CHAPTER-II

ACTIVE RC REALIZATION OF GROUNDED INDUCTOR WITH OPERATIONAL AMPLIFIERS

2.1 INTRODUCTION

Inductor simulation in active RC schemes has been of considerable significance for designing analogue signal processing network of special types and sinewave oscillators for use in the areas of instrumentation, communication and control. Considerable number of literatures are now available on realization of specific type of inductors such as linear, bilinear, ideal etc[1-4]. These schemes use different types of active blocks for the realization. A single scheme realizing the different forms of inductors through minor modification in passive or active parameters should however be a welcome addition to the simulation method. Such a scheme becomes not only very versatile, its usefulness is also widened to a large extent. Such a scheme is presented here which uses two operational amplifiers, one used as an integrating amplifier of low closed loop gain and the other as a negative immittance converter (n.i.c). The proposed circuit has been analysed to yield linear, bilinear inductor and also ideal inductor of arbitrarily large inductance value. The presence of an n.i.c. in an appropriate position allows the circuit to behave as an oscillator with a suitable capacitor at the input port of the scheme thus demonstrating the usefulness of the realized inductor scheme. The experimental results have fully corroborated the theoretical calculations for the scheme as an inductor or as an oscillator.

2.2 THE RL IMPEDANCE[5*]

The basic circuit of the proposed scheme is shown in figure 2.1.

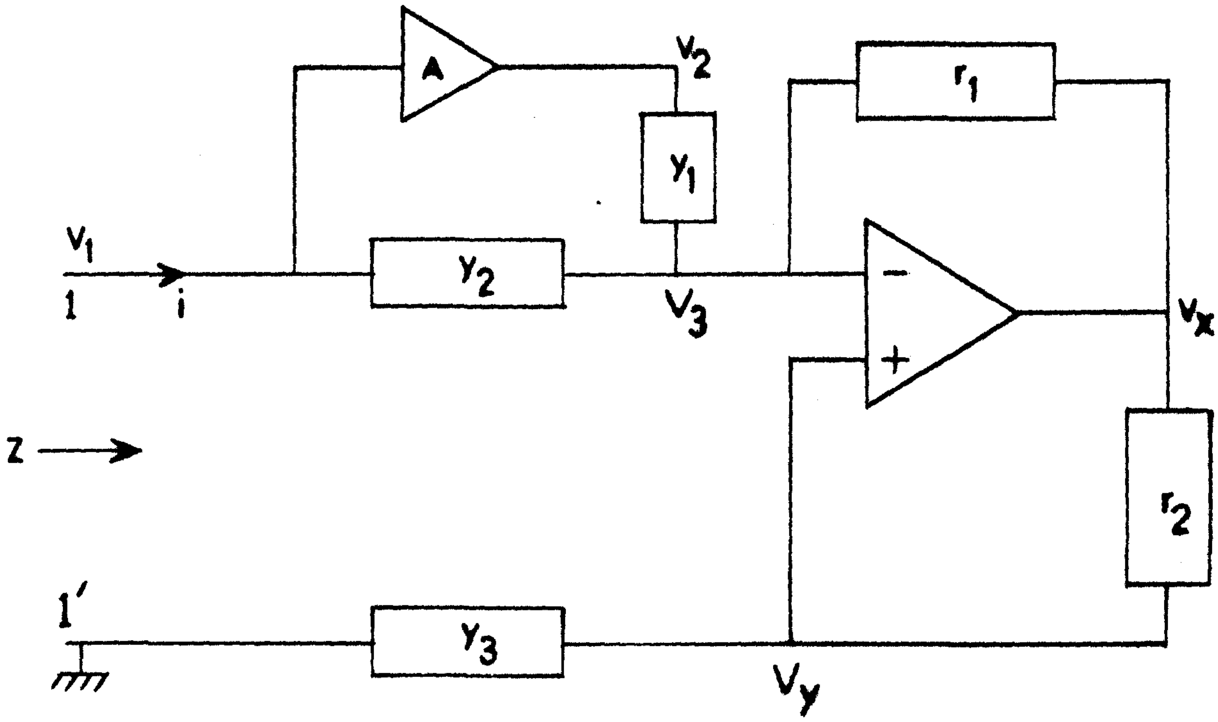

Fig. 2.1 The schematic circuit

Following equation are obtained from this circuit,

$$i = (V_1 - V_3)y_2 \tag{2.1a}$$

$$(V_1 - V_3)y_2 + (V_2 - V_3)y_1 = (V_3 - V_x)\frac{1}{r_1} \tag{2.1b}$$

$$(V_x - V_y)\frac{1}{r_2} = V_3 y_3 \tag{2.1c}$$

and

$$V_2 = AV_1 \tag{2.2}$$

Since $V_y = V_3$ from equation (2.1c) one may write

$$V_x = V_3(y_3 r_2 + 1) \tag{2.3}$$

Substituting V_2 and V_x from equation (2.2) and (2.3) into equation (2.1b) one obtains

$$V_3 = \frac{y_2 + Ay_1}{y_1 + y_2 - \frac{r_2}{r_1} y_3} V_1 \tag{2.4}$$

Finally with this V_3, equation (2.1a) changes to

$$i = V_1 \left[\frac{y_1 y_2 (1-A) - \frac{r_2}{r_1} y_2 y_3}{y_1 + y_2 - \frac{r_2}{r_1} y_3} \right]$$

Such that the input impedance Z is given by

$$Z = \frac{V_1}{I} = \frac{y_1 + y_2 - \frac{r_2}{r_1} y_3}{y_2 \left\{ y_1(1-A) - \frac{r_2}{r_1} y_3 \right\}} \tag{2.5}$$

If the admittances y_1, y_2 and y_3 are of the form

$$\left.\begin{aligned} y_i &= a_i s + b_i \\ a_i &= C_i \\ b_i &= \frac{1}{R_i} \end{aligned}\right\} \quad i = 1,\ 2 \text{ and } 3 \tag{2.6}$$

and $(1-A) = m \gg 1$, equation (2.5) may be rewritten as

$$Z = \frac{(a_1 + a_2 - \frac{r_2}{r_1} a_3)s + (b_1 + b_2 - \frac{r_2}{r_1} b_3)}{(a_2 s + b_2)\left\{(ma_1 - \frac{r_2}{r_1} a_3)s + (mb_1 - \frac{r_2}{r_1} b_3)\right\}} \qquad (2.7)$$

2.2.1 THE BILINEAR RL IMPEDANCE

For a bilinear inductive impedance function Z can be represented as

$$Z = \frac{Bs + D}{Hs + G} \qquad (2.8)$$

with a constraint $\frac{D}{B} < \frac{G}{H}$.

Comparing equations (2.7) and (2.8) the following identifications can be made

$$\left.\begin{aligned} (a_1 - \frac{r_2}{r_1} a_3) &= B \\ (b_1 + b_2 - \frac{r_2}{r_1} b_3) &= D \\ b_2(ma_1 - \frac{r_2}{r_1} a_3) &= H \\ b_2(mb_1 - \frac{r_2}{r_1} b_3) &= G \\ \text{and} \qquad a_2 &= 0 \end{aligned}\right\} \qquad (2.9)$$

It may be noted here that for equation (2.7) to represent bilinear impedance function, either a_2 may be made zero or $(ma_1 - \frac{r_2}{r_1} a_3)$ may be made zero. When a_2 is made zero there is reduction in circuit components and no rigid constraint is put on m, r_2, r_1, a_1 and a_3. This obviously is an advantage from the point of view of realization.

Where as for the other alternative $(ma_1 - \frac{r_2}{r_1} a_3) = 0$ and $a_2 > 0$, to make the denominator of equation (2.7) compatible, generally leads to extra complexity and this may not always be advisable for a stable driving point impedance function realization.

Combining equations (2.6) and (2.9) the impedance function Z is obtained in terms of the circuit parameters. Thus

$$Z = \frac{R_1R_2R_3(C_1 - \frac{r_2}{r_1} C_3)s + R_3(R_1 + R_2) - \frac{r_2}{r_1} R_1R_2}{R_1R_3(mC_1 - \frac{r_2}{r_1} C_3)s + mR_3 - \frac{r_2}{r_1} R_1} \tag{2.10}$$

Choosing now,

$$\left.\begin{aligned} R &= \frac{R_1}{\beta} = \frac{R_2}{\gamma} = R_3 \\ \text{and} \quad C &= \frac{C_1}{\delta} = C_3 \end{aligned}\right\} \tag{2.11}$$

equation (2.10) may be rewritten as

$$Z = R\,\frac{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma) + s\beta\gamma RC(\frac{r_2}{r_1} - \delta)}{\frac{r_2}{r_1}\beta - m + s\beta RC(\frac{r_2}{r_1} - m\delta)} \tag{2.12}$$

which may be factored into real and imaginary parts as

$$Z = R\,\frac{\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}(\frac{r_2}{r_1}\beta - m) + \omega^2\beta^2\gamma R^2C^2(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1} - m\delta)}{(\frac{r_2}{r_1}\beta - m)^2 + \omega^2\beta^2R^2C^2(\frac{r_2}{r_1} - m\delta)^2}$$

$$+ j\omega CR^2\,\frac{\beta\gamma(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1}\beta - m) - \beta(\frac{r_2}{r_1} - m\delta)\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}}{(\frac{r_2}{r_1}\beta - m)^2 + \omega^2\beta^2R^2C^2(\frac{r_2}{r_1} - m\delta)^2}$$

$$= R_b + j\omega L_b \tag{2.13}$$

From equation (2.13) the resistive and inductive components are obtained as

$$R_b = R\,\frac{\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}(\frac{r_2}{r_1}\beta - m) + \omega^2\beta^2\gamma R^2C^2(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1} - m\delta)}{(\frac{r_2}{r_1}\beta - m)^2 + \omega^2\beta^2R^2C^2(\frac{r_2}{r_1} - m\delta)^2} \qquad (2.14a)$$

and

$$L_b = CR^2\,\frac{\beta\gamma(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1}\beta - m) - \beta(\frac{r_2}{r_1} - m\delta)\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}}{(\frac{r_2}{r_1}\beta - m)^2 + \omega^2\beta^2R^2C^2(\frac{r_2}{r_1} - m\delta)^2} \qquad (2.14b)$$

Both R_b and L_b the resistance and inductance of the impedance Z are frequency dependent. It can be shown that suitable sets of m, β, γ, δ and $\frac{r_2}{r_1}$ may be found for stable bilinear RL impedance function Z. Figure 2.2 shows R_b - frequency and L_b - frequency curver as obtained from equation (2.14a) and (2.14b) for an identical set of the parameters m, β, γ, δ and two values of $\frac{r_2}{r_1}$; and R = 5K ohms and C = 0.094μF.

The Q factor of this impedance Z is given by

$$Q_b = \omega CR\,\frac{\beta\gamma(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1}\beta - m) - \beta(\frac{r_2}{r_1} - m\delta)\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}}{\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}(\frac{r_2}{r_1}\beta - m) + \omega^2\beta^2\gamma R^2C^2(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1} - m\delta)} \qquad (2.15)$$

Q_b may be increased by adjusting $\frac{r_2}{r_1}$, m, β, γ and δ maintaining the condition indicated by equation (2.8). For specific parameters $\frac{r_2}{r_1}$, m, β, γ and δ , Q_b is maximum at an angular frequency

$$\omega_{b_{max}} = \frac{1}{CR}\sqrt{\frac{\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}(\frac{r_2}{r_1}\beta - m)}{\beta^2\gamma(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1} - m\delta)}} \qquad (2.16)$$

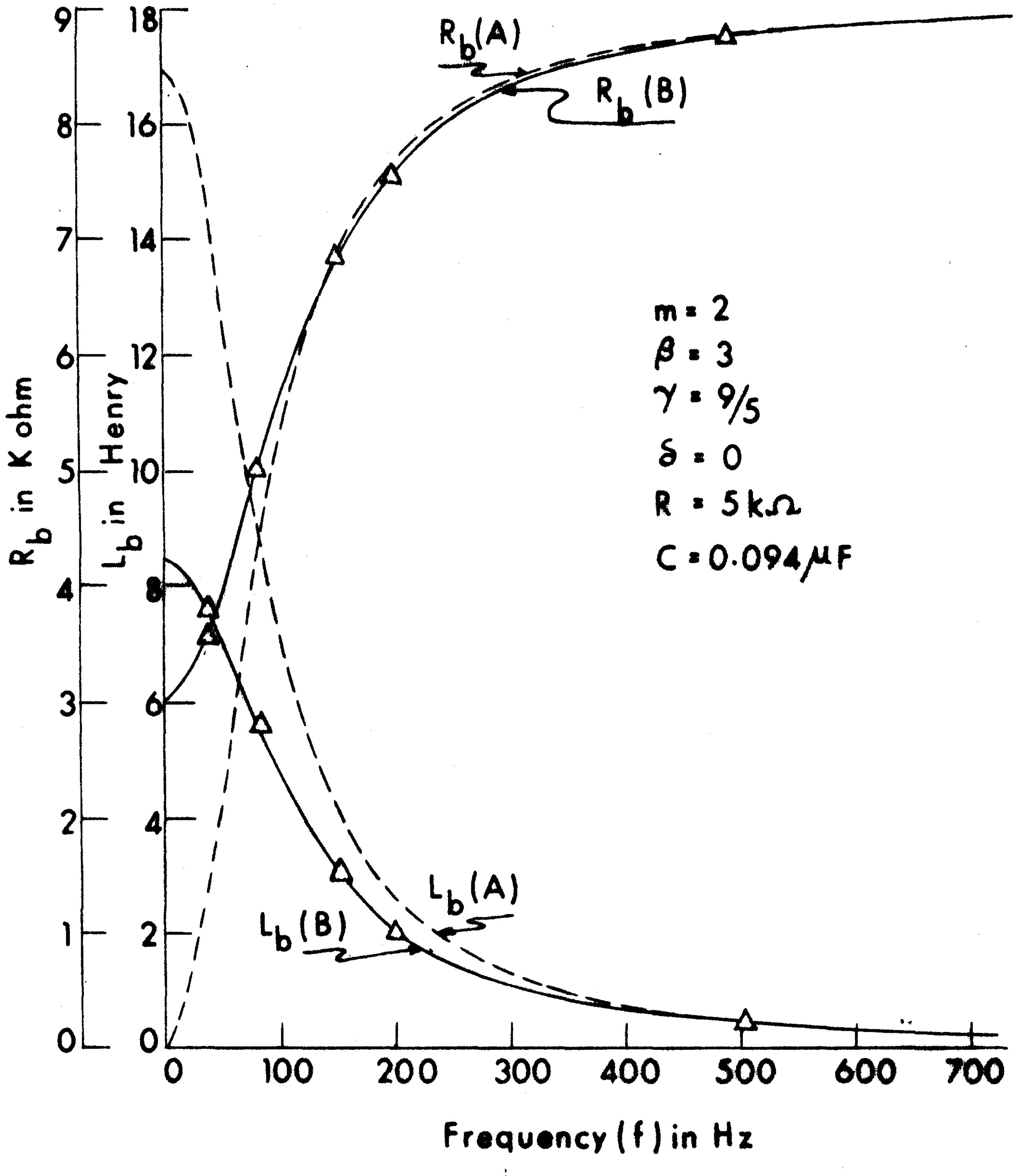

Fig.2.2. The R_b – Frequency and L_b – Frequency curves for bilinear case: (A) $r_2/r_1 = \frac{24}{27}$ (B) $r_2/r_1 = 1$

and the corresponding $Q_{b_{max}}$ is given by

$$Q_{b_{max}} = \frac{1}{2}\,\frac{\gamma(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1}\beta - m) - (\frac{r_2}{r_1} - m\delta)\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}}{\sqrt{\gamma(\frac{r_2}{r_1} - \delta)(\frac{r_2}{r_1} - m\delta)(\frac{r_2}{r_1}\beta - m)\left\{\frac{r_2}{r_1}\beta\gamma - (\beta+\gamma)\right\}}} \tag{2.17}$$

2.2.2 THE LINEAR RL IMPEDANCE

It is seen from equation (2.12) that if the capacitance C_1 is adjusted such that

$$\delta = \frac{r_2}{r_1 m} \tag{2.18}$$

the impedance Z is given by

$$Z = R\delta\,\frac{\beta\gamma - \frac{r_1}{r_2}(\beta+\gamma)}{(\beta\delta - 1)} + sCR^2\,\frac{\beta\gamma\delta\frac{r_1}{r_2}(\frac{r_2}{r_1} - \delta)}{(\beta\delta - 1)} \tag{2.19}$$

Equation (2.19) is linear in R and L and the resistance and inductance values are then given respectively by

$$R_1 = R\delta\,\frac{\beta\gamma - \frac{r_1}{r_2}(\beta+\gamma)}{(\beta\delta - 1)} \tag{2.20a}$$

and

$$L_1 = CR^2\,\frac{\beta\gamma\delta\frac{r_1}{r_2}(\frac{r_2}{r_1} - \delta)}{(\beta\delta - 1)} \tag{2.20b}$$

The Q factor of the realized inductor is therefore

$$Q_1 = CR\,\frac{\omega\beta\gamma\frac{r_1}{r_2}(\frac{r_2}{r_1} - \delta)}{\beta\gamma - \frac{r_1}{r_2}(\beta+\gamma)} \tag{2.21}$$

Q_1 is a linear function of frequency and is therefore maximum at $\omega = \alpha C$. However both R_1 and L_1 are independent of frequency and the simulated inductor has the characteristics of real inductors. Adjusting the parameters β and γ and maintaining $\beta\delta > 1$ the resistive part of the inductor can be made arbitrarily small and the Q factor at any frequency may be made arbitrarily large. Condition (2.18) can be realized by varying the phase inverter gain, keeping C_1 constant.

2.2.3 THE IDEAL INDUCTOR

From equation (2.19) it may be observed that if,

$$\left(\frac{1}{\beta} + \frac{1}{\gamma} \right) = \frac{r_2}{r_1} \qquad (2.22)$$

the simulated inductor is ideal with its inductance value

$$L_i = CR^2 \frac{\beta^2 \left(\frac{r_1}{r_2} \right)^2 \left(\frac{r_2}{r_1} - \delta \right)}{\left(\beta - \frac{1}{\delta}\right)\left(\beta - \frac{r_1}{r_2}\right)} \qquad (2.23)$$

The ideal inductor has infinite Q-factor at all frequencies. However its inductance value can be altered by suitable choice of parameters. It is interesting to note that the inductance can have very large value including infinity for reasonable values of the parameters.

Figure 2.3 shows the $L_i - \beta$ curves for three different values of δ and $\frac{r_2}{r_1} = 1$. The parameters β, γ and δ can be adjusted within reasonable limits to realize a very large value of L_i.

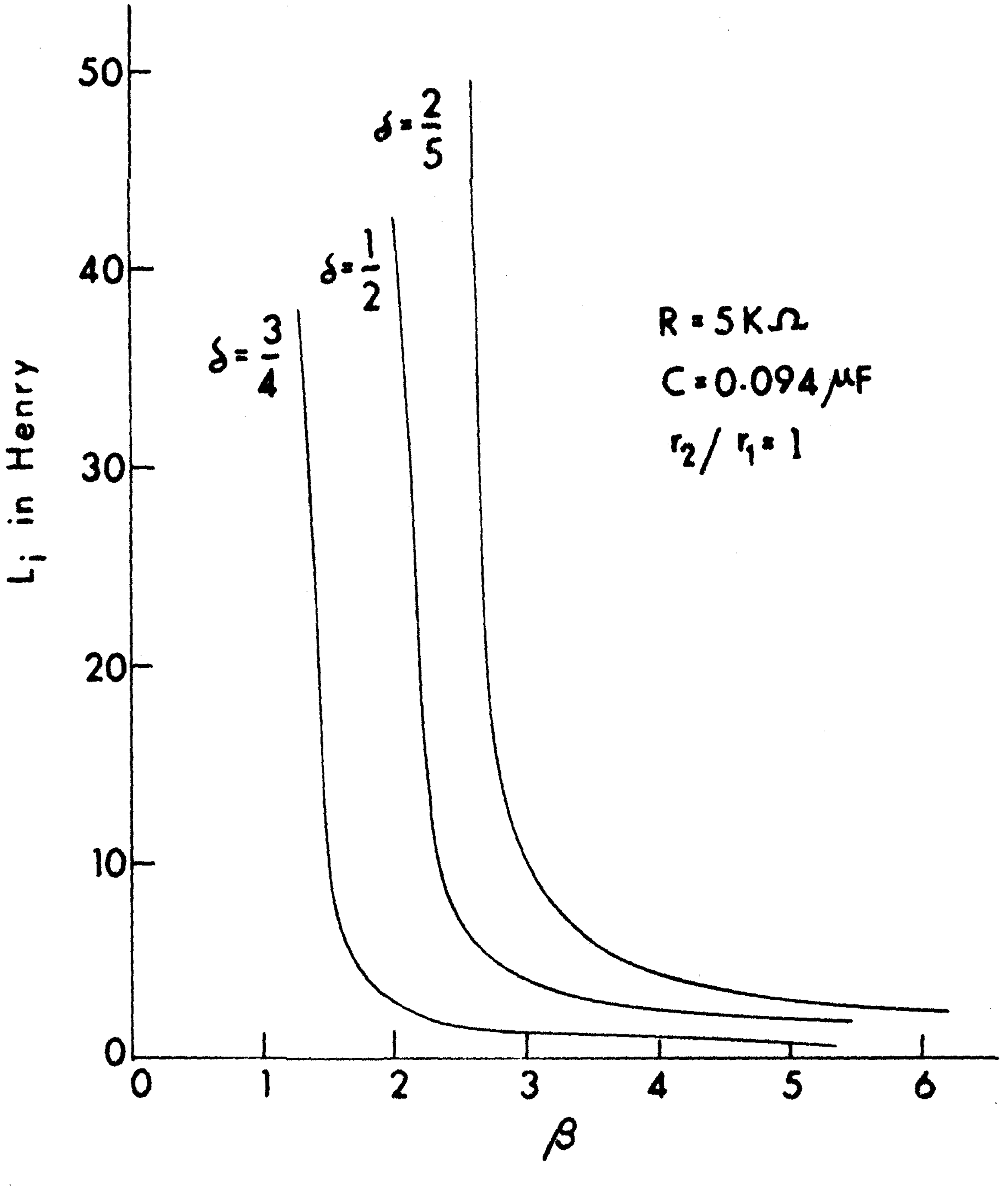

Fig.2.3. The $L_i - \beta$ curves for three different δ.

2.2.4 SENSITIVITY

The sensitivity study of the realized inductor becomes very important as the usefulness the inductor depends much on this sensitivity. For brevity the sensitivity of the ideal inductor is only considered. Instead of calculating the sensitivities to different active and passive parameters, the parasitic concept is introduced[6] to obtain the rationalized deviation of the impedance when ideal inductor realization constraints are tolerated. From equations (2.12) and (2.22) the constraints are,

$$\left.\begin{aligned} \frac{r_2}{r_1} &= m\delta \\ \text{and} \quad \frac{r_2}{r_1} &= \frac{1}{\beta} + \frac{1}{\gamma} \end{aligned}\right\} \qquad (2.24)$$

for an ideal inductor of value

$$L_i = CR^2 \frac{\beta\gamma\left(\frac{r_2}{r_1} - \delta\right)}{\frac{r_2}{r_1}\beta - m} \qquad (2.25)$$

We now introduce the parasitics

$$\left.\begin{aligned} \epsilon_1 &= \frac{r_2}{r_1} - m\delta \\ \text{and} \quad \epsilon_2 &= \frac{r_2}{r_1} - \left(\frac{1}{\beta} + \frac{1}{\gamma}\right) \end{aligned}\right\} \qquad (2.26)$$

so that

$$Z = R \frac{\epsilon_2 + s\beta\gamma CR\left(\frac{r_2}{r_1} - \delta\right)}{\frac{r_2}{r_1}\beta - m + s\beta CR\epsilon_1}$$

$$= R \frac{\epsilon_2 + j\gamma\omega_o\left(\frac{r_2}{r_1} - \delta\right)}{\frac{r_2}{r_1}\beta - m + j\omega_o\epsilon_1} \qquad (2.27)$$

where $\omega\beta CR = \omega_o$

Ideally ϵ_1 and ϵ_2 are nonexistent (i.e.zero). For their tolerance, the inductor becomes bilinear. Following the definition of the magnitude sensitivity function

$$MS_{\epsilon}^{f(\epsilon)} = \frac{\partial f(\epsilon)}{\partial \epsilon} \cdot \frac{\epsilon}{f(\epsilon)} \qquad (2.28)$$

one gets

$$MS_{\epsilon_1}^{Z(\epsilon_1)}\Big|_{\epsilon_2=0} = \frac{\omega_o\epsilon_1}{\omega_o^2\epsilon_1^2 + \left(\frac{r_2}{r_1}\beta - m\right)^2} \qquad (2.29)$$

which has a maximum value of unity with $\omega_o \rightarrow \infty$ and

$$MS_{\epsilon_2}^{Z(\epsilon_2)}\Big|_{\epsilon_1=0} = \frac{\epsilon_2}{\sqrt{\epsilon_2^2 + \omega_o^2\gamma^2\left(\frac{r_2}{r_1} - \delta\right)^2}} \qquad (2.30)$$

which has a maximum value of unity with $\omega_o \longrightarrow 0$. This indicate that the sensitivity of the scheme is only nominal. It is interesting to note that the sensitivity of the inductance value L_i is given by

$$MS_{\epsilon_{1,2}}^{L_i(\epsilon_{1,2})} = \frac{2\omega_o^2\epsilon_1^2}{\left(\frac{r_2}{r_1}\beta - m\right)^2 + \omega_o^2\epsilon_1^2} \qquad (2.31)$$

So that, with $\omega_o^2\epsilon_1^2 \ll \left(\frac{r_2}{r_1}\beta - m\right)^2$, MS^{L_i} is quite small. The

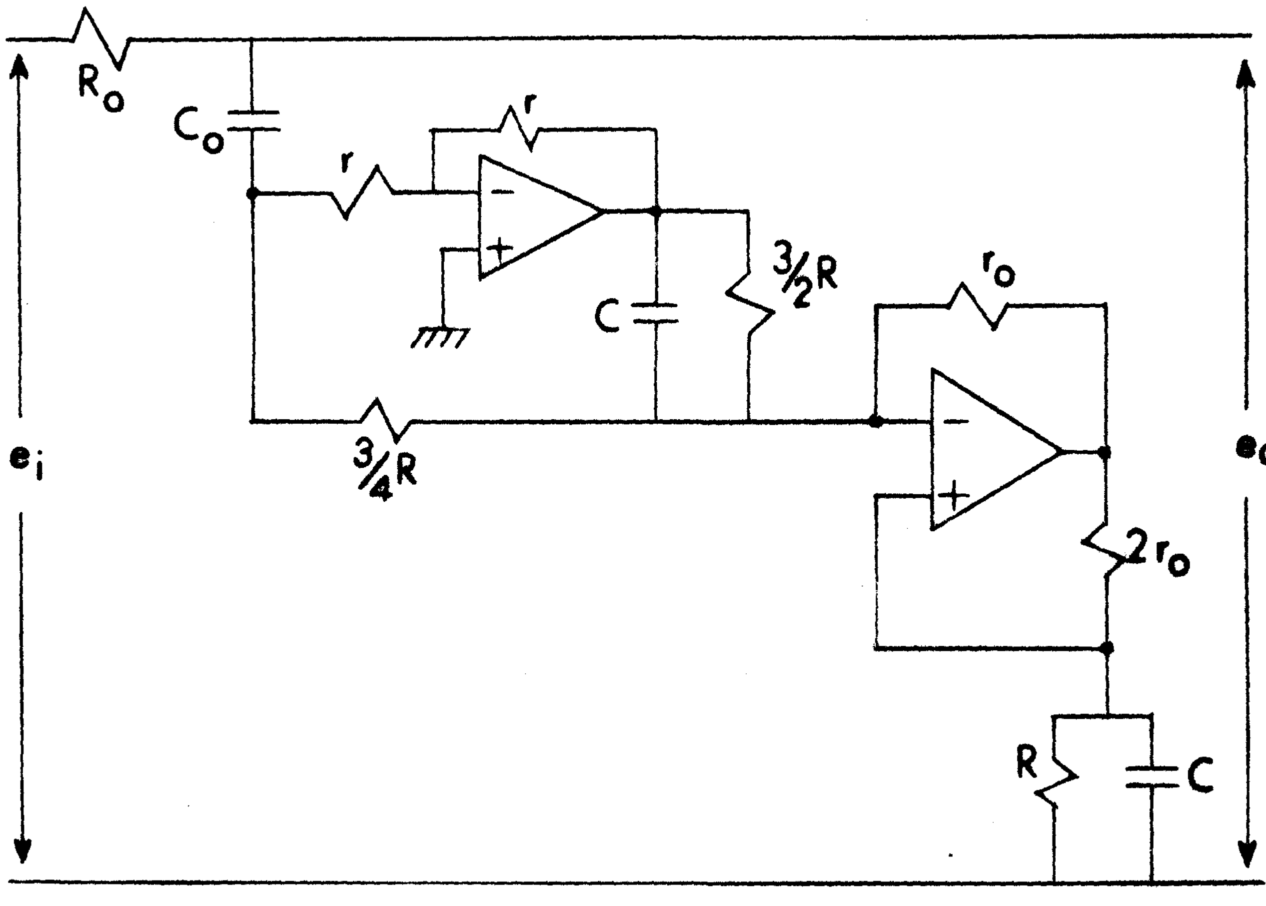

Fig. 2.4 A notch filter

maximum value of MS^{L_i} is 2 when $\omega \rightarrow \infty$.

2.3 FILTER REALIZATION

To demonstrate the application of the simulated inductor a notch filter is realized as shown in figure 2.4.

The transfer function is easily calculated as

$$\frac{V_o}{V_i}(s) = \frac{s^2 \frac{9}{4} CC_oR^2 + 1}{s^2 \frac{9}{4} CC_oR^2 + sC_oR_o + 1} \tag{2.32}$$

such that the notch frequency and the selectivity Q are given respectively by

$$\omega_n = \frac{2}{3R} \frac{1}{\sqrt{CC_o}} \tag{2.33}$$

and

$$Q = \frac{3}{2} \frac{R}{R_o}\sqrt{\frac{C}{C_o}} \tag{2.34}$$

It is interesting to note that the Q of such a filter is controlable independent of ω_n by adjusting R_o .

2.4 THE OSCILLATOR

When the realized inductor is ideal ($Q = \infty$) and has a sufficiently large value, a capacitor C' connected across port $1-1'$ (figure 2.1) would lead to realization of low frequency oscillator with a frequency of oscillation,

$$\begin{aligned} f &= \frac{1}{2\pi\sqrt{L_i C'}} \\ &= \frac{1}{2\pi\beta\frac{r_1}{r_2} R} \sqrt{\frac{(\beta - \frac{1}{\delta})(\beta - \frac{r_1}{r_2})}{(\frac{r_2}{r_1} - \delta)\, CC'}} \end{aligned} \tag{2.35}$$

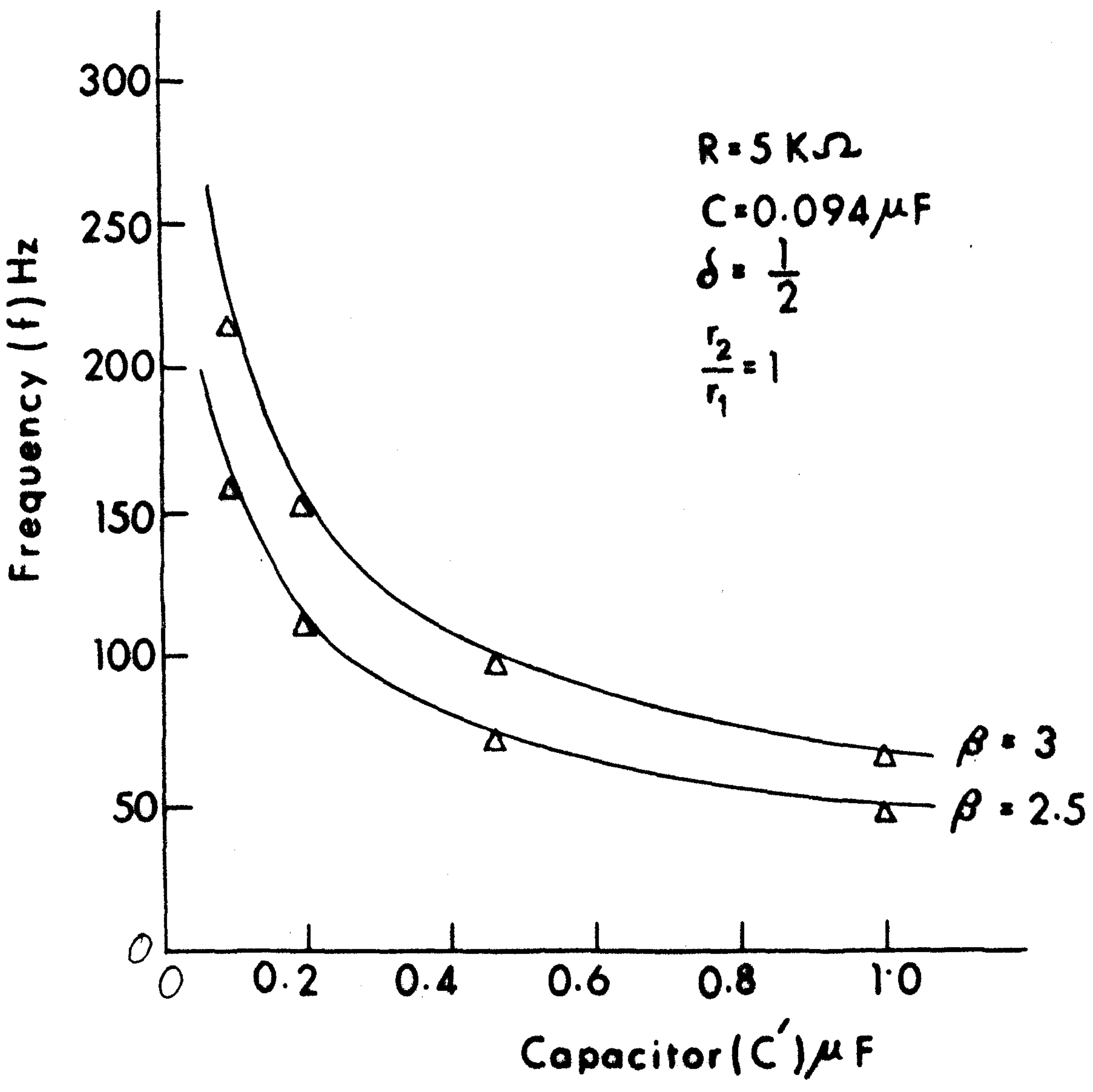

Fig.2.5. The f−C′ curves for two different β
(Experimental points: Δ)

A separate active device is not necessary but it is necessary that the passive parameters adapted in the circuit should lead to the condition of oscillation in terms of the circuit stability. This however is not difficult to realize in practice and in fact is obtained by interchanging the input terminals of the operational amplifier used as n.i.c.(cf. figure 2.4). The f-C' curves for two different values of β are shown in figure 2.5.

2.5 RESULTS

The measured values of L_b and R_b with varying f, for the bilinear case are shown by triangles in figure 2.2 for different parameter values along with the theoretical curves.

With the simulated ideal inductor an oscillator was set-up by connecting capacitor C' of different values across port 1-1$'$ (figure 2.1). The measured frequencies indicated by triangles are plotted in the f-C' coordinates along with the theoretical curves in figure 2.5 for two different values of β . The results plotted in the curves show good agreement with the calculated values.

2.6 CONCLUSION

The proposed circuit is a very generalized scheme of inductor realization as may be seen that with passive parameter adjustment it is possible to realize bilinear, linear and ideal inductor. In the case of ideal inductor it is seen that it is possible to simulate inductance of any value including infinity. Besides, realizing typical filters with the simulated inductor, a variable low frequency sine-wave oscillator very often required in instrumentation system can also be conveniently constructed with it. The tolerance in adjustment and matching of components needed, is also shown by the sensitivity analysis to be not very high. The circuit is well adaptable for integrated circuit form, with only a few external components required to adjust the operation in the desired mode.

The grounded inductor, non-ideal or ideal is not suitable to realize an arbitrary filter for signal processing purposes. Floating inductor is much more versatile in this respect. A method of realizing a low loss floating inductor is described in the next chapter.

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* Chapter-II is based mainly on this publication