REPRINTS and LETTERS

Int. J. Non-Linear Mechanics, Vol. 27, No. 6, pp. 1007–1014, 1992
Printed in Great Britain.

0020-7462/92 $5.00 + .00

LARGE DEFLECTIONS OF RHOMBIC PLATES—A NEW APPROACH

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(*Received* 6 *June* 1991; *in revised form* 16 *January* 1992)

Abstract—Non-linear static behaviour of rhombic plates has been analysed following Banerjee's hypothesis (B. Banerjee, Large deflections of polygonal plates under non-stationary temperature. *J. Thermal Stresses* 7, 285–292 (1984)). Calculations have been carried out for different skew angles. To test the accuracy of the theoretical results so obtained, experiments were carried out on copper and steel rhombic plates. The theoretical results were found to be in excellent agreement with those obtained from an analysis of the experimental data.

INTRODUCTION

Skew or oblique panels find wide applications in the aircraft and spaceship industry; hence, a study of the non-linear behaviour of skew plates is of great importance. In contrast to the non-linear behaviour analysis of elastic plates of geometries like circular, rectangular, triangular and elliptic, skew plates have not received much attention. This may be due to their relatively difficult mathematical models.

The most important work in this field is due to Nowinski [2], who analysed the large-amplitude oscillations of oblique panels having initial curvature. Two more interesting papers on non-linear vibration problems of skew plates are by Sathyamoorthy and Pandalai [3, 4]. They have analysed the non-linear flexural vibrations of simply supported skew plates of isotropic as well as anisotropic materials, using Berger's equation. In contrast to works on non-linear vibration problems of skew plates, the literature on non-linear deflection problems of skew plates is scanty. In this field three interesting papers could be located. Kennedy and Simon [5] carried out non-linear analysis of skew plates by the perturbation method. Srinivasan and Ramachandran [6] analysed the large deflections of skew plates of variable thickness using the Newton–Raphson procedure. Ashton's [7] work is on the linear static analysis of anisotropic skew plates. It is interesting to note that most of these investigations are carried out on skew plates of clamped edges only and the case of simply supported edges has not received proper attention.

In this paper large deflections of simply supported rhombic plates are studied following Banerjee's approach. A set of uncoupled differential equations has been obtained in oblique coordinates and solved by applying the Galerkin technique. The case of a simply supported rhombic plate is discussed in detail. To test the accuracy of the method, experiments were carried out on copper and steel rhombic plates. The details of the experiments are given in the Appendix. The numerical results obtained from the theoretical and the experimental analysis are compared. The present method appears to be more acceptable from the practical point of view.

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Contributed by J. N. Reddy.

ANALYSIS

Consider a rhombic plate of an elastic, isotropic material, having uniform thickness of h. Let the size of each side of the skew plate be a which is sufficiently large compared to h. The origin of the rectangular Cartesian coordinate (x, y) is located at one of the corners of the skew plate (see Fig. 1). The plate is considered to be simply supported along its edges and loaded uniformly all over.

Following Banerjee's hypothesis [1], the differential equations, referred to the system of rectangular Cartesian coordinates are:

$$\nabla^4\omega - \frac{12A}{h^2}\left(\frac{\partial^2\omega}{\partial x^2} + \nu\frac{\partial^2\omega}{\partial y^2}\right) - \frac{6\lambda}{h^2}\left\{\nabla^2\omega\left[\left(\frac{\partial\omega}{\partial x}\right)^2 + \left(\frac{\partial\omega}{\partial y}\right)^2\right]\right.$$
$$\left. + 2\left[\frac{\partial^2\omega}{\partial x^2}\left(\frac{\partial\omega}{\partial x}\right)^2 + \frac{\partial^2\omega}{\partial y^2}\left(\frac{\partial\omega}{\partial y}\right)^2\right] + 4\left(\frac{\partial^2\omega}{\partial x\partial y}\right)\left(\frac{\partial\omega}{\partial x}\right)\left(\frac{\partial\omega}{\partial y}\right)\right\} = \frac{q}{D} \tag{1}$$

where

ω = the deflection normal to the middle plane of the plate
ν = Poisson's ratio of the material of the plate
$\lambda = \nu^2$
q = load per unit area acting on the plate
D = the flexural rigidity of the plate = $Eh^3/12(1-\nu^2)$
E = the modulus of elasticity of the material of the plate

$$A = \frac{1}{2}\left\{\left(\frac{\partial\omega}{\partial x}\right)^2 + \nu\left(\frac{\partial\omega}{\partial y}\right)^2\right\} + \frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} \tag{2}$$

which is a constant depending on the surface and edge conditions of the plate, and ∇^2 is the Laplacian operator.

For a skew plate, let us adopt a system of oblique coordinates (x_1, y_1, θ), as shown in Fig. 1, θ being the skew angle.

Clearly,

$$x = x_1\cos\theta, \qquad y = y_1 + x_1\sin\theta \tag{3}$$

are the coordinate transformation equations. Hence the partial differential operators become

$$\frac{\partial}{\partial x} \equiv \sec\theta\left(\frac{\partial}{\partial x_1} - \sin\theta\frac{\partial}{\partial y_1}\right), \qquad \frac{\partial}{\partial y} \equiv \frac{\partial}{\partial y_1}$$

$$\frac{\partial^2}{\partial x^2} \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\frac{\partial^2}{\partial x_1\partial y_1}\right) + \tan^2\theta\frac{\partial^2}{\partial y_1^2}$$

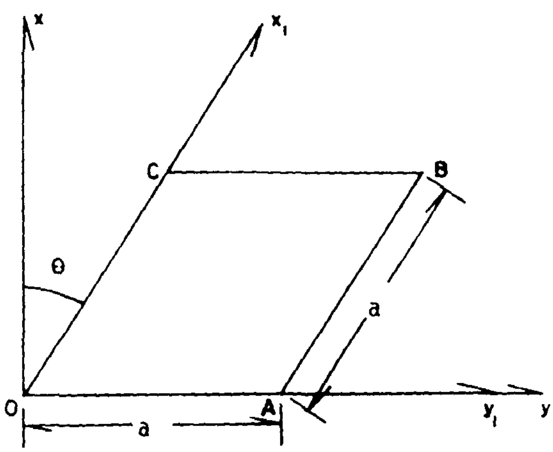

Fig. 1. Plan form of skew plate.

$$\frac{\partial^2}{\partial y^2} \equiv \frac{\partial^2}{\partial y_1^2}, \qquad \frac{\partial^2}{\partial x \partial y} \equiv \sec\theta\left(\frac{\partial^2}{\partial x_1 \partial y_1} - \sin\theta \frac{\partial^2}{\partial y_1^2}\right)$$

$$\nabla^2 \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta \frac{\partial^2}{\partial x_1 \partial y_1} + \frac{\partial^2}{\partial y_1^2}\right)$$

and

$$\nabla^4 \equiv \sec^4\theta\left[\frac{\partial^4}{\partial x_1^4} - 4\sin\theta\left(\frac{\partial^4}{\partial x_1^3 \partial y_1} + \frac{\partial^4}{\partial x_1 \partial y_1^3}\right) + 2(1 + 2\sin^2\theta)\frac{\partial^4}{\partial x_1^2 \partial y_1^2} + \frac{\partial^4}{\partial y_1^4}\right]. \tag{3a}$$

Using these operators, transforming the differential equations (1) and (2) in oblique coordinates, we arrive at the following set of transformed differential equations:

$$\begin{aligned}
&\sec^4\theta\left[\frac{\partial^4\omega}{\partial x_1^4} - 4\sin\theta\left(\frac{\partial^4\omega}{\partial x_1^3 \partial y_1} + \frac{\partial^4\omega}{\partial x_1 \partial y_1^3}\right) + 2(1 + 2\sin^2\theta)\frac{\partial^4\omega}{\partial x_1^2 \partial y_1^2} + \frac{\partial^4\omega}{\partial y_1^4}\right] \\
&- \frac{12A}{h^2}\left[\sec^2\theta\left(\frac{\partial^2\omega}{\partial x_1^2} - 2\sin\theta\frac{\partial^2\omega}{\partial x_1 \partial y_1}\right) + \tan^2\theta\frac{\partial^2\omega}{\partial y_1^2} + \nu\frac{\partial^2\omega}{\partial y_1^2}\right] \\
&- \frac{6\lambda}{h^2}\left\{\sec^4\theta\left(\frac{\partial^2\omega}{\partial x_1^2} - 2\sin\theta\frac{\partial^2\omega}{\partial x_1 \partial y_1} + \frac{\partial^2\omega}{\partial y_1^2}\right)\left[\left(\frac{\partial\omega}{\partial x_1}\right)^2 + \left(\frac{\partial\omega}{\partial y_1}\right)^2\right.\right. \\
&\left. - 2\sin\theta\left(\frac{\partial\omega}{\partial x_1}\right)\left(\frac{\partial\omega}{\partial y_1}\right)\right] + 2\left[\sec^4\theta\left(\frac{\partial^2\omega}{\partial x_1^2} - 2\sin\theta\frac{\partial^2\omega}{\partial x_1 \partial y_1} + \sin^2\theta\frac{\partial^2\omega}{\partial y_1^2}\right)\right. \\
&\times\left.\left(\frac{\partial\omega}{\partial x_1} - \sin\theta\frac{\partial\omega}{\partial y_1}\right)^2 + \frac{\partial^2\omega}{\partial y_1^2}\left(\frac{\partial\omega}{\partial y_1}\right)^2\right] + 4\sec^2\theta\left(\frac{\partial^2\omega}{\partial x_1 \partial y_1} - \sin\theta\frac{\partial^2\omega}{\partial y_1^2}\right) \\
&\times\left.\left(\frac{\partial\omega}{\partial x_1} - \sin\theta\frac{\partial\omega}{\partial y_1}\right)\left(\frac{\partial\omega}{\partial y_1}\right)\right\} = \frac{q}{D}
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
A = {}&\frac{1}{2}\left\{\sec^2\theta\left[\left(\frac{\partial\omega}{\partial x_1}\right)^2 - 2\sin\theta\left(\frac{\partial\omega}{\partial x_1}\right)\left(\frac{\partial\omega}{\partial y_1}\right) + \sin^2\theta\left(\frac{\partial\omega}{\partial y_1}\right)^2\right] + \nu\left(\frac{\partial\omega}{\partial y_1}\right)^2\right\} \\
&+ \sec\theta\left(\frac{\partial u}{\partial x_1} - \sin\theta\frac{\partial u}{\partial y_1}\right) + \nu\frac{\partial v}{\partial y_1}.
\end{aligned} \tag{5}$$

Now to solve the problem, let us assume

$$\omega = \omega_0 \sin\frac{\pi x_1}{a}\sin\frac{\pi y_1}{a} \tag{6}$$

ω_0 being the maximum central deflection.

For the value of A, let us integrate equation (5) over the whole area of the plate. Then we have

$$\begin{aligned}
\int_0^a\int_0^a A\cos\theta\, dx_1\, dy_1 = {}&\frac{1}{2}\int_0^a\int_0^a\left\{\sec^2\theta\left[\left(\frac{\partial\omega}{\partial x_1}\right)^2 + \sin^2\theta\left(\frac{\partial\omega}{\partial y_1}\right)^2\right.\right. \\
&\left.\left. - 2\sin\theta\left(\frac{\partial\omega}{\partial x_1}\right)\left(\frac{\partial\omega}{\partial y_1}\right)\right] + \nu\left(\frac{\partial\omega}{\partial y_1}\right)^2\right\}\cos\theta\, dx_1\, dy_1 .
\end{aligned}$$

After integration, we get

$$A = \frac{\pi^2\omega_0^2}{8a^2}(1 + \nu + 2\tan^2\theta). \tag{7}$$

Here, it is to be noted that, since the normal displacements are our primary interest, the in-plane displacements have been eliminated through integration by choosing suitable expressions for them, compatible with their boundary conditions.

Now, applying Galerkin's method of approximation to the transformed differential equation (4) and keeping in mind the value of A from equation (7), we get the following cubic equation determining β ($= \omega_0/h$).

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+\frac{3}{8}\{[(1+\nu)+(1-\nu)\sin^2\theta]^2 + \nu^2(5+\sin^2\theta)\}\left(\frac{\omega_0}{h}\right)^3 = \frac{4}{\pi^6}\left(\frac{qa^4}{Dh}\right)\cos^4\theta. \tag{8a}$$

Adopting the well known equation [8], of Berger with e_2[1] neglected, the corresponding cubic equation determining the central deflection parameter (for immovable edges only) takes the following form (after applying Galerkin's technique):

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+1.5\left(\frac{\omega_0}{h}\right)^3 = \frac{4}{\pi^6}\left(\frac{qa^4}{Dh}\right)\cos^4\theta. \tag{8b}$$

NUMERICAL CALCULATIONS

For a steel plate we have $E = 2\times10^{12}$ dyne/cm^2 and $\nu = 0.3$, for which equation (8a) becomes

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+\frac{3}{8}[(1.3+0.7\sin^2\theta)^2 + 0.09(5+\sin^2\theta)]\left(\frac{\omega_0}{h}\right)^3 = 22.66\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \tag{9a}$$

whereas for a copper plate we have $E = 1.25\times10^{12}$ dyne/cm^2 and $\nu = 0.333$, so that equation (8a) becomes

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+\frac{3}{8}[(1.333+0.667\sin^2\theta)^2 + 0.11(5+\sin^2\theta)]\left(\frac{\omega_0}{h}\right)^3 = 35.46\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta. \tag{9b}$$

Also, for a steel plate equation (8b) becomes

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+1.5\left(\frac{\omega_0}{h}\right)^3 = 22.66\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta \tag{9c}$$

and for a copper plate it becomes

$$(1+\sin^2\theta)\left(\frac{\omega_0}{h}\right)+1.5\left(\frac{\omega_0}{h}\right)^3 = 35.46\times10^{-15}\left(\frac{qa^4}{h^4}\right)\cos^4\theta. \tag{9d}$$

Tables 1 and 2 present a comparative view of the various theoretical and experimental values of the central deflection parameter β ($= \omega_0/h$) for different values of the load function Q ($= qa^4/Dh$), for the case of steel plate and copper plate, respectively. For movable edge conditions the value of A will be zero. (The experimental method is explained in the Appendix.)

OBSERVATIONS

It is observed from the two tables that the results of the present study are in excellent agreement with those obtained from the experimental analysis. It is well known that Berger's method fails [9] miserably under movable-edge conditions. The results for simply supported immovable edges, obtained by Berger's method (as shown in the Tables 1 and 2) show that this method is not even acceptable from the practical point of view. It is worth noting that Berger's method always gives less deflections for a given load. The errors of Berger's method (as shown in Tables 1 and 2) are certainly questionable from the view point of safety design.

Table 1. The central deflection parameter ($\beta = \omega_0/h$) vs the load function ($Q = qa^4/Dh$) for steel plate (a = 16 cm for skew angle $\theta = 15°$ and a = 14 cm for $\theta = 30°$; h = 0.1343 cm)

	β (for $\theta = 15°$)							
	Movable edges			Immovable edges			Percentage error	
Q qa^4/Dh	Banerjee's hypothesis	Experimental	Percentage error	Berger's method	Banerjee's hypothesis	Experimental	Berger's method	Banerjee's hypothesis
111.72	0.3716	0.3872	4%	0.3285	0.3454	0.36485	9.96%	5.33%
223.44	0.7038	0.7372	4.5%	0.5378	0.5914	0.6329	15%	6.56%
335.16	0.98592·	1.0201	3.3%	0.6843	0.7703	0.8414	18.67%	8.45%
446.88	1.22484	1.2882	4.9%	0.7982	0.9107	0.9903	19.4%	8%
558.6	1.4303	1.5115	5.4%	0.8924	1.027	1.1244	20.6%	8.66%

	β (for $\theta = 30°$)							
	Movable edges			Immovable edges			Percentage error	
Q qa^4/Dh	Banerjee's hypothesis	Experimental	Percentage error	Berger's method	Banerjee's hypothesis	Experimental	Berger's method	Banerjee's hypothesis
65.5	0.12208	0.134	8.9%	0.1202	0.1209	0.12658	5%	4.49%
131	0.2427	0.25316	4.13%	0.2301	0.2346	0.25316	9.11%	7.33%
196.5	0.3604	0.37975	5.1%	0.3254	0.3369	0.36485	10.8 %	7.66%
262	0.4742	0.4989	5%	0.40782	0.4276	0.46165	11.66%	7.4%
327.5	0.5835	0.61802	5.59%	0.4795	0.5081	0.55845	14.2%	9%

The average percentage error from Banerjee's hypothesis is only around 6% for skew angles of $\theta = 15°$, 30° whereas from Berger's method it is around 17% for $\theta = 15°$ and 10% for $\theta = 30°$.

Table 2. The central deflection parameter ($\beta = \omega_0/h$) vs the load function ($Q = qa^4/Dh$) for copper plate (a = 16 cm for skew angle $\theta = 15°$ and a = 14 cm for $\theta = 30°$; h = 0.0789 cm)

	β (for $\theta = 15°$)							
	Movable edges			Immovable edges			Percentage error	
Q qa^4/Dh	Banerjee's hypothesis	Experimental	Percentage error	Berger's method	Banerjee's hypothesis	Experimental	Berger's method	Banerjee's hypothesis
1467.53	2.3727	2.4208	2%	1.36820	1.57802	1.673	18.22%	5.68%
2935.06	3.2506	3.308	1.7%	1.79580	2.08753	2.23067	19.5%	6.4%
4402.59	3.8437	3.9924	3.72%	2.0891	2.43578	2.6109	19.98%	6.7%
5870.12	4.307	4.4867	4%	2.32003	2.7095	2.90241	20.07%	6.65%
7337.65	4.6935	4.90494	4.3%	2.5138	2.93893	3.1559	20.35%	6.9%
	β (for $\theta = 30°$)							
	Movable edges			Immovable edges			Percentage error	
Q qa^4/Dh	Banerjee's hypothesis	Experimental	Percentage error	Berger's method	Banerjee's hypothesis	Experimental	Berger's method	Banerjee's hypothesis
860.2	1.2602	1.2801	1.55%	0.8553	0.9283	0.9886	13.5%	6.1%
1720.4	1.9429	2.0279	4.2%	1.1901	1.3095	1.40684	15.5%	6.92%
2580.6	2.4064	2.5095	4.1%	1.4156	1.5657	1.6857	16%	7.12%
3440.8	2.765	2.9404	6%	1.5913	1.7649	1.90114	16.3%	7.17%
4301	3.0617	3.2319	5.3%	1.73760	1.9307	2.09125	16.91%	7.7%

The average percentage error from Banerjee's hypothesis is only around 5% for skew angles $\theta = 15°$, $30°$ whereas from Berger's method it is around 20% for $\theta = 15°$ and around 15% for $\theta = 30°$. The errors are calculated considering the experimental results as standard (sacrificing instrumental and personal errors).

It is observed that deflections for movable edges are always greater than those for immovable edges. This is quite expected from the practical point of view, because movable edge conditions give stress-free boundary and, hence, there are large energy changes in the boundary.

Here the results for skew angles $\theta = 15^\circ$ and 30° only have been considered, because for greater values of the skew angles the effect of non-linearity does not play important role in design, and the study of linear analysis serves the practical purpose.

CONCLUSIONS

Von Karman's classical equations are in the coupled form and the transformations of these coupled equations in oblique coordinates will involve much mathematical complexity. So this entails difficulty in solution. Berger's equations, although decoupled are questionable. Thus, the present method seems to be more advantageous. The main advantages are:

(1) the differential equations are uncoupled and easy to solve;

(2) it gives accurate results both for movable and immovable edge conditions; and

(3) from a single cubic equation determining $\beta\,(=\omega_0/h)$ the results could be obtained for movable as well as immovable edge conditions.

Thus, to study non-linear behaviour of skew plates, the present method seems to be more acceptable.

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APPENDIX

Experimental arrangement

A sketch of the apparatus used for the experimental purpose is shown in Fig. 2. Two skew boxes with upper side open are constructed and each of the four side walls are made of steel. Each vertical wall of one box is 16 cm and of the other is 14 cm. The upper side of each wall is made sharp (knife edge), care being taken to see that all the knife edges lie on the same horizontal plane. The walls of the box with sides 16 cm long are welded in a manner that the

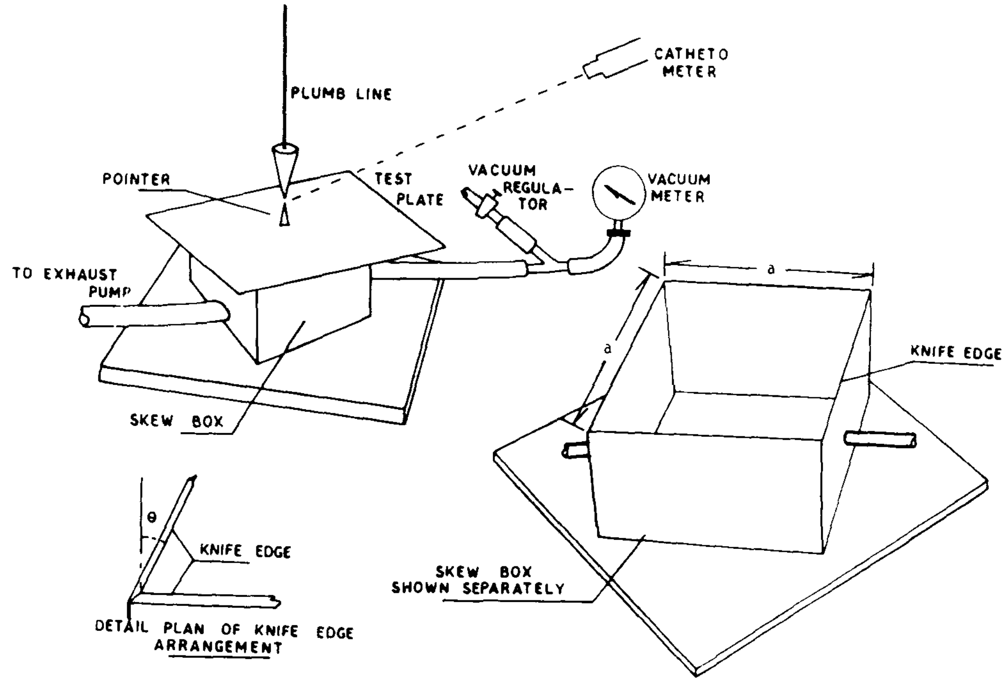

Fig. 2. The experimental arrangement.

two opposite angles are each 75° and the other two opposite angles are each 105°. Two opposite angles of the second box with sides 14 cm long are each 60° and the other two opposite angles are each 120°. Two holes are drilled on two opposite sides of each box and fitted with short metal pipes, one of which acts as an air inlet and the other as an air outlet.

For the experiment with the first skew box, the centre of the box is first found and then a plumb line is set as an indicator along the vertical line on which the centre of the box lies. For the free movable boundary conditions one test plate (which is approximately mirror surfaced) is symmetrically placed on the knife edges of the box and a pointer is fixed on the upper surface of the test plate with some adhesive along the plumb line. The outlet pipe is then joined to an exhaust pump by rubber tubing and the inlet pipe is joined to a standard vacuum meter and an air pressure regulator (as shown in the sketch). Along the contact line beneath the test plate some thick grease is used to make the box perfectly airtight. (Grease does not apply any appreciable tension on the plate.) When the exhaust pump operates, the box becomes evacuated, thereby causing the depression of the test plate by the excess outside air pressure, which is uniform all over the effective skew part of the test plate. The central deflection of the test plate is easily measured with the help of a precision cathetometer set at a distance of approximately 1.5 m from the pointer.

To make the free boundaries of a skew plate immovable, four pieces of steel collars are taken whose lengths are equal to the length of outer boundary line of the skew plate. The collars are kept outside the box in contact with the lower surface of the plate and with the side walls of the box and then the collars are tightly clamped with the test plate using nuts and bolts in sufficient number well outside the boundary of skew section.

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"Nonlinear behaviours of clamped rhombic plates" - a new approach
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Int. J. Solids Structures Vol. 00, No. 00, pp. 0000–0000, 1993
Printed in Great Britain

0020-7683/93 $6.00+ .00

NON-LINEAR ANALYSIS OF HEATED RHOMBIC PLATES

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(*Received* 24 *September* 1992; *in revised form* 15 *June* 1993)

Abstract—This paper concerns a new approach to the investigation of non-linear behaviours of heated rhombic plates. A new set of differential equations in oblique co-ordinates have been derived in this investigation. Numerical results showing central deflection parameters versus thermal load functions have been computed for different skew angles θ. For $\theta = 0°$ the results obtained in the present study are in excellent agreement with the known results. It is believed that the results obtained for other different skew angles are completely new.

INTRODUCTION

Determination of thermal deflections in thin elastic plates, is of vital importance in cases where the thermal stresses play a significant role. Although thermal deflections of thin elastic plates have been investigated by many authors (Aleck, 1949; Zizicas, 1952; Schneider, 1955; Boley and Weiner, 1960; Forray and Newmann, 1960; Nowacki, 1962; Katayama *et al.*, 1967; Sarkar, 1968; Kaiuk and Pavlenko, 1971, 1972; Roychowdhury, 1972; Prabhu and Durvasula, 1974; Matumoto and Sekiya, 1975), the literature on the large thermal deflections is somewhat sparse. The most interesting papers in this field are Williams (1955, 1958) who quite elegantly carried out large deflection analysis for a plate strip subjected to normal pressure and heating. Biswas investigated the large deflection of heated circular plates under non-constant temperature (Biswas, 1974) and large deflections of heated elastic plates under uniform load (Biswas, 1975). The author followed Berger's equation in his investigations. Another interesting paper in this field is Banerjee and Dutta (1979), investigation of non-linear behaviours of heated elastic plates under non-constant temperatures. The authors utilized a conformal mapping technique along with Berger's hypothesis. Later on Banerjee proposed a new approach to the Large Deflection analysis of thin elastic plates (Banerjee and Dutt, 1981) and afterwards carried out quite elegantly the non-linear behaviours of polygonal plates under non-constant temperatures (Banerjee, 1984). Following Banerjee's approach, another interesting paper is by Sinharay and Banerjee (1985) on non-linear behaviours of heated spherical and cylindrical shells, where the authors have achieved satisfactory results from the practical point of view. Also, the works of Kamiya (1978) on the large thermal bending of sandwich plates are very attractive and useful too.

All the investigations mentioned above deal with plate geometry either circular or rectangular or in the shape of regular polygons. Only five papers (Katayama *et al.*, 1967; Kaiuk and Pavlenko, 1971, 1972; Prabhu and Durvasula, 1974; Matumoto and Sekiya, 1975) concerned with the study of thermal behaviours of skew plates are found in the literature. But these papers do not consider the large deflections of plates. To the authors' knowledge, no paper has been devoted to the investigations of non-linear behaviours of

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heated elastic skew plates having various applications in modern design, especially in the space industry.

In this paper non-linear behaviours of simply-supported heated skew plates (taken in rhombic form for simplicity of calculation) are investigated. Various numerical results have been calculated showing central deflection parameters versus thermal load functions. Whereas the results for skew angles other than 0° are believed to be new, the results for a 0°-skew angle are found to be in remarkable agreement with the already known results [see Biswas (1975)].

ANALYSIS

Let us consider a rhombic plate of skew angle θ whose uniform thickness is h and edge-length $2a$. The material of the plate is considered isotropic having mass density ρ, Young's modulus E and Poisson's ratio ν. The origin of the co-ordinates is located at the geometric centre of the plate. The deflections are consideed to be of the same order of magnitude as the plate thickness, the edge-length being sufficiently large compared to the thickness.

Now the uncoupled set of differential equations in rectangular Cartesian co-ordinates, governing the thermal behaviours of elastic plates [see Banerjee (1984)] is given by

pl. insert 6 (six) in place of σ

$$\nabla^4 w-\frac{12A}{h^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)-\frac{\sigma\lambda}{h^2}\left[\nabla^2 w\left\{\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right\}\right.$$
$$\left.+2\left\{\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2\right\}+4\frac{\partial^2 w}{\partial x\,\partial y}\cdot\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right]$$
$$+\frac{12\alpha\tau_0}{h^2}\sqrt{\lambda(1-\nu^2)}\cdot\nabla^2 w+(1+\nu)\alpha_t\nabla^2\tau=\frac{q}{D}, \quad (1)$$

H: OK? It is o.k. i.e. α_t

where

$$A=\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}+\frac{1}{2}\left\{\left(\frac{\partial w}{\partial x}\right)^2+\nu\left(\frac{\partial w}{\partial y}\right)^2\right\}-(1+\nu)\alpha_t\tau_0, \quad (2)$$

$\lambda=\nu^2$ for simply-supported elastic plates, and $D=Eh^3/12(1-\nu^2)$, the flexural rigidity of the material of the elastic plate.

It is to be noted that in the derivation of eqns (1) and (2) in rectangular Cartesian co-ordinates, the expression

$$(1-\nu^2)\left[\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right)^2\cdot\frac{1}{2(1+\nu)}$$

in the total P.E. of the elastic plate (Banerjee, 1984) has been replaced by

$$\frac{\lambda}{4}\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]^2.$$

As a consequence the partial differential equations governing the deflection of the plate have become uncoupled and the two decoupled differential equations (1) and (2) have been obtained.

In the present problem, the temperature is assumed to vary linearly w.r.t. the thickness direction z. We also note that

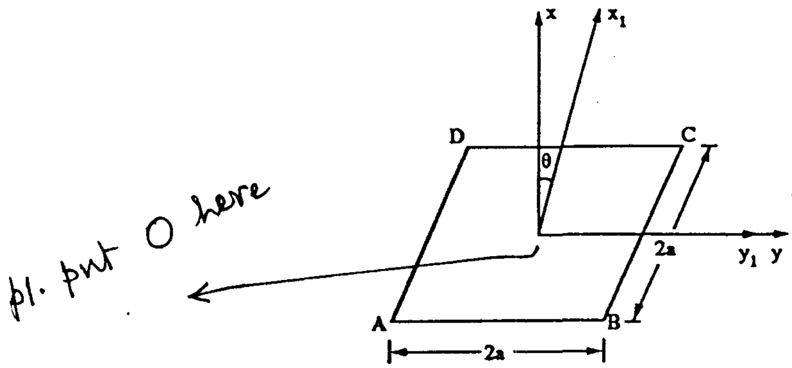

Fig. 1. Plan form of skew plate and co-ordinate system.

$$T(x,y,z) = \tau_0(x,y) + z\tau(x,y),$$

in which

$$\tau_0 = \tfrac{1}{2}(T_1 + T_2), \quad \tau = \frac{1}{h}(T_1 - T_2),$$

$$T_1 = T\left(x, y, \frac{h}{2}\right) \quad \text{and} \quad T_2 = T\left(x, y, -\frac{h}{2}\right) \quad \text{(Banerjee, 1984).}$$

Clearly τ_0 is the temperature in the middle plane and τ varies along the thickness of the plate and hence $\tau \neq \tau_0$.

The plan of the skew co-ordinates (x_1, y_1, θ) is shown in Fig. 1. Clearly

$$x = x_1 \cos\theta$$

$$\text{and} \quad y = y_1 + x_1 \sin\theta \tag{3}$$

are the co-ordinate transformation equations. Hence we have the following partial differential operators in oblique co-ordinates:

$$\frac{\partial}{\partial x} \equiv \sec\theta\left(\frac{\partial}{\partial x_1} - \sin\theta\frac{\partial}{\partial y_1}\right), \quad \frac{\partial}{\partial y} \equiv \frac{\partial}{\partial y_1},$$

$$\frac{\partial^2}{\partial x^2} \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\frac{\partial^2}{\partial x_1\,\partial y_1} + \sin^2\theta\frac{\partial^2}{\partial y_1^2}\right),$$

$$\frac{\partial^2}{\partial y^2} \equiv \frac{\partial^2}{\partial y_1^2}, \quad \frac{\partial^2}{\partial x\,\partial y} \equiv \sec\theta\left(\frac{\partial^2}{\partial x_1\,\partial y_1} - \sin\theta\frac{\partial^2}{\partial y_1^2}\right),$$

$$\nabla^2 \equiv \sec^2\theta\left(\frac{\partial^2}{\partial x_1^2} - 2\sin\theta\frac{\partial^2}{\partial x_1\,\partial y_1} + \frac{\partial^2}{\partial y_1^2}\right)$$

and

$$\nabla^4 \equiv \sec^4\theta\left\{\frac{\partial^4}{\partial x_1^4} - 4\sin\theta\left(\frac{\partial^4}{\partial x_1^3\,\partial y_1} + \frac{\partial^4}{\partial x_1\,\partial y_1^3}\right) + 2(1 + 2\sin^2\theta)\frac{\partial^4}{\partial x_1^2\,\partial y_1^2} + \frac{\partial^4}{\partial y_1^4}\right\}. \tag{4}$$

We now transform eqn (2) in oblique co-ordinates. For simply-supported plates the boundary conditions are pl. delete dot overhead of y_1

$$w = 0 \quad \text{at} \quad x_1 = \pm a \quad \text{and at} \quad y_1 = \pm a,$$

$$\frac{\partial^2 w}{\partial x_1^2} = 0 \quad \text{at} \quad x_1 = \pm a \quad \text{and} \quad \frac{\partial^2 w}{\partial y_1^2} = 0 \quad \text{at} \quad y_1 = \pm a.$$

Then let us choose the deflection function for the simply-supported plate as

$$w = w_0 \cos \frac{\pi x_1}{2a} \cos \frac{\pi y_1}{2a}, \tag{5}$$

which clearly satisfies the above-mentioned boundary conditions.

Now putting (5) in eqn (2) transformed in oblique co-ordinates and then integrating the relation thus obtained, over the entire surface of the plate, we obtain the value of A in the following form:

$$A = \frac{\pi^2 w_0^2}{32a^2}(1+\nu+2\tan^2\theta) - (1+\nu)\alpha_t \tau_0. \tag{6}$$

(As the normal displacement w is our primary interest, the in-plane displacements u, v have been eliminated through integration by the choice of appropriate functions for such displacements.) Again transforming eqn (1) in oblique co-ordinates, introducing eqns (5) and (6) in the transformed equation and then applying Galerkin's error minimizing technique we get the following equation determining the central deflection parameter w_0/h depending on the thermal load function $q'a^4/Eh^4$:

pl. transpose dot and bracket i.e. in place of .) put).

$$\left[(1+2\tan^2\theta)\sec^2\theta - \frac{6S}{(1+\nu)\pi^2}\{2\sqrt{\lambda(1-\nu^2)}\cdot\sec^2\theta + (1+\nu)(1+\nu+2\tan^2\theta)\}\right]\left(\frac{w_0}{h}\right) + \tfrac{3}{8}[(1+\nu+2\tan^2\theta)^2 + \frac{\lambda}{4}(8+49\tan^2\theta+29\tan^4\theta)]\left(\frac{w_0}{h}\right)^3 = \frac{768(1-\nu^2)}{\pi^6}\left(\frac{q'a^4}{Eh^4}\right), \tag{7}$$

where

$$S = 2\left(\frac{a}{h}\right)^2(1+\nu)\alpha_t\tau_0$$

and

$$q' = q - D\alpha_t(1+\nu)\nabla^2\tau.$$

Equation (7) is applicable for the immovable edge condition of the simply-supported skew plate. For the movable edge condition we have $A = 0$, so that eqn (7) takes the form:

$$\left[(1+2\tan^2\theta)\sec^2\theta - \frac{12S}{(1+\nu)\pi^2}\sqrt{\lambda(1-\nu^2)}\cdot\sec^2\theta\right]\left(\frac{w_0}{h}\right) + \frac{3\lambda}{32}(8+49\tan^2\theta+29\tan^4\theta)]\left(\frac{w_0}{h}\right)^3 = \frac{768(1-\nu^2)}{\pi^6}\left(\frac{q'a^4}{Eh^4}\right). \tag{8}$$

pl. delete the bracket→]

Table 1. $S = 0$, i.e. $\tau_0 = 0$

$\frac{q'a^4}{Eh^4}$	w_0/h by present method						w_0/h by Berger's method†		
	$\theta = 0°$		$\theta = 15°$		$\theta = 30°$		(Biswas, 1975) $\theta = 0°$	$\theta = 15°$	$\theta = 30°$
	Movable edge	Immovable edge	Movable edge	Immovable edge	Movable edge	Immovable edge	Immovable edge	Immovable edge	Immovable edge
2	1.30156	0.91435	1.08167	0.82069	0.6269	0.53604	0.9013	0.79972	0.53671
4	2.1909	1.3131	1.85443	1.20857	1.14734	0.84631	1.29017	1.16888	0.848
8	3.23354	1.78866	2.8581	1.67119	1.89675	1.22355	1.75406	1.60902	1.2266
10	3.73498	1.9613	3.2243	1.83866	2.17977	1.3597	1.92254	1.76847	1.36324

† Berger's method has been applied to the present problem by neglecting e_2, the second strain invariant in the expression for total P.E. of the plate.

NUMERICAL RESULTS

Numerical results are presented here (Tables 1 and 2) in the tabular forms for $S = 0$, 0.1; $\theta = 0°$, 15°, 30° and $q'a^4/Eh^4 = 2, 4, 8, 10$.

Pl. put the Tables 1 and 2 here in succession

OBSERVATIONS AND CONCLUSIONS

From the numerical analysis of the undertaken problem the following observations are made:

(i) The nature of the central deflection of a skew plate under thermal loading is the same as that of the plate under mechanical loading, i.e. the central deflection increases continuously with the increase of loading for any edge condition of the skew plate, whether movable or immovable.

(ii) The central deflection for the movable edge condition of the skew plate is always greater than that for the immovable edge condition of the plate, for the same loading.

(iii) Irrespective of the edge condition, the central deflection decreases with the increase in the skew angle.

(iv) The results for immovable edge conditions of the skew plate obtained by the present method, agree well with the results obtained by Berger's method. It is to be noted that Berger's method is a purely approximate method based on the neglect of e_2. But the present study is based on Banerjee's hypothesis which suggests a modified strain–energy expression, and thus this model embraces less approximation (Banerjee and Dutt, 1981) than that of Berger. Again Berger's method is meaningful only for immovable edge conditions of the plates.

(v) The deflections increase with τ_0.

The present method seems to be more advantageous than any other method found in open literature. The main advantages are:

Table 2. $S = 0.1$, i.e. $\tau_0 \neq 0$

$\frac{q'a^4}{Eh^4}$	w_0/h by present method						w_0/h by Berger's method ($e_2 = 0$)		
	$\theta = 0°$		$\theta = 15°$		$\theta = 30°$		(Biswas, 1975) $\theta = 0°$	$\theta = 15°$	$\theta = 30°$
	Movable edge	Immovable edge	Movable edge	Immovable edge	Movable edge	Immovable edge	Immovable edge	Immovable edge	Immovable edge
2	1.32786	0.94985	1.10168	0.83899	0.63597	0.55925	0.94058	0.83515	0.56109
4	2.22082	1.34324	1.87831	1.20992	1.1604	0.86901	1.32336	1.19954	0.87185
8	3.35106	1.81316	2.88067	1.65221	1.9111	1.24302	1.781	1.63412	1.24706
10	3.76118	1.98415	3.24585	1.81269	2.19385	1.37799	1.94764	1.79188	1.38247

(1) The differential equations are decoupled and easy to solve;
(2) from a single cubic equation determining w_0/h, the results could be obtained for movable as well as immovable edge conditions; and
(3) unlike Berger's method it gives accurate results both for movable and immovable edge conditions. Based on Banerjee's hypothesis a good number of works have been carried out and in each case sufficiently accurate results have been obtained [e.g. Banerjee and Dutt (1981), Banerjee (1984), Sinharay and Banerjee (1985) and Ray *et al.* (1992, 1993)]. So in the present case also, the same degree of accuracy was expected.

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BRIEF NOTE

Nonlinear Analysis of Skewed Sandwich Plates

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(Received: 5 June 1992; accepted in revised form: 5 July 1993)

Abstract. Nonlinear static and dynamic behaviours of freely supported Rhombic sandwich plates have been studied following Banerjee's hypothesis. Numerical results for 0° skew angle are compared with other known results. Results for other skew angles are believed to be new.

Sommario. Si studia, seguendo l'ipotesi di Banerjee, il comportamento nonlineare statico e dinamico di [illegible] rombiche semplicemente appoggiate. Si presentano risultati numerici relativi a piastre rombiche e rettangolari; questi ultimi vengono paragonati a risultati giá noti, mentre i primi si ritengono nuovi.

Key words: Skew plates; sandwich plates; nonlinear analysis.

1. Introduction

Sandwich plates find wide applications in technology and modern design. Outstanding investigations [1–8] on large deflections as well as large amplitude vibrations of such plates are limited to the rectangular form only. No attempt on the nonlinear behaviours of skewed sandwich plates has been reported as yet.

In this paper an attempt has been made to analyse the nonlinear behaviours of freely supported skewed sandwich plates having an isotropic core within isotropic upper and lower faces and under both static and dynamic loadings. For the sake of simplicity, a skewed plate in the form of a rhombus has been considered. Following the modified strain energy expression proposed by B. Banerjee, [8] a new set of decoupled differential equations for skewed sandwich plates has been derived. The final equations have been solved by Galerkin's method. Numerical results are computed and those for the 0° skew angle are compared with the other known results. The results for other skew angles are completely new.

2. Governing Equations

Let each side of a rhombic sandwich plate be [illegible] having an isotropic core of thickness h [illegible] isotropic upper and lower faces of identical thickness t_1 where $t_1 \ll h$.

Now let us set one rectangular cartesian coordinate system (x, y, z) and one o[illegible] coordinate system (x_1, y_1, θ) at the same corner of the plate, x, y being in the middle [illegible] of the core; Z the thickness direction; x_1, y_1 are parallel to the sides of the plate and θ, the skew angle (see Figure 1). Clearly, the coordinate transformation equations are

$$x = x_1 \cos \theta \text{ and } y = y_1 + x_1 \sin \theta \tag{1}$$

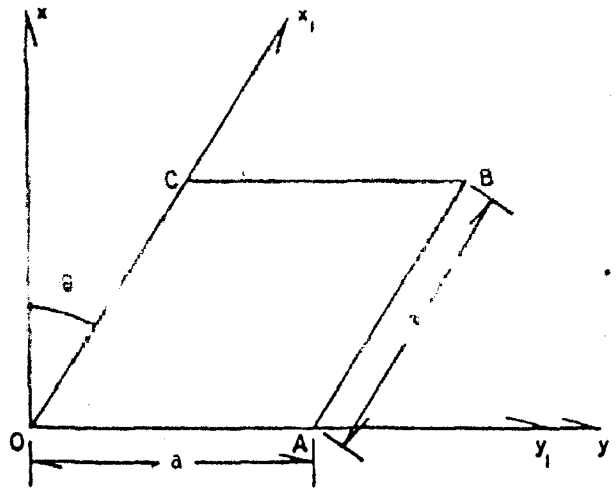

Fig. 1. Plan form of skew plate.

Following Banerjee's hypothesis, [8] the differential equations in rectangular coordinate system governing the deflections and vibrations of sandwich plates are

$$\left[\frac{Et_1}{2(1-\nu^2)}\nabla^2 - \frac{G'}{h}\right]\left[\frac{2Et_1}{(1-\nu^2)G'}I_1^m\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)\right.$$
$$+h\nabla^2 W + \frac{Et_1\lambda}{(1-\nu^2)G'}\left|\left\{\left(\frac{\partial W}{\partial x}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2\right\}\nabla^2 W\right.$$
$$\left.\left.+2\left\{\frac{\partial^2 W}{\partial x^2}\left(\frac{\partial W}{\partial x}\right)^2+\frac{\partial^2 W}{\partial y^2}\left(\frac{\partial W}{\partial y}\right)^2+2\frac{\partial^2 W}{\partial x\partial y}\cdot\frac{\partial W}{\partial x}\cdot\frac{\partial W}{\partial y}\right\}\right|+\xi\right]$$
$$+G'\nabla^4 W = 0 \qquad (1)$$

where $\xi = q/G'$ for nonlinear static deflections,

$$= -\frac{(\rho_1 t_1+\rho_2 h)}{G'}\frac{\partial^2 W}{\partial t^2} \text{ for nonlinear elastic vibrations,} \qquad (2)$$

$$\text{and } I_1^m = \frac{1}{2}\left\{\left(\frac{\partial W}{\partial x}\right)^2+\nu\left(\frac{\partial W}{\partial y}\right)^2\right\}+\frac{\partial P}{\partial x}+\nu\frac{\partial Q}{\partial y}$$

= constant, for nonlinear static deflections (3)

= $Cf(t)$ for nonlinear elastic vibrations,

C being a constant depending on θ . (3a)

In the above equations W is the transverse deflection function; q, the lateral load distribution function; P, Q are the in-plane displacements along x and y axes respectively; E, the Young's Modulus of elasticity of the material of the upper and lower faces; G', the shear modulus of the core material; ν , the poisson's ratio of the face material, $\lambda = \nu^2$; ρ_1, ρ_2 are the surface density and core density respectively, and $f(t)$, $F(t)$ are the functions of time such that $f(t) = F'(t)$.

It is to be noted that the strain-energy expression in ref. [8] has been modified by ... Banerjee's hypothesis, which states that the stretching of the plate is proportional to

$$\left[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right]^2.$$

As a consequence of this assumption, a set of uncoupled differential equations has ... obtained as given above.

3. Analysis

3.1. NON-LINEAR STATIC BEHAVIOUR OF FREELY SUPPORTED SKEWED SANDWICH PLATES

To find normal displacement W, the inplane displacements of the upper and lower faces are being eliminated through integration by choosing suitable expressions for them in the form of trigonometric functions compatible with the boundary conditions of the plate [8]. Then transforming equation (3a) in oblique coordinates, choosing

$$W = \bar{W} \sin \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a}$$

and

$$q = \bar{q} \sin \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a}$$

and then integrating the transformed equation over the whole domain of the plate we get

$$I_1^m = \frac{\pi^2 \bar{W}^2}{8a^2}(1 + \nu + 2 \tan^2 \theta).$$

Again transforming equation (2) with $\xi = q/G'$, in oblique coordinates, introducing equations (4a), (4b) and (4c) in the transformed equation and then applying Galerkin procedure, we arrive at the following cubic equation determining W, the central deflection of a freely supported rhombic sandwich plate

$$\frac{\pi^4 t_1}{4(1-\nu^2)}\left[\frac{\pi^2 E t_1 \mathrm{Sec}^2\theta}{(1-\nu^2)G'a^2}\{(1+\nu+2\tan^2\theta)(1+\nu+4\tan^2\theta) + \lambda(5+17\tan^2\theta+12\tan^4\theta)\} + \frac{1}{h}\{(1+\nu+2\tan^2\theta)^2 + \lambda(5+11\tan^2\theta+6\tan^4\theta)\}\right]\left(\frac{\bar{W}}{h}\right)^3 + \left[\frac{2\pi^4 t_1 \mathrm{Sec}^2\theta}{(1-\nu^2)h}(1+2\tan^2\theta)\right]\left(\frac{\bar{W}}{h}\right) = \frac{\bar{q}a^4}{Eh^4}\left[1+\frac{\pi^2 E h t_1 \mathrm{Sec}^2\theta}{(1-\nu^2)G'a^2}\right].$$

4. Numerical Results

Table 1 shows different numerical results of the central deflections of a (0.254 m × 0.254 m) rhombic plate having $t_1 = 6.35 \times 10^{-4}\ m, h = 1.7135 \times 10^{-2}\ m$. [5, 8].

Table I. Showing $\bar{W}/h$ vs θ. $E = 16.2 \times 10^9$ psm, $G' = 9.3 \times 10^6$ psm, $\nu = 0.3$, $\frac{qa^4}{Eh^4} = 10$ [5,8]

	Value of $\bar{W}/h$				
	Immovable edge			Movable edge	
Value of θ	Calculated value	Other known value [8]	Other known value [6]	Calculated value	Other known value[8]
0°	1.4988	1.53	1.30	2.3223	2.588
15°	1.3644	—	—	2.1563	—
30°	1.0328	—	—	1.6360	—
45°	0.6651	—	—	1.0414	—

Note: For movable edge condition of the freely supported plate $I_1^m = 0$.

4.1. NONLINEAR DYNAMIC BEHAVIOURS OF FREELY SUPPORTED SKEWED SANDWICH PLATES

Let us now consider free vibrations of skewed sandwich plates. Neglecting in-plane inertia for obvious reasons, transforming equation (3b) in oblique coordinates, choosing

$$W = \bar{W} \sin \frac{\pi x_1}{a} \sin \frac{\pi y_1}{a} F(t) \tag{6a}$$

for fundamental mode of vibration and then integrating the transformed equation over the whole domain of the plate we get

$$I_1^m = \frac{\pi^2 \bar{W}^2}{8a^2}(1 + \nu + 2 \tan^2 \theta) F^2(t). \tag{6b}$$

Now transforming equation (2) with $\xi = -\frac{(\rho_1 t_1 + \rho_2 h)}{G'} \frac{\partial^2 W}{\partial t^2}$ in oblique coordinates, substituting equations (6a) and (6b) in the transformed equation and then applying Galerkin's procedure we get the following equation for time function

$$\left[\frac{\pi^2(\rho_1 t_1 + \rho_2 h)E t_1}{(1-\nu^2)G' a^2} \operatorname{Sec}^2 \theta - \frac{(\rho_1 t_1 + \rho_2 h)}{h}\right] \ddot{F}$$
$$+ \left[\frac{2\pi^4 E t_1 h}{(1-\nu^2)a^4}(1 + 2 \tan^2 \theta) \operatorname{Sec}^2 \theta\right] F$$
$$+ \left[\frac{\pi^4 E t_1 \bar{W}^2}{4(1-\nu^2)a^4} \left\{\frac{\pi^2 E t_1 \operatorname{Sec}^2 \theta}{(1-\nu^2)G' a^2} \middle|(1 + \nu + 2 \tan^2 \theta)(1 + \nu + 4 \tan^2 \theta)\right.\right.$$
$$\left.\left. + \lambda(5 + 17 \tan^2 \theta + 12 \tan^4 \theta) \middle| + \frac{1}{h} \middle| (1 + \nu + 2 \tan^2 \theta)^2 \right.\right.$$
$$\left.\left. + \lambda(5 + 11 \tan^2 \theta + 6 \tan^4 \theta) \middle| \right\}\right] F^3 = 0, \tag{7}$$

which may be turned in the form $\ddot{F} + AF + BF^3 = 0$, the familiar Duffing's equation. With the initial conditions $F(0) = 1$ and $\dot{F}(0) = 0$, the solution of equation (7) is well known

Table 2. Showing ω_1^*/ω_1 vs θ

Value of θ	Value of $\frac{W}{2h_1}$	Value of ω_1^*/ω_1				
		Immovable edge			Movable edge	
		Calculated value	Other known value [8]	[7]	Calculated value	Other known value [8]
0°	—	1.15028	1.12	1.14	1.03342	1.024
15°	—	1.16621	—	—	1.03365	—
30°	0.5	1.22803	—	—	1.04313	—
45°	—	1.36556	—	—	1.06261	—
0°	—	1.51413	1.42	1.48	1.12774	1.094
15°	—	1.56211	—	—	1.1286	—
30°	1.0	1.74133	—	—	1.1630	—
45°	—	2.11166	—	—	1.2315	—

elliptic integral $F(t) = C_n(\omega_1^*, t, k)$. Then the ratio of nonlinear frequency ω_1^* to l[...]ear frequency ω_1 is given by

$$\frac{\omega_1^*}{\omega_1} = \left[1 + \frac{h\cos^2\theta}{8(1+2\tan^2\theta)}\left(\frac{W}{2h_1}\right)^2\left(1+\frac{2t_1}{h}\right)^2\left\{\frac{\pi^2 E t_1 \operatorname{Sec}^2\theta}{(1-\nu^2)G'a^2}\right|\right.$$
$$(1+\nu+2\tan^2\theta)(1+\nu+4\tan^2\theta)+\lambda(5+17\tan^2\theta+12\tan^4\theta)\Big|$$
$$\left.+\frac{1}{h}\left|(1+\nu+2\tan^2\theta)^2+\lambda(5+11\tan^2\theta+6\tan^4\theta)\right|\Big\}\right]^{\frac{1}{2}}, \qquad (8)$$

where $h_1 = t_1 + \frac{h}{2}$, $\omega_1 = \sqrt{A}$ and $\omega_1^* = \sqrt{A+B}$.

5. Numerical Results

Numerical results of the ratio ω_1^*/ω_1 are shown in Table 2. For calculations, the s[...] data which are used in the study of static behaviours of sandwich plates are used here al[...]

6. Observations

From the calculated results, the following observations are easily made.

1. The results of both static and dynamic behaviours of a sandwich plate having sk[...]angle $\theta = 0^\circ$ and aspect ratio 1 are in excellent agreement with those obtained by I[...] and Banerjee [8].
2. It is seen that the central deflection gradually decreases with the increase in sk[...] angle for both movable as well as immovable edge conditions.
3. For any assumed skew angle the central deflection is greater for the movable edge [...]ition than for the immovable edge condition. This is quite expected from a practic[...] of view.

4. In the dynamic case, the frequency ratio ω_1^*/ω_1 increases continuously with [illegible] angle θ, for both movable as well as immovable edge conditions of a skewed [illegible] ratio for immovable edge condition being always greater than that for the [illegible] condition.

7. Conclusions

1. Greater deflections, obtained in the present theoretical study in comparison to [illegible] tions obtainable from the other theories in open literature, indicate acceptibility of the present method for practical purposes.
2. It is advantageous from the point of view that following this method results, [illegible] immovable as well as movable edge conditions of the plate, can be derived from [illegible] cubic equation.
3. The governing differential equations, being de-coupled, are simple and easy to [illegible] but, are able to yield results with considerable accuracy.
4. The great advantage of the present method lies in the fact that the accuracy of the [illegible] does not depend on any correction factor and thus holds good for sandwich [illegible] of different geometry.

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