		Present Method	Sinha-Banerjee Method	Present Method	Sinha-Banerjee Method	Present Method	Sinha-Banerjee Method	Present Method	Sinha-Banerjee Method
Immovable Edge	0.25	1.02477	1.02477	1.02450	1.02447	1.02374	1.02350	1.02253	1.0220
	0.50	1.09573	1.09573	1.09473	1.09520	1.09187	1.09099	1.08734	1.0853
	0.75	1.20474	1.20474	1.20270	1.2025	1.19683	1.19600	1.18451	1.18330
	1.00	1.34257	1.34257	1.33932	1.33890	1.32992	1.32700	1.31500	1.3082
Movable Edge	0.25	1.00526	1.00526	1.00522	1.00526	1.00509	1.00514	1.00490	1.00493
	0.50	1.02088	1.02088	1.02071	1.02088	1.02022	1.02040	1.01946	1.01950
	0.75	1.04640	1.04640	1.04600	1.04640	1.04495	1.04530	1.04327	1.04340
	1.00	1.08110	1.08110	1.08040	1.08110	1.07860	1.07930	1.07572	1.07600

TABLE 3

Showing Dynamic Characteristics of Rhombic Plates, $\theta = 15^\circ$, $\theta = 22.5^\circ$, and $\theta = 30^\circ$

Skew Angle θ	w_o/h_o	ω^*/ω Immovable Edge p = 0	p = 0.1	p = 0.2	p = 0.3	Movable Edge p = 0	p = 0.1	p = 0.2	p = 0.3
15°	0.25	1.02463	1.02438	1.02365	1.02250	1.00500	1.00496	1.00484	1.00467
	0.50	1.09520	1.09425	1.09152	1.08721	1.01984	1.01968	1.01924	1.01854
	0.75	1.20366	1.20172	1.19612	1.18724	1.04410	1.04377	1.04279	1.04125
	1.00	1.34084	1.33774	1.32880	1.31462	1.07716	1.07658	1.07489	1.07224
22.5°	0.25	1.02454	1.02430	1.02362	1.02254	1.00472	1.0046	1.00439	1.00433
	0.50	1.09486	1.09388	1.09142	1.08737	1.01876	1.01863	1.01823	1.01760
	0.75	1.20297	1.20115	1.19591	1.18758	1.04174	1.04144	1.04057	1.03920
	1.00	1.33974	1.33684	1.32847	1.31511	1.07308	1.07256	1.07106	1.06868
30°	0.25	1.02452	1.02431	1.02370	1.02272	1.00442	1.00439	1.00430	1.00417
	0.50	1.09481	1.09401	1.09700	1.08802	1.01756	1.01745	1.01711	1.01658
	0.75	1.20286	1.20120	1.19650	1.19189	1.03910	1.03885	1.03810	1.03692
	1.00	1.33957	1.33695	1.32940	1.31726	1.06853	1.06810	1.06680	1.06476

OBSERVATIONS

For static behaviour of a rhombic plate of varying thickness, it is observed that

(i) with the increase of skew angle,central deflection decredases for the same loading whether the edge conditions of the plate are movable or immovable,

(ii) for any assumed skew angle,the central deflection is greater for movable edge conditions than that for immovable edge conditions,the load reamining same in both the cases,

(iii) increasing thickness parameter decreases the central delfection.

All the above observations are quite expected from practical point of view.

As regards dynamic behaviour of a variable thickness rhombic plate, the following observations are made:

(i) The frequency ratio decreases with increasing thickness variation parameter irrespective of the edge conditions.

(ii) The frequency ratio increases with the non-dimensional amplitude.

(iii) The frequency ratio gradually decreases with the increase of p, the edge conditions being movable or immovable. This is an expected result, because the thickness is minimum at the centre and maximum at the edge of the tapered plate,

(iv) The frequency ratio decreases with the increase of skew angle for lower values of p. For comparatively higher values of p, the vibration character tends to change in the case of immovable edge conditions. But for movable edge conditions the vibration character does not show such irregularity. This situation demands further investigation.

CHAPTER III

PAPER I

NON-LINEAR ANALYSIS OF HEATED RHOMBIC PLATES

PAPER I

NON-LINEAR ANALYSIS OF HEATED RHOMBIC PLATES *

ABSTRACT

This paper concerns a new approach to the investigation of non-linear behaviours of heated rhombic plates. A set of differential equations in oblique co-ordinates have been derived in this investigation. Numerical results showing central deflection parameters versus thermal load functions have been computed for different skew angles θ. For $\theta = 0^{\circ}$ the results obtained in the present study are in excellent agreement with the known results. It is believed that the results obtained for other different skew angles are completely new.

ANALYSIS

Let us consider a rhombic plate of skew angle 'θ' whose uniform thickness is 'h' and edge-length '2a'. The material of the plate is considered isotropic having mass density 'ρ', Young's Modulus 'E' and Poissn's Ratio 'ν'. The origin of the co-ordinates is located at the geometric centre of the plate

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(vide Fig.3 in Paper II, Chapter I). The deflections are considered to be of the same order of magnitude of the plate thickness, the edge-length being sufficiently large compared to the thickness.

Now the uncoupled set of differential equations in rectangular Cartesian co-ordinates, governing the thermal behaviours of elastic plates (vide Ref. 167) are given by

$$\nabla^4 W - \frac{12A}{h^2}\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) - \frac{6\lambda}{h^2}\Big[\nabla^2 W\Big\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\Big\}$$
$$+ 2\Big\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2\Big\} + 4\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial y}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right)\Big]$$
$$+ \frac{12\alpha_t \tau_0}{h^2}\sqrt{\lambda(1-\nu^2)}\ \nabla^2 W + (1+\nu)\alpha_t \nabla^2 \tau = \frac{q}{D} \quad . \tag{1}$$

where

$$A = \frac{1}{2}\Big\{\left(\frac{\partial W}{\partial x}\right)^2 + \nu\left(\frac{\partial W}{\partial y}\right)^2\Big\} + \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y}$$
$$- (1+\nu)\alpha_t \tau_0 \quad . \tag{2}$$

It is to be noted that in the derivation of the above equations (1) and (2) in rectangular Cartesian co-ordinates, the expression

$$(1-\nu^2)\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial W}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial W}{\partial x}\cdot\frac{\partial W}{\partial y}\right)^2 \cdot \frac{1}{2(1+\nu)} \quad .$$

in the total P.E. of the elastic plate (Ref.167) has been replaced by

$$\frac{\lambda}{4}\left[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right]^2 .$$

As a consequence the partial differential equations governing the deflection of the plate have become uncoupled and the two decoupled differential equations (1) and (2) have been obtained.

In the present problem, the temperature is assumed to vary linearly with respect to the thickness direction z. We also note that (Ref.167)

$$T(x,y,z) = \tau_0(x,y) + z\,\tau(x,y)$$

in which

$$\tau_0 = \frac{1}{2}(T_1 + T_2)\ ; \qquad \tau = \frac{1}{h}(T_1 - T_2)\ ;$$

where

$$T_1 = T(x,y,+\frac{h}{2}) \quad \text{and} \quad T_2 = T(x,y,-\frac{h}{2}).$$

Clearly τ_0 is the temperature in the middle plane and τ varies along the thickness of the plate and hence $\tau \neq \tau_0$.

The plan of the skew co-ordinates (x_1, y_1, θ) is shown in Fig.3 in the Paper II, Chapter I.

We now transform the equation (2) in oblique co-ordintes. For Simply-Supported plates the boundary conditions are

$$W = 0 \quad \text{at} \quad x_1 = \pm a \quad \text{and at} \quad y_1 = \pm a\ ;$$
$$\frac{\partial^2 W}{\partial x_1^2} = 0 \quad \text{at} \quad x_1 = \pm a \quad \text{and} \quad \frac{\partial^2 W}{\partial y_1^2} = 0 \quad \text{at} \quad y_1 = \pm a\ .$$

Then let us choose the deflection function for the Simply-Supported plate as

$$W = W_0 \cos\frac{\pi x_1}{2a}\cos\frac{\pi y_1}{2a} \tag{3}$$

which clearly satisfies the above-mentioned boundary conditions.

Now putting (3) in the transformed form of equation (2) in oblique co-ordinates and then integrating it over the entire surface of the plate, we obtain the value of A in the following form

$$A = \frac{\pi^2 W_0^2}{32a^2}(1+\nu+2\tan^2\theta) - (1+\nu)\alpha_t \tau_0 \,. \tag{4}$$

(As normal displacement W is our primary interest, the in-plane displacements u,v have been eliminated through integration by the choice of appropriate functions for such displacements). Again transforming the equation (1) in oblique co-ordinates, introducing (3) and (4) in the transformed equation and then applying the Galerkin's error minimising technique we get the following equation determining central deflection parameter W_o/h depending on thermal load function $q'a^4/Eh^4$

$$\Big[(1+2\tan^2\theta)\sec^2\theta - \frac{6S}{(1+\nu)\pi^2}\Big\{(1+\nu)(1+\nu+2\tan^2\theta) + 2\sqrt{\lambda(1-\nu^2)}\cdot\sec^2\theta\Big\}\Big]\left(\frac{W_0}{h}\right) + \frac{3}{8}\Big[(1+\nu+2\tan^2\theta)^2 + \frac{\lambda}{4}(8+49\tan^2\theta+29\tan^4\theta)\Big]\left(\frac{W_0}{h}\right)^3 = \frac{768(1-\nu^2)}{\pi^6}\left(\frac{q'a^4}{Eh^4}\right). \tag{5}$$

where

$$S = 2(a/h)^2(1+\nu)\alpha_t\tau_0$$

and

$$q' = q - D\alpha_t(1+\nu)\nabla^2\tau \,.$$

The equation (5) is applicable for the immovable edge conditions of the Simply-Supported skew plate. For the movable edge conditions we have A = 0, so that the equation (5) takes the form

$$\left[(1+2\tan^2\theta)\sec^2\theta - \frac{12S}{(1+\nu)\pi^2}\sqrt{\lambda(1-\nu^2)}\cdot\sec^2\theta\right]\left(\frac{W_0}{h}\right) + \frac{3\lambda}{32}(8+49\tan^2\theta+29\tan^4\theta)\left(\frac{W_0}{h}\right)^3 = \frac{768(1-\nu^2)}{\pi^6}\left(\frac{q'a^4}{Eh^4}\right). \tag{6}$$

NUMERICAL RESULTS

Numerical results are presented here in tabular forms (Tables 1 and 2) for S = 0, 0.1, $\theta = 0^\circ, 15^\circ, 30^\circ$ and $q'a^4/Eh^4$ = 2,4,8,10.

TABLE -1

S = 0, i.e. $\tau_o = 0$

$\frac{q'a^4}{Eh^4}$	W_o/h by Present Method						W_o/h by Berger's Method *		
	θ = 0°		θ = 15°		θ = 30°		(Ref.134) θ = 0°	θ = 15°	θ = 30°
	Movable Edge	Immovable Edge	Movable Edge	Immovable Edge	Movable Edge	Immovable Edge	Immovable Edge	Immovable Edge	Immovable Edge
2	1.30156	0.91435	1.08167	0.82069	0.6269	0.53604	0.9013	0.79972	0.53671
4	2.1909	1.3131	1.85443	1.20857	1.14734	0.84631	1.29017	1.16888	0.848
8	3.23354	1.78866	2.8581	1.67119	1.89675	1.22355	1.75406	1.60902	1.2266
10	3.73498	1.9613	3.2243	1.83866	2.17977	1.3597	1.92254	1.76847	1.36324

* Berger's method has been applied to the present problem by neglecting e_2, the second strain invariant, in the expression for total P.E. of the plate.

TABLE - 2

$S = 0.1$, i.e. $\tau_o \neq 0$

$\frac{q'a^4}{Eh^4}$	W_o/h by Present Method						W_o/h by Berger's Method *		
	$\theta = 0^o$		$\theta = 15^o$		$\theta = 30^o$		(Ref. 134) $\theta = 0^o$	$\theta = 15^o$	$\theta = 30^o$
	Movable Edge	Immovable Edge	Movable Edge	Immovable Edge	Movable Edge	Immovable Edge	Immovable Edge	Immovable Edge	Immovable Edge
2	1.32786	0.94985	1.10168	0.83899	0.63597	0.55925	0.94058	0.83515	0.56109
4	2.22082	1.34324	1.87831	1.20992	1.1604	0.86901	1.32336	1.19954	0.87185
8	3.35106	1.81316	2.88067	1.65221	1.9111	1.24302	1.781	1.63412	1.24706
10	3.76118	1.98415	3.24585	1.81269	2.19385	1.37799	1.94764	1.79188	1.38247

* $e_2 = 0$ according to Berger's method.

OBSERVATIONS

From the numerical analysis of the undertaken problem the following observations are made :

(i) The nature of the central deflection of a skew plate under thermal loading is the same as that of the plate under mechanical loading, i.e. the central deflection increases continuously with the increase of loading for any edge conditions of the skew plate, whether movable or immovable.

(ii) Central deflection for movable edge conditions of the skew plate is always greater than that for immovable edge conditions of the plate, for the same loading in the two cases.

(iii) Irrespective of the edge conditions, the central deflection decreases with the increase of the skew angle.

(iv) The results for immovable edge conditions of the skew plate obtained by the present method, agree well with the results obtained by Berger's method. It is to be noted that Berger's method is a purely approximate method based on the neglect of e_2. But present study is based on Banerjee's hypothesis which suggests a modified strain-energy expression, and thus this model embraces less approximation (Ref.162) than that of Berger. Again Berger's method is meaningful only or immovable edge conditions of the plates.

and

(v) The deflections increase with τ_0.

CHAPTER IV

LARGE DEFLECTION ANALYSES OF SKEWED SANDWICH PLATES

PAPER I

NON-LINEAR ANALYSES OF SKEWED SANDWICH PLATES*

ABSTRACT

Investigations on finite deformation of sandwich plates are gaining importance day by day due to its wide applications in modern design. Outstanding research workers who carried out interesting investigations in this field are E.Reissner, A.M. Alwan and J.L.Mowinski and H.Ohnabe [19,67,122]. N.Kamiya [140] has offered a new set of governing equations by using Berger's approximation to study the non-linear static behaviours of sandwich plates. The author has analysed in detail the case of rectangular sandwich plates. The accuracy of this method depends on a correction factor F(b/a).

In this paper an attempt has been made to analyse the non-linear behaviours of Simply—Supported skewed sandwich plates having an isotropic core within isotropic upper and lower faces and under both static and dynamic loadings. For the sake of simplicity, skewed plate in the form of rhombus has been considered. Following the modified strain energy expression proposed by B.Banerjee [162], a new set of decoupled differential equations for sandwich plates, in rectangular cartesian co-ordinate system

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has been derived (Ref.196). Then this set of differential equations has been transformed in oblique co-ordinates to suit them for skewed plates and solved by the Galerkin's technique. Numerical results for rhombic sandwich plates under mechanical as well as dynamical loadings are presented and the results of the special case, where the skew angle $\theta = 0^{\circ}$, are compared with the other known results. The results for other skew angles are believed to be completely new.

GOVERNING EQUATIONS

Let us consider a rhombic sandwich plate of sides 'a', having an isotropic core as well as isotropic upper and lower-faces of identical thickness 't_1' (vide Fig.1). The faces respond to the bending and membrane actions of the plate; the core is assumed to transfer only shear deformations. Moreover compared with the core thickness 'h', the face thickness 't_1' is supposed to be thin ehough to ignore a variation of stress in the thickness direction of the faces.

Now let us set a rectangular cartesian co-ordinate system (x,y,z); x,y being in the middle plane of the core and z the thickness direction, positive downwards. Also let us set an oblique co-ordinate system (x_1,y_1,θ) at the same origin (one of the corners of the skew plate), x_1,y_1 being parallel to the sides of the plate, and θ, the skew angle of the plate (vide Fig.1 in Paper I, Chapter I).

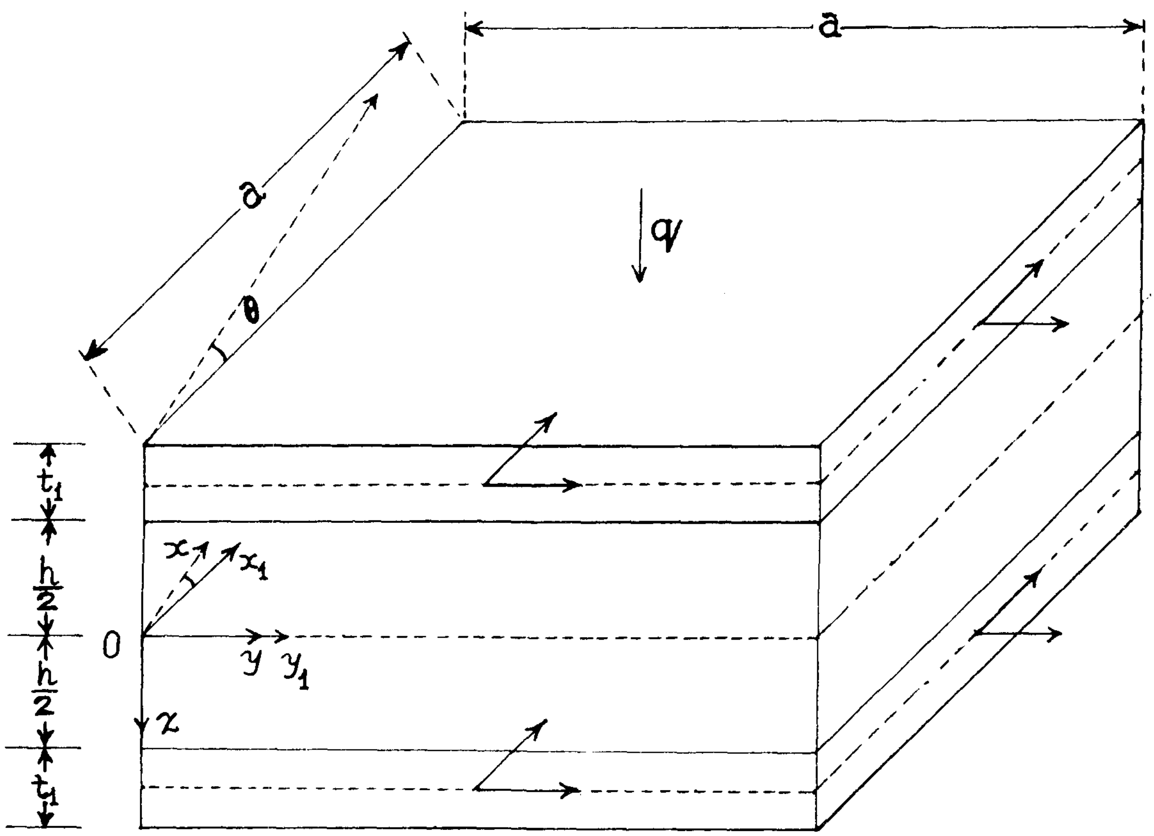

Fig.-1: ELEMENT OF SKEWED (RHOMBIC) SANDWICH PLATE .

Now following the Banerjee's hypothesis, the differential equations in rectangular cartesian co-ordinate system governing the deflections and vibrations of sandwich plates, (Ref.196) are

$$\left[\frac{Et_1}{2(1-\nu^2)}\nabla^2 - \frac{G'}{h}\right]\left[-\frac{2Et_1}{(1-\nu^2)G'} I_1^m\left(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\right) + h\nabla^2 W + \frac{Et_1\lambda}{(1-\nu^2)G'}\left|\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right\}\nabla^2 W + 2\left\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2 + 2\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial y}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right)\right\}\right| + \xi\right] + G'\nabla^2 W = 0 \quad (1)$$

where, $\xi = q/G'$ for non-linear static deflections,

$$= -\frac{(\rho_1 t_1 + \rho_2 h)}{G'}\frac{\partial^2 W}{\partial t^2}$$ for non-linear elastic vibrations,

and

$$I_1^m = \frac{1}{2}\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \nu\left(\frac{\partial W}{\partial y}\right)^2\right\} + \frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} \quad (2a)$$

= constant, for non-linear static deflections,

= Cf(t) for non-linear elastic vibrations, C being a constant depending on θ. (2b)

In the above equations

W is the transverse deflection function,

q, the lateral load distribution function,

E, the Young's Modulus of elasticity of the material of the upper and lower faces,

G', the shear Modulus of the core material,

ν, the Poisson's Ratio of the Face material,

ρ_1 , ρ_2 are the surface density and core density respectively, and f(t), F(t) are the functions of time such that $f(t) = F^2(t)$.

It is to be noted that the strain-energy expression in Ref.196 , has been modified by using Banerjee's hypothesis, which states that the stretching of the plate is proportional to

$$\left[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right]^2$$

As a consequence of this assumption, a set of uncoupled differential equations has been obtained as given above.

ANALYSIS

(A) Non-linear static behaviours of Simply-Supported skewed sandwich plates —

To find the normal displacement W, the in-plane displacements u,and v of the upper and lower faces of the sandwich plate are eliminated here, for obvious reasons. Now with the help of the co-ordinate transformation equations

$x = x_1 \cos\theta$ and $y = y_1 + x_1 \sin\theta$

we transform the equations (1) and (2a) in oblique co-ordinates, as usual.

Let us choose $W = W_o \sin\frac{\pi x_1}{a}\sin\frac{\pi y_1}{a}$ (3a)

Substituting(3a)in the transformed form of equation (2a) in oblique co-ordinates and integrating over the whole area of the plate we get

$$I_1^m = \frac{\pi^2 W_o^2}{8a^2}(1+\nu+2\tan^2\theta)$$

(3b)

Again choosing $q = q_o \sin\frac{\pi x_1}{a}\sin\frac{\pi y_1}{a}$ (3c)

we introduce (3a), (3b) and (3c) in the transformed form of equation (1) with $\xi = q/G'$. Now applying Galerkin procedure, we arrive at the following cubic equation determining W_o, the central deflection of a Simply-Supported rhombic sandwich plate

$$\frac{\pi^4 t_1}{4(1-\nu^2)}\left[\frac{\pi^2 E t_1 \sec^2\theta}{(1-\nu^2)G'a^2}\left\{(1+\nu+2\tan^2\theta)(1+\nu+4\tan^2\theta)\right.\right.$$
$$\left.+\lambda(5+17\tan^2\theta+12\tan^4\theta)\right\}+\frac{1}{h}\left\{(1+\nu+2\tan^2\theta)^2\right.$$
$$\left.\left.+\lambda(5+11\tan^2\theta+6\tan^4\theta)\right\}\right]\left(\frac{W_o}{h}\right)^3+\left[\frac{2\pi^4 t_1 \sec^2\theta}{(1-\nu^2)h}\right(1$$
$$\left.+2\tan^2\theta)\right]\left(\frac{W_o}{h}\right)=\frac{q_o a^4}{Eh^4}\left[1+\frac{\pi^2 E h t_1 \sec^2\theta}{(1-\nu^2)G'a^2}\right]. \quad (4)$$

NUMERICAL RESULTS

Table 1 shows different numerical results of the central deflections of a (0.254m. X 0.254m.) rhombic plate having

$t_1 = 6.35 \times 10^{-4}$m, $h = 1.7135 \times 10^{-2}$m.

TABLE 1

Showing W_o/h Vs θ .

$E = 16.2 \times 10^9$ psm, $G' = 9.3 \times 10^6$ psm, $\nu = 0.3$, $q_o a^4/Eh^4 = 10$

Value of θ	Value of W_o/h				
	IMMOVABLE EDGE			MOVABLE EDGE	
	Calculated value	Other known value [196]	Other known value [140]	Calculated value	Other known value [196]
0^o	1.4988	1.53	1.30	2.3223	2.588
15^o	1.3644	-	-	2.1563	-
30^o	1.0328	-	-	1.6360	-
45^o	0.6651	-	-	1.0414	-
60^o	0.3450	-	-	0.5051	-

Note - For movable edge conditions of the Simply-Supported plate $I_1^m = 0$.

(B) Non-linear dynamic behaviours of Simply-Supported skewed sandwich plates -

Let us now consider free vibrations of skewed sandwich plates. In this case also, we eliminate in-plane inertia. Then transforming equation (2b) in oblique co-ordinates, choosing $W = W_o \sin\frac{\pi x_1}{a} \sin\frac{\pi y_1}{a} F(t)$ (5a)

for fundamental mode of vibration and then integrating the transformed equation over the whole domain of the plate we get

$$I_1^m = \frac{\pi^2 W_o^2}{8a^2}\left(1+\nu+2\tan^2\theta\right)F^2(t) \, . \tag{5b}$$

Now transforming equation (1) with $\xi = -\frac{(\rho_1 t_1 + \rho_2 h)}{G'}\frac{\partial^2 W}{\partial t^2}$ in oblique co-ordinates, inserting (5a) and (5b) in the transformed equation and then applying the Galerkin's procedure, we get the following equation for the time function.

$$\left[\frac{\pi^2(\rho_1 t_1 + \rho_2 h)}{(1-\nu^2)G'a^2} E t_1 \sec^2\theta - \frac{(\rho_1 t_1 + \rho_2 h)}{h}\right]\ddot{F}$$
$$+\left[\frac{2\pi^4 E t_1 h}{(1-\nu^2)a^4}\left(1+3\tan^2\theta+2\tan^4\theta\right)\right]F + \left[\frac{\pi^4 E t_1 W_o^2}{4(1-\nu^2)a^4}\left\{\frac{1}{h}\right|\left(1+\nu\right.\right.$$
$$\left.+2\tan^2\theta\right)^2+\lambda\left(5+11\tan^2\theta+6\tan^4\theta\right)\Big| + \frac{\pi^2 E t_1 \sec^2\theta}{(1-\nu^2)G'a^2}\Big|\left(1+\nu\right.$$
$$\left.+2\tan^2\theta\right)\left(1+\nu+4\tan^2\theta\right)+\lambda\left(5+17\tan^2\theta+12\tan^4\theta\right)\Big|\Big\}\Big]F^3$$
$$= 0 \, . \tag{6}$$

The above equation can be put in the form

$$\ddot{F} + AF + BF^3 = 0,$$

the familiar Duffing's Equation.

With the initial conditions $F(0) = 1$ and $\dot{F}(0) = 0$, the solution of equation (6) is the well-known elliptic integral $F(t) = C_n(\omega_1^*, t, k)$. The ratio of the non-linear frequency ω_1^* to the linear frequency ω_1 is given by

$$\frac{\omega_1^*}{\omega_1} = \Big[1 + \frac{h\cos^2\theta}{8(1+2\tan^2\theta}\Big(\frac{W_0}{2h_1}\Big)^2\Big(1+\frac{2t_1}{h}\Big)^2\Big\{\frac{\pi^2 E t_1 \sec^2\theta}{(1-\nu^2)G' a^2}\Big|(1+\nu + 2\tan^2\theta)(1+\nu+4\tan^2\theta)+\lambda(5+17\tan^2\theta+12\tan^4\theta)\Big| + \frac{1}{h}\Big|(1+\nu+2\tan^2\theta)^2+\lambda(5+11\tan^2\theta+6\tan^4\theta)\Big|\Big\}\Big]^{\frac{1}{2}} \qquad (7)$$

where $h_1 = t_1 + \frac{h}{2}$, $\omega_1 = \sqrt{A}$ and $\omega_1^* = \sqrt{A + B}$

NUMERICAL RESULTS

Numerical results of the ratio ω_1^*/ω_1 are shown in Table 2. For calculations, the same data which are used in the study of static behaviours of sandwich plates, are used here also.

TABLE 2

Value of θ	Value of $W_o/2h_1$	Value of ω_1^*/ω_1				
		IMMOVABLE EDGE			MOVABLE EDGE	
		Calculated value	Other known value [196]	Other known value [149]	Calculated value	Other known value [196]
0°	0.5	1.15028	1.12	1.14	1.03342	1.024
15°		1.16621	-	-	1.03365	-
30°		1.22803	-	-	1.04313	-
45°		1.36556	-	-	1.06261	-
60°		1.70764	-	-	1.11940	-
0°	1.0	1.51413	1.42	1.48	1.12774	1.094
15°		1.56211	-	-	1.12860	-
30°		1.74133	-	-	1.16300	-
45°		2.11166	-	-	1.23150	-
60°		2.9435	-	-	1.41850	-

Note - For movable edge conditions of the Simply-Supported plate $I_1^m = 0$, as usual.

OBSERVATIONS

From the calculated results, the following observations are made :

(i) The results of both static and dynamic behaviours of a sandwich plate having skew angle $\theta = 0^{\circ}$ and aspect ratio 1 are in excellent agreement with those obtained by Dutta, S. and Banerjee, B.[196] .

(ii) It is seen that the central deflection gradually decreases with the increase in skew angle for both movable as well as immovable edge conditions.

(iii) For any assumed skew angle the central deflection is greater for movable edge conditions than that for immovable edge conditions. This is quite expected from the practical point of view.

(iv) In the dynamic case, the frequency ratio ω_1^*/ω_1 increases continuously with the skew angle θ, for both movable as well as immovable edge conditions of a skewed plate, the ratio for immovable edge conditions being always greater than that for movable edge conditions.

Greater deflections, obtained in the present study in comparison to the deflections obtained from the other theories in open literature, indicate acceptibility of the present method for practical purposes.

The great advantage of the present method lies in the fact that the accuracy of this method does not depend on any correction factor [140] and thus holds good for sandwich plates of different geometry.

CHAPTER V

LARGE AMPLITUDE FREE VIBRATIONS OF SKEW PLATES INCLUDING TRANSVERSE SHEAR DEFORMATION AND ROTATORY INERTIA - A NEW APPROACH

PAPER I

LARGE AMPLITUDE FREE VIBRATIONS OF SKEW PLATES INCLUDING TRANSVERSE SHEAR DEFORMATION AND ROTATORY INERTIA - A NEW APPROACH *

ABSTRACT

In the present paper, an attempt has been made to study the large amplitude flexural vibrations of clamped and Simply-Supported homogeneous, transversely isotropic elastic rhombic plates including the effects of transverse shear deformation and rotatory inertia. Employing Banerjee's hypothesis[162,179], a new set of decoupled differential equations have been formed in oblique co-ordinates and the final equation for the time function has been obtained for a rhombic plate by the use of well-known Galerkin technique. Computations are, however, restricted to the fundamental mode of flexural vibrations, which are usually considered sufficient for practical and engineering purposes. The governing equations derived here, agree with those given in reference 179, when the skew angle tends to zero. A good number of numerical results for clamped and Simply-Supported rhombic plates having immovable as well as movable edge conditions are carefully computed. Some results for clamped rhombic plates are

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compared with those available in the references 146 and 151 To the author's opinion the agreement is good. The results for Simply-Supported rhombic plates could not be compared due to the absence of similar results in open literature. It has been found that the influences of skew angle, transverse shear deformation and rotatory inertia on the large amplitude vibrations of elastic plates are so dominant as not to be set aside during analyses of their non-linear vibration characters.

ANALYSIS

Let us consider a rhombic plate of skew angle θ whose uniform thickness is h and edge-length 2a. The material of the plate is homogeneous, transversely isotropic having mass density ρ Young's Modulus E and Poisson's Ratio ν . The origin of the co-ordinates is located at the geometric centre of the plate (vide Fig.3, Paper II, Chapter II). The deflections are considered to be of the same order of magnitude of the plate thickness.

We now consider the uncoupled set of differential equations proposed by R.Bhattacharjee and B.Banerjee[179] . These equations in rectangular cartesian co-ordinates governing the vibrations of an elastic plate take the following forms

$$\nabla^4 W + \frac{6}{5(1-\nu^2)} K\left(\frac{E}{G_e}\right) \frac{\bar{\alpha}^2 h^2}{12} \tau^2(t) \nabla^2 \left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right)$$

$$+\frac{3\lambda}{5(1-\nu^2)}K\left(\frac{E}{G_c}\right)\nabla^2\left[\nabla^2 W\left\{\left(\frac{\partial W}{\partial x}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2\right\}+2\left\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2\right.\right.$$
$$\left.\left.+\frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2\right\}+4\frac{\partial W}{\partial x}\cdot\frac{\partial W}{\partial y}\cdot\frac{\partial^2 W}{\partial x\partial y}\right]-\frac{6}{5}\left(\frac{\rho}{G_c}\right)\frac{\partial^2}{\partial t^2}(\nabla^2 W)$$
$$-\bar{\alpha}^2\tau(t)\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)-\frac{6\lambda}{h^2}\left[\nabla^2 W\left\{\left(\frac{\partial W}{\partial x}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2\right\}\right.$$
$$\left.+2\left\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2+\frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2\right\}+4\frac{\partial W}{\partial x}\cdot\frac{\partial W}{\partial y}\cdot\frac{\partial^2 W}{\partial x\partial y}\right]+\frac{12}{h^2 C_p^2}\frac{\partial^2 W}{\partial t^2}=0\,. \quad (1)$$

where

$$\frac{\bar{\alpha}^2 h^2}{12}\tau^2(t)=\frac{1}{2}\left\{\left(\frac{\partial W}{\partial x}\right)^2+\nu\left(\frac{\partial W}{\partial y}\right)^2\right\}+\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}\,. \quad (2)$$

$\bar{\alpha}$ being the coupling parameter of the system,

and $C_p = \left[E/(1-\nu^2)\rho\right]^{\frac{1}{2}}$ the speed of wave propagation along the surface of the plate.

We are hereby interested in the fundamental mode of transverse vibrations of elastic plates only, for obvious reasons.

With the help of the co-ordinate transformation equations, $x = x_1 \cos\theta$, $y = y_1 + x_1 \sin\theta$, the equations (1) and (2) are transformed in oblique co-ordinates. The transformed equations are respectively

$$\sec^4\theta\left[\frac{\partial^4 W}{\partial x_1^4}-4\sin\theta\left(\frac{\partial^4 W}{\partial x_1^3\partial y_1}+\frac{\partial^4 W}{\partial x_1\partial y_1^3}\right)+2(1+2\sin^2\theta)\frac{\partial^4 W}{\partial x_1^2\partial y_1^2}\right.$$
$$\left.+\frac{\partial^4 W}{\partial y_1^4}\right]+\frac{6}{5(1-\nu^2)}K\left(\frac{E}{G_c}\right)\frac{\bar{\alpha}^2 h^2}{12}\tau^2(t)\sec^2\theta\left(\frac{\partial^2}{\partial x_1^2}-2\sin\theta\frac{\partial^2}{\partial x_1\partial y_1}\right.$$
$$\left.+\frac{\partial^2}{\partial y_1^2}\right)\left[\sec^2\theta\left(\frac{\partial^2 W}{\partial x_1^2}-2\sin\theta\frac{\partial^2 W}{\partial x_1\partial y_1}\right)+(\nu+\tan^2\theta)\frac{\partial^2 W}{\partial y_1^2}\right]$$
$$+\frac{3\lambda}{5(1-\nu^2)}K\left(\frac{E}{G_c}\right)\sec^2\theta\left(\frac{\partial^2}{\partial x_1^2}-2\sin\theta\frac{\partial^2}{\partial x_1\partial y_1}+\frac{\partial^2}{\partial y_1^2}\right)\left[\sec^4\theta\left(\frac{\partial^2 W}{\partial x_1^2}\right.\right.$$
$$\left.-2\sin\theta\frac{\partial^2 W}{\partial x_1\partial y_1}+\frac{\partial^2 W}{\partial y_1^2}\right)\left\{\left(\frac{\partial W}{\partial x_1}\right)^2-2\sin\theta\frac{\partial W}{\partial x_1}\cdot\frac{\partial W}{\partial y_1}+\left(\frac{\partial W}{\partial y_1}\right)^2\right\}$$

$$+2\left\{\sec^4\theta\left(\frac{\partial^2 W}{\partial x_1^2}-2\sin\theta\frac{\partial^2 W}{\partial x_1\partial y_1}+\sin^2\theta\frac{\partial^2 W}{\partial y_1^2}\right)\left(\frac{\partial W}{\partial x_1}-\sin\theta\frac{\partial W}{\partial y_1}\right)^2\right.$$

$$\left.+\frac{\partial^2 W}{\partial y_1^2}\left(\frac{\partial W}{\partial y_1}\right)^2+2\sec^2\theta\left(\frac{\partial^2 W}{\partial x_1\partial y_1}-\sin\theta\frac{\partial^2 W}{\partial y_1^2}\right)\left(\frac{\partial W}{\partial x_1}-\sin\theta\frac{\partial W}{\partial y_1}\right)\frac{\partial W}{\partial y_1}\right\}\Bigg]$$

$$-\frac{6}{5}\left(\frac{\rho}{G_c}\right)\sec^2\theta\frac{\partial^2}{\partial t^2}\left[\frac{\partial^2 W}{\partial x_1^2}-2\sin\theta\frac{\partial^2 W}{\partial x_1\partial y_1}+\frac{\partial^2 W}{\partial y_1^2}\right]-\bar{\alpha}^2\tau^2(t)\times$$

$$\times\left[\sec^2\theta\left(\frac{\partial^2 W}{\partial x_1^2}-2\sin\theta\frac{\partial^2 W}{\partial x_1\partial y_1}\right)+(\nu+\tan^2\theta)\frac{\partial^2 W}{\partial y_1^2}\right]-\frac{6\lambda}{h^2}\Bigg[\sec^4\theta\times$$

$$\times\left(\frac{\partial^2 W}{\partial x_1^2}-2\sin\theta\frac{\partial^2 W}{\partial x_1\partial y_1}+\frac{\partial^2 W}{\partial y_1^2}\right)\left\{\left(\frac{\partial W}{\partial x_1}\right)^2-2\sin\theta\frac{\partial W}{\partial x_1}\cdot\frac{\partial W}{\partial y_1}+\left(\frac{\partial W}{\partial y_1}\right)^2\right\}$$

$$+2\left\{\sec^4\theta\left(\frac{\partial^2 W}{\partial x_1^2}-2\sin\theta\frac{\partial^2 W}{\partial x_1\partial y_1}+\sin^2\theta\frac{\partial^2 W}{\partial y_1^2}\right)\left(\frac{\partial W}{\partial x_1}-\sin\theta\frac{\partial W}{\partial y_1}\right)^2\right.$$

$$\left.+\frac{\partial^2 W}{\partial y_1^2}\left(\frac{\partial W}{\partial y_1}\right)^2+2\sec^2\theta\left(\frac{\partial^2 W}{\partial x_1\partial y_1}-\sin\theta\frac{\partial^2 W}{\partial y_1^2}\right)\left(\frac{\partial W}{\partial x_1}-\sin\theta\frac{\partial W}{\partial y_1}\right)\frac{\partial W}{\partial y_1}\right\}\Bigg]$$

$$+\frac{12}{h^2 C_p^2}\frac{\partial^2 W}{\partial t^2}=0 \quad . \tag{3}$$

and

$$\frac{\bar{\alpha}^2 h^2}{12}\tau^2(t)=\frac{1}{2}\left[\sec^2\theta\left\{\left(\frac{\partial W}{\partial x_1}\right)^2-2\sin\theta\frac{\partial W}{\partial x_1}\cdot\frac{\partial W}{\partial y_1}\right\}\right.$$

$$\left.+(\nu+\tan^2\theta)\left(\frac{\partial W}{\partial y_1}\right)^2\right]+\sec\theta\left(\frac{\partial u}{\partial x_1}-\sin\theta\frac{\partial u}{\partial y_1}\right)+\nu\frac{\partial v}{\partial y_1} \, . \tag{4}$$

Now choosing the deflection function for the fundamental mode of vibrations as

$$W=\frac{1}{4}A_{oo}\,\tau(t)\,\left(1+\cos\frac{\pi X_1}{2a}\right)\left(1+\cos\frac{\pi Y_1}{2a}\right) \tag{5}$$

for a clamped rhombic plate, and as

$$W=A_{oo}\,\tau(t)\,\cos\frac{\pi X_1}{2a}\cos\frac{\pi Y_1}{2a} \tag{6}$$

for a Simply-Supported rhombic plate, and then integrating the transformed equation (4) over the whole area of the plates, we respectively get

$$\bar{\alpha}^2=\frac{9\pi^2 A_{oo}^2}{32a^2h^2}\left(1+\nu+2\tan^2\theta\right) \, . \tag{7}$$

for the clamped rhombic plate, and

$$\bar{\alpha}^2 = \frac{3\pi^2 A_{00}^2}{8a^2h^2}\left(1+\nu+2\tan^2\theta\right) \quad . \tag{8}$$

for the Simply-Supported rhombic plate.

Here it is to be noted that for transverse vibration the normal displacement $W(x_1, y_1, t)$ is our primary interest and hence the in-plane displacements u,v in equation (4) have been eliminated through integration by choosing suitable expressions for them in the forms

$$u = \sum_{k=1}^{\infty} g_k(y_1)\sin\frac{k\pi x_1}{a}\,\tau^2(t) \quad \text{and} \quad v = \sum_{k=0}^{\infty} h_k(y_1)\cos\frac{k\pi x_1}{a}\,\tau^2(t)$$

Also it is to be noted that the deflection function in the form (5) satisfies the boundary conditions

$$W = 0 \text{ along } x_1 = \pm a \text{ and } y_1 = \pm a \;;$$

$$\frac{\partial W}{\partial x_1} = \frac{\partial W}{\partial y_1} = 0 \text{ along } x_1 = \pm a \text{ and } y_1 = \pm a$$

for a skew plate clamped along all the four edges and the deflection function in the form (6) satisfies the boundary conditions

$$W = 0 \text{ along } x_1 = \pm a \text{ and } y_1 = \pm a \;;$$

$$\frac{\partial^2 W}{\partial x_1^2} = \frac{\partial^2 W}{\partial y_1^2} = 0 \text{ along } x_1 = \pm a \text{ and } y_1 = \pm a$$

for a Simply-Supported skew plate.

Again introducing equations (5) and (7) or (6) and (8), (as the case may be), in the transformed equation (3) and then applying the Galerkin's error minimising technique we get the following differential equations for the time function $\tau(t)$

$$\left[1+\frac{4\pi^2\sec^2\theta}{15(1-\nu^2)}\mu\delta^2\right]\ddot{\tau}+\frac{4}{9}\frac{\pi^4}{a^4}\left(\frac{D}{\rho h}\right)\left[(2+\sin^2\theta)\sec^4\theta\right]\tau$$
$$+\frac{4}{9}\frac{\pi^4}{a^4}\left(\frac{D}{\rho h}\right)\Big[\frac{3}{128}\left\{9(1+\nu+2\tan^2\theta)^2+5\lambda(13+31\tan^2\theta+18\tan^4\theta)\right\}$$
$$+\frac{9}{16}\left\{\frac{1}{5}(1+\nu+2\tan^2\theta)(1+\nu+3\tan^2\theta)+\frac{3\lambda}{2}(1+3\tan^2\theta\right.$$
$$\left.+2\tan^4\theta)\right\}\frac{\pi^2\sec^2\theta}{(1-\nu^2)}\mu\delta^2\Big]\bar{\beta}^2\tau^3=0 \quad . \tag{9}$$

for the clamped rhombic plate, and

$$\left[1+\frac{\pi^2\sec^2\theta}{5(1-\nu^2)}\mu\delta^2\right]\ddot{\tau}+\frac{\pi^4}{4a^4}\left(\frac{D}{\rho h}\right)\left[(1+\sin^2\theta)\sec^4\theta\right]\tau$$
$$+\frac{\pi^4}{4a^4}\left(\frac{D}{\rho h}\right)\Big[\frac{3}{8}\left\{(1+\nu+2\tan^2\theta)^2+\lambda(5+11\tan^2\theta+6\tan^4\theta)\right\}$$
$$+\frac{3}{40}\left\{(1+\nu+2\tan^2\theta)(1+\nu+4\tan^2\theta)+\lambda(5+17\tan^2\theta\right.$$
$$\left.+12\tan^4\theta)\right\}\frac{\pi^2\sec^2\theta}{(1-\nu^2)}\mu\delta^2\Big]\bar{\beta}^2\tau^3=0 \quad . \tag{10}$$

for the Simply-Supported rhombic plate, where $(D/\rho h) = h^2c_p^2/12$

when $D = Eh^3/12(1-\nu^2)$ the Bending Rigidity of the plates, $\delta = h/2a$ the thickness-to-span ratio, $\mu = K(E/G_c)$, (K being 1 for isotropic elastic plates[179]), the quantity signifying transverse shear deformation and rotatory inertia and $\bar{\beta} = A_{oo}/h$ the non-dimensional amplitude, The equations (9) and (10) are the familiar Duffing's equations. The solutions of these equations subject to the initial conditions $\tau(0) = 1$ and $\dot{\tau}(0) = 0$ are well-known and are obtained in terms of Jacobic elliptic functions.

NUMERICAL RESULTS

Numerical results are presented here in the tabular forms, both for immovable as well as movable edge conditions of moderately thick isotropic skew plates . It is to be borne in mind that for movable edge conditions $\bar{\alpha} = 0$. This is because movable edge implies stress-free boundary.

The ratios of the non-linear period T* of vibrations including the effects of transverse shear deformation and rotatory ineritia to the corresponding linear period T of vibrations not including those effects are computed for skew angles $\theta = 0^\circ$, 15°, 30° and 45°; thickness parameters $\delta = 0.1$, 0.05, 0.03 and 0.025; Poisson's Ratio $\nu = 0.3$; $\mu = 2.5$, 20 and 30 and at non-dimensional amplitudes of vibration $\bar{\beta} = 0$, 0.2, 0.4, 0.6, 0.8 and 1 (Tables 1 - 7). In the Table 8, the results of the present study are compared with those obtained from the references 146 and 151.

TABLE 1

Clamped Square Plate ($\theta = 0^{\circ}$)

δ	$\bar{\beta}$	T*/T for Immovable Edge			T*/T for Movable Edge		
		μ= 2.5	μ= 20	μ= 30	μ= 2.5	μ= 20	μ= 30
1/10	0	1.0356	1.2565	1.3669	1.0356	1.2565	1.3669
	0.2	1.0300	1.2438	1.3494	1.0331	1.2509	1.3591
	0.4	1.0138	1.2079	1.3007	1.0259	1.2344	1.3365
	0.6	0.9886	1.1546	1.2304	1.0142	1.2084	1.3012
	0.8	0.9562	1.0907	1.1490	0.9985	1.1746	1.2564
	1.0	0.9191	1.0224	1.0648	0.9795	1.1352	1.2052
1/20	0	1.0090	1.0699	1.1032	1.0090	1.0699	1.0132
	0.2	1.0041	1.0634	1.0958	1.0069	1.0671	1.0999
	0.4	0.9898	1.0447	1.0744	1.0005	1.0587	1.0903
	0.6	0.9673	1.0156	1.0414	0.9902	1.0451	1.0748
	0.8	0.9384	0.9788	1.0001	0.9763	1.0270	1.0542
	1.0	0.9048	0.9370	0.9536	0.9593	1.0051	1.0294
1/30	0	1.0040	1.0317	1.0471	1.0040	1.0317	1.0471
	0.2	0.9992	1.0262	1.0413	1.0019	1.0293	1.0446
	0.4	0.9853	1.0103	1.0243	0.9957	1.0222	1.0370
	0.6	0.9633	0.9855	0.9977	0.9857	1.0107	1.0247
	0.8	0.9350	0.9536	0.9639	0.9721	0.9953	1.0082
	1.0	0.9020	0.9170	0.9252	0.9555	0.9765	1.9882
1/40	0	1.0023	1.0179	1.0268	1.0023	1.0179	1.0268
	0.2	0.9975	1.0128	1.0214	1.0002	1.0157	1.0244
	0.4	0.9837	0.9979	1.0059	0.9940	1.0090	1.0175
	0.6	0.9619	0.9745	0.9817	0.9841	0.9983	1.0063
	0.8	0.9338	0.9444	0.9504	0.9706	0.9838	0.9912
	1.0	0.9011	0.9096	0.9144	0.9541	0.9661	0.9728

TABLE 2

Clamped Rhombic Plate with Skew Angle $\theta = 15^{\circ}$

δ	$\bar{\beta}$	T*/T for Immovable Edge			T*/T for Movable Edge		
		$\mu = 2.5$	$\mu = 20$	$\mu = 30$	$\mu = 2.5$	$\mu = 20$	$\mu = 30$
1/10	0	1.0381	1.2730	1.3895	1.0381	1.2730	1.3895
	0.2	1.0323	1.2590	1.3700	1.0356	1.2670	1.3810
	0.4	1.0156	1.2198	1.3163	1.0283	1.2493	1.3566
	0.6	0.9994	1.1621	1.2395	1.0165	1.2217	1.3418
	0.8	0.9567	1.0937	1.1518	1.0007	1.1859	1.2708
	1.0	0.9186	1.0213	1.0624	0.9814	1.1444	1.2163
1/20	0	1.0097	1.0748	1.1103	1.0097	1.0748	1.1103
	0.2	1.0046	1.0680	1.1024	1.0075	1.0718	1.1069
	0.4	0.9900	1.0483	1.0797	1.0011	1.0632	1.0969
	0.6	0.9674	1.0179	1.0449	0.9909	1.0494	1.0809
	0.8	0.9375	0.9796	1.0015	0.9770	1.0309	1.0596
	1.0	0.9033	0.9362	0.9530	0.9601	1.0085	1.0341
1/30	0	1.0043	1.0339	1.0504	1.0043	1.0339	1.0504
	0.2	0.9999	1.0282	1.0443	1.0022	1.0315	1.0478
	0.4	0.9857	1.0118	1.0266	0.9961	1.0243	1.0401
	0.6	0.9638	0.9862	0.9991	0.9860	1.0128	1.0276
	0.8	0.9340	0.9535	0.9641	0.9725	0.9973	1.0109
	1.0	0.9005	0.9159	0.9242	0.9560	0.9783	0.9906
1/40	0	1.0029	1.0192	0.0287	1.0029	1.0192	1.0287
	0.2	0.9979	1.0139	1.0232	1.0004	1.0170	1.0263
	0.4	0.9838	0.9986	1.0071	0.9943	1.0103	1.0194
	0.6	0.9613	0.9746	0.9821	0.9843	0.9995	1.0081
	0.8	0.9327	0.9438	0.9501	0.9710	0.9850	0.9929
	1.0	0.8995	0.9083	0.9132	0.9546	0.9673	0.9744

TABLE 3

Clamped Rhombic Plate with Skew Angle θ = 30°

δ	$\bar{\beta}$	T*/T for Immovable Edge			T*/T for Movable Edge		
		$\mu = 2.5$	$\mu = 20$	$\mu = 30$	$\mu = 2.5$	$\mu = 20$	$\mu = 30$
1/10	0	1.0471	1.3311	1.4690	1.0471	1.3311	1.4690
	0.2	1.0407	1.3126	1.4421	1.0446	1.3236	1.5481
	0.4	1.0221	1.2614	1.3698	1.0370	1.3020	1.4269
	0.6	0.9933	1.1883	1.2706	1.0247	1.2682	1.3792
	0.8	0.9568	1.1046	1.1622	1.0083	1.2252	1.3200
	1.0	0.9175	1.0192	1.0567	0.9883	1.1759	1.2541
1/20	0	1.0120	1.0922	1.1355	1.0120	1.0922	1.1355
	0.2	1.0066	1.0843	1.1260	1.0115	1.0891	1.1317
	0.4	0.9911	1.0615	1.0989	1.0035	1.0797	1.1205
	0.6	0.9677	1.0266	1.0586	0.9933	1.0647	1.1131
	0.8	0.9365	0.9832	1.0074	0.9794	1.0447	1.0789
	1.0	0.8995	0.9348	0.9523	0.9625	1.0206	1.0507
1/30	0	1.0054	1.0420	1.0624	1.0054	1.0420	1.0624
	0.2	1.0002	1.0357	1.0555	1.0033	1.0395	1.0596
	0.4	0.9849	1.0176	1.0355	0.9972	1.0321	1.0515
	0.6	0.9616	0.9903	1.0046	0.9873	1.0201	1.0383
	0.8	0.9313	0.9537	0.9659	0.9739	1.0041	1.0207
	1.0	0.8964	0.9132	0.9221	0.9575	0.9846	0.9993
1/40	0	1.0039	1.0238	1.0356	1.0039	1.0238	1.0356
	0.2	0.9985	1.0181	1.0295	1.0010	1.0216	1.0331
	0.4	0.9839	1.0015	1.0119	0.9950	1.0148	1.0260
	0.6	0.9598	0.9757	0.9925	0.9888	1.0039	1.0144
	0.8	0.9299	0.9427	0.9499	0.9720	0.9892	0.9988
	1.0	0.8953	0.9050	0.9103	0.9558	0.9712	0.9798

TABLE 4

Clamped Rhombic Plate with Skew Angle $\theta = 45^{\circ}$

δ	$\bar{\beta}$	T*/T for Immovable Edge			T*/T for Angle Edge		
		$\mu = 2.5$	$\mu = 20$	$\mu = 30$	$\mu = 2.5$	$\mu = 20$	$\mu = 30$
1/10	0	1.0699	1.4690	1.6543	1.0699	1.4690	1.6543
	0.2	1.0621	1.4381	1.6070	1.0670	1.4575	1.6366
	0.4	1.0395	1.3563	1.4870	1.0585	1.4247	1.5870
	0.6	1.0049	1.2468	1.3360	1.0448	1.3749	1.5136
	0.8	0.9620	1.1308	1.1855	1.0266	1.3133	1.4264
	1.0	0.9147	1.0196	1.0596	1.0045	1.2452	1.3838
1/20	0	1.0180	1.1355	1.1976	1.0180	1.1355	1.1976
	0.2	1.0140	1.1250	1.1840	1.0158	1.1316	1.1926
	0.4	0.9947	1.0949	1.1622	1.0093	1.1202	1.1780
	0.6	0.9679	1.0498	1.0903	0.9988	1.1020	1.1550
	0.8	0.9338	0.9954	1.0246	0.9847	1.0779	1.1249
	1.0	0.8949	0.9364	0.9554	0.9695	1.0492	1.0895
1/30	0	1.0080	1.0624	1.0922	1.0080	1.0624	1.0922
	0.2	1.0024	1.0548	1.0835	1.0060	1.0596	1.0890
	0.4	0.9899	1.0354	1.0584	0.9999	1.0514	1.0796
	0.6	0.9666	0.9997	1.0204	0.9900	1.0382	1.0644
	0.8	0.9292	0.9580	0.9736	0.9766	1.0206	1.0442
	1.0	0.8910	0.9120	0.9221	0.9603	0.9992	1.0199
1/40	0	1.0045	1.0356	1.0529	1.0045	1.0356	1.0529
	0.2	0.9990	1.0290	1.0457	1.0025	1.0331	1.0502
	0.4	0.9843	1.0100	1.0249	0.9965	1.0260	1.0424
	0.6	0.9591	0.9806	0.9939	0.9869	1.0111	1.0298
	0.8	0.9262	0.9436	0.9520	0.9788	0.9990	1.0130
	1.0	0.8897	0.9027	0.9081	0.9599	0.9802	0.9925

TABLE 5

Simply-Supported Rhombic Plate with Skew Angle $\theta = 15^{\circ}$

δ	$\bar{\beta}$	T*/T for Immovable Edge			T*/T for Movable Edge		
		$\mu = 2.5$	$\mu = 20$	$\mu = 30$	$\mu = 2.5$	$\mu = 20$	$\mu = 30$
1/10	0	1.0287	1.2104	1.3031	1.0287	1.2104	1.3031
	0.2	1.0158	1.1892	1.2765	1.0261	1.2062	1.2976
	0.4	0.9801	1.1318	1.2057	1.0183	1.1934	1.2817
	0.6	0.9284	1.0525	1.1104	1.0058	1.1731	1.2564
	0.8	0.8681	0.9652	1.0086	0.9891	1.1463	1.2235
	1.0	0.8056	0.8795	0.9112	0.9689	1.1145	1.1848
1/20	0	1.0072	1.0566	1.0838	1.0072	1.0566	1.0838
	0.2	0.9952	1.0426	1.0686	1.0048	1.0537	1.0807
	0.4	0.9617	1.0039	1.0270	0.9976	1.0453	1.0716
	0.6	0.9128	0.9483	0.9674	0.9858	1.0318	1.0570
	0.8	0.8556	0.8841	0.8993	0.9702	1.0137	1.0375
	1.0	0.7957	0.8180	0.8298	0.9511	0.9919	1.0140
1/30	0	1.0033	1.0255	1.0381	1.0033	1.0255	1.0318
	0.2	0.9914	1.0128	1.0249	1.0008	1.0229	1.0354
	0.4	0.9582	0.9774	0.9882	0.9937	1.0153	1.0274
	0.6	0.9099	0.9261	0.9351	0.9821	1.0029	1.0146
	0.8	0.8532	0.8663	0.8735	0.9666	0.9864	0.9974
	1.0	0.7939	0.8041	0.8098	0.9477	0.9663	0.9766
1/40	0	1.0018	1.0144	1.0216	1.0018	1.0144	1.0216
	0.2	0.9900	1.0021	1.0090	0.9994	1.0120	1.0190
	0.4	0.9570	0.9679	0.9741	0.9923	1.0045	1.0115
	0.6	0.9089	0.9181	0.9232	0.9808	0.9926	0.9992
	0.8	0.8523	0.8598	0.8640	0.9653	0.9765	0.9829
	1.0	0.7932	0.7991	0.7763	0.9465	0.9571	0.9630

TABLE 6

Simply-Supported Rhombic Plate with Skew Angle $\theta = 30^{\circ}$

δ	$\bar{\beta}$	T*/T for Immovable Edge			T*/T for Movable Edge		
		$\mu = 2.5$	$\mu = 20$	$\mu = 30$	$\mu = 2.5$	$\mu = 20$	$\mu = 30$
1/10	0	1.0356	1.2565	1.3669	1.0356	1.2565	1.3669
	0.2	1.0223	1.2309	1.3332	1.0331	1.2519	1.3604
	0.4	0.9855	1.1627	1.2456	1.0261	1.2383	1.3417
	0.6	0.9323	1.0710	1.1319	1.0146	1.2167	1.3122
	0.8	0.8707	1.9728	1.0151	0.9992	1.1883	1.2741
	1.0	0.8069	0.8800	0.9207	0.9804	1.1546	1.2299
1/20	0	1.0090	1.0699	1.1032	1.0090	1.0699	1.1032
	0.2	0.9969	1.0551	1.0867	1.0068	1.0672	1.1002
	0.4	0.9632	1.0140	1.0413	1.0003	1.0593	1.0914
	0.6	0.9141	0.9554	0.9772	0.9898	1.0465	1.0772
	0.8	0.8566	0.8883	0.9047	0.9757	1.0293	1.0583
	1.0	0.7966	0.8197	0.8315	0.9584	1.0085	1.0353
1/30	0	1.0040	1.0317	1.0471	1.0040	1.0317	1.0471
	0.2	0.9922	1.0186	1.0333	1.0019	1.0293	1.0446
	0.4	0.9590	0.9823	0.9952	0.9955	1.0223	1.0373
	0.6	0.9107	0.9153	0.9402	0.9852	1.0110	1.0253
	0.8	0.8539	0.8686	0.8767	0.9713	0.9957	1.0094
	1.0	0.7946	0.8054	0.8113	0.9542	0.9772	0.9899
1/40	0	1.0023	1.0180	1.0268	1.0023	1.0180	1.0268
	0.2	0.9905	1.0055	1.0139	1.0001	1.0157	1.0245
	0.4	0.9575	0.9707	0.9782	0.9938	1.0090	1.0176
	0.6	0.9094	0.9203	0.9264	0.9835	0.9982	1.0064
	0.8	0.8530	0.8614	0.8661	0.9697	0.9836	0.9914
	1.0	0.7939	0.8001	0.8035	0.9528	0.9658	0.9731

TABLE 7

Simply-Supported Rhombic Plate with Skew Angle θ 45°

δ	$\bar{\beta}$	T*/T for Immovable Edge			T*/T for Movable Edge		
		μ = 2.5	μ = 20	μ = 30	μ = 2.5	μ = 20	μ = 30
1/10	0	1.0529	1.3669	1.5174	1.0529	1.3669	1.5174
	0.2	1.0390	1.3363	1.4757	1.0506	1.3610	1.5090
	0.4	1.0006	1.2561	1.3692	1.0439	1.3437	1.4846
	0.6	0.9453	1.1497	1.2340	1.0330	1.3165	1.4466
	0.8	0.8814	1.0381	1.0982	1.0182	1.2810	1.3980
	1.0	0.8157	0.9332	0.9755	1.0003	1.2395	1.3424
1/20	0	1.0135	1.1032	1.1514	1.0135	1.1032	1.1514
	0.2	1.0012	1.0872	1.1332	1.0115	1.1005	1.1482
	0.4	0.9668	1.0433	1.0835	1.0057	1.0925	1.1387
	0.6	0.9168	0.9808	1.0137	0.9963	1.0794	1.1235
	0.8	0.8584	0.9097	0.9356	0.9836	1.0620	1.1033
	1.0	0.7975	0.8376	0.8574	0.9679	1.0408	1.0789
1/30	0	1.0060	1.0476	1.0699	1.0060	1.0476	1.0699
	0.2	0.9940	1.0335	1.0554	1.0041	1.0449	1.0675
	0.4	0.9603	0.9957	1.0151	0.9985	1.0383	1.0603
	0.6	0.9113	0.9412	0.9574	0.9893	1.0276	1.0487
	0.8	0.8539	0.8781	0.8911	0.9769	0.01132	1.0331
	1.0	0.7950	0.8131	0.8233	0.9617	0.9956	1.0141
1/40	0	1.0034	1.0268	1.0399	1.0034	1.0268	1.0399
	0.2	0.9914	1.0140	1.0266	1.0015	1.0247	1.0377
	0.4	0.9580	0.9782	0.9895	0.9959	1.0186	1.0313
	0.6	0.9100	0.9265	0.9360	0.9859	1.0087	1.0209
	0.8	0.8542	0.8663	0.8739	0.9746	0.9953	1.0069
	1.0	0.7942	0.8038	0.8098	0.9575	0.9789	0.9897

It is to be noted that for $\theta = 0^{\circ}$ in the case of Simply-Supported isotropic plate, the results of the present study are in exact agreement with those obtained in reference 179 .

TABLE 8

Some clamped plate results compared with those obtained from references 146 and 151; $\mu = 2.5$, $\nu = 0.25$, $\delta = 0.1$.

$\bar{\beta}$	T*/T							
	Immovable Edge				Movable Edge			
	$\theta = 15^{\circ}$		$\theta = 30^{\circ}$		$\theta = 15^{\circ}$		$\theta = 30^{\circ}$	
	Present Approach	Ref.146	Present Approach	Ref.146	Present Approach	Ref.151	Present Approach	Ref.151
0	1.03690	1.0300	1.04578	1.0330	1.0369	1.0300	1.04578	1.0330
0.2	1.03217	1.0250	1.04036	1.0233	1.0352	1.0260	1.0440	1.0293
0.6	0.9963	0.9744	0.99980	0.9800	1.0220	0.9938	1.03015	1.0066
1.0	0.93466	0.8925	0.95154	0.9056	0.9968	0.9470	1.00410	0.9678

OBSERVATIONS

The non-linear behaviours of isotropic elastic thick skew plates at large amplitude transverse vibrations are evident form the numerical results presented here. The combined effects of transverse shear deformation and rotatory inertia on the large amplitude vibrations of skew plates are shown by a constant increase in the non-linear period T* with μ, δ remaining constant. The increase in non-linear period is less for vibrations at moderately large amplitudes, but throughout the range of amplitude $\bar{\beta}$, the effect of μ is very significant. For the same value of μ , as the thickness of the plate decreases, T*/T decreases due to the decreasing influences of transverse shear deformation and rotatory inerita. Those effects are insignificant for $\delta < 1/30$.

The period-amplitude relationship is of the hardening type i.e. the period of non-linear vibration decreases with increasing amplitude irrespective of the boundary conditions whether movable or immovable. This is a well-known phenomenon[146,151] . Again for a given value of μ , the period ratio for a skew plate (whether clamped or Simply-Supported) with movable edge conditions is greater than that for a corresponding plate with immovable edge conditions.This trend is in agreement with that reported in open literature for isotropic rectangular and skew plates.

The period ratio varies significantly with skew angle θ. Square plates seem to be more responding to the influences of transverse shear deformation and rotatory inertia at small amplitudes than at relatively large amplitudes, but, reverse is the case with skew plates[151]. For the same values of μ, $\bar{\beta}$ and δ, T*/T generally increases with θ whether the plate is clamped or Simply-Supported. As θ tends to zero, the present results agree exactly with those of Bhattacharjee and Banerjee[179] in the case of Simply-Supported thick plate. It is found also that for the same values of θ, μ, and δ, T*/T is greater for clamped plate than for Simply-Supported plate.

A comparative study of the results presented in Table 8 speaks for itself. The larger values of T*/T by the present approach indicate the influences of transverse shear deformation and rotatory inertia more prominently than by other approaches in open literature.

CONCLUSION OF THE THESIS

The present project is an humble attempt to offer a new set of uncoupled differential equations in oblique co-ordinates to study the non-linear behaviours of different rhombic plates (skew plates having aspect ratio 1), under'Static', 'Dynamic' and'Thermal' loadings. It is observed from the numerical results of the present study, [as shown in the different tables for different rhombic plates (viz. thin rhombic plates of uniform thickness, rhombic plates of variable thickness, rhombic sandwich plates and thick rhombic plates of uniform thickness)] , that, the non-linear behaviours of skew plates can be predicted with ease and accuracy by applying the present set of differential equations. Moreover results for Clamped and Simply-Supported plates with immovable as well as movable edge conditions can be obtained from the same set of differential equations. This is an additional advantage over Berger's equation used by different authors in analysing non-linear behaviours of elastic skew plates. Furthermore, unlike Von-Kármán's coupled differential equations in oblique co-ordinates, the proposed differential equations offer reasonably good results from the practical point of view, with minimum computational labour, because of its uncoupled form.

Thus considering the simplicity, versatility and practicability, it may be concluded that the proposed differential equations presented in tne thesis are quite efficient to fill up void in the non-linear theory of skew plates.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. J.L. Lagrange, "History of the theory of elasticity", 1811.

2. G.R. Kirchhoff, Crelle J. Vol.40, No. 51, 1850.

3. F. Gehring, "De Acquationibus differentialibus quibus acquilibrium et motus laminae crystallinae difiniuntur", 1860.

4. G.R. Kirchhoff, " Vorlesungen Uber Mathematische, Physik, Mechanik, 2nd Edn., 1883.

5. Lord Rayleigh, "On the Free Vibrations of an Inflinite Plate of Homogeneous Isotropic Elastic Matter", Proc. London Math. Soci., London, Vol.10, 1889, pp 225-234.

6. A. Foppl, Vorlesungen Uber Technische Mechanik, Bd.5, Leipzig, 24, 1907, p-132.

7. T. Von-Karman, "Festigkeitsprobleme in Maschinenbau", Encyklopadie der Mathematischem Wissenschaften, Bd. IV 2, II, Leipzig, 8, 1910, pp 348-351.

8. H. Lamb, "On Waves in an Elastic Plate", Proc. Royal Soci. London, Series A, Vol.93, 1917, pp 114-128.

9. S. Way, "Bending of Circular Plates with Large Deflections", J. Appl. Mech. TRANS. ASME. Vol.59, 1934, p-637.

10. S. Timoshenko, "Vibration Problems in Engineering", 2nd Edn. D. Van Nostrand Company, Inc. N.Y., 1937.

11. S. Way, "Uniformly Loaded Clamped Rectangular Plates with Large Deflections", Proc. 5th Int. Cong. Appl. Mech., Cambridge, Mass, 1938, p-123.

12. S. Levy, "Bending of Rectangular plates with large deflections, NACA TR 737, 1942.

13. S. Levy, "Square plate with clamped edges under normal pressure producing large deflections", NACA TR 740, 1942.

14. S. Levy and S. Greenman, "Bending with large deflections of a clamped rectangular plate with length-width ratio of 1.5 under normal pressure, NACA TN 853, 1942.

15. E. Reissner, "The Effect of Transverse Shear Deformation on the Bending of Elastic plates", J. Appl. Mech.,Trans ASME Vol. 67, 1945, pp 69-77.

16. A.E.H. Love, "A Treatise on the Mathematical Theory of Elasticity", 4th Edn., Dover Publications N.Y., 1944.

17. E. Reissner, " On the bending of elastic plates", Qly. of Appl. Math. Vol. 5, 1947, pp 55 - 68

18. W.Z. Chien, "Large Deflection of a Circular Clamped Plate under Uniform Pressure", Chinese J.Physics, Vol.7, 1947 pp 102-113

19. E. Reissner, "Finite deflections of Sandwich Plates", J. Aero Sci., Vol.15, July 1948, pp 435-440.

20. Chi-Teh Wang, " Non-Linear Large Deflection Boundary Value Problems of Rectangular Plates", NACA TN 1462, 1948.

21. Chi-Teh Wang, " Bending of Rectangular Plates with Large Deflections", NACA TN 1462, 1948.

22. Y.S. Uflyand, "The Propagation of Waves in tne Transverse Vibrations of Bars and Plates", (in Russian), Akad, Nauk SSSR Prikl, Mat. Mch. Vol. 12, 1948, pp 287-300.

23. A.E. Green, "On Reissner's Theory of Bending of Elastic Plates", Qly. of Appl. Math. Vol.7, 1949, pp 223-225.

24. B.J. Aleck, "Thermal Stresses in a Rectangular Plate Clamped along an Edge", J.Appl. Mech. Trans. ASME, Vol.71, 1949.

25. E. Reissner, "On a Variational Theorem in Elasticity", J.Math. Phys., Vol.29, 1950, pp 90-95.

26. R.D. Mindlin, "Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic Elastic Plates", J.Appl. Mech. Trans. ASME, Vol.18, March 1951, pp 31-38.

27. J. Guest, "The Buckling of Uniformly Compressed Parallelogram Plates having all Edges Clamped", Aero. Research Lab. Rept. S.M.172, Australia, 1951.

28. C.T. Wang, "Principle and Application of Complementary Energy Method for Thin Homogeneous Sandwich Plates and Shells with Finite Deflections", NACA TN, 2620, 1952, pp 1-33.

29. G.A. Zizicas, "Transient Thermal Stresses in Thin Isotropic Plates", ICLA, Engg.Rept. Vol.52, No.7, 1952.

30. W.H. Wittrick, "Buckling of Oblique Plates with Clamped Edges under Uniform Compression", Aero. Quart.4, Part2, Feb. 1953, pp 151-163.

31. F.S. Shaw, F.Phole and N.Perrone, "The Non-Linear Deflection of Infinite Strip Plates", PIBAL Rept. 255, June 1953 .

32. V.V. Novozhilov, "Foundations of the Non-Linear Theory of Elasticity", Graylock Press, 1953.

33. W.H. Wittrick, "Buckling of Oblique Plates with Clamped Edges under Uniform Shear",Aero. Quart.5, Part 1, May 1954.

34. M. Hasegawa, "On Buckling of a Clamped Rhombic Thin Plate in Shear", J. Aero. Sci.21,720, 1954.

35. G.Herrmann, "Influences of Large Amplitudes on the Free Flexural Motions of Elastic Plates", NACA TN 3578, 1955.

36. H.M. Berger, "A New Approach to the Analysis of Large Deflections of Plates", J. Appl. Mech. ASME. Vol.22, Dec. 1955, pp 465-472.

37. P.J. Schneider, "Variation of Maximum Thermal Stress in Free Plates", J. Aero. Sci., Vol.22, 1955.

38. M.L. Williams, "Large Deflection Analysis for a Plate Strip Subjected to Normal Pressure and Heat", J. Appl. Mech.Trans. ASME Vol.22, No.4, Dec.1955, pp 458-464.

39. E.I. Grigoliuk, "Vibration of a Shallow Circularly Cylindrical Panel exhibiting Finite Deflections", Prikl. Mat. Mekh.19, 1955, pp 376-382.

40. H.N. Chu and G. Herrmann, "Influence of Large Amplitudes on Free Flexural Vibrations of Rectangular Elastic Plates", J.Appl. Mech.Trans. ASME Vol.23, No.4, Dec.1956, pp 532-540.

41. A.S. Volmir, "Flexible Plates and Shells", Govt. Tech. Publn. Theo. Lit. Moscow, 1956, pp 58-94, 112 and 208.

42. Z. Dzygadlo, "Stability of Freely Supported Oblique Plates under Compression", Bul. Mil. Tekhn. Akad 5, 1956,pp 81-92.

43. N.A. Weil and N.M. Newmark, "Large Deflections of Ellipitc Plates", J. Appl. Mech. Trans. ASME, Vol.23,1956,pp 21-26.

44. T. Iwinski and J.L. Nowinski, "The Problem of Large Deflections of Orthotropic Plates", Arch. Mech. Stos, Vol.9,1957, pp 593-604.

45. J.J. Stoker, "Non-Linear Vibrations", Interscience Pub. (N.Y.) Chap.3, 1957.

46. S.G. Lekhnitski, "Certain Approximate Analysis of the Non-Linear Behaviour of Anisotropic Plates", O.G.I.Z. Moscow, 1957, p.375.

47. M.L. Williams, "Further Large Deflection Analysis for a Plate Strip Subjected to Normal Pressure and Heating", J. Appl. Mech. Trans. ASME, Vol.25, No.2, June 1958.

48. S.J. Medwadowski, "A Refined Theory of Elastic Orthotropic Plates", J. Appl. Mech. Trans. ASME, Vol.25, 1958 pp 437-443.

49. W.A. Nash and J.R. Modeer, "Certain Approximate Analysis of the Non-Linear Behaviour of Plates and Shallow Shells", Proc. Symp. Theory of Thin Elastic Shells, Interscience, Tech. Rept.1, Depart. Engg. Mech. Univ. Florida, 1959.

50. W.A. Nash and I.D. Cooley, "Large Deflections of a Clamped Elliptical Plate Subjected to Uniform Pressure", J. Appl. Mech. Trans. ASME, Vol.81 (26), Series E, 1959, pp 291-293.

51. B.A. Boley and J.H. Weiner, "Theory of Thermal Stresses", 2nd Edn. Wiley, N.Y., 1960.

52. M. Forray and M. Newmann, " Axisymmetric Bending Stresses in Solid Circular Plates with Thermal Gradient", J. Aero. Sci., Vol.27, No.9, 1960,

53. W. Nowacki, "Thermoelasticity", Pergamon Press, Oxford, 1962.

54. J.E. Greenspon, "A Simplified Expression for the Period of Non-Linear Oscillations of Curved and Flat Panels", J. Aero. Sci. Vol.27, No.2, Feb. 1960, pp 138-139.

55. R.E. Kelly, "A Simplified Expression on the Period of Non-Linear Oscillations of Curved and Flat Panels", J. Aero. Sci. Vol.27, No.8 (Reader's Forum) Aug. 1960, pp 621-622.

56. J.E. Hassert and J.L. Nowinski ,"Non-Linear Transverse Vibrations of a Flat Rectangular Orthotropic Plate Supported by Stiff Ribs", Proc.5th Int. Symp. on Space Tech. Sci. ,Tech. Rept.18, Univ. of Delaware 1962.

57. Y. Yoshimura and K. Iwata, "Buckling of Simply-Supported Oblique Plates", J.Appl. Mech. Trans. ASME, Vol.30(363)1963.

58. L.S.D. Morley, "Skew Plates and Structures", Pergamon Press, Oxford, 1963.

59. J.L. Nowinski, "Non-Linear Vibrations of Elastic Circular Plates Exhibiting Rectilinear Orthotropy", ZAMP, Vol.14, 1963, pp 112-124.

60. S.N. Sinha, "Large Deflections of Plates on Elastic Foundations", J. ASCE(U.S.A.),Vol.89, 1963, pp 1-24.

61. E.H. Mansfield, "Bending and Stretching of Plates", Addision Wesley Pub. Co.(N.Y.),1962.

62. T. Wah, "Large Amplitude Flexural Vibrations of Rectangular Plates", Int. J. Mech. Sci.,Vol.5, No.6,Dec. 1963,pp 425-438.

63. J.L. Nowinski, "Large Amplitude Oscillations of Oblique Panels with an initial Curvature", AIAA Journal, Vol.2,No.6, June 1964, pp 1025-1031.

64. N. Yamaki, "Influence of Large Amp. on Flexural Vibrations of El. Plates",Tohoku Univ. Inst. High Speed Mechanics Reports Vol.15,1963-64, pp 159-178.

65. T. Wah, "The Normal Modes of Vibration of certain Non-Linear Continuous System", Trans. ASME, 31 E(JAM), 1, (Brief Notes) March 1964, pp 139-140.

66. I.K. Ebcioglu, "A Large Deflection Theory of Anisotropic Plates", Ingenieur-Archiv. Vol.33, 1964, pp 396-403.

67. A.M. Alwan, "Large Deflection of Sandwich Plates with Orthotropic Cores", J. AIAA, Vol.2, Oct. 1964, pp 1820-1822.

68. V.S. Gontkevich, " Free Vibrations of Plates and Shells", Handbook(in Russian), Kiev Nenkova Dumka, 1964.

69. J.B.Kennedy, "On the Bending of Clamped Skewed Plates under Uniform Pressure", J. Royal Aero. Soci. May 1965.

70. G. Schmidt, "Non-Linear Parametric Vibrations of Sandwich Plates", Polska Akademia Nauk Proc. of Vibration Problems, Vol.6, No.2, 1965, pp 209-228.

71. J.B. Kennedy and Simon Ng., "Analysis of Skewed Plate Structures with Fixed Edges", Trans. Engg. Inst. Canada, Oct.1965,

72. J.L. Nowinski, "Some Static and Dynamic Problems Concerning Non-Linear Behaviour of Plates and Shallow Shells with Discontinuous Boundary Conditions", Paper Presented to the Cong. of Scholars and Scientists of Polish Background, N.Y. Nov. 1965.

73. G.A.Wempner and J.L. Baylor, "General Theory of Sandwich Plates with Dissimilar Facings", Int. J. of Solids and Structures Vol.1, 1965, pp 157-177.

74. J.L. Nowinski and I.A. Ismail, "Large Oscillations of an Anisotropic Triangular Plate", J. Franklin Institute,Vol.280 No.5, Nov.1965, pp 417-424.

75. S.P. Timoshenko and S. Woinowsky-Krieger, "Theory of Plates and Shells", (2nd Edn.), McGraw-Hill Book Company, 1962.

76. Y.Y. Yu, "Non-Linear Flexural Vibrations of Sandwich Plates", AFOSR 1310 (Poly, Inst., Brooklyn, Dept. Mech. Engg. TN 12), Feb. 1962, 19 pp.

77. Y.Y. Yu, "Non-Linear Flexural Vibrations of Sandwich Plates", J.Acoust. Soci. Ame. Vol.34, No.9 (Part 1), Sept. 1962, pp 1176-1183.

78. Y.Y. Yu, "Influence of Transverse Shear Deformation on Linear and Non-Linear Vibrations of Sandwich Plates", Technical Note No.15, Poly. Inst. Brooklyn, Dept. Mech. Engg., Aug. 1962.

79. Y.Y. Yu, "Application of Variational Equation of Motion to the Non-Linear Vibration Analysis of Homogeneous and Layered Plates and Shells", J.App. Mech. Trans. ASME, Vol.30, 1963, pp79-86.

80. Y.Y. Yu, and J.L. Lai, "Influence of Transverse Shear and Edge Conditions on Non-Linear Vibration and Dynamic Buckling of Homogeneous and Sandwich Plates", J. App. Mech. Trans. ASME, Vol.33, 1966, p.934.

81. S. Basuli, "Large Deflection of a Circular Plate under a Concentrated load at the Centre", J. App. Math. and Phys.(India) Vol.12, No.4, 1961.

82. L.M. Habip, "Theory of Elastic Plates in the Reference State", Int. J. Solids and Structures, Vol.2, 1966, pp 157-166,

83. R.K. Kaul, "Finite Thermal Oscillations of Thin Plates", Int. J. Solids and Structs. Vol2, 1966, pp 337-350.

84. F.C. Appl and N.R. Byers, "Fundamental Frequency of a Simply-

Supported Rectangular Plate with Linearly Varying Thickness", J. App. Mech. TRANS. ASME, Vol.31, 1965, pp 163-168.

85. J.B. Kennedy and Simon Ng., "Linear and Non-Linear Analyses of Skew Plates", J.Appl. Mech. Trans. ASME, Vol.34, 1967, pp 271-277.

86. J.B. Kennedy and Simon Ng., "Behaviour of Clamped Parallelogramic Panels with Large Deflections", J. Strain Analysis, The Inst. Mech. Engg., London, 1967.

87. B. Banerjee, "Large Amp. Free Vibrations of Elliptic Plates", J. Phys. Soci. of Japan, Vol.23, No.5, Nov. 1967,pp 1169-1172.

88. T. Katayama, E. Matumoto and T. Sekiya, "Fundamental Equations for Thermoelastic Deformation of Skew Plates", Bull. of the Univ. of OSAKA, Prefecture, Series A Vol.16, 1967,

89. S.R. Sarkar, "Quasi-Static Thermal Deflections in a Solid Circular Plate in the Axisymmetric Case", Aplikace Mathematiky Czechoslovakia, Vol.13, 1968.

90. R. Schmidt, "Large Deflections of a Clamped Circular Plate", J. ASCE(U.S.A.) Dec. 1968, p.1603.

91. S. Basuli, Indian J. Mech. and Math. Vol.VI, No.1, 1968.

92. M.C. Pal, "Large Amp. Free Vibrations of Rectilinear Plates", J. Sc. Engg. Research, India, Vol.11, No.2, July 1967, pp 317-326.

93. S.K. Sarkar, "Thermal Deflection of a Non-Homogeneous Rectangular Plate", Aplikace Mathematiky, Ceskoslovenka Akademic Ved. No.2, 1967.

94. G.Z. Harris and E.H. Mansfield, "On the Large Deflection Vibrations of Elastic Plates", Phil. Trans. Royal Soci. of London,

Series A Math. Phys. Sc., 261,1122, June 1967,pp 289-343.

95. B. Banerjee, "Large Amplitude Free Vibration of an Isosceles Right Angled Triangular Plate with Simply-Support Edges", J. Sc. Engg. Research, India Vol.12, No.1, Jan. 1968, pp 45-49.

96. H.F. Bauer, "Non-Linear Response of Elastic Plates to Pulse Excitations", J. Appl. Mech. Trans. ASME, Vol.35, 1968, pp 47-52.

97. C.I. Wu, "On the Lateral Vibrations of Rectangular Plates of Pyrolytic-Graphite-Type Materials", Master's Thesis, University of Delaware, 1968,.

98. J.E. Ashton, "Clamped Skew Plates of Orthotropic Materials under Transvers Load", Develop. in Mech. Vol.5, 1969, pp 297-306.

99. M.C. Pal, "Large Deflection of Heated Circular Plates", Acta. Mech. Vol.8, 1969, pp 82-103.

100. J.E. Ashton, "Natural Modes of Vibration of Tapered Plates", J. Structural Divn. ASCE (U.S.A.),Vol.95, No.ST-4, Proc. Paper 6489.

101. C.I. Wu and J.R. Vinson, "On the Non-Linear Oscillations of Plates Composed of Composite Materials", J. Compo. Materials, Vol.3, 1969, pp 548-561.

102. C.I. Wu and J.R. Vinson, "Influences of Large Amplitudes, Transverse Shear Deformation and Rotatory Inertia on Lateral Vibrations of Transversely Isotropic Plates", J.Appl. Mech. Trans. ASME, Vol.36, June 1969, pp 254-260.

103. R. Bolton, "Non-Linear Symmetrical Vibrations of a Circular Plate on Elastic Foundation", Ph.D. Thesis, 1967.

104 . G.Z. Harris and E.H. Mansfield, "On the Large Deflection Vibrations of Elastic Plates", Phil. Trans. Roy. Soci.,London , Series (A) Mathematical and Physical Science, Vol.261,No.1122, June 1967,pp 289-343.

105 . N.F. Morozov, "Non-Linear Oscillations of Thin Plates with Allowance for Rotational Inertia", (in Russion), Dok lady Aka-demii Nauk SSSR, Vol.176, No.3, Sept.1967, pp 522-525.

106 . J.M. Whitney and A.W. Leissa, "Analysis of heterogeneous anisotropic Plates", J.App. Mech. Trans. ASME, Vol.36, 1969, pp 261-266.

107 . A.W. Leissa, "Vibration of Plates", NASA SP-160, 1969.

108 . N.F. Morozov and M.E. Iudovin, "Convergence of the Bubnov - Galerkin Approximations in the Problems of Non-Linear Vibrations of a Thin Plate with Allowance for Rotational Inertia", (in Russion), Matematika, Aug. 1969, pp 47-51.

109 . M.Sathyamoorthy and K.A.V. Pandalai, "Non-Linear Flexural Vibrations of Orthotropic Rectangular Plates", J. Aero. Soci. (India) Vol.22, No.4, 1970, pp 264-266.

110 . K.A.V. Pandalai, "Large Amp. Flexural Vibration of Thin Elc. Flat Plates and Shells", Rept. Work Sponsored under NASA Grant NGR 09-010-053, Aug. 1971.

111 . F.La. Kaiuk and V.I. Pavlenko, "Thermal Buckling of Parallelogram Shaped Plates", (in Russian) Teplovye, Napriazheniia v Elementakh Konstruktsii, Vol.11, 1971, p.173.

112 . C.Y. Chia, " Non-Linear Theory of Heterogeneous and anisotropic Plates", Proc. 3rd Australasian Conf. on the Mechanics of Structures and Materials, Vol.II, Session B2-Plate Theory, Univ.,Auckland, 1971, pp 1-25.

113 . P.N. Singh, Y.C. Das and V. Sundarrajan,"Large Amplitude Vibration of Rectangular Plates", J. Sound and Vibration Vol.17, 1971, pp 235-240.

114 . J.L. Nowinski and H. Ohnabe, "On Certain Inconsistencies in Berger Equations for Large Deflections of Elastic Plates", Int. J. Mech. Sci.,Vol.14, 1972, pp 165-170.

115 . M. Sathyamoorthy and K.A.V. Pandalai, "Large Amp. Flexural Vibrations of Skew Plates", IIT(Madras) AE Rept. No.24, March 1972.

116 . T.C. Soong, "A Non-Linear Method Applied to Variable Thickness Plates", J. AIAA, Vol.10,Feb. 1972, pp 148-154.

117 . R. Bolton, "Stresses in Circular Plates on Elastic Foundation", J. ASCE, (U.S.A.), Vol.98, 1972, pp 629-640.

118 . F.La. Kaiuk and V.I. Pavlenko, "Thermal Stability of Skewed Plates", (in Russian) Voprosy Dinamiki i Prochnosti, Vol.22, 1972, p.159.

119 . M. Sathyamoorthy and K.A.V. Pandalai, "Governing Equations for the Large Amplitude Flexural Vibrations of Plates and Shells", IIT(Madras), India, AE Report, No.19, Feb. 1972.

120 . M. Sathyamoorthy and K.A.V. Pandalai, "Non-Linear Flexural Vibration of Orthotropic Skew Plates", J. Sound and Vibration Vol.24, No.1, Sept. 1972, pp 115-120.

121 . S.K. RoyChowdhury, "Some Problems on Thermoelasticity", Ph.D. Thesis Presented to Jadavpur University, India, 1972.

122 . J.L. Nowinski and H. Ohnabe, "Fundamental Equations for Large Deflections of Sandwich Plates with Orthotropic Core and Faces", Proc. 10th Int. Sympo. on Space Tech. and Sci. Tokyo, 1973, pp 311-318.

123 . M. Sathyamoorthy and K.A.V. Pandalai, "Large Amplitude Flexural Vibration of Simply-Supported Skew Plates", J. AIAA Vol.11, No.9, Sept. 1973, pp 1279-1282.

124 . M. Sathyamoorthy and K.A.V. Pandalai, "Non-Linear Vibration of Elastic Skew Plates Exhibiting Rectilinear Orthotropy", J. Franklin Instritute, Vol.296, No.5, Nov. 1973, pp 359-369.

125 . M. Sathyamoorthy and K.A.V. Pandalai, "Large Amplitude Vibrations of Certain Deformable Bodies (Part 2, Plates and Shells)", J. Aero. Soci. India, Vol.25, No.1, Feb. 1973, pp 1-10.

126 . M.C. Pal, "Static and Dynamic Behaviours of Heated Orthotropic Plate", Int. J. Non-Linear Mech. Vol.8, 1973, pp 489-504.

127 . J. Mazumdar and R. Jones, " A Simplified Approach to the Analysis of Large Deflection of Plates", J. Appl. Mech. Trans. ASME, Vol.41, 1974, pp 523 -524.

128 . P. Biswas, "Large Deflection of Heated Circular Plate under Non-Stationary Temperature", Bull. Cal. Math. Soci. Vol.66, 1974, pp 247-252.

129 . M.S.S. Prabhu and S. Durvasula, "Elastic Stability of Thermally Stressed Clamped-Clamped Skew Plates", J. Appl. Mech. Trans. ASME, Vol.41, No.3, 1974, pp 820-821.

130 . M. Sathyamoorthy and K.A.V. Pandalai, "Large Amplitude Vibrations of Variable Thickness Skew Plates", Proc. of Noise, Shock and Vibration Conf., Monash Univ.,Australia, 1974, pp 99-106.

131 . R.S. Srinivasan and S.V. Ramachandran, "Non-Linear Analysis of Skew Plates with Variable Thickness", J. AIAA. Vol.13, 1975, pp 843-844.

132 . S. Dutta, "Large Deflection of a Circular Plate on Elastic Foundation under a Concentrated Load at the Centre", J. Appl. Mech.,Trans. ASME, June 1975, pp 503-505.

133 . S. Dutta, "Large Deflections of Circular Plate under Symmetrical Load", J. Structural Mech,,Vol.3, No.4, 1975,

134 . P. Biswas, "Large Deflections of Heated Elastic Plates under Uniform Load", Mechanique Applique, Vol.20, No.4, 1975,

135 . E. Matumoto and T. Sekiya, "Elastic Stability of Thermally Stressed Parallelogram Panels", Trans. JSME, Vol.41, No.343, 1975, pp 736-745.

136 . J. Ramachandran, "Vibration of Variable Thickness Plates at Large Amplitudes", J. Franklin Institute, Vol.299, 1975, pp 359-362.

137 . C.P. Vendhan and Y.C. Das, "Application of Rayleigh-Ritz and Galerkin Methods to Non-Linear Vibrations of Plates", J.Sound and Vibration, Vol.39, 1975, pp 147-154.

138. M.M. Banerjee, "Note on the Large Deflections of Irregular Shaped Plates by the Method of Conformal Mapping", J. Appl. Mech. Trans. ASME, June 1976, pp 356-357.

139. S. Dutta, "Thermal Buckling of some Heated Plates Placed on Elastic Foundation", Defence Sci. Journal,Govt. of India, Vol.26, No.3, July 1976.

140. N. Kamiya, "Governing Equations for Large Deflections of Sandwich Plates", J. AIAA, Vol.14, No.2,1976,pp 250-253.

141. K. Kanakaraju and C.R.V. Rao, "Axisymmetric Vibrations of Circular Plates Including the Effects of Geometric Non-Linearity, Shear Deformation and Rotatory Inertia", J. Sound and Vibration, Vol.47, 1976, pp 179-184.

142. B.M. Karmakar, "Non-Linear Dynamic Behavours of Plates on Elastic Foundation", J. Sound and Vibration Vol.54, No.2, 1977, pp 265-271.

143. J. Mazumdar and R. Jones, "A Simplified Approach to the Large Amplitude Vibration of Plates and Membranes", J. Sound and Vibration, Vol.50, 1977, pp 389-397.

144. M.A. Sayed and R. Schmidt, "Large Deflections of Circular Plates under Concentrated Load", J. Industrial Math.,Vol.27, 1977, pp 87-91.

145. R. Schmidt, "A Refined Non-Linear Theory of Plates with Transverse Shear Deformation", J. Industrial Math.,Soci. Vol.27, 1977, pp 23-37.

146. M. Sathyamoorthy, "Shear and Rotatory Inertia Effects on Large Amp. Vibrations of Skew Plates", J. Sound and Vibra-

tions of Skew Plates", J. Sound and Vibration, Vol.52, 1977, pp 155-163.

147. M.K. Prabhakara and C.Y. Chia, "Non-Linear Bending of Symmetrically Laminated and Homogeneous Anisotropic Plates", Rozprawy Inzynierskie, Vol.24, 1977, pp 559-570.

148. N. Kamiya, "Analysis of the Large Thermal Bending of Sandwich Plates by a Modified Berger Method", J. Strain Analysis Vol.13, No.1, 1978, pp 17-22.

149. B.M. Karmakar, "Amplitude-Frequency Characteristics of Large Amp. Vibrations of Sandwich Plates", J. Appl. Mech., Trans, ASME, June 1978.

150. M. Sathyamoorthy, "Vibration of Plates Considering Shear and Rotatory Inertia", J. AIAA, Vol.16, 1978, pp 285-286.

151. M. Sathyamoorthy, "Vibration of Skew Plates at Large Amplitudes Including Shear and Rotatory Inertia Effects", Int. J. Solids and Structures, Vol.14, 1978, pp 869-880.

152. M. Sathyamoorthy, "Non-Linear Vibrations of Rectangular Plates", J. Appl. Mech.,Trans. ASME, Vol.46, 1979, pp 215-217.

153. B.M. Karmakar, "Amplitude-Frequency Characteristics of Non-Linear Vibrations of Clamped Elliptic Plate Carrying a Concentrated Mass", Int. J. Non-Linear Mech., Vol.13, 1979, pp 351-359.

154. M. Sathyamoorthy, "Large Deflections of Orthotropic Plates", J. Appl. Mech., Trans. ASME, Vol.46, 1979, pp 523-526.

155. B. Banerjee and S. Dutta, "Large Deflections of Elastic Plates under Non-Stationary Temperature", J. ASCE, EM-4, 1979, p 705.

156. M. Sathyamoorthy, "Effects of Large Amplitude, Shear and Rotatory Inertia on Vibration of Rectangular Plates", J. Sound and Vibration, Vol.63, 1979, pp 161-167.

157. C.Y. Chia and M. Sathyamoorthy, "Non-Linear Vibration of Anisotropic Skew Plates", Fibre Science and Technology,1979.

158. C.Y. Chia, "Non-Linear Analysis of Plates", McGraw-Hill Book Company, N.Y. 1980.

159. S. Dutta and B. Banerjee, "Large Deflections of Circular Plates under Symmetrical Loads", J. ASCE, Vol.106, No.4,1980.

160. M. Sathyamoorthy and C.Y. Chia, "Effects of Transverse Shear and Rotatory Inertia on Large Amplitude Vibrations of Anisotropic Skew Plates,(Parts I and II)", J. Appl. Mech., Trans. ASME, Vol.47, March 1980, pp 128-138.

161. J.N. Reddy and W.C. Chao, "Large Deflections and Large Amp-Free Vibrations of Laminated Composite Material Plates", Comput, Struct. Vol.13, 1981, pp 341-347.

162. B. Banerjee and S. Dutta, "A New Approach to the Analysis of Large Deflections of Thin Elastic Plates", Int. J. Non-Linear Mech.,Vol.16, 1981, pp 47- 52.

163. J.N. Reddy and C.L. Huang, "Large Amplitude Free Vibrations of Annular Plates of Varying Thickness", J. Sound and Vibration, Vol.29, 1981, pp 387-396.

164. B. Banerjee, "Large Deflections of Circular Plates of Variable Thickness", Int. J. Solids and Structures,(1993), 1982, pp 179-183.

165. B. Banerjee, "Large Deflection of an Elliptic Plate under a Concentrated Load", J. AIAA, Vol.20, No.1, 1983, pp 158-160.

166. S. Chaudhuri, "Large Deflection of an Equilateral Triangular Plate using Vonkármán Equations", Mech.,Research Communication, No.12, 1982, pp 112-115.

167. B. Banerjee, "Large Deflections of Plygonal Plates under Non-Stationary Temperature", J. Thermal Stresses (U.S.A.), Vol.7, 1984, pp 285-292.

168. B. Banerjee, "Large Amplitude Vibrations of Polygonal Plates - A New Approach", J. Appl. Mech.,Trans. ASME, Vol.51, March 1984, pp 211-213.

169. S.K. Chaudhuri, "Large Amp. Vibrations of Clamped Circular Plate of Variable Thickness", J. Appl. Mech.,Trans. ASME, Vol.51, March 1984, pp 207-210.

170. S. Das, "Linear and Non-Linear Analysis of Circular and Rectangular Plates", Ph. D. Thesis Submitted to the N.B.U. (India), 1984.

171. G.C. Sinharay and B. Banerjee, "A New Approach to Large Deflection Analysis of Spherical and Cylindrical Shells under Thermal Loading", Mech. Research Comm.(U.S.A.), Vol.12, No.2, 1985, pp 53-64.

172. G.C. Sinharay and B. Banerjee, "Large Deflections of Heated Thin Elastic Plates", Mech. Research Comm. 1986.

173. G.C. Sinharay and B. Banerjee,"Large Deflections of Thin Elc. Plates under Mechanical Loading", Int.J.Mech.Sc. 1986.

174. G.C. Sinharay and B. Banerjee, "Large Amp. Free Vibrations of Shells of Variable Thickness-A New Approach", J. AIAA, Vol.24, No.6, June 1986, pp 998-1004.

175. G.C. Sinharay and B. Banerjee, "Non-Linear Vibrations of Thin Elastic Plates", J. Sound and Vibration, Vol.107, No.2, July 1986.

176. T. Das, "Vibration and Buckling of Anisotropic Trapezoidal Plate on Non-Linear Elastic Foundation at Large Amplitude", Proc. Indian National Sc. Academy,Part-A, Vol.52, No.3, May 1986, pp 592-597.

177. K. Kanakaraju and C. Venkateswara Rao, "Vibrations of Isotropic Annular Plates with Edges Elastically Restrained Against Rotation", J. Sound and Vibration, Vol.109, No.2, Sept. 1986, pp 353-358.

178. E. Reissner, "On a Generalization of some Formulae of the Theory of Moderately Thick Elastic Plates", Int. J. Solids and Structures, Vol.23, No.6, 1987,pp 711-717.

179. R. Bhattacharjee and B. Banerjee, "Influences of Large Amplitudes, Transverse Shear Deformation and Rotatory Inertia on Free Lateral Vibrations of Transversely Isotropic Plates — A New Approach", Int. J. Non-Linear Mech., 1988.

180. D.J. Gorman, "Accurate Free Vibration Analysis of Rhombic Plates with Simply-Supported and Fully Clamped Edge Conditions", J. Sound and Vibration, Vol.125, No.2, Sept. 1988, pp 281-290.

181. G.L. Ostiguy and H. Nguyen, "Combination Resonances of Parametrically Excited Rectangular Plates", Proc. 3rd Int. Conf. on recent advances in Structural Dynamics, Southampton, U.K. 18-22 July 1988, Vol.2, pp819-829.

182. P.C. Dumir, "Large Deflection Axisymmetric Analysis of Orthotropic Annular Plates on Elastic Foundations", Int. J. Solids and Structures Vol.24, No.8, 1988, pp 777-787.

183. E. Hinton and B.S.Al. Janabi, "Vibrations of Plates with Concentrated Masses and Plates with Point Supports", Proc. 3rd Int. Conf. on recent advances in Structural Dynamics, Southampton, U.K.,18-22 July 1988, Vol.1, pp 101-110.

184. D.J. Gorman, "A General Computational Technique for the Free Vibration Analysis of Rectangular Plates with Classical Edge Support based on the Superposition Method", Proc. 3rd Int. Conf. on recent advances in Structural Dynamics, Southampton, U.K.,18-22 July 1988, Vol.1, pp 3-12.

185. H. Kobayashi and K. Sonoda, "Rectangular Mindlin Plates on Elastic Foundations", Int. J. Mech. Sci.(U.K.), Vol.31, No.9, 1989, pp 679-692.

186. S.F. Ng. and Y. Araar, "Free Vibrations and Buckling Analysis of Clamped Rectangular Plates of Variable Thickness by the Galerkin Method", J. Sound and Vibration, Vol.135, No.2, Dec. 1989, pp 263-274.

187. C.C. Lin and L.W.Chen, "Large Amplitude Vibration of an Initially Imperfect Moderately Thick Plate", J. Sound and Vibration, Vol.135, No.2, Dec. 1989, pp 213-224.

188. H.T. Yang and Y.C. Wu, "A Geometrically Non-Linear Tensorial Formulation of a Skewed Quadrilateral Thin Shell Finite Element", Int. J. Num. Methods Engg.(U. k.), Vol.28, No.12, Dec. 1989, pp 2855-2875.

189. H. Kobayashi and K. Sonoda, "Natural Frequencies of Rectangular Plates with Tapered Thickness", Mem. Fac. Eng. OSAKA City Univ. (Jap.), Vol.30, Dec. 1989, pp 123-144.

190. S. Huis-Shen, "Post Buckling Behaviour of Rectangular Plates under Combined Loading", Thin. Walled Struct.(U.K.), Vol.8, No.3, 1989, pp 203-216.

191. M. Mukhopadhyay, "A Simple Numerical Transient Vibration Analysis of Plate Structures", Engg. Software,(Navona Publg. House), New Delhi, India, Vol.4, No.7, Dec. 1989, pp 613-619.

192. Yeh Kai-Yuan, Zheng Xiao-Jing and Zhou You-he, "An Analytical Formula of the Exact Solution to Von Kármán's Equations of a Circular Plate under a Concentrated Load", Int. J. Non-Linear Mech. Vol.24, No.6, 1989, pp 551-560.

193. M. Gorji, Compos. Sci. Technol.(U.K.), Vol.34, No.3, 1989, pp 187-203.

194. K.M. Liew and K.Y. Lam, "Application of Two-dimensional Orthogonal Plate Function to Flexural Vibration of Skew Plates", J. Sound and Vibration, Vol.139, No.2, June 1990, pp 241-252.

195. R. Bhumbla, J.B. Kosmatka and J.N. Reddy, "Free Vibration Behaviour of Spinning Shear-Deformable Plates Composed of Composite Materials J.AIAA(U.S.A.), Vol.28, No.11, Nov. 1990, PP 1962-'70.

196. S. Dutta and B. Banerjee, "Governing Equations for Non-Linear Analysis of Sandwich Plates",-A New Approach", Int. J. Non-Linear Mech.,Vol.26, No.34, 1991, pp 313-318.

197. J.W. Zhang, "A Simplified Perturbation Solution to the Large Deflection Kármán Equations of Orthotropic Composite Plates in Axial Compression and Lateral Pressure", Arch. Appl. Mech.(Germany), Vol.61, Pt.3, May 1991, pp174-182.

198. B. Singh and S. Chakravorty, "Transverse Vibration of Circular and Elliptic Plates with Variable Thickness", Indian J. Pure and Appl. Math.,Vol.22, No.9, Sept. 1991, pp 787-803.

199. H. Kobayashi and K. Sonoda, "Vibration and Buckling of Tapered Rectangular Plates with Two Opposite Edges Simply-Supported and the other Two Edges Elastically Restrained against Rotation", J. Sound and Vibration(U.K.), Vol.146, No.2, April 1991, pp 323-337.

200. M. Ganapathi, T.K. Varadan and B.S. Sarma, "Non-Linear Flexural Vibrations of Laminated Orthotropic Plates", Comput. Struct., Vol.39, No.6, 1991, pp 685-688.

201. U.S. Gupta, R. Lal and S.K. Jain, "Buckling and Vibrations of Polar Orthotropic Circular Plates of Linearly Varying Thickness Resting on an Elastic Foundation", J. Sound and Vibration,Vol.147, No.3, June 1991, pp 423-434.

202. D.J. Gorman, "Accurate Analytical-Type Solutions for the Free Vibrations of Simply-Supported Parallelogram Plates", J. Appl. Mech.,Trans. ASME,Vol.58, No.1, March 1991, pp 203-208.

203. J.T. Katsikadelis, "Large Deflection Analysis of Plates on Elastic Foundation by the Boundary Element Method", Int.J. Solids and Structures, Vol.27, No.15, 1991, pp 1867-1878.

204. G.B. Chai, "Large Deflections of Laminated Composite Plates", Compos. Sci. Technol.(U.K.), Vol.42, No.4, 1991, pp 349-360.

205. Xiu-Xi Wang, Jiang Qian and Mao-Kuang Huang, "A Boundary Integral Equation Formulation for Large Amplitude Non-Linear Vibration of Thin Elastic Plates", Comput. Methods. Appl. Mech., Engg.(Netharlands),Vol.86, No.1, March 1991, pp 73-86.

206. F.H. Yeh and W.H. Liu ,"Non-Linear Analysis of Rectangular, Orthotropic Plates", Int. J. Mech. Sci.(U.K.), Vol.33, No.7, 1991, pp 563-578.

207. S.K. Ghosh, "Large Amplitude Vibrations of a Clamped Orthotropic Elliptic Plate", Iranian J. of Sc. and Technol. Vol.15, No.2, 1991, pp 106-115.

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LARGE DEFLECTIONS OF RHOMBIC PLATES—A NEW APPROACH

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Abstract—Non-linear static behaviour of rhombic plates has been analysed following Banerjee's hypothesis (B. Banerjee, Large deflections of polygonal plates under non-stationary temperature. *J. Thermal Stresses* 7, 285–292 (1984)). Calculations have been carried out for different skew angles. To test the accuracy of the theoretical results so obtained, experiments were carried out on copper and steel rhombic plates. The theoretical results were found to be in excellent agreement with those obtained from an analysis of the experimental data.

INTRODUCTION

Skew or oblique panels find wide applications in the aircraft and spaceship industry; hence, a study of the non-linear behaviour of skew plates is of great importance. In contrast to the non-linear behaviour analysis of elastic plates of geometries like circular, rectangular, triangular and elliptic, skew plates have not received much attention. This may be due to their relatively difficult mathematical models.

The most important work in this field is due to Nowinski [2], who analysed the large-amplitude oscillations of oblique panels having initial curvature. Two more interesting papers on non-linear vibration problems of skew plates are by Sathyamoorthy and Pandalai [3, 4]. They have analysed the non-linear flexural vibrations of simply supported skew plates of isotropic as well as anisotropic materials, using Berger's equation. In contrast to works on non-linear vibration problems of skew plates, the literature on non-linear deflection problems of skew plates is scanty. In this field three interesting papers could be located. Kennedy and Simon [5] carried out non-linear analysis of skew plates by the perturbation method. Srinivasan and Ramachandran [6] analysed the large deflections of skew plates of variable thickness using the Newton–Raphson procedure. Ashton's [7] work is on the linear static analysis of anisotropic skew plates. It is interesting to note that most of these investigations are carried out on skew plates of clamped edges only and the case of simply supported edges has not received proper attention.

In this paper large deflections of simply supported rhombic plates are studied following Banerjee's approach. A set of uncoupled differential equations has been obtained in oblique coordinates and solved by applying the Galerkin technique. The case of a simply supported rhombic plate is discussed in detail. To test the accuracy of the method, experiments were carried out on copper and steel rhombic plates. The details of the experiments are given in the Appendix. The numerical results obtained from the theoretical and the experimental analysis are compared. The present method appears to be more acceptable from the practical point of view.

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Contributed by J. N. Reddy.

ANALYSIS

Consider a rhombic plate of an elastic, isotropic material, having uniform thickness of h. Let the size of each side of the skew plate be a which is sufficiently large compared to h. The origin of the rectangular Cartesian coordinate (x, y) is located at one of the corners of the skew plate (see Fig. 1). The plate is considered to be simply supported along its edges and loaded uniformly all over.

Following Banerjee's hypothesis [1], the differential equations, referred to the system of rectangular Cartesian coordinates are:

$$\nabla^4\omega - \frac{12A}{h^2}\left(\frac{\partial^2\omega}{\partial x^2} + \nu\frac{\partial^2\omega}{\partial y^2}\right) - \frac{6\lambda}{h^2}\left\{\nabla^2\omega\left[\left(\frac{\partial\omega}{\partial x}\right)^2 + \left(\frac{\partial\omega}{\partial y}\right)^2\right]\right.$$
$$\left. + 2\left[\frac{\partial^2\omega}{\partial x^2}\left(\frac{\partial\omega}{\partial x}\right)^2 + \frac{\partial^2\omega}{\partial y^2}\left(\frac{\partial\omega}{\partial y}\right)^2\right] + 4\left(\frac{\partial^2\omega}{\partial x\partial y}\right)\left(\frac{\partial\omega}{\partial x}\right)\left(\frac{\partial\omega}{\partial y}\right)\right\} = \frac{q}{D} \qquad (1)$$

where

ω = the deflection normal to the middle plane of the plate
ν = Poisson's ratio of the material of the plate
$\lambda = \nu^2$
q = load per unit area acting on the plate
D = the flexural rigidity of the plate $= Eh^3/12(1 - \nu^2)$
E = the modulus of elasticity of the material of the plate

$$A = \frac{1}{2}\left\{\left(\frac{\partial\omega}{\partial x}\right)^2 + \nu\left(\frac{\partial\omega}{\partial y}\right)^2\right\} + \frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} \qquad (2)$$

which is a constant depending on the surface and edge conditions of the plate, and ∇^2 is the Laplacian operator.

For a skew plate, let us adopt a system of oblique coordinates (x_1, y_1, θ), as shown in Fig. 1, θ being the skew angle.

Clearly,

$$x = x_1\cos\theta, \qquad y = y_1 + x_1\sin\theta \qquad (3)$$

are the coordinate transformation equations. Hence the partial differential operators become

$$\frac{\partial}{\partial x} \equiv \sec\theta\left(\frac{\partial}{\partial x_1} - \sin\theta\frac{\partial}{\partial y_1}\right), \qquad \frac{\partial}{\partial y} \equiv \frac{\partial}{\partial y_1}$$

$$\frac{\partial^2}{\partial x^2} \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\frac{\partial^2}{\partial x_1\partial y_1}\right) + \tan^2\theta\frac{\partial^2}{\partial y_1^2}$$

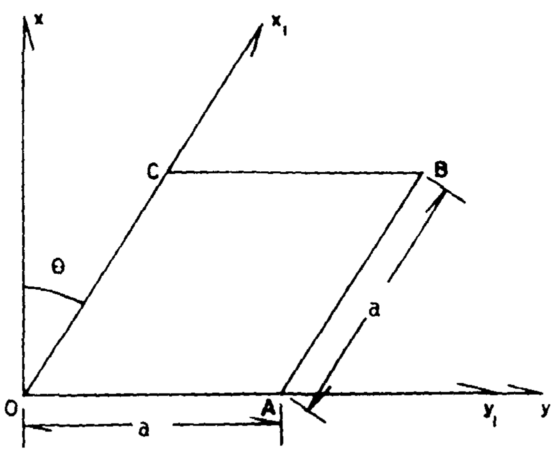

Fig. 1. Plan form of skew plate.

$$\frac{\partial^2}{\partial y^2} \equiv \frac{\partial^2}{\partial y_1^2}, \qquad \frac{\partial^2}{\partial x \partial y} \equiv \sec\theta\left(\frac{\partial^2}{\partial x_1 \partial y_1} - \sin\theta\frac{\partial^2}{\partial y_1^2}\right)$$

$$\nabla^2 \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\frac{\partial^2}{\partial x_1 \partial y_1} + \frac{\partial^2}{\partial y_1^2}\right)$$

and

$$\nabla^4 \equiv \sec^4\theta\left[\frac{\partial^4}{\partial x_1^4} - 4\sin\theta\left(\frac{\partial^4}{\partial x_1^3 \partial y_1} + \frac{\partial^4}{\partial x_1 \partial y_1^3}\right) + 2(1 + 2\sin^2\theta)\frac{\partial^4}{\partial x_1^2 \partial y_1^2} + \frac{\partial^4}{\partial y_1^4}\right]. \tag{3a}$$

Using these operators, transforming the differential equations (1) and (2) in oblique coordinates, we arrive at the following set of transformed differential equations:

$$\begin{aligned}
&\sec^4\theta\left[\frac{\partial^4\omega}{\partial x_1^4} - 4\sin\theta\left(\frac{\partial^4\omega}{\partial x_1^3\partial y_1} + \frac{\partial^4\omega}{\partial x_1\partial y_1^3}\right) + 2(1 + 2\sin^2\theta)\frac{\partial^4\omega}{\partial x_1^2\partial y_1^2} + \frac{\partial^4\omega}{\partial y_1^4}\right] \\
&- \frac{12A}{h^2}\left[\sec^2\theta\left(\frac{\partial^2\omega}{\partial x_1^2} - 2\sin\theta\frac{\partial^2\omega}{\partial x_1\partial y_1}\right) + \tan^2\theta\frac{\partial^2\omega}{\partial y_1^2} + \nu\frac{\partial^2\omega}{\partial y_1^2}\right] \\
&- \frac{6\lambda}{h^2}\left\{\sec^4\theta\left(\frac{\partial^2\omega}{\partial x_1^2} - 2\sin\theta\frac{\partial^2\omega}{\partial x_1\partial y_1} + \frac{\partial^2\omega}{\partial y_1^2}\right)\left[\left(\frac{\partial\omega}{\partial x_1}\right)^2 + \left(\frac{\partial\omega}{\partial y_1}\right)^2\right.\right. \\
&\left. - 2\sin\theta\left(\frac{\partial\omega}{\partial x_1}\right)\left(\frac{\partial\omega}{\partial y_1}\right)\right] + 2\left[\sec^4\theta\left(\frac{\partial^2\omega}{\partial x_1^2} - 2\sin\theta\frac{\partial^2\omega}{\partial x_1\partial y_1} + \sin^2\theta\frac{\partial^2\omega}{\partial y_1^2}\right)\right. \\
&\left.\times\left(\frac{\partial\omega}{\partial x_1} - \sin\theta\frac{\partial\omega}{\partial y_1}\right)^2 + \frac{\partial^2\omega}{\partial y_1^2}\left(\frac{\partial\omega}{\partial y_1}\right)^2\right] + 4\sec^2\theta\left(\frac{\partial^2\omega}{\partial x_1\partial y_1} - \sin\theta\frac{\partial^2\omega}{\partial y_1^2}\right) \\
&\left.\times\left(\frac{\partial\omega}{\partial x_1} - \sin\theta\frac{\partial\omega}{\partial y_1}\right)\left(\frac{\partial\omega}{\partial y_1}\right)\right\} = \frac{q}{D}
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
A = {}&\frac{1}{2}\left\{\sec^2\theta\left[\left(\frac{\partial\omega}{\partial x_1}\right)^2 - 2\sin\theta\left(\frac{\partial\omega}{\partial x_1}\right)\left(\frac{\partial\omega}{\partial y_1}\right) + \sin^2\theta\left(\frac{\partial\omega}{\partial y_1}\right)^2\right] + \nu\left(\frac{\partial\omega}{\partial y_1}\right)^2\right\} \\
&+ \sec\theta\left(\frac{\partial u}{\partial x_1} - \sin\theta\frac{\partial u}{\partial y_1}\right) + \nu\frac{\partial v}{\partial y_1}.
\end{aligned} \tag{5}$$

Now to solve the problem, let us assume

$$\omega = \omega_0 \sin\frac{\pi x_1}{a}\sin\frac{\pi y_1}{a} \tag{6}$$

ω_0 being the maximum central deflection.

For the value of A, let us integrate equation (5) over the whole area of the plate. Then we have

$$\begin{aligned}
\int_0^a\int_0^a A\cos\theta\,dx_1\,dy_1 = {}&\frac{1}{2}\int_0^a\int_0^a\left\{\sec^2\theta\left[\left(\frac{\partial\omega}{\partial x_1}\right)^2 + \sin^2\theta\left(\frac{\partial\omega}{\partial y_1}\right)^2\right.\right. \\
&\left.\left. - 2\sin\theta\left(\frac{\partial\omega}{\partial x_1}\right)\left(\frac{\partial\omega}{\partial y_1}\right)\right] + \nu\left(\frac{\partial\omega}{\partial y_1}\right)^2\right\}\cos\theta\,dx_1\,dy_1 .
\end{aligned}$$

After integration, we get

$$A = \frac{\pi^2\omega_0^2}{8a^2}(1 + \nu + 2\tan^2\theta). \tag{7}$$

Here, it is to be noted that, since the normal displacements are our primary interest, the in-plane displacements have been eliminated through integration by choosing suitable expressions for them, compatible with their boundary conditions.

Now, applying Galerkin's method of approximation to the transformed differential equation (4) and keeping in mind the value of A from equation (7), we get the following cubic equation determining β ($= \omega_0/h$).

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+\frac{3}{8}\{[(1+\nu)+(1-\nu)\sin^2\theta]^2 + \nu^2(5+\sin^2\theta)\}\left(\frac{\omega_0}{h}\right)^3 = \frac{4}{\pi^6}\left(\frac{qa^4}{Dh}\right)\cos^4\theta. \tag{8a}$$

Adopting the well known equation [8], of Berger with e_2[1] neglected, the corresponding cubic equation determining the central deflection parameter (for immovable edges only) takes the following form (after applying Galerkin's technique):

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+1.5\left(\frac{\omega_0}{h}\right)^3 = \frac{4}{\pi^6}\left(\frac{qa^4}{Dh}\right)\cos^4\theta. \tag{8b}$$

NUMERICAL CALCULATIONS

For a steel plate we have $E = 2\times10^{12}$ dyne/cm^2 and $\nu = 0.3$, for which equation (8a) becomes

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+\frac{3}{8}[(1.3+0.7\sin^2\theta)^2 + 0.09(5+\sin^2\theta)]\left(\frac{\omega_0}{h}\right)^3 = 22.66\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \tag{9a}$$

whereas for a copper plate we have $E = 1.25\times10^{12}$ dyne/cm^2 and $\nu = 0.333$, so that equation (8a) becomes

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+\frac{3}{8}[(1.333+0.667\sin^2\theta)^2 + 0.11(5+\sin^2\theta)]\left(\frac{\omega_0}{h}\right)^3 = 35.46\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta. \tag{9b}$$

Also, for a steel plate equation (8b) becomes

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+1.5\left(\frac{\omega_0}{h}\right)^3 = 22.66\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \tag{9c}$$

and for a copper plate it becomes

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+1.5\left(\frac{\omega_0}{h}\right)^3 = 35.46\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta. \tag{9d}$$

Tables 1 and 2 present a comparative view of the various theoretical and experimental values of the central deflection parameter β ($= \omega_0/h$) for different values of the load function Q ($= qa^4/Dh$), for the case of steel plate and copper plate, respectively. For movable edge conditions the value of A will be zero. (The experimental method is explained in the Appendix.)

OBSERVATIONS

It is observed from the two tables that the results of the present study are in excellent agreement with those obtained from the experimental analysis. It is well known that Berger's method fails [9] miserably under movable-edge conditions. The results for simply supported immovable edges, obtained by Berger's method (as shown in the Tables 1 and 2) show that this method is not even acceptable from the practical point of view. It is worth noting that Berger's method always gives less deflections for a given load. The errors of Berger's method (as shown in Tables 1 and 2) are certainly questionable from the view point of safety design.

Table 1. The central deflection parameter ($\beta = \omega_0/h$) vs the load function ($Q = qa^4/Dh$) for steel plate (a = 16 cm for skew angle $\theta = 15°$ and a = 14 cm for $\theta = 30°$; h = 0.1343 cm)

	β (for $\theta = 15°$)							
	Movable edges			Immovable edges			Percentage error	
Q qa^4/Dh	Banerjee's hypothesis	Experimental	Percentage error	Berger's method	Banerjee's hypothesis	Experimental	Berger's method	Banerjee's hypothesis
111.72	0.3716	0.3872	4%	0.3285	0.3454	0.36485	9.96%	5.33%
223.44	0.7038	0.7372	4.5%	0.5378	0.5914	0.6329	15%	6.56%
335.16	0.98592·	1.0201	3.3%	0.6843	0.7703	0.8414	18.67%	8.45%
446.88	1.22484	1.2882	4.9%	0.7982	0.9107	0.9903	19.4%	8%
558.6	1.4303	1.5115	5.4%	0.8924	1.027	1.1244	20.6%	8.66%

	β (for $\theta = 30°$)							
	Movable edges			Immovable edges			Percentage error	
Q qa^4/Dh	Banerjee's hypothesis	Experimental	Percentage error	Berger's method	Banerjee's hypothesis	Experimental	Berger's method	Banerjee's hypothesis
65.5	0.12208	0.134	8.9%	0.1202	0.1209	0.12658	5%	4.49%
131	0.2427	0.25316	4.13%	0.2301	0.2346	0.25316	9.11%	7.33%
196.5	0.3604	0.37975	5.1%	0.3254	0.3369	0.36485	10.8 %	7.66%
262	0.4742	0.4989	5%	0.40782	0.4276	0.46165	11.66%	7.4%
327.5	0.5835	0.61802	5.59%	0.4795	0.5081	0.55845	14.2%	9%

The average percentage error from Banerjee's hypothesis is only around 6% for skew angles of $\theta = 15°$, 30° whereas from Berger's method it is around 17% for $\theta = 15°$ and 10% for $\theta = 30°$.

Table 2. The central deflection parameter ($\beta = \omega_0/h$) vs the load function ($Q = qa^4/Dh$) for copper plate (a = 16 cm for skew angle $\theta = 15°$ and a = 14 cm for $\theta = 30°$; h = 0.0789 cm)

	β (for $\theta = 15°$)							
	Movable edges			Immovable edges			Percentage error	
Q qa^4/Dh	Banerjee's hypothesis	Experimental	Percentage error	Berger's method	Banerjee's hypothesis	Experimental	Berger's method	Banerjee's hypothesis
1467.53	2.3727	2.4208	2%	1.36820	1.57802	1.673	18.22%	5.68%
2935.06	3.2506	3.308	1.7%	1.79580	2.08753	2.23067	19.5%	6.4%
4402.59	3.8437	3.9924	3.72%	2.0891	2.43578	2.6109	19.98%	6.7%
5870.12	4.307	4.4867	4%	2.32003	2.7095	2.90241	20.07%	6.65%
7337.65	4.6935	4.90494	4.3%	2.5138	2.93893	3.1559	20.35%	6.9%
	β (for $\theta = 30°$)							
	Movable edges			Immovable edges			Percentage error	
Q qa^4/Dh	Banerjee's hypothesis	Experimental	Percentage error	Berger's method	Banerjee's hypothesis	Experimental	Berger's method	Banerjee's hypothesis
860.2	1.2602	1.2801	1.55%	0.8553	0.9283	0.9886	13.5%	6.1%
1720.4	1.9429	2.0279	4.2%	1.1901	1.3095	1.40684	15.5%	6.92%
2580.6	2.4064	2.5095	4.1%	1.4156	1.5657	1.6857	16%	7.12%
3440.8	2.765	2.9404	6%	1.5913	1.7649	1.90114	16.3%	7.17%
4301	3.0617	3.2319	5.3%	1.73760	1.9307	2.09125	16.91%	7.7%

The average percentage error from Banerjee's hypothesis is only around 5% for skew angles $\theta = 15°$, $30°$ whereas from Berger's method it is around 20% for $\theta = 15°$ and around 15% for $\theta = 30°$. The errors are calculated considering the experimental results as standard (sacrificing instrumental and personal errors).

It is observed that deflections for movable edges are always greater than those for immovable edges. This is quite expected from the practical point of view, because movable edge conditions give stress-free boundary and, hence, there are large energy changes in the boundary.

Here the results for skew angles $\theta = 15°$ and $30°$ only have been considered, because for greater values of the skew angles the effect of non-linearity does not play important role in design, and the study of linear analysis serves the practical purpose.

CONCLUSIONS

Von Karman's classical equations are in the coupled form and the transformations of these coupled equations in oblique coordinates will involve much mathematical complexity. So this entails difficulty in solution. Berger's equations, although decoupled are questionable. Thus, the present method seems to be more advantageous. The main advantages are:

(1) the differential equations are uncoupled and easy to solve;

(2) it gives accurate results both for movable and immovable edge conditions; and

(3) from a single cubic equation determining $\beta\,(=\omega_0/h)$ the results could be obtained for movable as well as immovable edge conditions.

Thus, to study non-linear behaviour of skew plates, the present method seems to be more acceptable.

REFERENCES

1. B. Banerjee, Large deflections of polygonal plates under non-stationary temperature. *J. Thermal Stresses* **7**, 285–292 (1984).
2. J. L. Nowinski, Large amplitude oscillations of oblique panels with initial curvature. *AIAA J.* **2**, 1025–1031 (1964).
3. M. Sathyamoorthy and K. A. V. Pandalai, Nonlinear flexural vibration of orthotropic skew plates. *J. Sound Vib.* **24**, 115–120 (1972).
4. M. Sathyamoorthy and K. A. V. Pandalai, Large amplitude flexural vibration of simply supported skew plates *AIAA J.* **2**(9), 1279–1282 (1973).
5. J. B. Kennedy and N. G. Simon, Linear and nonlinear analysis of skewed plates. *J. appl. Mech.* **34**, *Trans. ASME* 271–277 (1967).
6. R. S. Srinivasan. and S. V. Ramachandran, Nonlinear analysis of skew plates with variable thickness. *AIAA J.* **13**, 843–844 (1975).
7. J. E. Ashton, Clamped skew plates of orthotropic materials under transverse load. *Develop. Mech.* **5**, 297–306 (1969).
8. H. M. Berger, A new approach to the analysis of large deflection of plates. *J. appl. Mech.* **22**, *Trans. ASME* **77**, 465–472 (1955).
9. J. L. Nowinski and H. Ohunube, On certain inconsistencies in Berger equations for large deflections of elastic plates. *Int. J. Mech. Sci.* **14**, 165–170 (1972).
10. S. P. Timoshenko and S. Woinoswky-Krieger, *Theory of Plates and Shells.* McGraw-Hill, New York (1959).
11. L. S. D. Morley, *Skew Plates and Structures.* Pergamon Press, New York (1963).
12. J. L. Nowinski, Some static and dynamic problems concerning nonlinear behaviour of plates and shallow shells with discontinuous boundary conditions. Invited paper presented to the congress of scholars and scientists of Polish background, New York, November (1965).
13. S. Way, Bending of circular plates with large deflections. *Trans. ASME* **56**, 627–636 (1934).
14. W. A. Nash and I. D. Cooley, Large deflections of a clamped elliptical plate subjected to uniform pressure. *J. appl. Mech.* **26**, *Trans. ASME*, Series E **81**, 291–293 (1959).
15. C. Y. Chia, *Nonlinear Analysis of Plates.* McGraw-Hill, New York (1980).
16. B. Banerjee and S. Dutta, A new approach to the analysis of large deflections of thin plates. *Int. J. Non-Linear Mech.* **76**, 47–52 (1981).
17. W. A. Nash and J. Modeer, Certain approximate analysis of the non-linear behaviour of plates and shallow shells. *Proc. Symp. Theory of Thin Elastic Shells.* Interscience, New York (1959).
18. J. N. Reddy and W. C. Chao, Large deflection and large amplitude free vibrations of laminated compositive material plates. *Comput. Struct.* **13**, 341–347 (1981).

APPENDIX

Experimental arrangement

A sketch of the apparatus used for the experimental purpose is shown in Fig. 2. Two skew boxes with upper side open are constructed and each of the four side walls are made of steel. Each vertical wall of one box is 16 cm and of the other is 14 cm. The upper side of each wall is made sharp (knife edge), care being taken to see that all the knife edges lie on the same horizontal plane. The walls of the box with sides 16 cm long are welded in a manner that the

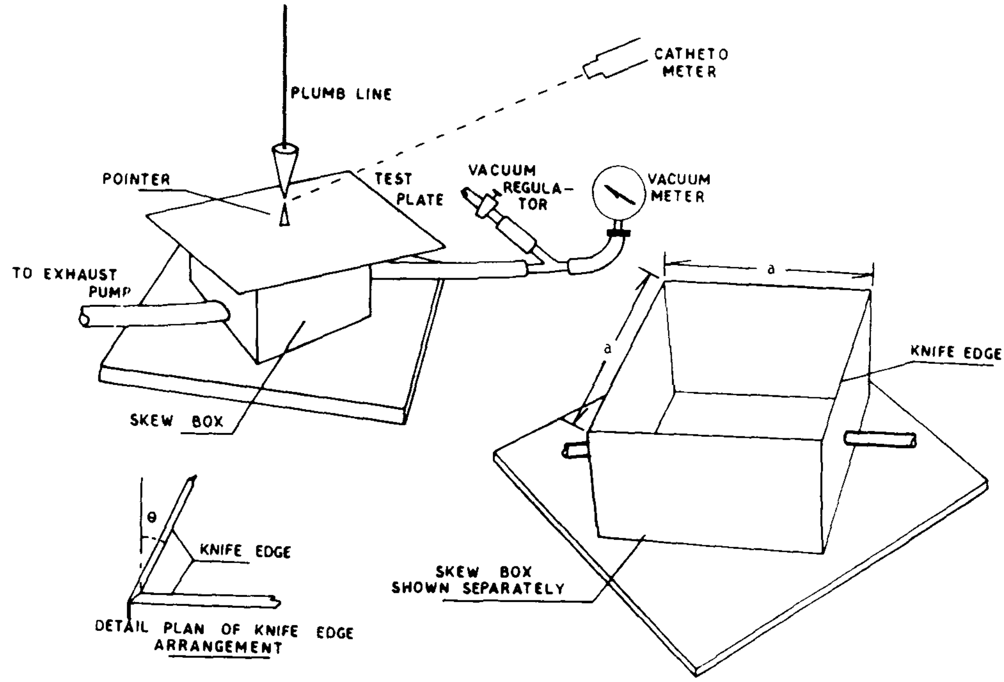

Fig. 2. The experimental arrangement.

two opposite angles are each 75° and the other two opposite angles are each 105°. Two opposite angles of the second box with sides 14 cm long are each 60° and the other two opposite angles are each 120°. Two holes are drilled on two opposite sides of each box and fitted with short metal pipes, one of which acts as an air inlet and the other as an air outlet.

For the experiment with the first skew box, the centre of the box is first found and then a plumb line is set as an indicator along the vertical line on which the centre of the box lies. For the free movable boundary conditions one test plate (which is approximately mirror surfaced) is symmetrically placed on the knife edges of the box and a pointer is fixed on the upper surface of the test plate with some adhesive along the plumb line. The outlet pipe is then joined to an exhaust pump by rubber tubing and the inlet pipe is joined to a standard vacuum meter and an air pressure regulator (as shown in the sketch). Along the contact line beneath the test plate some thick grease is used to make the box perfectly airtight. (Grease does not apply any appreciable tension on the plate.) When the exhaust pump operates, the box becomes evacuated, thereby causing the depression of the test plate by the excess outside air pressure, which is uniform all over the effective skew part of the test plate. The central deflection of the test plate is easily measured with the help of a precision cathetometer set at a distance of approximately 1.5 m from the pointer.

To make the free boundaries of a skew plate immovable, four pieces of steel collars are taken whose lengths are equal to the length of outer boundary line of the skew plate. The collars are kept outside the box in contact with the lower surface of the plate and with the side walls of the box and then the collars are tightly clamped with the test plate using nuts and bolts in sufficient number well outside the boundary of skew section.

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NON-LINEAR ANALYSIS OF HEATED RHOMBIC PLATES

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Abstract—This paper concerns a new approach to the investigation of non-linear behaviours of heated rhombic plates. A new set of differential equations in oblique co-ordinates have been derived in this investigation. Numerical results showing central deflection parameters versus thermal load functions have been computed for different skew angles θ. For $\theta = 0°$ the results obtained in the present study are in excellent agreement with the known results. It is believed that the results obtained for other different skew angles are completely new.

INTRODUCTION

Determination of thermal deflections in thin elastic plates, is of vital importance in cases where the thermal stresses play a significant role. Although thermal deflections of thin elastic plates have been investigated by many authors (Aleck, 1949; Zizicas, 1952; Schneider, 1955; Boley and Weiner, 1960; Forray and Newmann, 1960; Nowacki, 1962; Katayama *et al.*, 1967; Sarkar, 1968; Kaiuk and Pavlenko, 1971, 1972; Roychowdhury, 1972; Prabhu and Durvasula, 1974; Matumoto and Sekiya, 1975), the literature on the large thermal deflections is somewhat sparse. The most interesting papers in this field are Williams (1955, 1958) who quite elegantly carried out large deflection analysis for a plate strip subjected to normal pressure and heating. Biswas investigated the large deflection of heated circular plates under non-constant temperature (Biswas, 1974) and large deflections of heated elastic plates under uniform load (Biswas, 1975). The author followed Berger's equation in his investigations. Another interesting paper in this field is Banerjee and Dutta (1979), investigation of non-linear behaviours of heated elastic plates under non-constant temperatures. The authors utilized a conformal mapping technique along with Berger's hypothesis. Later on Banerjee proposed a new approach to the Large Deflection analysis of thin elastic plates (Banerjee and Dutt, 1981) and afterwards carried out quite elegantly the non-linear behaviours of polygonal plates under non-constant temperatures (Banerjee, 1984). Following Banerjee's approach, another interesting paper is by Sinharay and Banerjee (1985) on non-linear behaviours of heated spherical and cylindrical shells, where the authors have achieved satisfactory results from the practical point of view. Also, the works of Kamiya (1978) on the large thermal bending of sandwich plates are very attractive and useful too.

All the investigations mentioned above deal with plate geometry either circular or rectangular or in the shape of regular polygons. Only five papers (Katayama *et al.*, 1967; Kaiuk and Pavlenko, 1971, 1972; Prabhu and Durvasula, 1974; Matumoto and Sekiya, 1975) concerned with the study of thermal behaviours of skew plates are found in the literature. But these papers do not consider the large deflections of plates. To the authors' knowledge, no paper has been devoted to the investigations of non-linear behaviours of

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heated elastic skew plates having various applications in modern design, especially in the space industry.

In this paper non-linear behaviours of simply-supported heated skew plates (taken in rhombic form for simplicity of calculation) are investigated. Various numerical results have been calculated showing central deflection parameters versus thermal load functions. Whereas the results for skew angles other than 0° are believed to be new, the results for a 0°-skew angle are found to be in remarkable agreement with the already known results [see Biswas (1975)].

ANALYSIS

Let us consider a rhombic plate of skew angle θ whose uniform thickness is h and edge-length $2a$. The material of the plate is considered isotropic having mass density ρ, Young's modulus E and Poisson's ratio ν. The origin of the co-ordinates is located at the geometric centre of the plate. The deflections are consideed to be of the same order of magnitude as the plate thickness, the edge-length being sufficiently large compared to the thickness.

Now the uncoupled set of differential equations in rectangular Cartesian co-ordinates, governing the thermal behaviours of elastic plates [see Banerjee (1984)] is given by

$$\nabla^4 w - \frac{12A}{h^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) - \frac{\sigma\lambda}{h^2}\left[\nabla^2 w\left\{\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right\}\right.$$
$$\left. + 2\left\{\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2\right\} + 4\frac{\partial^2 w}{\partial x\,\partial y}\cdot\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right]$$
$$+ \frac{12\alpha\tau_0}{h^2}\sqrt{\lambda(1-\nu^2)}\cdot\nabla^2 w + (1+\nu)\alpha_t\nabla^2\tau = \frac{q}{D}, \quad (1)$$

where

$$A = \frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} + \frac{1}{2}\left\{\left(\frac{\partial w}{\partial x}\right)^2 + \nu\left(\frac{\partial w}{\partial y}\right)^2\right\} - (1+\nu)\alpha_t\tau_0, \quad (2)$$

$\lambda = \nu^2$ for simply-supported elastic plates, and $D = Eh^3/12(1-\nu^2)$, the flexural rigidity of the material of the elastic plate.

It is to be noted that in the derivation of eqns (1) and (2) in rectangular Cartesian co-ordinates, the expression

$$(1-\nu^2)\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right)^2\cdot\frac{1}{2(1+\nu)}$$

in the total P.E. of the elastic plate (Banerjee, 1984) has been replaced by

$$\frac{\lambda}{4}\left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]^2.$$

As a consequence the partial differential equations governing the deflection of the plate have become uncoupled and the two decoupled differential equations (1) and (2) have been obtained.

In the present problem, the temperature is assumed to vary linearly w.r.t. the thickness direction z. We also note that

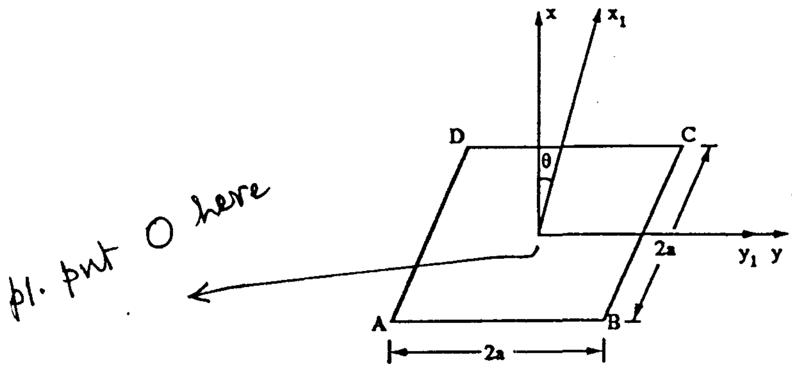

Fig. 1. Plan form of skew plate and co-ordinate system.

$$T(x,y,z) = \tau_0(x,y) + z\tau(x,y),$$

in which

$$\tau_0 = \tfrac{1}{2}(T_1+T_2), \quad \tau = \frac{1}{h}(T_1 - T_2),$$

$$T_1 = T\left(x, y, \frac{h}{2}\right) \quad \text{and} \quad T_2 = T\left(x, y, -\frac{h}{2}\right) \quad \text{(Banerjee, 1984).}$$

Clearly τ_0 is the temperature in the middle plane and τ varies along the thickness of the plate and hence $\tau \neq \tau_0$.

The plan of the skew co-ordinates (x_1, y_1, θ) is shown in Fig. 1. Clearly

$$x = x_1 \cos\theta$$

$$\text{and} \quad y = y_1 + x_1 \sin\theta \tag{3}$$

are the co-ordinate transformation equations. Hence we have the following partial differential operators in oblique co-ordinates:

$$\frac{\partial}{\partial x} \equiv \sec\theta\left(\frac{\partial}{\partial x_1} - \sin\theta\,\frac{\partial}{\partial y_1}\right), \quad \frac{\partial}{\partial y} \equiv \frac{\partial}{\partial y_1},$$

$$\frac{\partial^2}{\partial x^2} \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\,\frac{\partial^2}{\partial x_1\,\partial y_1} + \sin^2\theta\,\frac{\partial^2}{\partial y_1^2}\right),$$

$$\frac{\partial^2}{\partial y^2} \equiv \frac{\partial^2}{\partial y_1^2}, \quad \frac{\partial^2}{\partial x\,\partial y} \equiv \sec\theta\left(\frac{\partial^2}{\partial x_1\,\partial y_1} - \sin\theta\,\frac{\partial^2}{\partial y_1^2}\right),$$

$$\nabla^2 \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\,\frac{\partial^2}{\partial x_1\,\partial y_1} + \frac{\partial^2}{\partial y_1^2}\right)$$

and

$$\nabla^4 \equiv \sec^4\theta\left\{\frac{\partial^4}{\partial x_1^4} - 4\sin\theta\left(\frac{\partial^4}{\partial x_1^3\,\partial y_1} + \frac{\partial^4}{\partial x_1\,\partial y_1^3}\right) + 2(1+2\sin^2\theta)\,\frac{\partial^4}{\partial x_1^2\,\partial y_1^2} + \frac{\partial^4}{\partial y_1^4}\right\}. \tag{4}$$

We now transform eqn (2) in oblique co-ordinates. For simply-supported plates the boundary conditions are

$$w = 0 \quad \text{at} \quad x_1 = \pm a \quad \text{and at} \quad \dot{y}_1 = \pm a,$$

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$$\frac{\partial^2 w}{\partial x_1^2} = 0 \quad \text{at} \quad x_1 = \pm a \quad \text{and} \quad \frac{\partial^2 w}{\partial y_1^2} = 0 \quad \text{at} \quad y_1 = \pm a.$$

Then let us choose the deflection function for the simply-supported plate as

$$w = w_0 \cos\frac{\pi x_1}{2a}\cos\frac{\pi y_1}{2a}, \tag{5}$$

which clearly satisfies the above-mentioned boundary conditions.

Now putting (5) in eqn (2) transformed in oblique co-ordinates and then integrating the relation thus obtained, over the entire surface of the plate, we obtain the value of A in the following form:

$$A = \frac{\pi^2 w_0^2}{32a^2}(1+\nu+2\tan^2\theta)-(1+\nu)\alpha_t\tau_0. \tag{6}$$

(As the normal displacement w is our primary interest, the in-plane displacements u, v have been eliminated through integration by the choice of appropriate functions for such displacements.) Again transforming eqn (1) in oblique co-ordinates, introducing eqns (5) and (6) in the transformed equation and then applying Galerkin's error minimizing technique we get the following equation determining the central deflection parameter w_0/h depending on the thermal load function $q'a^4/Eh^4$:

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$$\left[(1+2\tan^2\theta)\sec^2\theta-\frac{6S}{(1+\nu)\pi^2}\{2\sqrt{\lambda(1-\nu^2)}\cdot\sec^2\theta + (1+\nu)(1+\nu+2\tan^2\theta)\}\right]\left(\frac{w_0}{h}\right)+\tfrac{3}{8}[(1+\nu+2\tan^2\theta)^2 + \frac{\lambda}{4}(8+49\tan^2\theta+29\tan^4\theta)]\left(\frac{w_0}{h}\right)^3 = \frac{768(1-\nu^2)}{\pi^6}\left(\frac{q'a^4}{Eh^4}\right), \tag{7}$$

where

$$S = 2\left(\frac{a}{h}\right)^2(1+\nu)\alpha_t\tau_0$$

and

$$q' = q - D\alpha_t(1+\nu)\nabla^2\tau.$$

Equation (7) is applicable for the immovable edge condition of the simply-supported skew plate. For the movable edge condition we have $A = 0$, so that eqn (7) takes the form:

$$\left[(1+2\tan^2\theta)\sec^2\theta-\frac{12S}{(1+\nu)\pi^2}\sqrt{\lambda(1-\nu^2)}\cdot\sec^2\theta\right]\left(\frac{w_0}{h}\right) + \frac{3\lambda}{32}(8+49\tan^2\theta+29\tan^4\theta)]\left(\frac{w_0}{h}\right)^3 = \frac{768(1-\nu^2)}{\pi^6}\left(\frac{q'a^4}{Eh^4}\right). \tag{8}$$

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Table 1. $S = 0$, i.e. $\tau_0 = 0$

	w_0/h by present method						w_0/h by Berger's method†		
	$\theta = 0^\circ$		$\theta = 15^\circ$		$\theta = 30^\circ$		(Biswas, 1975) $\theta = 0^\circ$	$\theta = 15^\circ$	$\theta = 30^\circ$
$\frac{q'a^4}{Eh^4}$	Movable edge	Immovable edge	Movable edge	Immovable edge	Movable edge	Immovable edge	Immovable edge	Immovable edge	Immovable edge
2	1.30156	0.91435	1.08167	0.82069	0.6269	0.53604	0.9013	0.79972	0.53671
4	2.1909	1.3131	1.85443	1.20857	1.14734	0.84631	1.29017	1.16888	0.848
8	3.23354	1.78866	2.8581	1.67119	1.89675	1.22355	1.75406	1.60902	1.2266
10	3.73498	1.9613	3.2243	1.83866	2.17977	1.3597	1.92254	1.76847	1.36324

† Berger's method has been applied to the present problem by neglecting e_2, the second strain invariant in the expression for total P.E. of the plate.

NUMERICAL RESULTS

Numerical results are presented here (Tables 1 and 2) in the tabular forms for $S = 0$, 0.1; $\theta = 0^\circ, 15^\circ, 30^\circ$ and $q'a^4/Eh^4 = 2, 4, 8, 10$.

Pl. put the Tables 1 and 2 here in succession

OBSERVATIONS AND CONCLUSIONS

From the numerical analysis of the undertaken problem the following observations are made:

(i) The nature of the central deflection of a skew plate under thermal loading is the same as that of the plate under mechanical loading, i.e. the central deflection increases continuously with the increase of loading for any edge condition of the skew plate, whether movable or immovable.

(ii) The central deflection for the movable edge condition of the skew plate is always greater than that for the immovable edge condition of the plate, for the same loading.

(iii) Irrespective of the edge condition, the central deflection decreases with the increase in the skew angle.

(iv) The results for immovable edge conditions of the skew plate obtained by the present method, agree well with the results obtained by Berger's method. It is to be noted that Berger's method is a purely approximate method based on the neglect of e_2. But the present study is based on Banerjee's hypothesis which suggests a modified strain–energy expression, and thus this model embraces less approximation (Banerjee and Dutt, 1981) than that of Berger. Again Berger's method is meaningful only for immovable edge conditions of the plates.

(v) The deflections increase with τ_0.

The present method seems to be more advantageous than any other method found in open literature. The main advantages are:

Table 2. $S = 0.1$, i.e. $\tau_0 \neq 0$

	w_0/h by present method						w_0/h by Berger's method ($e_2 = 0$)		
	$\theta = 0^\circ$		$\theta = 15^\circ$		$\theta = 30^\circ$		(Biswas, 1975) $\theta = 0^\circ$	$\theta = 15^\circ$	$\theta = 30^\circ$
$\frac{q'a^4}{Eh^4}$	Movable edge	Immovable edge	Movable edge	Immovable edge	Movable edge	Immovable edge	Immovable edge	Immovable edge	Immovable edge
2	1.32786	0.94985	1.10168	0.83899	0.63597	0.55925	0.94058	0.83515	0.56109
4	2.22082	1.34324	1.87831	1.20992	1.1604	0.86901	1.32336	1.19954	0.87185
8	3.35106	1.81316	2.88067	1.65221	1.9111	1.24302	1.781	1.63412	1.24706
10	3.76118	1.98415	3.24585	1.81269	2.19385	1.37799	1.94764	1.79188	1.38247

(1) The differential equations are decoupled and easy to solve;
(2) from a single cubic equation determining w_0/h, the results could be obtained for movable as well as immovable edge conditions; and
(3) unlike Berger's method it gives accurate results both for movable and immovable edge conditions. Based on Banerjee's hypothesis a good number of works have been carried out and in each case sufficiently accurate results have been obtained [e.g. Banerjee and Dutt (1981), Banerjee (1984), Sinharay and Banerjee (1985) and Ray *et al.* (1992, 1993)]. So in the present case also, the same degree of accuracy was expected.

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REFERENCES

Alec, B. J. (1949). Thermal stresses in a rectangular plate clamped along an edge. *J. Appl. Mech., Trans ASME* **71**, ●–●.
Banerjee, B. (1984). Large deflections of polygonal plates under nonstationary temperature. *J. Thermal Stresses (U.S.A.)* **7**, 285–292.
Banerjee, B. and Dutt, S. (1979). Large deflections of elastic plates under non-stationary temperature. *J. ASCE* **4**, 705.
Banerjee, B. and Dutt, S. (1981). A new approach to an analysis of large deflections of thin elastic plates. *Int. J. Non-Lin. Mech.* **16**, 47–52.
Biswas, P. (1974). Large deflection of heated circular plate under non-stationary temperature. *Bull. Cal. Math. Soc.* **66**, 247–252.
Biswas, P. (1975). Large deflections of heated elastic plates under uniform load. *Mechanique Applique* **20**(4).
Boley, B. A. and Weiner, J. H. (1960). *Theory of Thermal Stresses* (2nd Edn). Wiley, New York.
Forray, M. and Newmann, M. (1960). Axisymmetric bending stresses in solid circular plates with thermal gradient. *J. Aerospace Sci.* **27**(9), ●–●.
Kaiuk, Ia. F. and Pavlenko, V. I. (1971). Thermal buckling of parallelogram shaped plates (in Russian). *Teplovye Napriazheniia v Elementakh Konstruktsii* **11**, 173.
Kaiuk, Ia. F. and Pavlenko, V. I. (1972). Thermal stability of skewed plates (in Russian). *Voprosy Dinamiki i Prochnosti* **22**, 159.
Kamiya, N. (1978). Analysis of the large thermal bending of sandwich plates by a modified Berger method. *J. Strain Anal.* **13**(1), 17–22.
Katayama, T., Matumoto, E. and Sekiya, T. (1967). Fundamental equations for thermoelastic deformation of skew plates. *Bulletin of the University of OSAKA Prefecture, Series A* **16**, ●–●.
Matumoto, E. and Sekiya, T. (1975). Elastic stability of thermally stressed parallelogram panels. *Trans. JSME* **41**(343), 736–745.
Nowacki, W. (1962). *Thermoelasticity*. Pergamon Press, Oxford.
Prabhu, M. S. S. and Durvasula, S. (1974). Elastic stability of thermally stressed clamped–clamped skew plates. *J. Appl. Mech., Trans ASME* **41**(3), 820–821.
Ray, A. K., Banerjee, B. and Bhattacharjee, B. (1992). Large deflections of rhombic plates—a new approach. *Int. J. Non-Lin. Mech.* **27**(6), 1007–1014.
Ray, A. K., Banerjee, B. and Bhattacharjee, B. (1993). *Meccanica*. (accepted paper). Au update
Roychowdhury, S. K. (1972). Some problems on thermoelasticity. Ph.D. Thesis presented to the Jadavpur University, India.
Sarkar, S. R. (1968). Quasi-static thermal deflections in a solid circular plate in the axisymmetric case. *Aplikace Mathematiky, Czechoslovakia* **13**, ●–●.
Schneider, P. J. (1955). Variation of maximum thermal stress in free plates. *J. Aerospace Sci.* **22**, ●–●.
Sinharay, G. C. and Banerjee, B. (1985). A new approach to large deflection analysis of spherical and cylindrical shells under thermal loading. *Mech. Res. Comm. (U.S.A.)* **12**(2), 53–64.
Williams, M. L. (1955). Large deflection analysis for a plate strip subjected to normal pressure and heat. *J. Appl. Mech., Trans ASME* **22**(4), ●–●.
Williams, M. L. (1958). Further large deflection analysis for a plate strip subjected to normal pressure and heating. *J. Appl. Mech., Trans ASME* **25**(2), ●–●.
Zizicas, G. A. (1952). Transient thermal stresses in thin isotropic plates. *ICLA Engng Report* **52**(7), ●–●.

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BRIEF NOTE

Nonlinear Analysis of Skewed Sandwich Plates

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Abstract. Nonlinear static and dynamic behaviours of freely supported Rhombic sandwich plates have been studied following Banerjee's hypothesis. Numerical results for 0° skew angle are compared with other known results. Results for other skew angles are believed to be new.

Sommario. Si studia, seguendo l'ipotesi di Banerjee, il comportamento nonlineare statico e dinamico di [illegible] rombiche semplicemente appoggiate. Si presentano risultati numerici relativi a piastre rombiche e retta[illegible] questi ultimi vengono paragonati a risultati giá noti, mentre i primi si ritengono nuovi.

Key words: Skew plates; sandwich plates; nonlinear analysis.

1. Introduction

Sandwich plates find wide applications in technology and modern design. Outstanding investigations [1–8] on large deflections as well as large amplitude vibrations of such plates are limited to the rectangular form only. No attempt on the nonlinear behaviours of skewed sandwich plates has been reported as yet.

In this paper an attempt has been made to analyse the nonlinear behaviours of freely supported skewed sandwich plates having an isotropic core within isotropic upper and lower faces and under both static and dynamic loadings. For the sake of simplicity, a skewed plate in the form of a rhombus has been considered. Following the modified strain energy expression proposed by B. Banerjee, [8] a new set of decoupled differential equations for skewed sandwich plates has been derived. The final equations have been solved by Galerkin's method. Numerical results are computed and those for the 0° skew angle are compared with the other known results. The results for other skew angles are completely new.

2. Governing Equations

Let each side of a rhombic sandwich plate be a, having an isotropic core of thickness [illegible] isotropic upper and lower faces of identical thickness t_1 where $t_1 \ll h$.

Now let us set one rectangular cartesian coordinate system (x, y, z) and one o[illegible] coordinate system (x_1, y_1, θ) at the same corner of the plate, x, y being in the middle [illegible] of the core; Z the thickness direction; x_1, y_1 are parallel to the sides of the plate and θ, the skew angle (see Figure 1). Clearly, the coordinate transformation equations are

$$x = x_1 \cos\theta \text{ and } y = y_1 + x_1 \sin\theta \qquad (1)$$

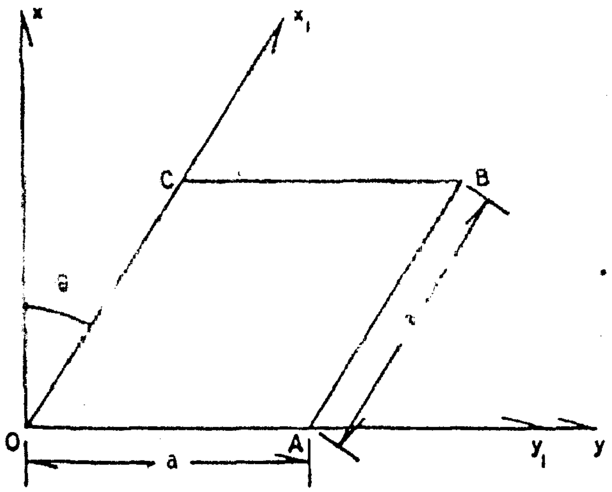

Fig. 1. Plan form of skew plate.

Following Banerjee's hypothesis, [8] the differential equations in rectangular coordinate system governing the deflections and vibrations of sandwich plates are

$$\left[\frac{Et_1}{2(1-\nu^2)}\nabla^2 - \frac{G'}{h}\right]\left[\frac{2Et_1}{(1-\nu^2)G'}I_1^m\left(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\right)\right.$$
$$+h\nabla^2 W + \frac{Et_1\lambda}{(1-\nu^2)G'}\left|\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right\}\nabla^2 W\right.$$
$$\left.\left.+2\left\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2 + 2\frac{\partial^2 W}{\partial x \partial y}\cdot\frac{\partial W}{\partial x}\cdot\frac{\partial W}{\partial y}\right\}\right| + \xi\right]$$
$$+G'\nabla^4 W = 0 \tag{1}$$

where $\xi = q/G'$ for nonlinear static deflections,

$$= -\frac{(\rho_1 t_1 + \rho_2 h)}{G'}\frac{\partial^2 W}{\partial t^2} \text{ for nonlinear elastic vibrations,} \tag{2}$$

and $I_1^m = \frac{1}{2}\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \nu\left(\frac{\partial W}{\partial y}\right)^2\right\} + \frac{\partial P}{\partial x} + \nu\frac{\partial Q}{\partial y}$

$=$ constant, for nonlinear static deflections (4a)

$= Cf(t)$ for nonlinear elastic vibrations,

C being a constant depending on θ . (3a)

In the above equations W is the transverse deflection function; q, the lateral load distribution function; P, Q are the in-plane displacements along x and y axes respectively; E, the Young's Modulus of elasticity of the material of the upper and lower faces; G', the shear modulus of the core material; ν , the poisson's ratio of the face material, $\lambda = \nu^2$; ρ_1, ρ_2 are the surface density and core density respectively, and $f(t)$, $F(t)$ are the functions of time such that $f(t) = F'(t)$.

It is to be noted that the strain-energy expression in ref. [8] has been modified by ... Banerjee's hypothesis, which states that the stretching of the plate is proportional to

$$\left[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right]^2.$$

As a consequence of this assumption, a set of uncoupled differential equations has ... obtained as given above.

3. Analysis

3.1. Non-linear static behaviour of freely supported skewed sandwich plates

To find normal displacement W, the inplane displacements of the upper and lower faces are being eliminated through integration by choosing suitable expressions for them in the form of trigonometric functions compatible with the boundary conditions of the plate [8]. Then transforming equation (3a) in oblique coordinates, choosing

$$W = \bar{W} \sin \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a}$$

and

$$q = \bar{q} \sin \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a}$$

and then integrating the transformed equation over the whole domain of the plate we get

$$I_1^m = \frac{\pi^2 \bar{W}^2}{8a^2}(1 + \nu + 2 \tan^2 \theta).$$

Again transforming equation (2) with $\xi = q/G'$, in oblique coordinates, introducing equations (4a), (4b) and (4c) in the transformed equation and then applying Galerkin procedure, we arrive at the following cubic equation determining $\bar{W}$, the central deflection of a freely supported rhombic sandwich plate

$$\frac{\pi^4 t_1}{4(1-\nu^2)} \left[\frac{\pi^2 E t_1 \mathrm{Sec}^2\theta}{(1-\nu^2)G'a^2}\{(1+\nu+2\tan^2\theta)(1+\nu+4\tan^2\theta) + \lambda(5+17\tan^2\theta+12\tan^4\theta)\} + \frac{1}{h}\{(1+\nu+2\tan^2\theta)^2 + \lambda(5+11\tan^2\theta+6\tan^4\theta)\}\right]\left(\frac{\bar{W}}{h}\right)^3 + \left[\frac{2\pi^4 t_1 \mathrm{Sec}^2\theta}{(1-\nu^2)h}(1+2\tan^2\theta)\right]\left(\frac{\bar{W}}{h}\right) = \frac{\bar{q}a^4}{Eh^4}\left[1 + \frac{\pi^2 E h t_1 \mathrm{Sec}^2\theta}{(1-\nu^2)G'a^2}\right].$$

4. Numerical Results

Table 1 shows different numerical results of the central deflections of a (0.254 m × 0.254 m) rhombic plate having $t_1 = 6.35 \times 10^{-4}\ m$, $h = 1.7135 \times 10^{-2}\ m$. [5, 8].

Table 1. Showing $\bar{W}/h$ vs θ. $E = 16.2 \times 10^9$ psm, $G' = 9.3 \times 10^6$ psm, $\nu = 0.3$, $\frac{q a^4}{E h^4} = 10$ [5,8]

Value of θ	Immovable edge: Calculated value	Immovable edge: Other known value [8]	Immovable edge: Other known value [6]	Movable edge: Calculated value	Movable edge: Other known value[8]
0°	1.4988	1.53	1.30	2.3223	2.588
15°	1.3644	—	—	2.1563	—
30°	1.0328	—	—	1.6360	—
45°	0.6651	—	—	1.0414	—

Note: For movable edge condition of the freely supported plate $I_1^m = 0$.

4.1. NONLINEAR DYNAMIC BEHAVIOURS OF FREELY SUPPORTED SKEWED SANDWICH PLATES

Let us now consider free vibrations of skewed sandwich plates. Neglecting in-plane inertia for obvious reasons, transforming equation (3b) in oblique coordinates, choosing

$$W = \bar{W} \sin \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a} F(t) \tag{6a}$$

for fundamental mode of vibration and then integrating the transformed equation over the whole domain of the plate we get

$$I_1^m = \frac{\pi^2 \bar{W}^2}{8a^2}(1 + \nu + 2 \tan^2 \theta)F^2(t). \tag{6b}$$

Now transforming equation (2) with $\xi = -\frac{(\rho_1 t_1 + \rho_2 h)}{g} \frac{\partial^2 W}{\partial t^2}$ in oblique coordinates, inserting equations (6a) and (6b) in the transformed equation and then applying Galerkin's procedure we get the following equation for time function

$$\begin{aligned}&\left[\frac{\pi^2(\rho_1 t_1 + \rho_2 h)E t_1}{(1-\nu^2)gG' a^2} \operatorname{Sec}^2 \theta - \frac{(\rho_1 t_1 + \rho_2 h)}{h}\right] \ddot{F} \\ &+ \left[\frac{2\pi^4 E t_1 h}{(1-\nu^2)a^4}(1 + 2\tan^2 \theta) \operatorname{Sec}^2 \theta\right] F \\ &+ \left[\frac{\pi^4 E t_1 \bar{W}^2}{4(1-\nu^2)a^4}\left\{\frac{\pi^2 E t_1 \operatorname{Sec}^2 \theta}{(1-\nu^2)G' a^2}\left|(1+\nu+2\tan^2\theta)(1+\nu+4\tan^2\theta)\right.\right.\right. \\ &\left.+\lambda(5 + 17\tan^2\theta + 12\tan^4\theta)\right| + \frac{1}{h}\left|(1+\nu+2\tan^2\theta)^2\right. \\ &\left.\left.\left.+\lambda(5 + 11\tan^2\theta + 6\tan^4\theta)\right|\right\}\right] F^3 = 0,\end{aligned} \tag{7}$$

which may be turned in the form $\ddot{F} + AF + BF^3 = 0$, the familiar Duffing's equation. With the initial conditions $F(0) = 1$ and $\dot{F}(0) = 0$, the solution of equation (7) ... known

Table 2. Showing ω_1^*/ω_1 vs θ

Value of θ	Value of $\frac{W}{2h_1}$	Value of ω_1^*/ω_1				
		Immovable edge			Movable edge	
		Calculated value	Other known value [8]	[7]	Calculated value	Other known value [8]
0°	—	1.15028	1.12	1.14	1.03342	1.024
15°	—	1.16621	—	—	1.03365	—
30°	0.5	1.22803	—	—	1.04313	—
45°	—	1.36556	—	—	1.06261	—
0°	—	1.51413	1.42	1.48	1.12774	1.094
15°	—	1.56211	—	—	1.1286	—
30°	1.0	1.74133	—	—	1.1630	—
45°	—	2.11166	—	—	1.2315	—

elliptic integral $F(t) = C_n(\omega_1^*, t, k)$. Then the ratio of nonlinear frequency ω_1^* to the linear frequency ω_1 is given by

$$\frac{\omega_1^*}{\omega_1} = \left[1 + \frac{h\cos^2\theta}{8(1+2\tan^2\theta)}\left(\frac{W}{2h_1}\right)^2\left(1+\frac{2t_1}{h}\right)^2\left\{\frac{\pi^2 E t_1 \operatorname{Sec}^2\theta}{(1-\nu^2)G'a^2}\right|\right.$$
$$(1+\nu+2\tan^2\theta)(1+\nu+4\tan^2\theta) + \lambda(5+17\tan^2\theta+12\tan^4\theta)\Big|$$
$$\left.\left.+\frac{1}{h}\left|(1+\nu+2\tan^2\theta)^2 + \lambda(5+11\tan^2\theta+6\tan^4\theta)\right|\right\}\right]^{\frac{1}{2}}, \qquad (8)$$

where $h_1 = t_1 + \frac{h}{2}$, $\omega_1 = \sqrt{A}$ and $\omega_1^* = \sqrt{A+B}$.

5. Numerical Results

Numerical results of the ratio ω_1^*/ω_1 are shown in Table 2. For calculations, the same data which are used in the study of static behaviours of sandwich plates are used here also.

6. Observations

From the calculated results, the following observations are easily made.

1. The results of both static and dynamic behaviours of a sandwich plate having skew angle $\theta = 0°$ and aspect ratio 1 are in excellent agreement with those obtained by [illegible] and Banerjee [8].
2. It is seen that the central deflection gradually decreases with the increase in skew angle for both movable as well as immovable edge conditions.
3. For any assumed skew angle the central deflection is greater for the movable edge condition than for the immovable edge condition. This is quite expected from a practical point of view.

4. In the dynamic case, the frequency ratio ω_1^*/ω_1 increases continuously with t[illegible] [illegible]w angle θ, for both movable as well as immovable edge conditions of a skewed [illegible] [illegible]he ratio for immovable edge condition being always greater than that for the mova[illegible] [illegible]ge condition.

7. Conclusions

1. Greater deflections, obtained in the present theoretical study in comparison to th[illegible] [illegible]ec-tions obtainable from the other theories in open literature, indicate acceptabil[illegible] [illegible] the present method for practical purposes.
2. It is advantageous from the point of view that following this method results, [illegible] [illegible]oth immovable as well as movable edge conditions of the plate, can be derived from [illegible] [illegible]me cubic equation.
3. The governing differential equations, being de-coupled, are simple and easy to s[illegible] [illegible]but, are able to yield results with considerable accuracy.
4. The great advantage of the present method lies in the fact that the accuracy of th[illegible] [illegible]od does not depend on any correction factor and thus holds good for sandwich [illegible] of different geometry.

References

1. Reissner, E., 'Finite deflections of sandwich plates', *Journal of Aeronautical Sciences*, **15** (1948) [illegible]
2. Wang, C.T.' Principle and application of complementary energy method for thin homogeneous sand[illegible] [illegible]ates and shells with finite deflections', *NACA TN*, **2620** (1952) 1–33.
3. Alwan, A.M., 'Large deflection of sandwich plates with orthotropic cores', *AIAA Journal*, **2** (1964) [illegible] 32[illegible]
4. Wempner, G.A. and Baylor, J.L., 'General theory of sandwich plates with dissimilar facings', *Int* [illegible] *[illegible]olids and Structures*, **1** (1965) 157–177.
5. Nowinski, J.L. and Ohnabe, H., 'Fundamental equations for large deflections of sandwich plates wi[illegible] [illegible]trop-ic core and faces', *Proc. 10th. Int. Symp. on Space Technology and Sciences, Tokyo*, (1973), 311 [illegible]
6. Kamiya, N. 'Governing equations for large deflections of sandwich plates', *AIAA J.*, **14**(2) (1976 [illegible] 5[illegible]
7. Karmakar, B.M., 'Amplitude-frequency characteristics of large amplitude vibrations of sandwi[illegible] [illegible] *Appl. Mech.*, (1978).
8. Dutta, S. and Banerjee, B. 'Governing equations for non-linear analysis of sandwich plates, a ne[illegible] [illegible] *Int. J. Non-linear Mech.*, **2[illegible]**(34) 313–318