

CHAPTER - III.

INFLUENCES OF LARGE AMPLITUDES, TRANSVERSE
SHEAR DEFORMATION AND ROTATORY INERTIA ON
FREE VIBRATIONS OF TRANSVERSELY ISOTROPIC
PLATES - A NEW APPROACH.

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ABSTRACT

In this Chapter the non-linear static and dynamic behaviours of moderately thick plates of different shapes have been analysed with the help of a new set of uncoupled differential equations proposed in the chapter I. Numerical results for different plates with different edge conditions have been computed and compared with other known results.

A. Non-linear analysis of square, elliptical and isosceles right angled triangular plates.

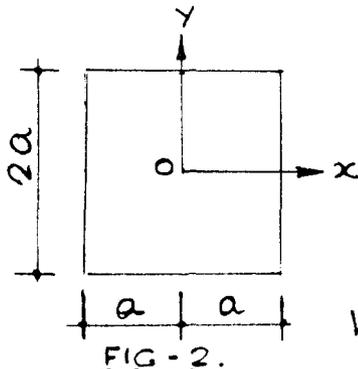
Analysis : -

Let us consider the free vibrations of thick plates of thickness h in cartesian co-ordinate system. The material is transversely isotropic (such as pyrolytic graphite, for example). The origin of co-ordinates is located at the centre of the square plate of side $2a$ and at the centre of the elliptic plate with semi-axes a and b . For isosceles

right angled triangular plate of equal side a , it is located at one corner. The deflections are considered to be of the same order of magnitude as the plate thickness.

*

(i) Square plate :



For square plate of side $2a$, let us choose the deflection function in the following form

$$W = A_0 \gamma(t) \cos \frac{\pi x}{a} \cdot \cos \frac{\pi y}{a} \quad \dots (41)$$

for fundamental mode of vibrations. Clearly this form of W satisfies the following simply supported edge conditions

$$W = 0 \quad \text{at } x = \pm a$$

$$W = 0 \quad \text{at } y = \pm a$$

$$\frac{\partial^2 W}{\partial x^2} = 0 \quad \text{at } x = \pm a$$

$$\frac{\partial^2 W}{\partial y^2} = 0 \quad \text{at } y = \pm a.$$

Putting (41) in (18b) of the first chapter and integrating over the area of the plate one gets

$$\bar{\alpha}^2 = \frac{3}{8} \cdot \frac{A_0^2 \pi^2 (1+\nu)}{a^2 h^2} \quad \dots (42)$$

For transverse vibration the normal displacement is our primary interest. So the inplane displacements have been eliminated through integration by choosing suitable expression for them compatible with their boundary conditions i.e. $u_0 = 0$, $v_0 = 0$ on the boundary.

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Now inserting (41) in (18a) of the 1st chapter.

Considering (42) and applying Galerkin's error minimising technique one gets the following differential equation for the time function $\tau(t)$

$$\begin{aligned} & \left[\frac{12}{h^2 C_p^2} + \frac{3}{5} \frac{\pi^2 \rho}{a^2 G_c} \right] \ddot{\tau}(t) + \frac{\pi^4}{4a^4} \tau(t) + \left[\frac{15}{32} \frac{\pi^4}{a^4} \lambda \left(\frac{A_0}{h} \right)^2 \right. \\ & + \frac{3}{32} \frac{\pi^4 (1+\nu)^2}{a^4} \left(\frac{A_0}{h} \right)^2 + \frac{3}{640} \frac{\pi^6}{a^6} \frac{h^2 (1+\nu)^2}{(1-\nu^2)^2} k \left(\frac{E}{G_c} \right) \left(\frac{A_0}{h} \right)^2 \\ & \left. + \frac{3}{128} \lambda \frac{\pi^6}{a^6} \frac{h^2}{(1-\nu^2)^2} k \left(\frac{E}{G_c} \right) \left(\frac{A_0}{h} \right)^2 \right] \tau^3(t) = 0 \quad \dots (43) \end{aligned}$$

The above equation is of the form

$$\ddot{\tau}(t) + \alpha_1 \tau(t) + \beta_1 \tau^3(t) = 0 \quad \dots (44)$$

The solution of equation (44) subject to the initial conditions

$$\tau(0) = 1$$

$$\dot{\tau}(0) = 0$$

is well-known and is obtained in terms of Jacobi's elliptic function. The ratio of the nonlinear and linear time periods is

$$\frac{T^*}{T} = \frac{2K}{\pi} \left[\frac{1 + \frac{\pi^2}{20(1-\nu^2)} \cdot \frac{E}{G_c} \cdot \frac{h^2}{a^2}}{1 + \frac{15}{8} \lambda \bar{\beta}^2 + \frac{3}{8} (1+\nu)^2 \bar{\beta}^2 + \frac{3}{32(1-\nu^2)^2} k \left(\frac{E}{G_c} \right) \frac{\pi^2 h^2 \bar{\beta}^2}{a^2} + \frac{3}{160} k \left(\frac{E}{G_c} \right) \frac{\pi^2 h^2 (1+\nu)^2}{a^2 (1-\nu^2)^2} \bar{\beta}^2} \right]^{1/2} \quad \dots (45)$$

where $\bar{\beta} = \frac{A_0}{h}$ is the ratio of the static deflection to the thickness of the plate.

RATIO OF NON-LINEAR TO LINEAR PERIOD FOR THE FUNDAMENTAL
MODE OF VIBRATION OF A SIMPLY SUPPORTED SQUARE PLATE.

($\nu = 0.3$, $\lambda = \nu^2 [18]$)

IMMOVABLE EDGES. (TABLE 4 TO 7)

PRESENT FINDINGS.

REF. [38]

TABLE - 4

$\frac{h}{2a} = \frac{1}{10}$	$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$							
		$k\left(\frac{E}{G_c}\right)$				$k\left(\frac{E}{G_c}\right)$			
		2.5	20	30	50	2.5	20	30	50
	0	1.0268	1.1976	1.2850	1.4440	1.0268	1.1976	1.2850	1.4440
	0.2	1.0140	1.1774	1.2602	1.4092	1.0037	1.1606	1.2397	1.3806
	0.4	0.9785	1.1228	1.1940	1.3187	0.9418	1.0663	1.1290	1.2290
	0.6	0.9270	1.0469	1.1066	1.2012	0.8606	0.9577	0.9978	1.0656
	0.8	0.8624	0.9636	1.0113	1.0819	0.7758	0.8422	0.8710	0.9159
	1.0	0.8055	0.8809	0.9123	0.9678	0.6976	0.7449	0.7648	0.7948

TABLE - 5

\bar{B} $= \frac{A_0}{h}$	$\frac{T^*}{T}$									
	$k \left(\frac{E}{Gc} \right)$					$k \left(\frac{E}{Gc} \right)$				
	2.5	20	30	50		2.5	20	30	50	
0	1.0067	1.0529	1.0785	1.1274		1.0067	1.0529	1.0785	1.1274	
0.2	0.9947	1.0391	1.0635	1.1100		0.9846	1.0173	1.0511	1.0966	
0.4	0.9610	1.0009	1.0227	1.0644		0.9270	0.9617	0.9810	1.0176	
0.6	0.9121	0.9460	0.9643	0.9905		0.8487	0.8757	0.8963	0.9175	
0.8	0.8548	0.8825	0.8968	0.9251		0.7670	0.7869	0.7973	0.8166	
1.0	0.7948	0.8170	0.8273	0.8503		0.6900	0.7049	0.7119	0.7255	

$$\frac{h}{2a} = \frac{1}{20}$$

PRESENT FINDINGS

REF. [38]

TABLE-6

$\frac{h}{2a}$ $= \frac{1}{30}$	$\bar{\beta} = \frac{A_c}{h}$	$\frac{T^*}{T}$							
		$k\left(\frac{E}{G_c}\right)$				$k\left(\frac{E}{G_c}\right)$			
		2.5	20	30	50	2.5	20	30	50
0		1.0030	1.0239	1.0355	1.0585	1.0030	1.0239	1.0355	1.0585
0.2		0.9912	1.0111	1.0225	1.0445	0.9811	1.0005	1.0113	1.0221
0.4		0.9578	0.9759	0.9860	1.0058	0.9172	0.9393	0.9482	0.9656
0.6		0.9093	0.9247	0.9334	0.9501	0.8464	0.8586	0.8653	0.8784
0.8		0.8525	0.8641	0.8722	0.8855	0.7630	0.7742	0.7791	0.7885
1.0		0.7930	0.8031	0.8087	0.8197	0.6889	0.6952	0.6980	0.7052

PRESENT STUDY | REF [38] —
 CLASSICAL THIN PLATE THEORY.

TABLE - 7.

$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$	$\frac{T^*}{T}$
	$\frac{E}{G_c} = 0$	$\frac{E}{G_c} = 0$
0	1	1
0.2	0.9882	0.9782
0.4	0.9552	0.9210
0.6	0.9072	0.8446
0.8	0.8507	0.7640
1.0	0.7917	0.6878

PRESENT FINDINGS FOR MOVABLE EDGES. (TABLE 8 TO 11)

TABLE - 8

$\frac{h}{2a} = \frac{1}{10}$	$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T} \quad (\nu = 0.3, \lambda = \nu^2 [18])$			
		$k \left(\frac{E}{G_c} \right)$			
		2.5	20	30	50
	0	1.02680	1.19760	1.2850	1.4440
	0.2	1.02406	1.19325	1.2796	1.4366
	0.4	1.01601	1.18056	1.2641	1.4147
	0.6	1.00298	1.1604	1.2394	1.3802
	0.8	0.9857	1.13376	1.2071	1.3362
	1.0	0.9647	1.1022	1.1693	1.2852

TABLE - 9

$\frac{h}{2a} = \frac{1}{20}$	$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T} \quad (\nu = 0.3, \lambda = \nu^2 [18])$			
		$k \left(\frac{E}{G_c} \right)$			
		2.5	20	30	50
	0	1.0067	1.0523	1.07850	1.12754
	0.2	1.0042	1.0499	1.0752	1.1239
	0.4	0.9966	1.0412	1.0659	1.1134
	0.6	0.9844	1.0291	1.0509	1.0963
	0.8	0.9678	1.0089	1.0310	1.0738
	1.0	0.9480	0.9859	1.0070	1.0468

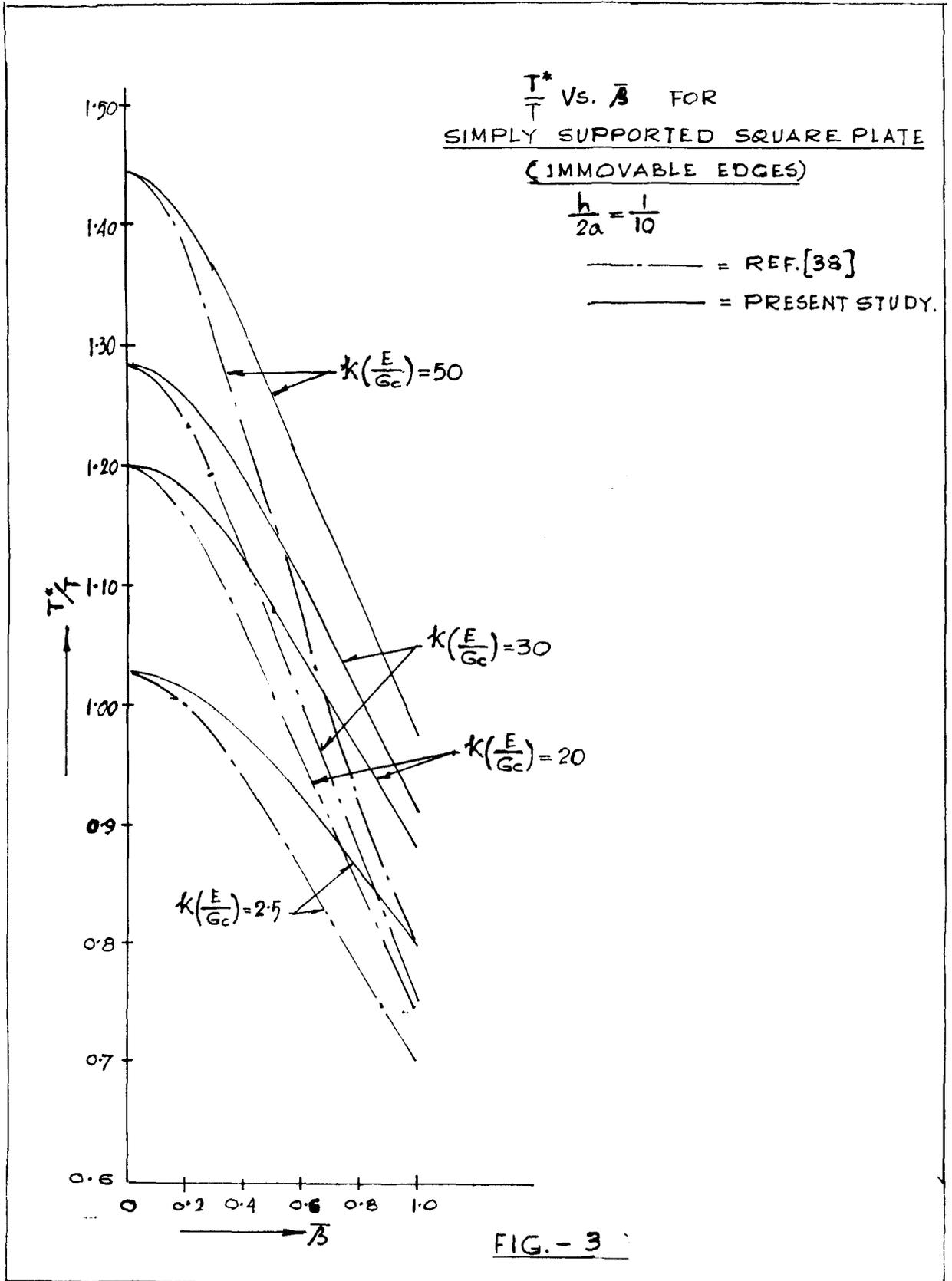
Table 10

	$\bar{\beta} = \frac{A_0}{h}$	$\frac{I^*}{T}$			
		$k\left(\frac{E}{Gc}\right)$			
		2.5	20	30	50
$\frac{h}{2a} = \frac{1}{30}$	0	1.0030	1.0239	1.0355	1.0585
	0.2	1.0005	1.0213	1.0327	1.0555
	0.4	0.9930	1.0132	1.0245	1.0467
	0.6	0.9808	1.0003	1.0110	1.0326
	0.8	0.9647	0.9831	0.9934	1.0139
	1.0	0.9449	0.9622	0.9720	0.9910

Table 11

	$\frac{I^*}{T}$	
	$\bar{\beta} = \frac{A_0}{h}$	
$k\left(\frac{E}{Gc}\right) = 0$	0	1
	0.2	0.9975
	0.4	0.9900
	0.6	0.9779
	0.8	0.9616
	1.0	0.9416

Note that absurd results are obtained by Berger's method for movable edge conditions.



$\frac{I^*}{T}$ VS β FOR
SIMPLY SUPPORTED SQUARE PLATE
(IMMOVABLE EDGES)

----- = REF [3B]
————— = PRESENT STUDY.

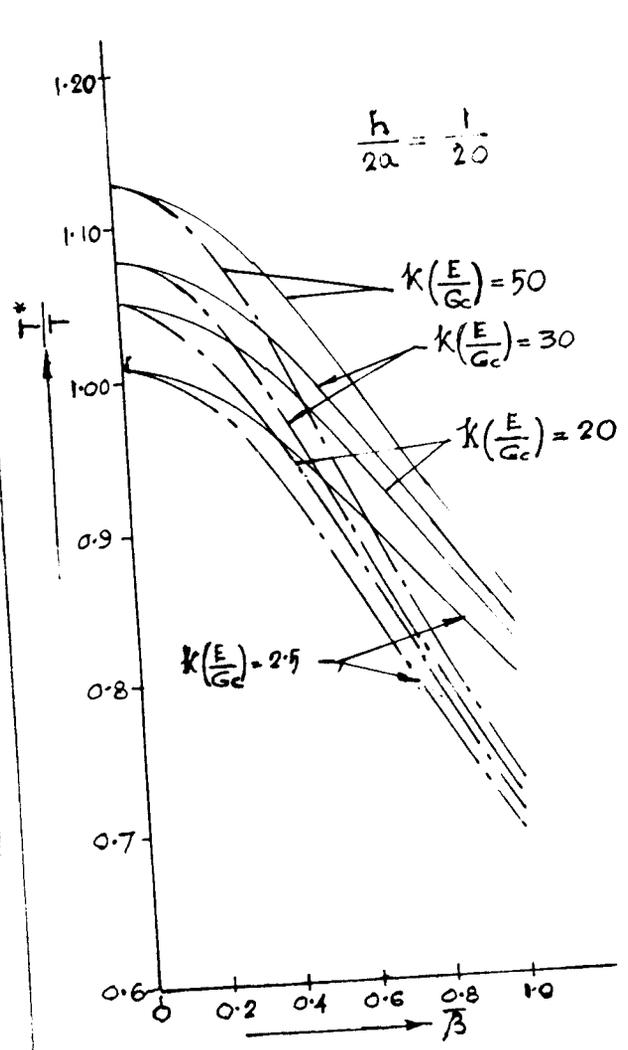


FIG. - 4

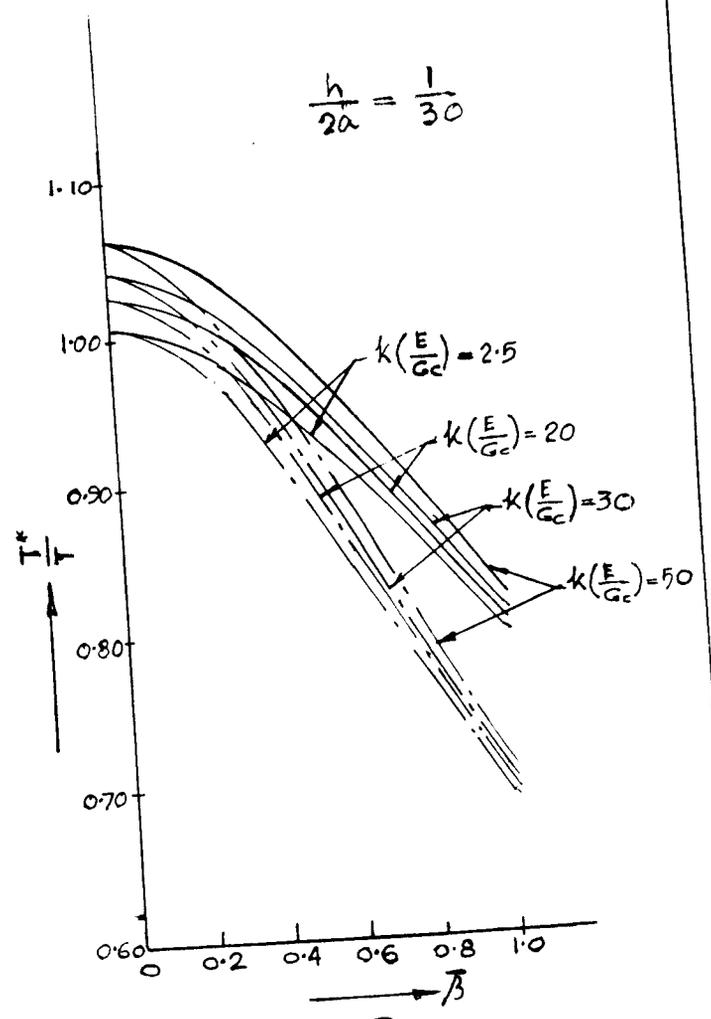
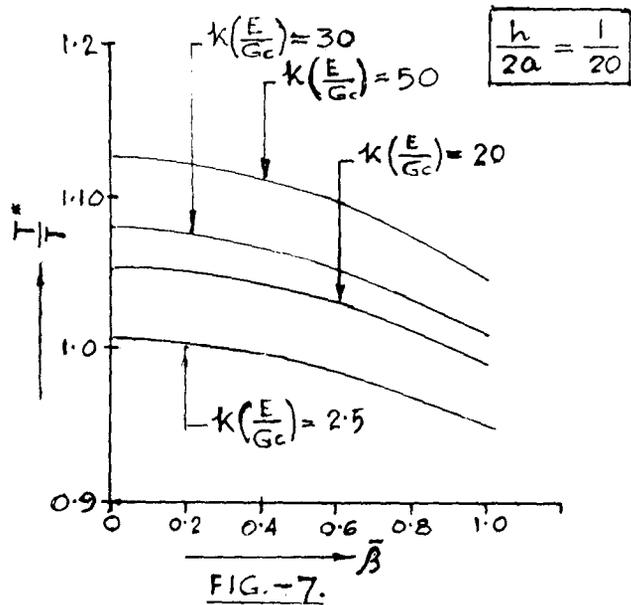
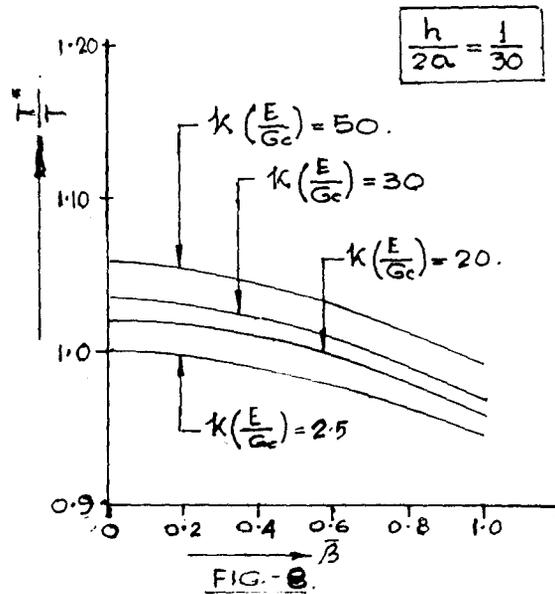
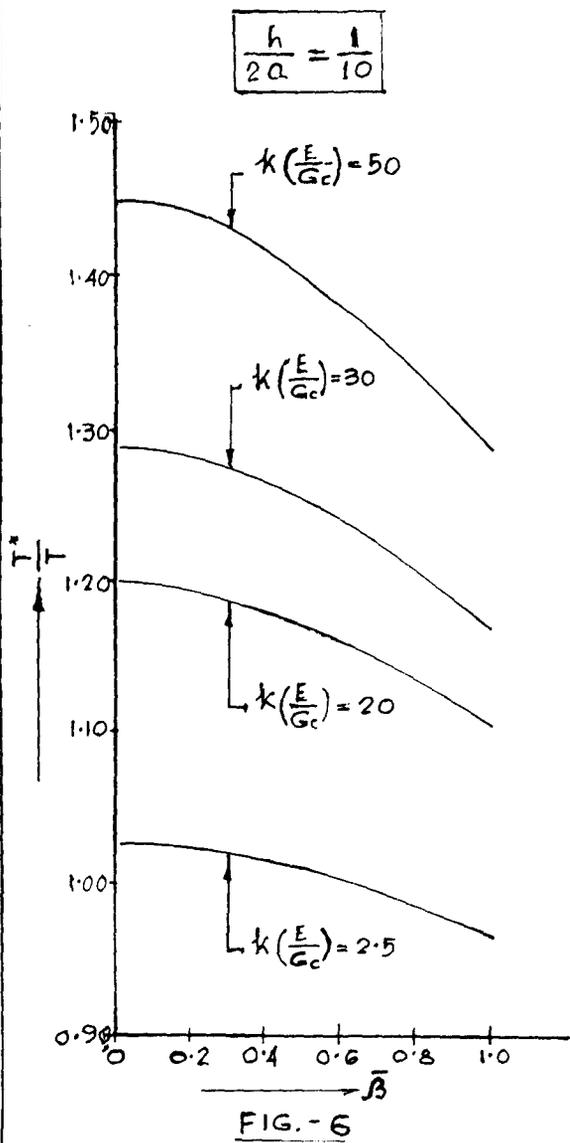
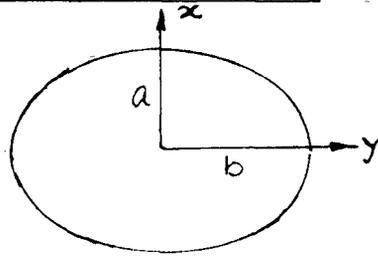


FIG. - 5

$\frac{I^*}{T}$ VS $\bar{\beta}$ FOR
SIMPLY SUPPORTED SQUARE PLATE.
(MOVABLE EDGES.)



(ii) Large deflections of uniformly loaded elliptical plate with clamped edges :



The plate geometry and co-ordinate system are shown in the fig.9

FIG.-9

The elliptical plate with semi-axes a and b is clamped along the boundary whose equation is given by

$$1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \quad \dots (46)$$

For static deflection let us rewrite the differential equation 18(a) and 18(b) in the following forms by replacing the inertial term by the corresponding term due to mechanical loading.

$$\begin{aligned} & \nabla^4 W + \frac{6}{5(1-\nu^2)} \cdot k \left(\frac{E}{G_c} \right) \cdot \frac{\bar{\alpha}^2 h^2}{12} \nabla^2 \left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2} \right) \\ & + \frac{3\lambda}{5(1-\nu^2)} \cdot k \left(\frac{E}{G_c} \right) \nabla^2 \left[\nabla^2 W \left\{ \left(\frac{\partial W}{\partial x} \right)^2 + \left(\frac{\partial W}{\partial y} \right)^2 \right\} \right. \\ & \left. + 2 \left\{ \frac{\partial^2 W}{\partial x^2} \left(\frac{\partial W}{\partial x} \right)^2 + \frac{\partial^2 W}{\partial y^2} \left(\frac{\partial W}{\partial y} \right)^2 \right\} + 4 \cdot \frac{\partial^2 W}{\partial x \partial y} \cdot \frac{\partial W}{\partial y} \cdot \frac{\partial W}{\partial x} \right] \\ & - \bar{\alpha}^2 \left[\frac{\partial^2 W}{\partial x^2} + \nu \cdot \frac{\partial^2 W}{\partial y^2} \right] - \frac{6\lambda}{h^2} \left[\nabla^2 W \left\{ \left(\frac{\partial W}{\partial x} \right)^2 + \left(\frac{\partial W}{\partial y} \right)^2 \right\} + \right. \\ & \left. 2 \left\{ \frac{\partial^2 W}{\partial x^2} \left(\frac{\partial W}{\partial x} \right)^2 + \frac{\partial^2 W}{\partial y^2} \left(\frac{\partial W}{\partial y} \right)^2 \right\} + 4 \frac{\partial^2 W}{\partial x \partial y} \cdot \frac{\partial W}{\partial x} \cdot \frac{\partial W}{\partial y} \right] = \frac{q_0}{D} \quad \dots (47) \end{aligned}$$

where q_0 is the intensity of continuously distributed load,
and

$$\frac{\bar{\alpha}^2 h^2}{12} = \frac{\partial u_0}{\partial x} + \nu \frac{\partial v_0}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + \frac{\nu}{2} \left(\frac{\partial w}{\partial y} \right)^2 \quad \dots (48)$$

For movable edge condition

$$\bar{\alpha} = 0$$

Let us assume the deflection function in the following form

$$W = W_0 \left[1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right]^2 \quad \dots (49)$$

Clearly this form of W satisfies the clamped edge conditions of the plate.

Now putting (49) in (48) and integrating over the area of the plate we get,

$$\bar{\alpha}^2 = \frac{4W_0^2}{h^2} \left(\frac{1}{a^2} + \frac{\nu}{b^2} \right) \quad \dots (50)$$

Inserting (49) in (47), remembering (50) and applying Galerking's technique, as before, we get the cubic equation determining the deflection function $\frac{W_0}{h}$

$$\frac{W_0}{h} + \left[\frac{W_0}{h} \right]^3 \frac{1}{\frac{1}{a^4} + \frac{1}{b^4} + \frac{2}{3a^2b^2}} \left[\frac{2h^2}{5(1-\nu^2)} \cdot k \left(\frac{E}{G_c} \right) \left(\frac{1}{a^2} + \frac{\nu}{b^2} \right) \right. \\ \left. \cdot \left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{1+\nu}{3a^2b^2} \right) + \frac{18\lambda h^2}{25(1-\nu^2)} \cdot k \left(\frac{E}{G_c} \right) \left(\frac{1}{a^6} + \frac{1}{b^6} + \frac{7}{9a^4b^2} + \frac{7}{9a^2b^4} \right) \right]$$

$$+ \frac{1}{3} \left(\frac{1}{a^2} + \frac{\nu}{b^2} \right)^2 + \frac{24\lambda}{35} \left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{2}{3a^2b^2} \right) \Bigg]$$

$$= \frac{q_0 (1-\nu^2)}{2 Eh^4} \cdot \frac{1}{\frac{1}{a^4} + \frac{1}{b^4} + \frac{2}{3a^2b^2}}$$

.... (51)

(iii) Large deflections of simply supported isosceles right angled triangular plate :

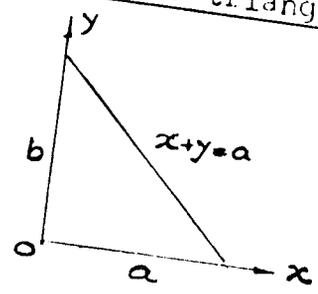


FIG-10

The plate geometry and co-ordinate system are shown in the fig.10

Here the deflection function is chosen in the following form

$$W = W_0 \left[\sin \frac{\pi x}{a} \cdot \sin \frac{2\pi y}{a} + \sin \frac{\pi y}{a} \cdot \sin \frac{2\pi x}{a} \right]$$

.... (52)

This form of W satisfies the following simply supported edge conditions, namely -

$$\left. \begin{aligned} W &= 0 \text{ at } x = 0, a \\ W &= 0 \text{ at } y = 0, a \\ \frac{\partial^2 W}{\partial x^2} &= 0 \text{ at } x = 0, a \\ \frac{\partial^2 W}{\partial y^2} &= 0 \text{ at } y = 0, a \\ W &= 0 \text{ at } x + y = a \\ \frac{\partial^2 W}{\partial \nu^2} &= 0 \text{ at } x + y = a \end{aligned} \right\}$$

.... (53)

and
where

$$\frac{\partial}{\partial \nu} \equiv \frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right)$$

Putting (52) in (48) and using the same method as in the case of elliptic plate we get

$$\bar{\alpha}^2 = \frac{15}{2} \cdot \frac{W_0^2 \pi^2}{a^2 h^2} \cdot (1+\nu) \quad \dots (54)$$

Now inserting (52) in (48), remembering (54) and applying Galerkin's technique as before we get the following cubic equation determining $\left(\frac{W_0}{h}\right)$

$$\begin{aligned} \frac{W_0}{h} + \left(\frac{W_0}{h}\right)^3 \left[6.8665 \kappa \left(\frac{E}{G_c}\right) \cdot \frac{h^2}{a^2} + 4.875 \lambda + \right. \\ \left. 1.2675 + 18.6086 \lambda \kappa \left(\frac{E}{G_c}\right) \cdot \frac{h^2}{a^2} \right] \\ = \frac{48(1-\nu^2)}{25\pi^6} \cdot \frac{q_0 a^4}{E h^4} \quad \dots (55) \end{aligned}$$

Numerical Results :

Numerical results are presented here in tabular forms both for movable as well as immovable edges for different moderately thick isotropic plates and compared with other known results. The results of the isosceles right angled triangular plates are new.

For free vibrations the ratios of the non-linear period T^* of vibrations including the effects of transverse shear deformation to the corresponding linear period T of the classical plate (not including transverse shear and rotatory inertia) are computed for various thickness parameter and material constants at different nondimensional amplitudes of vibration. It is to be noted that the effects of rotatory inertia have been neglected in each case because these are considered to

be small compared with effects due to transverse shear deformation as the plate is undergoing flexural vibrations.

To study the non-linear static behaviours of the plates the nondimensional deflection functions at the centre $\frac{w_0}{h}$ have been obtained for different values of the nondimensional load parameter $\frac{q_0 a^4}{Dh}$.

It is observed that for moderately thick plates, the non-linear periods are dependent on the thickness parameter whereas they are independent of the same for thin plates.

STATIC DEFLECTIONS OF CLAMPED ELLIPTICAL PLATE.

TABLE - 12.

$\lambda = 0.18 [18], \kappa \left(\frac{E}{G_c} \right) = 1.$

$\frac{a}{b} = 1$	$\frac{W_0}{h}$		$\frac{a}{b} = 1.5$	$\frac{W_0}{h}$		$\frac{a}{b} = 2.0$	$\frac{W_0}{h}$				
	REF. [46]	IMMOVABLE EDGES. PRESENT STUDY		MOVABLE EDGES. PRESENT STUDY	REF. [46]		IMMOVABLE EDGES. PRESENT STUDY	MOVABLE EDGES. PRESENT STUDY	REF. [46]	IMMOVABLE EDGES. PRESENT STUDY	MOVABLE EDGES. PRESENT STUDY
3.2756	0.5	0.5123	0.5392	9.2855	0.5	0.5220	0.5385	24.1379	0.5	0.5263	0.5382
8.6227	1.0	1.0619	1.2350	24.4213	1.0	1.1138	1.2320	63.4326	1.0	1.1404	1.2281
18.1126	1.5	1.6235	2.0346	51.2577	1.5	1.7365	2.0270	133.0412	1.5	1.7975	2.0170
33.8169	2.0	2.1885	2.8548	95.6450	2.0	2.3622	2.8420	248.1205	2.0	2.4595	2.8253

$$\frac{W_0}{h} \text{ vs. } \frac{q_0 a^4}{Eh^4} \text{ FOR}$$

STATIC DEFLECTIONS OF CLAMPED
ELLIPTICAL PLATE.

$$\lambda = 0.18, \quad k\left(\frac{E}{E_c}\right) = 1$$

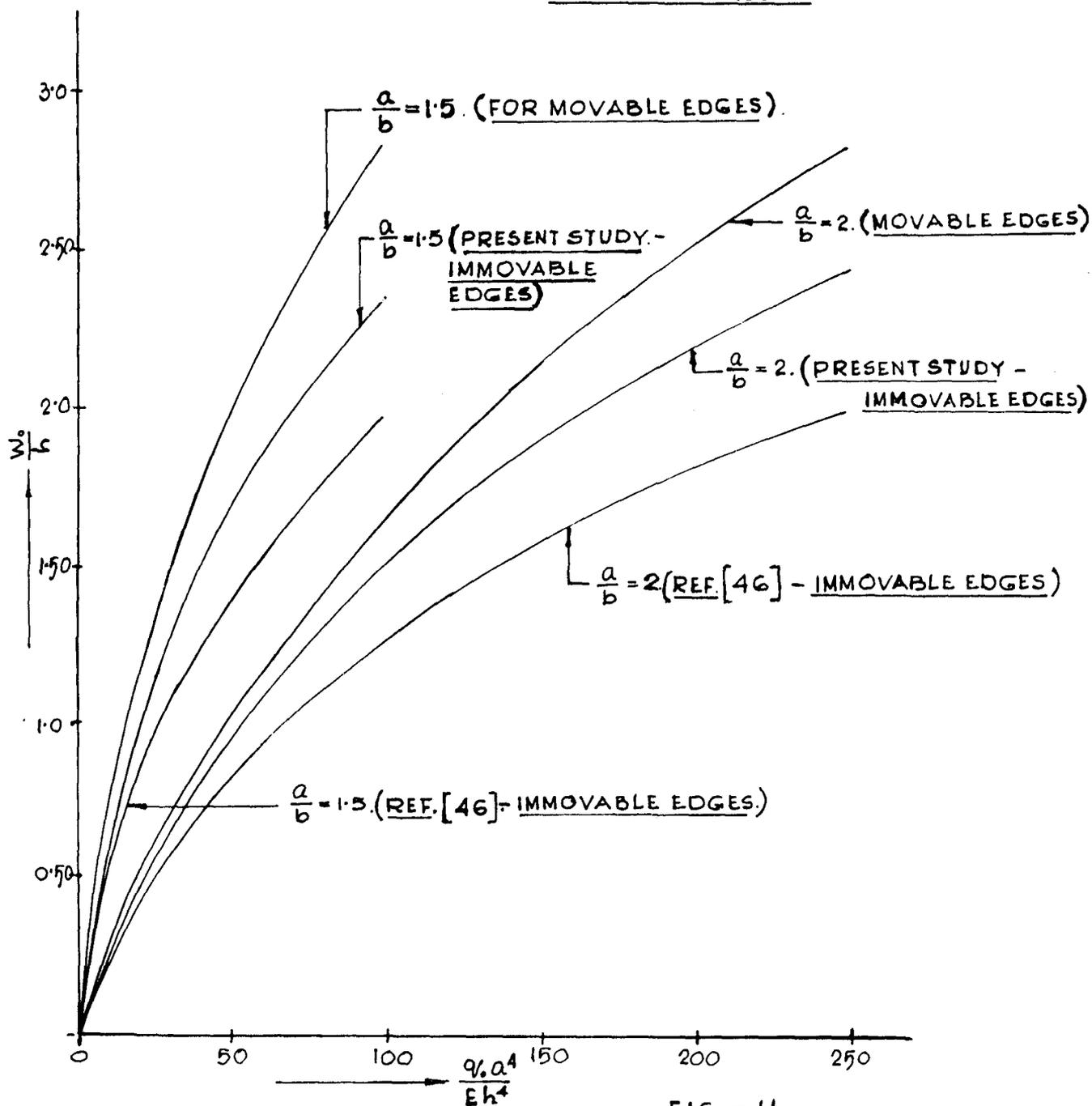


FIG. - 11

STATIC DEFLECTIONS OF SIMPLY SUPPORTED
ISOSCELES RIGHT ANGLED TRIANGULAR PLATE.

TABLE - 13 $\lambda = 0.09 [18], \kappa \left(\frac{E}{G_c}\right) = 1.$

$\frac{q_0 a^4}{Eh^4}$	$\frac{w_0}{h}$	
	IMMOVABLE EDGES.	MOVABLE EDGES.
500	0.5735	0.7322
1000	0.8232	1.1428
1500	0.9909	1.4225
2000	1.1210	1.6393

$$\frac{w_0}{h} \text{ VS } \frac{q_0 a^4}{E h^4} \text{ FOR}$$

STATIC DEFLECTIONS OF SIMPLY SUPPORTED
RIGHT ANGLED ISOSCELES TRIANGULAR PLATE.

$$\lambda = 0.18,$$

$$k \left(\frac{E}{G_c} \right) = 1.$$

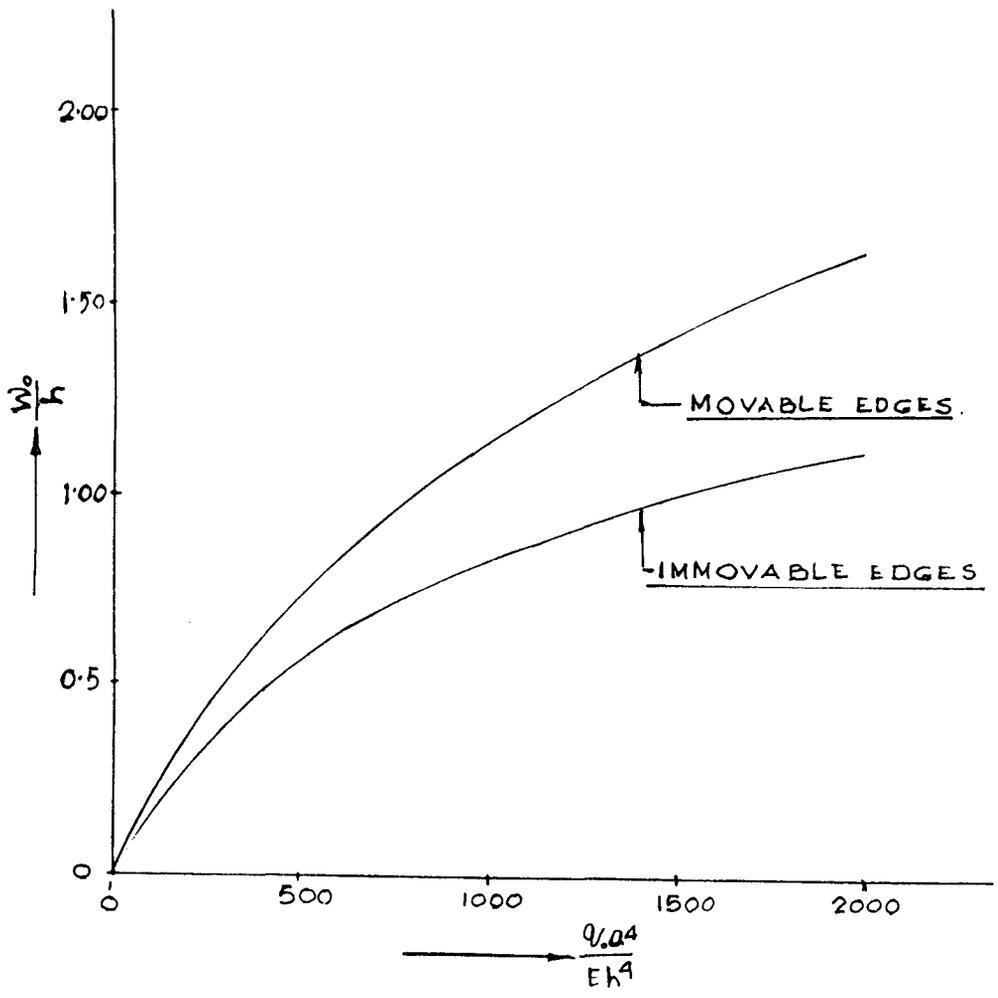


FIG. - 12.

B. *Vibrations of clamped circular plates.

Let us consider a thick circular plate of radius a . The origin is located at the centre of the plates. The polar co-ordinates are chosen in the analysis. The deflection of the plate is of the same order of magnitude as the thickness of the plate.

For circular plate of radius a , let us choose the deflection function in the following form

$$W = A_0 \tau(t) \left[1 - \frac{r^2}{a^2} \right]^2 \quad \dots (56)$$

clearly this form of W satisfies the following clamped edge conditions,

$$(W)_{r=a} = 0$$

and

$$\left(\frac{\partial W}{\partial r} \right)_{r=a} = 0$$

To evaluate the coupling parameter $\bar{\alpha}^2$, let us now recall our attention to equation 19(b) of Chapter I. Multiplying this equation by the integrating factor r^ν , putting (56) in this exact equation and finally integrating the equation between the limits 0 and a , the constant $\bar{\alpha}^2$ is obtained in the following form

$$\bar{\alpha}^2 = \frac{1536\nu}{a^{1+\nu}(3+\nu)(5+\nu)(7+\nu)} \cdot \frac{A_0^2}{h^2} \quad \dots (57)$$

Putting (56) in 19(a) of 1st Chapter, considering (57) and applying Galerkin's error minimising technique one gets the following differential equation for the time function $\tau(t)$

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133(1), PP. 185 - 188, 1989.

$$\begin{aligned}
& \left[\frac{6}{5} \cdot \frac{a^2}{h^2 c_p^2} + \frac{4}{5} \frac{\rho}{G_c} \right] \ddot{\zeta}(t) + \frac{32}{3a^2} \cdot \zeta(t) \\
& + \left[864 \cdot 79872 \cdot k \left(\frac{E}{G_c} \right) \frac{\nu}{(1-\nu^2)(\nu+5)(\nu+7)} \cdot \frac{A_0^2}{h^2} \cdot \frac{h^2}{a^4} \right. \\
& + 1541 \cdot 2224 \cdot \frac{A_0^2}{a^2 h^2} \cdot \frac{\nu}{(\nu+3)(\nu+5)(\nu+7)} \\
& \left. + 10 \cdot 24 \lambda k \left(\frac{E}{G_c} \right) \cdot \frac{1}{(1-\nu^2)} \cdot \frac{A_0^2}{h^2} \cdot \frac{h^2}{a^4} + 7 \cdot 3142 \frac{\lambda}{a^2} \cdot \frac{A_0^2}{h^2} \right] \zeta^3(t) = 0
\end{aligned}$$

.... (58)

The ratio of the non-linear and linear time period is obtained as before in the form

$$\begin{aligned}
\frac{T^*}{T} = \frac{2K}{\pi} & \left[\frac{1}{\left\{ 1 + 81 \cdot 1255 k \left(\frac{E}{G_c} \right) \frac{h^2}{a^2} \frac{\nu}{(1-\nu^2)(\nu+5)(\nu+7)} \bar{\beta}^2 \right.} \right. \\
& + 144 \cdot 4986 \cdot \frac{\nu}{(\nu+3)(\nu+5)(\nu+7)} \bar{\beta}^2 \\
& \left. \left. + 0 \cdot 9606 \frac{\lambda}{(1-\nu^2)} \cdot k \left(\frac{E}{G_c} \right) \frac{h^2}{a^2} \bar{\beta}^2 + 0 \cdot 6857062 \lambda \bar{\beta}^2 \right\} \right]^{1/2}
\end{aligned}$$

.... (59)

Numerical results : -

Numerical results have been computed here in tabular form both for movable as well as immovable edge conditions as in the previous case.

RATIO OF NON-LINEAR TO LINEAR PERIOD FOR THE FUNDAMENTAL
MODE OF VIBRATION OF A CLAMPED CIRCULAR PLATE.

$\nu = 0.3, \lambda = 0.18$

IMMOVABLE EDGES. (TABLE 14 TO 19)

PRESENT STUDY

REF. [40]

TABLE - 14

$\frac{T^*}{T}$					$\frac{T^*}{T}$				
$\bar{\beta} = \frac{A_0}{h}$	$\frac{h}{a} = 0.2$ $k(\frac{E}{Gc}) = 8.1971$	$\frac{h}{a} = 0.15$ $k(\frac{E}{Gc}) = 8.8133$	$\frac{h}{a} = 0.10$ $k(\frac{E}{Gc}) = 10.4869$	$\frac{h}{a} = 0.05$ $k(\frac{E}{Gc}) = 19.3165$	$\bar{\beta} = \frac{A_0}{h}$	$\frac{h}{a} = 0.20$ $k(\frac{E}{Gc}) = 8.1971$	$\frac{h}{a} = 0.15$ $k(\frac{E}{Gc}) = 8.8133$	$\frac{h}{a} = 0.10$ $k(\frac{E}{Gc}) = 10.4869$	$\frac{h}{a} = 0.05$ $k(\frac{E}{Gc}) = 19.3165$
0	1.0000	1.0000	1.0000	1.0000	0	1.0000	1.0000	1.0000	1.0000
0.20	0.9891	0.9907	0.9919	0.9926	0.2	0.9921	0.9924	0.9927	0.9928
0.40	0.9584	0.9661	0.9688	0.9716	0.4	0.9699	0.9710	0.9718	0.9722
0.60	0.9138	0.9251	0.9339	0.9394	0.6	0.9366	0.9388	0.9402	0.9410
0.80	0.8576	0.8774	0.8908	0.8993	0.8	0.8965	0.8995	0.9015	0.9026
1.00	0.8029	0.8257	0.8435	0.8547	1.0	0.8533	0.8568	0.8591	0.8603

TABLE - 15

THIN PLATE.

PRESENT STUDY

REF. [40]

$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}, K\left(\frac{E}{G_c}\right) = 0$	$\frac{T^*}{T}$
0	1.0000	1.0000
0.20	0.9933	0.9928
0.40	0.9739	0.9724
0.60	0.9441	0.9413
0.80	0.9068	0.9029
1.00	0.8648	0.8607

TABLE - 16

$\bar{\beta} = \frac{A_0}{h}$	$k\left(\frac{E}{G_c}\right) = 8.1971$			
	$\frac{h}{a} = 0.2$ $\frac{T^*}{T}$	$\frac{h}{a} = 0.15$ $\frac{T^*}{T}$	$\frac{h}{a} = 0.10$ $\frac{T^*}{T}$	$\frac{h}{a} = 0.05$ $\frac{T^*}{T}$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9890	0.9909	0.9922	0.9930
0.40	0.9584	0.9651	0.9699	0.9729
0.60	0.9138	0.9263	0.9361	0.9421
0.80	0.8576	0.8793	0.8943	0.9036
1.00	0.8029	0.8283	0.8480	0.8605

TABLE - 17

$\bar{\beta} = \frac{A_0}{h}$	$k\left(\frac{E}{G_c}\right) = 10.4869$			
	$\frac{h}{a} = 0.2$ $\frac{T^*}{T}$	$\frac{h}{a} = 0.15$ $\frac{T^*}{T}$	$\frac{h}{a} = 0.10$ $\frac{T^*}{T}$	$\frac{h}{a} = 0.05$ $\frac{T^*}{T}$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9879	0.9962	0.9919	0.9929
0.40	0.9542	0.9626	0.9688	0.9726
0.60	0.9051	0.9216	0.9339	0.9415
0.80	0.8477	0.8721	0.8908	0.9027
1.00	0.7879	0.8189	0.8435	0.8593

TABLE - 18

$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$ FOR $\frac{h}{a} = 0.20$			
	$k\left(\frac{E}{G_c}\right) = 8.1971$	$k\left(\frac{E}{G_c}\right) = 8.8133$	$k\left(\frac{E}{G_c}\right) = 10.4869$	$k\left(\frac{E}{G_c}\right) = 19.3165$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9890	0.9887	0.9879	0.9834
0.40	0.9584	0.9572	0.9542	0.9385
0.60	0.9138	0.9110	0.9051	0.8758
0.80	0.8576	0.8564	0.8477	0.8061
1.00	0.8029	0.7988	0.7879	0.7369

TABLE - 19

$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$ FOR $\frac{h}{a} = 0.10$			
	$k\left(\frac{E}{G_c}\right) = 8.1971$	$k\left(\frac{E}{G_c}\right) = 8.8133$	$k\left(\frac{E}{G_c}\right) = 10.4869$	$k\left(\frac{E}{G_c}\right) = 19.3165$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9922	0.9921	0.9919	0.9967
0.40	0.9699	0.9696	0.9688	0.9647
0.60	0.9361	0.9355	0.9339	0.9255
0.80	0.8943	0.8933	0.8908	0.8781
1.00	0.8480	0.8467	0.8435	0.8267

MOVABLE EDGES (TABLE 20 TO 24)

TABLE - 20.

$\bar{\beta} = \frac{A_0}{h}$	$k\left(\frac{E}{G_c}\right) = 8.1971$			
	$\frac{h}{a} = 0.20$	$\frac{h}{a} = 0.15$	$\frac{h}{a} = 0.10$	$\frac{h}{a} = 0.05$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9972	0.9976	0.9979	0.9981
0.40	0.9891	0.9914	0.9918	0.9924
0.60	0.9759	0.9793	0.9817	0.9833
0.80	0.9583	0.9461	0.9683	0.9708
1.00	0.9371	0.9456	0.9518	0.9555

TABLE - 21.

$\bar{\beta} = \frac{A_0}{h}$	$k\left(\frac{E}{G_c}\right) = 8.8133$			
	$\frac{h}{a} = 0.20$	$\frac{h}{a} = 0.15$	$\frac{h}{a} = 0.10$	$\frac{h}{a} = 0.05$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9972	0.9976	0.9979	0.9981
0.40	0.9888	0.9905	0.9917	0.9924
0.60	0.9753	0.9790	0.9816	0.9832
0.80	0.9573	0.9635	0.9680	0.9707
1.00	0.9357	0.9447	0.9514	0.9554

TABLE - 22

$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$ FOR $\frac{h}{a} = 0.20$			
	$k\left(\frac{E}{G_c}\right) = 8.1971$	$k\left(\frac{E}{G_c}\right) = 8.8133$	$k\left(\frac{E}{G_c}\right) = 10.4869$	$k\left(\frac{E}{G_c}\right) = 19.3165$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9972	0.9971	0.9969	0.9959
0.40	0.9890	0.9887	0.9880	0.9842
0.60	0.9759	0.9753	0.9737	0.9655
0.80	0.9583	0.9573	0.9547	0.9410
1.00	0.9371	0.9357	0.9318	0.9123

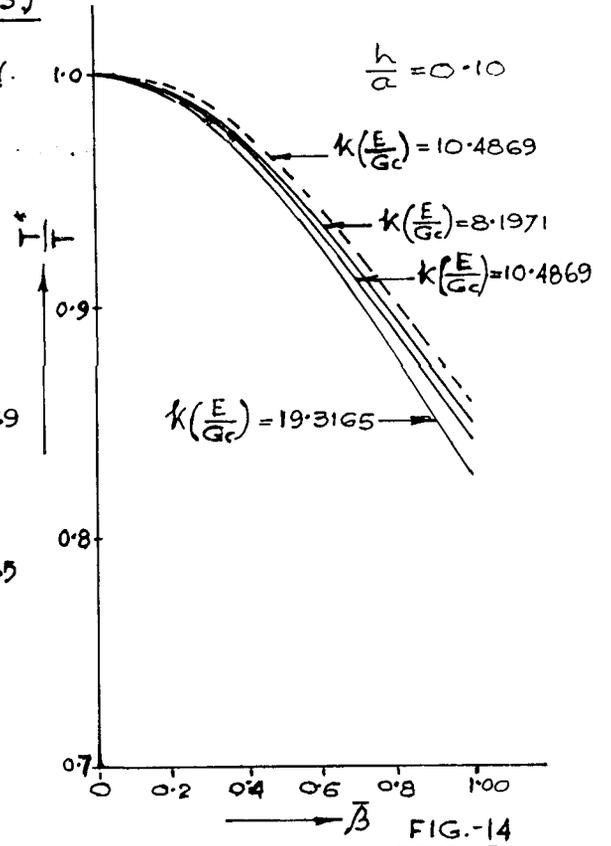
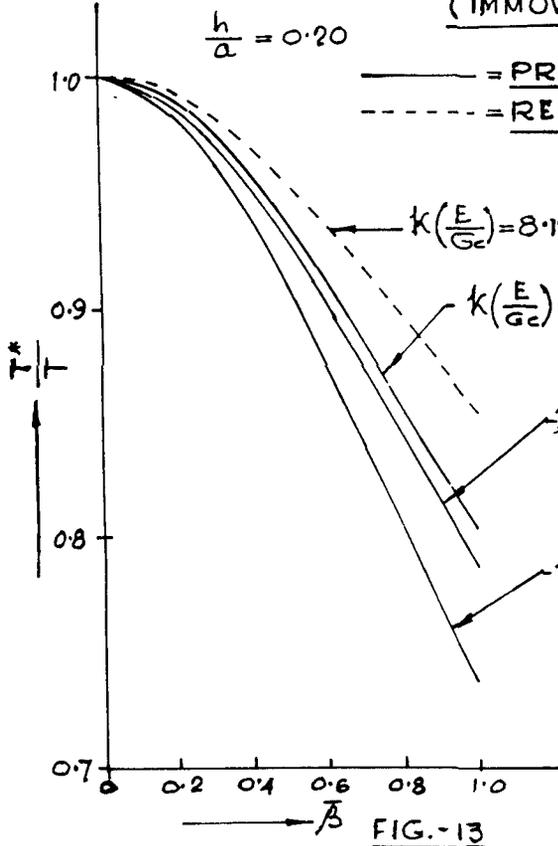
TABLE - 23

$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$ FOR $\frac{h}{a} = 0.10$			
	$k\left(\frac{E}{G_c}\right) = 8.1971$	$k\left(\frac{E}{G_c}\right) = 8.8133$	$k\left(\frac{E}{G_c}\right) = 10.4869$	$k\left(\frac{E}{G_c}\right) = 19.3165$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9979	0.9979	0.9978	0.9976
0.40	0.9918	0.9917	0.9915	0.9905
0.60	0.9817	0.9816	0.9812	0.9791
0.80	0.9683	0.9680	0.9673	0.9542
1.00	0.9518	0.9514	0.9504	0.9450

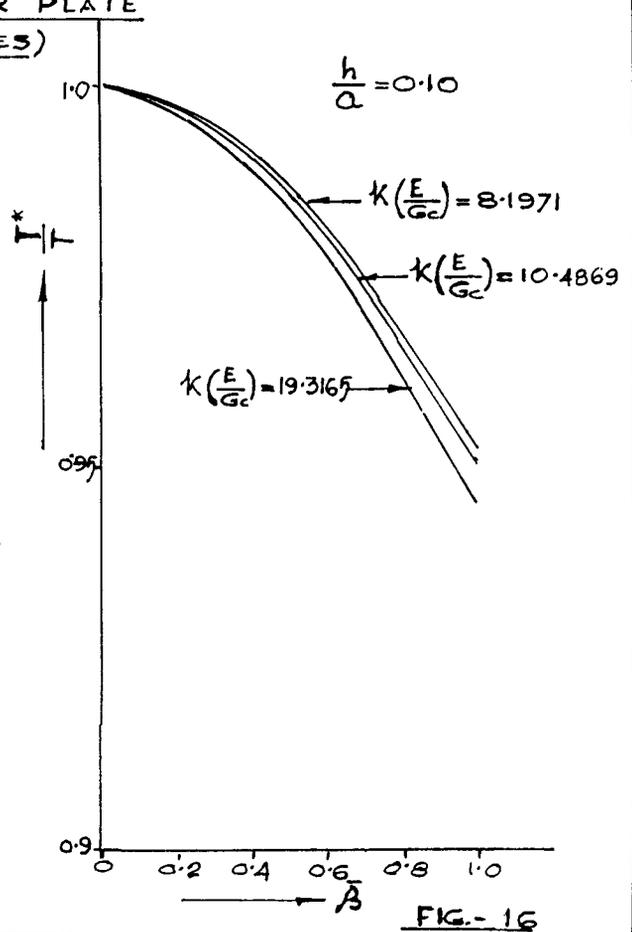
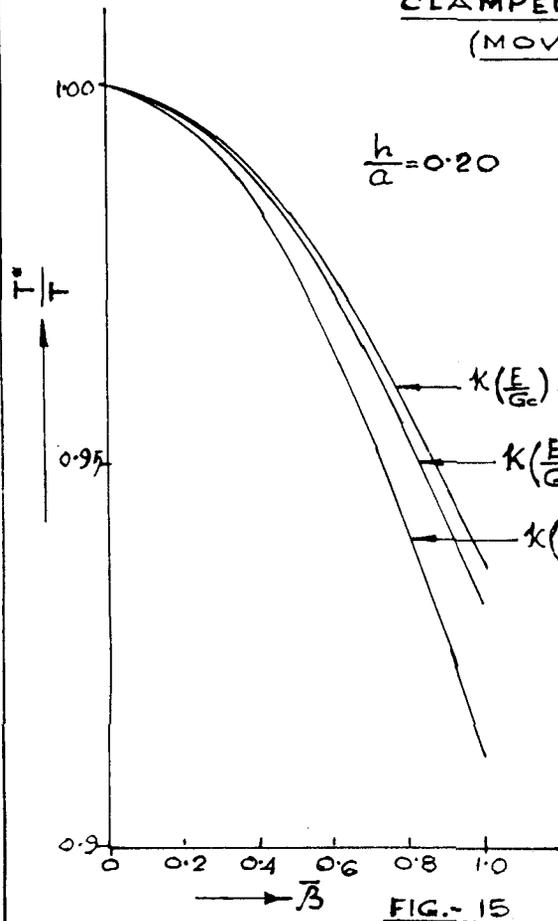
TABLE - 24

$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$ FOR $\nu = 0.3, \lambda = 0.18$			
	$\frac{h}{a} = 0.20$ $k(\frac{E}{G_c}) = 8.1971$	$\frac{h}{a} = 0.15$ $k(\frac{E}{G_c}) = 8.8133$	$\frac{h}{a} = 0.10$ $k(\frac{E}{G_c}) = 10.4869$	$\frac{h}{a} = 0.05$ $k(\frac{E}{G_c}) = 19.3165$
0	1.0000	1.0000	1.0000	1.0000
0.20	0.9972	0.9976	0.9978	0.9980
0.40	0.9890	0.9904	0.9915	0.9921
0.60	0.9759	0.9789	0.9812	0.9826
0.80	0.9583	0.9635	0.9673	0.9697
1.00	0.9371	0.9447	0.9504	0.9538

**CLAMPED CIRCULAR PLATE
(IMMOVABLE EDGES)**



**CLAMPED CIRCULAR PLATE
(MOVABLE EDGES)**



C. Large Amplitudes, Transverse shear Deformation and Rotatory Inertia on Free Vibrations of Moderately Thick Polygonal Plates.*

Formulation of the differential equation :

Let us consider the free vibrations of thick polygonal plates of thickness h .

In a complex co-ordinate system $z = x + iy$, $\bar{z} = x - iy$ the equations 18(a) and 18(b) of the 1st chapter change. Let

$$z = f(\xi) \quad \dots \quad \dots \quad \dots \quad (60)$$

be the analytic function which maps the given shape in the z -plane on to a unit circle in the ξ -plane. Substituting the relation (60) into the transformed equations in (z, \bar{z}) the following set of differential equations in $(\xi, \bar{\xi})$ co-ordinates have been obtained:

$$\begin{aligned} & 16 \left[\frac{\partial^4 W}{\partial \xi^2 \partial \bar{\xi}^2} \cdot \left(\frac{dz}{d\xi} \right)^3 \cdot \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^3 - \frac{\partial^3 W}{\partial \xi^2 \partial \bar{\xi}} \cdot \frac{d^2 \bar{z}}{d\bar{\xi}^2} \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^2 \left(\frac{dz}{d\xi} \right)^3 \right. \\ & \left. - \frac{\partial^3 W}{\partial \bar{\xi}^2 \partial \xi} \cdot \frac{d^2 z}{d\xi^2} \left(\frac{dz}{d\xi} \right)^2 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^3 + \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \cdot \frac{d^2 z}{d\xi^2} \cdot \frac{d^2 \bar{z}}{d\bar{\xi}^2} \cdot \left(\frac{dz}{d\xi} \right)^2 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^2 \right] \\ & + \frac{2}{5(1-\nu^2)} k \left(\frac{E}{G} \right) \alpha^2 h^2 \gamma^2(t) \left[(1-\nu) \left\{ \frac{\partial^4 W}{\partial \xi^3 \partial \bar{\xi}} \cdot \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^4 \cdot \left(\frac{dz}{d\xi} \right)^2 \right. \right. \\ & \left. \left. - 3 \frac{\partial^3 W}{\partial \xi^2 \partial \bar{\xi}} \cdot \frac{d^2 z}{d\xi^2} \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^4 \cdot \frac{dz}{d\xi} + 3 \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \left(\frac{d^2 z}{d\xi^2} \right)^2 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^4 \right. \right. \\ & \left. \left. - \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \cdot \frac{d^3 z}{d\xi^3} \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^4 \cdot \frac{d\bar{z}}{d\bar{\xi}} \right\} + (1-\nu) \left\{ \frac{\partial^4 W}{\partial \bar{\xi}^3 \partial \xi} \cdot \left(\frac{dz}{d\xi} \right)^4 \cdot \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^2 \right. \right. \end{aligned}$$

* Accepted for publication in the Journal of Applied Mechanics (ASME) - U.S.A., June 1990.

$$\begin{aligned}
& -3 \frac{\partial^3 W}{\partial \xi^2 \partial \bar{\xi}} \cdot \frac{d^2 \bar{z}}{d \xi^2} \left(\frac{dz}{d \xi} \right)^4 \frac{d \bar{z}}{d \bar{\xi}} + 3 \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \left(\frac{d^2 \bar{z}}{d \xi^2} \right)^2 \left(\frac{dz}{d \xi} \right)^4 \\
& - \left. \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \cdot \frac{d^3 \bar{z}}{d \xi^3} \cdot \left(\frac{dz}{d \xi} \right)^4 \cdot \frac{d \bar{z}}{d \bar{\xi}} \right\} + 2(1+\nu) \left\{ \frac{\partial^4 W}{\partial \xi^2 \partial \bar{\xi}^2} \left(\frac{dz}{d \xi} \right)^3 \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^3 \right. \\
& - \frac{\partial^3 W}{\partial \xi^2 \partial \bar{\xi}} \cdot \frac{d^2 \bar{z}}{d \xi^2} \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 \left(\frac{dz}{d \xi} \right)^3 - \frac{\partial^3 W}{\partial \bar{\xi}^2 \partial \xi} \cdot \frac{d^2 z}{d \bar{\xi}^2} \left(\frac{dz}{d \xi} \right)^2 \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^3 \\
& \left. + \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \cdot \frac{d^2 z}{d \xi^2} \cdot \frac{d^2 \bar{z}}{d \bar{\xi}^2} \cdot \left(\frac{dz}{d \xi} \right)^2 \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 \right] \\
& + \frac{96\lambda}{5(1-\nu^2)} k \left(\frac{E}{G_0} \right) \left[4 \left\{ \frac{\partial^4 W}{\partial \xi^2 \partial \bar{\xi}^2} \cdot \frac{\partial W}{\partial \xi} \cdot \frac{\partial W}{\partial \bar{\xi}} \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 \left(\frac{dz}{d \xi} \right)^2 \right. \right. \\
& - \frac{\partial^3 W}{\partial \xi^2 \partial \bar{\xi}} \cdot \frac{\partial W}{\partial \xi} \cdot \frac{\partial W}{\partial \bar{\xi}} \cdot \frac{d^2 \bar{z}}{d \bar{\xi}^2} \cdot \frac{d \bar{z}}{d \bar{\xi}} \cdot \left(\frac{dz}{d \xi} \right)^2 \\
& - \left. \frac{\partial^3 W}{\partial \bar{\xi}^2 \partial \xi} \cdot \frac{\partial W}{\partial \xi} \cdot \frac{\partial W}{\partial \bar{\xi}} \cdot \frac{d^2 z}{d \xi^2} \cdot \frac{dz}{d \xi} \cdot \left(\frac{dz}{d \xi} \right)^2 \right. \\
& \left. + \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \cdot \frac{\partial W}{\partial \xi} \cdot \frac{\partial W}{\partial \bar{\xi}} \cdot \frac{d^2 z}{d \xi^2} \cdot \frac{d^2 \bar{z}}{d \bar{\xi}^2} \cdot \frac{dz}{d \xi} \cdot \frac{d \bar{z}}{d \bar{\xi}} \right\} \\
& + \left\{ \frac{\partial^4 W}{\partial \bar{\xi}^3 \partial \xi} \left(\frac{\partial W}{\partial \xi} \right)^2 \left(\frac{dz}{d \xi} \right)^2 \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 - 3 \frac{\partial^3 W}{\partial \bar{\xi}^2 \partial \xi} \left(\frac{\partial W}{\partial \xi} \right)^2 \frac{d^2 \bar{z}}{d \bar{\xi}^2} \left(\frac{dz}{d \xi} \right)^2 \cdot \frac{d \bar{z}}{d \bar{\xi}} \right. \\
& \left. + 3 \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \left(\frac{\partial W}{\partial \xi} \right)^2 \left(\frac{d^2 \bar{z}}{d \bar{\xi}^2} \right)^2 \left(\frac{dz}{d \xi} \right)^2 - \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \left(\frac{\partial W}{\partial \xi} \right)^2 \frac{d^3 \bar{z}}{d \bar{\xi}^3} \left(\frac{dz}{d \xi} \right)^2 \frac{d \bar{z}}{d \bar{\xi}} \right\} \\
& + \left\{ \frac{\partial^4 W}{\partial \xi^3 \partial \bar{\xi}} \left(\frac{\partial W}{\partial \bar{\xi}} \right)^2 \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 \left(\frac{dz}{d \xi} \right)^2 - 3 \frac{\partial^3 W}{\partial \xi^2 \partial \bar{\xi}} \left(\frac{\partial W}{\partial \bar{\xi}} \right)^2 \frac{d^2 z}{d \xi^2} \cdot \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 \cdot \frac{dz}{d \xi} \right. \\
& \left. + 3 \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \left(\frac{\partial W}{\partial \bar{\xi}} \right)^2 \left(\frac{d^2 z}{d \xi^2} \right)^2 \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 - \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \left(\frac{\partial W}{\partial \bar{\xi}} \right)^2 \frac{d^3 z}{d \xi^3} \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 \cdot \frac{dz}{d \xi} \right\} \\
& + 4 \left(\frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \right)^3 \cdot \left(\frac{dz}{d \xi} \right)^2 \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 + 6 \left\{ \frac{\partial^3 W}{\partial \bar{\xi}^2 \partial \xi} \cdot \frac{\partial^2 W}{\partial \xi^2} \cdot \frac{\partial W}{\partial \bar{\xi}} \left(\frac{dz}{d \xi} \right)^2 \left(\frac{d \bar{z}}{d \bar{\xi}} \right)^2 \right.
\end{aligned}$$

$$\begin{aligned}
& \left. -3 \frac{\partial W}{\partial \xi} \left(\frac{\partial W}{\partial \xi} \right)^2 \left(\frac{d^2 z}{d\xi^2} \right)^2 \frac{d^2 \bar{z}}{d\bar{\xi}^2} \frac{d\bar{z}}{d\bar{\xi}} + \frac{\partial W}{\partial \xi} \left(\frac{\partial W}{\partial \xi} \right)^2 \frac{d^3 z}{d\xi^3} \frac{d^2 \bar{z}}{d\bar{\xi}^2} \frac{d\bar{z}}{d\bar{\xi}} \frac{dz}{d\xi} \right\} \\
& - \frac{24}{5} \frac{\rho}{Gc} \ddot{\gamma}(t) \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \left(\frac{dz}{d\xi} \right)^4 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^4 - \bar{\alpha}^2 \gamma^2(t) \left[(1-\nu) \left\{ \frac{\partial^2 W}{\partial \xi^2} \left(\frac{dz}{d\xi} \right)^3 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^5 \right. \right. \\
& \left. \left. - \frac{\partial W}{\partial \xi} \frac{d^2 z}{d\xi^2} \left(\frac{dz}{d\xi} \right)^2 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^5 \right\} + (1-\nu) \left\{ \frac{\partial^2 W}{\partial \bar{\xi}^2} \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^3 \left(\frac{dz}{d\xi} \right)^5 - \frac{\partial W}{\partial \bar{\xi}} \frac{d^2 \bar{z}}{d\bar{\xi}^2} \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^2 \left(\frac{dz}{d\xi} \right)^5 \right\} \right. \\
& \left. + 2(1+\nu) \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \left(\frac{dz}{d\xi} \right)^4 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^4 \right] \\
& - \frac{48\lambda}{h^2} \left[4 \left\{ \frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} \frac{\partial W}{\partial \xi} \frac{\partial W}{\partial \bar{\xi}} \left(\frac{dz}{d\xi} \right)^3 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^3 \right\} + \frac{\partial^2 W}{\partial \xi^2} \left(\frac{\partial W}{\partial \xi} \right)^2 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^3 \left(\frac{dz}{d\xi} \right)^3 \right. \\
& \left. + \frac{\partial^2 W}{\partial \bar{\xi}^2} \left(\frac{\partial W}{\partial \bar{\xi}} \right)^2 \left(\frac{dz}{d\xi} \right)^3 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^3 - \frac{\partial W}{\partial \bar{\xi}} \left(\frac{\partial W}{\partial \xi} \right)^2 \frac{d^2 \bar{z}}{d\bar{\xi}^2} \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^2 \left(\frac{dz}{d\xi} \right)^3 \right. \\
& \left. - \frac{\partial W}{\partial \xi} \left(\frac{\partial W}{\partial \xi} \right)^2 \frac{d^2 z}{d\xi^2} \left(\frac{dz}{d\xi} \right)^2 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^3 \right] + \frac{12}{h^2 G_0} \ddot{\gamma}(t) W(\xi, \bar{\xi}) \left(\frac{dz}{d\xi} \right)^5 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^5 = 0 \\
& \dots (61)
\end{aligned}$$

where $\bar{\alpha}^2$ is obtained from the following equation

$$\begin{aligned}
\frac{\bar{\alpha}^2 h^2}{12} \left\{ \left(\frac{dz}{d\xi} \right) \left(\frac{d\bar{z}}{d\bar{\xi}} \right) \right\}^2 \gamma^2(t) &= \frac{1}{2} (1-\nu) \left(\frac{\partial W}{\partial \xi} \right)^2 \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^2 + \frac{1}{2} (1-\nu) \left(\frac{\partial W}{\partial \bar{\xi}} \right)^2 \left(\frac{dz}{d\xi} \right)^2 \\
&+ \frac{\partial u_0}{\partial \xi} \frac{dz}{d\xi} \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^2 + \frac{\partial u_0}{\partial \bar{\xi}} \frac{d\bar{z}}{d\bar{\xi}} \left(\frac{dz}{d\xi} \right)^2 \\
&+ \nu i \left\{ \frac{\partial v_0}{\partial \xi} \frac{dz}{d\xi} \left(\frac{d\bar{z}}{d\bar{\xi}} \right)^2 - \frac{\partial v_0}{\partial \bar{\xi}} \frac{d\bar{z}}{d\bar{\xi}} \left(\frac{dz}{d\xi} \right)^2 \right\} \\
&+ (1+\nu) \frac{\partial W}{\partial \xi} \frac{\partial W}{\partial \bar{\xi}} \frac{dz}{d\xi} \frac{d\bar{z}}{d\bar{\xi}} \\
&\dots (62)
\end{aligned}$$

Here $\xi = r.e^{i\theta}$, $\bar{\xi} = r.e^{-i\theta}$, r being the radius of the circle. Values for λ have been obtained from the condition $\frac{\partial v}{\partial \lambda} = 0$, for minimum potential energy.

For regular polygons the mapping function is

$$Z = L\xi + \lambda_2 \xi^5 \quad \dots (63)$$

where values L and λ_2 are given in a separate table.

Let us choose the deflection function in the following form

$$W = A_0 \tau(t) [1 - \xi \bar{\xi}] \left[1 - \frac{1}{3} \xi \bar{\xi} + \frac{1}{2} (\xi^2 + \bar{\xi}^2) (1 - \xi \bar{\xi})^2 \right] \quad \dots (64)$$

clearly W is θ dependent and satisfies the simply supported edge conditions, namely,

$$W = 0 \text{ at } r = 1.$$

$$\frac{\partial^2 W}{\partial \xi \partial \bar{\xi}} = 0 \text{ at } r = 1.$$

Substituting equation (63) and (64) in (61) the error function $\epsilon(\xi, \bar{\xi}, t)$ is obtained. Galerkin's technique requires

$$\int_0^{2\pi} \int_0^1 \epsilon(\xi, \bar{\xi}, t) W(\xi, \bar{\xi}, t) r dr d\theta = 0 \quad \dots (65)$$

The constant $\bar{\alpha}$ is determined by putting (64) in (62) using (63) and integrating over the area of the plate.

It is to be noted that for transverse vibrations the normal displacement $w(\xi, \bar{\xi}, t)$ is our primary interest. So, the in-plane displacements u_0 and v_0 in equations (62) have been eliminated through integration by choosing suitable expressions for them compatible with their boundary conditions, namely, $u_0 = 0, v_0 = 0$ on the boundary for immovable edges.

$$\text{For movable edges } \bar{\alpha} = 0 \quad \dots (66)$$

Evaluating the integrals in (65) and considering the values of $\bar{\alpha}$ obtained from (62) (after integrating over the area of the plate) one obtains the Duffing's equation as in the previous cases in the form

$$\ddot{\tau}(t) + \alpha_1 \tau(t) + \beta_1 \tau^3(t) = 0 \quad \dots (67)$$

Here the β_1 consists of a huge number of terms. So these terms have not been shown. Numerical results coming out from these terms have been presented in the tables.

The ratio of the nonlinear time period and linear time period in this case is

$$\frac{T^*}{T} = \frac{\frac{2K}{\kappa}}{\left[1 + \frac{\beta_1}{\alpha_1} \bar{\beta}^2\right]^{1/2}} \quad \dots (68)$$

where $\bar{\beta} = \frac{A_0}{h}$

Numerical results : -

Numerical results are presented here in the tabular form for movable as well as immovable edges, for moderately thick polygonal plates. If the mapping function is known, the nonlinear behaviours of thick plates of any shape can be studied with ease and accuracy by using the proposed differential equations.

TABLE - 25.
MAPPING FUNCTION COEFFICIENTS. [57]

POLYGONS	L	λ_2
SQUARE	1.08a	-0.11a
PENTAGON	1.053a	-0.07a
HEXAGON	1.038a	-0.05a
HEPTAGON	1.029a	-0.036a
OCTAGON	1.022a	-0.028a

TABLE - 26
LINEAR TIME PERIOD.

$$T_L^* (\text{THICK PLATE}) = \frac{2\pi}{\sqrt{\alpha}} \left(\frac{E}{G_c} \neq 0 \right), \quad T_L (\text{THIN PLATE}) = \frac{2\pi}{\sqrt{\alpha}} \left(\frac{E}{G_c} = 0 \right)$$

POLYGONS	$T_L^* \left(\frac{h}{a} = 0.2, \frac{E}{G_c} = 2.5 \right)$	$\frac{T_L^*}{T_L}$
SQUARE	1.5613	1.0261
PENTAGON	1.2121	1.0285
HEXAGON	1.1185	1.0296
HEPTAGON	1.0722	1.0303
OCTAGON	1.0469	1.0308

RATIO OF NON-LINEAR TO LINEAR PERIOD FOR THE FUNDAMENTAL MODE OF VIBRATION OF SIMPLY SUPPORTED POLYGONAL PLATES (SQUARE OF SIDE 2a).

TABLE - 27

$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$ FOR IMMOVABLE EDGES. ($\nu=0.3, \lambda=\nu^2 [18], \frac{h}{2a} = \frac{1}{10}$)										$\frac{T^*}{T}$ FOR MOVABLE EDGES. ($\nu=0.3, \lambda=\nu^2 [18], \frac{h}{2a} = \frac{1}{10}$)									
	THIN PLATE		$k(\frac{E}{G_c})=2.5$ REF. [51]	$k(\frac{E}{G_c})=20$ REF. [51]	$k(\frac{E}{G_c})=30$ REF. [51]	$k(\frac{E}{G_c})=50$ REF. [51]	THIN PLATE		$k(\frac{E}{G_c})=2.5$ REF. [51]	$k(\frac{E}{G_c})=20$ REF. [51]	$k(\frac{E}{G_c})=30$ REF. [51]	$k(\frac{E}{G_c})=50$ REF. [51]								
	$k(\frac{E}{G_c})=0$	REF. [51]					$k(\frac{E}{G_c})=0$	REF. [51]												
0.60	0.9143	0.9072	0.9346	0.9270	1.0582	1.0469	1.1177	1.1066	1.2131	1.2012	0.9850	0.9779	1.0105	1.0029	1.1717	1.1604	1.2505	1.2334	1.3931	1.3802
0.80	0.8613	0.8507	0.8784	0.8624	0.9797	0.9636	1.0268	1.0113	1.1046	1.0819	0.9722	0.9616	1.0017	0.9857	1.1498	1.1337	1.2226	1.2071	1.3589	1.3362
1.00	0.8050	0.7917	0.8191	0.8055	0.9006	0.8809	0.9372	0.9123	0.9959	0.9678	0.9550	0.9416	0.9782	0.9647	1.1213	1.1022	1.1942	1.1693	1.3133	1.2852

RATIO OF NON-LINEAR TO LINEAR PERIOD FOR THE FUNDAMENTAL MODE OF VIBRATION OF DIFFERENT POLYGONS.

TABLE - 28

	$\bar{\beta} = \frac{A_0}{h}$	$\frac{T^*}{T}$ ($\nu=0.3, \lambda=\nu^2 [18], \frac{h}{2a} = \frac{1}{10}$) [2a is a dimension in length and related to the side of each polygon]																			
		PENTAGON					HEXAGON					HEPTAGON					OCTAGON				
		$k(\frac{E}{G_c})$					$k(\frac{E}{G_c})$					$k(\frac{E}{G_c})$					$k(\frac{E}{G_c})$				
		0	2.5	20	30	50	0	2.5	20	30	50	0	2.5	20	30	50	0	2.5	20	30	50
IMMOVABLE EDGES.	0.60	0.9341	0.9546	1.0794	1.1397	1.2391	0.9534	0.9747	1.0994	1.1607	1.2591	0.9784	0.9994	1.1204	1.1808	1.2791	0.9954	1.0194	1.1414	1.2008	1.3001
	0.80	0.8814	0.8995	1.0007	1.0476	1.1267	0.9016	0.9207	1.0207	1.0691	1.1467	0.9201	0.9417	1.0417	1.0891	1.1677	0.9429	0.9627	1.0627	1.1101	1.1887
	1.00	0.8255	0.8411	0.9209	0.9577	1.0170	0.8453	0.8621	0.9429	0.9777	1.0381	0.8632	0.8821	0.9640	0.9987	1.0594	0.8830	0.9031	0.9843	1.0197	1.0804
MOVABLE EDGES.	0.60	1.0050	1.0305	1.1917	1.2708	1.4182	1.0250	1.0515	1.2116	1.2909	1.4383	1.0451	1.0716	1.2317	1.3110	1.4584	1.0652	1.0917	1.2518	1.3311	1.4785
	0.80	0.9922	1.0227	1.1699	1.2436	1.3784	1.0124	1.0427	1.1900	1.2636	1.3989	1.0352	1.0627	1.2103	1.2837	1.4190	1.0567	1.0827	1.2303	1.3041	1.4391
	1.00	0.9762	0.9923	1.1416	1.2145	1.3336	0.9962	1.0126	1.1619	1.2346	1.3539	1.0177	1.0329	1.1822	1.2549	1.3742	1.0391	1.0532	1.2025	1.2752	1.3945

Observations :

Numerical results obtained from different tables of this chapter show that the new approach presented in the present study can be conveniently applied to study the static as well as dynamic behaviours of different thick plates of different shapes under different edge conditions with ease and accuracy.