3.1 **INTRODUCTION**

Although active RC grounded ideal inductors are now easily realizable in practice in some applications, such as a type of filters, they are unsuitable. Floating ideal inductor realization with RC elements then started receiving attention. Lately, there has been some efforts to realize a floating lossless inductor with RC elements and operational amplifiers or other active devices, which are in turn realizable by operational amplifiers. Rollett\(^1\) obtained a floating inductance from a floating capacitance via an active 3-port circulator. Reddy\(^2\) developed an approach to realize a floating inductance from a floating capacitance starting from a type of two operational amplifier grounded-inductor-realization scheme using the complementing properties of input and ground in active filters\(^3\). Sewell\(^4\) developed Rollett's approach further to produce ungrounded inductance from two grounded capacitances. Holt and Taylor\(^5\) suggested a scheme with two ideal gyrators and a grounded capacitance. A circuit with one grounded capacitance was also proposed by Deboo\(^6\) in a somewhat arbitrary manner. In this chapter two new systematic approaches to realize an ideal floating inductor with a single grounded capacitance and three operational amplifiers are presented. The operational amplifiers are connected as Current type negative immittance converter (INIC). It has also been shown that the non-unity gains may be allowed in the INIC's which then are required to be compensated by controlling the conductance parameter of the circuit.
3.2 REALIZATION APPROACHES

One approach follows from the 3-port gyrator model proposed by Sewell\(^8\). The nodal equations are first written from his block schematic and these equations are then expanded. The expanded equations lead straight right to the implementation of the proposed scheme.

The second approach is through a step by step process starting from a five-element ladder section. From the admittance matrix of such a ladder the elements that are to be negative for the desired realization are first listed. The possibility of obtaining these negative parameters through minimal actual passive and active elements is then explored. Both the approaches are discussed in brief in the following.

3.2.1 APPROACH FROM THE 3-PORT GYRATOR MODEL

The block schematic of the proposed scheme is shown in figure 3.1. From this figure and the admittance matrix

\[
\begin{bmatrix}
Y
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & g \\
0 & 0 & -g \\
-g & g & 0
\end{bmatrix}
\]  

(3.1)

of the 3-port gyrator (Sewell\(^8\)), the nodal equations are easily written as

\[
I_1 = gE_3 
\]  

(3.2a)

\[
I_2 = -gE_3 
\]  

(3.2b)

\[
I_3 = -gE_1 + gE_2 
\]  

(3.2c)
Fig. 3.1 Block of a gyrator

Fig. 3.2 Circuit schematic of the gyrator with capacitive termination
Fig. 3.3 Realization scheme of the circuit of Fig. 3.2

Fig. 3.4 The OA version of Fig. 3.3
Without affecting the gyrator admittance matrix, equations (3.2) may be expanded as

\[ I_1 = (-g)(E_1 - E_3) + gE_1 \]  \hspace{1cm} (3.3a)
\[ I_2 = g(E_2 - E_3) + (-g)E_2 \]  \hspace{1cm} (3.3b)
\[ I_3 = (-g)(E_1 - E_3) + g(E_2 - E_3) \]  \hspace{1cm} (3.3c)

For circuit implementation of equations (3.3), voltage differences in the right hand side of the third one of equations (3.3) should appear as \((E_3 - E_1)\) and \((E_3 - E_2)\). The changes made are however taken care of, in the floating ideal inductor realization, by terminating port 3 with a negative capacitance. Then from equations (3.3) one easily obtains the circuit scheme as shown in figure 3.2. The presence of the negative conductance at port 2 make this circuit realizable by three INIC's as shown in figure 3.3. The operational amplifier version of figure 3.3 is shown in figure 3.4.

3.2.2 APPROACH FROM THE LADDER NETWORK

The five element ladder section is shown in figure 3.5. Analysis yields its admittance parameter matrix as

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\end{bmatrix} =
\begin{bmatrix}
2y_2 \\
-y_2y_4 \\
\end{bmatrix}
\begin{bmatrix}
y_1 + y_2 - \frac{y_2}{y_2 + y_3 + y_4} \\
\frac{-y_2y_4}{y_2 + y_3 + y_4} \\
\end{bmatrix}
\begin{bmatrix}
y_4 + y_5 - \frac{y_4}{y_2 + y_3 + y_4} \\
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
\end{bmatrix}
\]

(3.4)

One notes that a lossless floating inductor with a grounded capacitor is realizable in the ladder of figure 3.5 for
Fig. 3.5 The five element ladder section
(a) \( y_1 + y_2 = y_4 + y_5 = y_2 + y_4 = 0 \) \hspace{1cm} (3.5a)
(b) \( y_2^2 = y_4^2 = y_2y_4 \) \hspace{1cm} (3.5b)
and
(c) \( y_3 = s\epsilon ; \ y_j = g_j, (j \neq 3) \) \hspace{1cm} (3.5c)

along with the further conditions that all \( g_j \)'s are identical and either \( y_2, y_3 \) and \( y_5 \) are negative or \( y_1, y_3 \) and \( y_4 \) are negative.

With the above conditions the ladder network of figure 3.5 can be easily seen to be transformed to that of figure 3.2. From figure 3.2 similar steps lead to the final scheme of figure 3.4.

3.3 THE REALIZED INDUCTANCE

Either from the three port admittance matrix of equation (3.1) coupled with capacitor termination and transforming it into a two port form as discussed earlier or from the matrix of equation (3.4), the admittance parameter matrix of the ideal floating inductance is obtained as

\[
\begin{bmatrix}
Y_L \\
\end{bmatrix} = \frac{g^2}{s\epsilon} \begin{bmatrix}
1 & -1 \\
-1 & 1 \\
\end{bmatrix}
\] (3.6)

such that the realized ideal floating inductor is given as

\[
L = \frac{C}{g^2}
\] (3.7)

3.4 NONIDEALITY OF INDUCTANCE DUE TO COMPONENT TOLERANCE

For nonunity gains of the INIC's and tolerance in the values of the conductances, the admittance matrix tends to become complicated. However, the transmission matrix can be more easily derived from figure 3.3 in such a situation. If in figure 3.3, the INIC's from
right to left (port 2 to port 1) are considered to have gains $k_1$, $k_2$ and $k_3$ respectively, with $k$'s as the ratios of the resistances $r_1$'s and $r_2$'s, and the conductances from the left to right (port 1 to port 2) are marked as $g_1$, $g_2$, $g_3$ and $g_4$, then the transmission matrix between ports 1 and 2 is easily obtained as

$$
[A]_{12} = \begin{bmatrix}
(1 + \frac{SC}{g_2})(1 - k_1 \frac{g_4}{g_3}) + \frac{1}{g_2}(k_1 k_2 g_4) & \frac{1}{g_3}(1 + \frac{SC}{g_2}) - \frac{k_2}{g_2} \\
\left(g_1(1 + \frac{SC}{g_2}) - k_3 g_2 - k_3 sC \right)(1 - k_1 \frac{g_4}{g_3}) + \left(\frac{g_1}{g_2} - k_3\right) k_1 k_2 g_4 & \frac{1}{g_3}(g_1 + g_1 \frac{SC}{g_2} - k_3 sC) - \left(\frac{g_1}{g_2} - k_3\right) k_2
\end{bmatrix}
$$

(3.8)

The realizability condition from equation (3.8) is now obtained as

$$
g_1 = g_3 = k_3 g_2 = k_2 k_3 g_3 = k_1 k_2 k_3 g_4
$$

(3.9)

The inductance has a value

$$
L = \frac{C}{g_2 g_3}
$$

(3.10)

in such a situation.

Equation (3.9) shows that only two conductances need to be equal in value, the other two can be adjusted to compensate for the nonunity gains of the INIC's. This increases the design flexibility to a certain extent.
3.5 **SENSITIVITY**

In an ideal realization the sensitivity has to be calculated for \( L = C/g^2 \) only. This yields

\[
S_C^L = \frac{1}{2} S_1/g = 1
\]

(3.11)

indicating sufficiently low sensitivity figures. However, with equation (3.9) observed sensitivities from the parasitic consideration may also be obtained. The floating impedance is

\[
z =\frac{sC}{g_2g_3} + \frac{1}{g_3} - \frac{k_2}{g_2}
\]

\[
= \frac{sC}{g_2g_3} + \frac{\pm \varepsilon}{g_2g_3}
\]

(3.12)

where \( \varepsilon \) is the parasitic \( g_2 \sim k_2g_3 \). The sensitivity of \( Z \) is given by

\[
MS_Z(\varepsilon) \approx \frac{\varepsilon}{\omega C}
\]

(3.13)

where \( M \) stands for 'magnitude of'. In obtaining equation (3.13) second order of \( \varepsilon \) has been neglected. It appears from equation (3.13) that at very low frequencies \( MS_Z(\varepsilon) \) may be quite large unless \( \varepsilon \) is of the order of ppm. As a parasitic however \( \varepsilon \) is usually not larger than that, hence the sensitivity always tends to be quite low.

3.6 **CIRCUIT STABILITY**

The circuit stability in this context means the stability of the INIC's. It is simple to stabilize this circuit by properly choosing the input terminals of the operational amplifiers. With
r_1 > r_2$, the connections in the operational amplifiers with the indicated polarity as in figure 3.4 are proper for stability.

3.7 EXPERIMENTAL RESULTS

Experimental studies have confirmed the theoretical analysis. The circuit of figure 3.4 was then actually used to design a low pass filter using low tolerance (1%) passive components and operational amplifiers with 1 MHz gain band width with an attempt for proper stop band trimming, the filter function being

$$\frac{e_0}{e_1} = \frac{1 + hs^2}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{3.14}$$

where

- $h = LC$
- $a_3 = LR_1(C_1 + C_2 + C_1 C_2)$
- $a_2 = L (C + C_2)$
- $a_1 = R_1(C_1 + C_2)$
- $a_0 = 1$

The cut-off frequency obtained is at 10.8 kHz. Deviation of the experimental result was less than 5% from the calculated values. The experimental filter scheme is shown in figure 3.6, and the theoretical response curve along with the experimental plot are shown in figure 3.7.

3.8 CONCLUSION

Two systematic approaches for floating inductor realization through a grounded capacitor three operational amplifier scheme
Fig. 3.6. The Low-pass filter

Fig. 3.7. The LP filter response

- Theoretical
- Experimental

$L = 100 \text{ mH}$

$C = 220 \text{ pf}$

$C_1 = 220 \text{ pf}$

$C_2 = 1200 \text{ pf}$

$R_1 = 10 \text{ K } \Omega$
are presented in this chapter. Although two different approaches have been shown in Section 3.2, the basic structure obtained through them come out to be the same. A grounded capacitor scheme has been considered because of convenience in integrability. Of the many schemes, now known, the presented method has comparatively less cancellation constraints. A low-pass filter has been designed using the developed prescribed method of realization and it has been shown that the realization approaches the ideal values very well with 1% tolerance components.

The realized inductors can be used to design filters of desired response characteristics. The values of realized inductor and their Q-factor may be called to determine the frequency range and the filter selectivity and it is not difficult to have the filter operating at low frequency as required in instrumentation system. However inductorless filters may directly be designed with active RC elements only. In such situations inductors are inherently provided in the system. In the following chapter a generalized approach is presented to obtain such a scheme in which all types of response can be shown to be obtained by minor changes in the passive RC networks. The basic realization approach however, remains unchanged by these changes.
REFERENCES


* Chapter-III is based mainly on this publication.